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Cell Models for Non-Newtonian Fluid Past a Semipermeable Sphere

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Abstract
This paper is focused on investigating the boundary effects of the steady translational motion of a semipermeable sphere located at the center of a spherical envelope filled with an incompressible micropolar fluid. Stokes equations of micropolar fluid are employed inside the spherical envelope and Darcy's law governs in semipermeable region. On the surface of semipermeable sphere, the boundary conditions used are continuity of normal velocity, vanishing of tangential velocity of micropolar fluid, and continuity of pressure. On the surface of the spherical envelope, the Happel's, Kuwabara's, Kvashnin's, and Cunningham's boundary conditions, are used along with no spin boundary condition. The expression for the hydrodynamic normalized drag force acting on the semipermeable sphere is obtained. The limiting cases of drag expression exerted on the semipermeable sphere and impermeable solid sphere in cell models filled with Newtonian fluid are obtained. Also, in absence of envelope, the drag expression for the micropolar fluid past a semipermeable sphere is obtained.

Keywords- Semipermeable sphere, Micropolar fluid, Darcy's law, Drag force.

1. Introduction

Eringen (Eringen, 1966; 2001) introduced the theory of micropolar fluids. This theory describes the correct behaviour of fluids with microstructure. It has many industrial applications in biology, science and engineering. Ariman et al. (1974) discussed the applications of microcontinuum. Rao and Rao (1970) examined the micropolar fluid past a sphere. Ramkisson and Majumdar (1976) derived solution of the creeping flow of axially symmetric bodies in micropolar fluids. Hoffmann et al. (2007) extended the result (Rao and Rao, 1970; Ramkisson and Majumdar, 1976) by using spin condition for the microrotation. Rao and Iyengar (1981) investigated the slow flow of micropolar fluid past a spheroid. Iyengar and Srinivasacharya (1993) studied the slow flow of micropolar fluid past an approximate sphere. Iyengar and Radhika (2011; 2015) investigated micropolar fluid flow past a porous spheroid and prolate spheroidal shell containing a solid core using spheroidal coordinate system. They have numerically studied the variation of drag exerted on the particle with respect to the geometric parameters.

Considerable amount of work has been done on the viscous fluid flow through porous particles in an unbounded medium. Leonov (1962), Joseph and Tao (1964), Shapovalov (2009) investigated viscous fluid flow through a permeable sphere in different situations. In many practical applications, the particles like solid sphere, semipermeable sphere, porous sphere, spherical droplet are not isolated and surrounding fluid is bounded by a wall. Therefore, one has interest to investigate whether the boundary wall will affect the movement of a particle. Summaries of the flow relative to assemblage of particles can be founded in Happel and Brenner (1965). These boundary value problems can be solved by using cell model technique. In literature, four different cell models: Happel's model (Happel, 1958), Kuwabara's model (Kuwabara, 1959), Kvashnin's model (Kvashnin, 1979), and Cunningham's model (Cunningham, 1910) are used by many

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researchers. Dassios et al. (1995) studied the creeping flow of a spheroidal particle in a Happel and Kuwabara cell models. Prakash et al. (2011) investigated Stokes flow of an assemblage of porous spherical particles using stress jump condition. Srinivasacharya and Prasad (2013) studied the slow motion of a porous approximate sphere in an approximate spherical container.

Recently, cell model technique is applied for fluids with microstructure. Saad (2008) investigated the Stokes axisymmetrical flow past a spheroid in an envelope filled with micropolar fluid. Saad (2012) discussed micropolar fluid flow past a viscous fluid sphere and vice-versa. Translational motion of a slip sphere and spheroidal particle-in-cell-models are investigated by Sherief et al. (2015). Prasad and Kaur (2017, 2018) examined the boundary effects of viscous fluid spheroidal droplet in a micropolar fluid spheroidal cavity and micropolar fluid spheroidal droplet in a viscous spheroidal cavity. Prasad and Tina (2019) investigated the effect of magnetic field on the steady viscous fluid flow around a semipermeable sphere.

The purpose of present paper is to extended previous investigation of Shapovalov (2009) by presenting an analytical solution for the axially symmetric problem of semipermeable sphere in cell models filled micropolar fluid. The hydrodynamic resistance force exerted on the semipermeable sphere enclosed in spherical envelope is calculated. Variation of normalized drag force versus geometric parameters is shown graphically. Deduction of drag force agrees with results available in literature.

2. Assemblage of a Semipermeable Sphere Using Darcy's law

Consider an incompressible flow of a semipermeable sphere of radius a and permeability k in a concentric spherical envelope of radius b, which contains a micropolar fluid. The semipermeable sphere is at rest while the spherical envelope moves in the negative Z direction with uniform velocity , as illustrated in Figure 1. The flow is creep, steady and axisymmetric. Semipermeable region and micropolar fluid regions are denoted by I and II, respectively. Stokesian approximation is assumed in the liquid region and Darcy’s law in the semipermeable region.

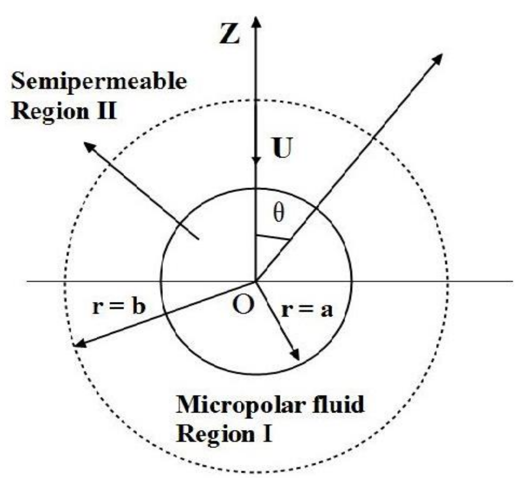

Figure 1. Geometry of the problem

The field equations for Stokesian approximation of micropolar fluid (Eringen, 1966; 2001; Ariman et al., 1974) are

$$\nabla \cdot \mathbf{v}^{(1)} = 0, \tag{1}$$

$$\nabla \mathrm{p}^{(1)} + (\mu + \kappa)\nabla \times \nabla \times \mathbf{v}^{(1)} - \kappa \nabla \times \boldsymbol{\nu} = 0, \tag{2}$$

$$\kappa \nabla \times \mathbf{v}^{(1)} - 2\kappa \boldsymbol{\nu} - \gamma_0 \nabla \times \nabla \times \boldsymbol{\nu} + (\alpha_0 + \beta_0 + \gamma_0)\nabla\nabla \cdot \boldsymbol{\nu} = 0, \tag{3}$$

where $\mathbf{v}^{(1)}, \boldsymbol{\nu}$, and $\mathrm{p}^{(1)}$ are velocity vector, microrotation vector and pressure, μ is the viscosity coefficient, κ is the vortex viscosity coefficient. The constants α_0, β_0 and γ_0 are the gyroviscosity coefficients.

The field equations for semipermeable region are (Darcy, 1856)

$$\nabla \cdot \mathbf{v}^{(2)} = 0, \tag{4}$$

$$\mathbf{v}^{(2)} = -\frac{\mathrm{k}}{\mu}\nabla \mathrm{p}^{(2)} \tag{5}$$

where $\mathbf{v}^{(2)}$ is the velocity vector, $\mathrm{p}^{(2)}$ is the pressure, and k is the permeability of the porous medium.

Let $(\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_\phi)$ be unit base vectors of spherical polar co-ordinate system (r, θ, ϕ). The flow generated is axially symmetric, velocity vectors and microrotation vector are independent of ϕ. Hence, one can write

$$\boldsymbol{v}^{(i)} = v_r^{(i)}(r,\theta)\boldsymbol{e}_r + v_\theta^{(i)}(r,\theta)\boldsymbol{e}_\theta, i = 1,2, \tag{6}$$

$$v = v_\phi(r,\theta)\boldsymbol{e}_\phi. \tag{7}$$

Introduce stream functions $\psi^{(i)}, i = 1,2$ by

$$v_r^{(i)} = \frac{1}{r^2}\frac{\partial \psi^{(i)}}{\partial \xi}, v_\theta^{(i)} = \frac{1}{r\sqrt{(1-\xi^2)}}\frac{\partial \psi^{(i)}}{\partial r}, i = 1,2 \tag{8}$$

where $\xi = cos\theta$.

We get partial differential equations of sixth, fourth and second order respectively by eliminating pressure term from Eq. (2) and (5)

$$E^4(E^2 - \lambda^2)\psi^{(1)} = 0, \tag{9}$$

$$\nu_\phi = \frac{1}{2r\sqrt{1-\xi^2}}\left(E^2\psi^{(1)} + \frac{(2+\tau)}{\tau\lambda^2}E^4\psi^{(1)}\right), \tag{10}$$

$$E^2\psi^{(2)} = 0. \tag{11}$$

Where

$$E^2 = \frac{\partial^2}{\partial r^2} + \frac{(1-\xi^2)}{r^2}\frac{\partial^2}{\partial \zeta^2},\ \lambda^2 = \left(\frac{\gamma_0}{a^2\kappa}\right)^{-1}\frac{(2+\tau)}{(1+\tau)}, \tau = \frac{\kappa}{\mu}, \alpha^2 = \frac{a^2}{k}. \tag{12}$$

3. Boundary Conditions

To solve Eq. (9)-(11), on boundary of semipermeable sphere, normal velocity and pressure are continuous and vanishing of tangential velocity, no spin condition (Joseph and Tao, 1964; Saad, 2008; 2012; Shapovalov, 2009; Iyengar and Radhika, 2011; 2015) are used. On cell surface, radial velocity is continuous, no spin condition, Happel's (Happel, 1958), Kuwabara's (Kuwabara, 1959), Kvashnin's (Kvashnin, 1979), and Cunningham's (Cunningham, 1910) boundary conditions, are valid.

On semipermeable sphere $r = 1$ are

$$\frac{\partial \psi^{(1)}}{\partial \xi} = \frac{\partial \psi^{(2)}}{\partial \xi}, \tag{13}$$

$$\frac{\partial \psi^{(1)}}{\partial r} = 0, \tag{14}$$

$$p^{(1)} = p^{(2)}, \tag{15}$$

$$v_\phi = 0, \tag{16}$$

While on the cell surface $r = \frac{1}{\eta}, \eta = \frac{a}{b}$

$$\frac{\partial \psi^{(1)}}{\partial \xi} = -r^2 \xi, \tag{17}$$

$$v_\phi = 0, \quad 0 \le \eta < 1 \tag{18}$$

(i) Happel model :

$$2r \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi^{(1)}}{\partial r}\right) - E^2\psi^{(1)} - \lambda^{-2}E^4\psi^{(1)} = 0, \tag{19}$$

(ii) Kuwabara model :

$$E^2\psi^{(1)} = 0, \tag{20}$$

(iii) Kvashnin model :

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \psi^{(1)}}{\partial r}\right) = 0, \tag{21}$$

(iv) Cunningham model :

$$\frac{\partial \psi^{(1)}}{\partial r} = r(1 - \xi^2). \tag{22}$$

4. General Solution of the Problem

The solutions of (9) - (11) (Happel and Brenner, 1965) are

$$\psi^{(1)} = \left[Ar^2 + Br^{-1} + Cr^4 + Dr + E\sqrt{r}K_{\frac{3}{2}}(\lambda r) + F\sqrt{r}I_{\frac{3}{2}}(\lambda r)\right]\vartheta_2(\xi), \tag{23}$$

$$\upsilon_\phi = \frac{1}{r\sqrt{1-\xi^2}}\left[5Cr^2 - Dr^{-1} + E\lambda^2(1+\tau)\tau^{-1}\sqrt{r}K_{\frac{3}{2}}(\lambda r) + F\lambda^2(1+\tau)\tau^{-1}\sqrt{r}I_{\frac{3}{2}}(\lambda r)\right]\vartheta_2(\xi), \quad (24)$$

$$\psi^{(2)} = Gr^2\vartheta_2(\xi)\ . \quad (25)$$

where arbitrary constants $A, B, C, D, E, F,$ and G are obtained by using boundary conditions (13)-(18) and one condition of different cell models (19)-(22). $\vartheta_2(\xi)$ is Gegenbauer function, I and K are modified Bessel functions, respectively.

5. Drag on the Semipermeable Sphere

The drag force exerted on the semipermeable surface in a cell by the micropolar fluid is

$$F = 2\pi a^2 \int_0^\pi r^2 \left(t_{rr}^{(1)} cos\theta - t_{r\theta}^{(1)} sin\theta\right)\Big|_{r=a} \sin\theta d\theta = 2\pi a U\mu(2+\tau)D \quad (26)$$

Consider the ratio of particle to cell volume is equal to the particle volume fraction $\delta(=\eta^3)$ in the cell model. For comparison purpose, when the cell is absent $(\delta = 0)$, so that the fluid is infinite.

$$F_\infty = -2\pi a U\mu(2+\tau)\left[\frac{6\,K_{\frac{3}{2}}(\lambda)(\tau+1)\alpha^2\lambda}{K_{\frac{3}{2}}(\lambda)\lambda(\tau+1)(\tau+4\alpha^2+2) - 2K_{\frac{1}{2}}(\lambda)\alpha^2\tau}\right] \quad (27)$$

Expression (27) is the new result for the drag force exerted on semipermeable sphere in micropolar fluid when envelope is absent.

For the slow flow of a semipermeable sphere in a viscous fluid $(\tau = 0)$, the drag expression reduces to

$$F_\infty = -4\pi a U\mu\left[\frac{3\alpha^2}{2\alpha^2+1}\right] \quad (28)$$

which is due to Shapovalov (2009).

5.1 Limiting Cases

5.1.1 First Case

If $\alpha \to \infty$,

$$F_\infty = -6\pi a U\mu(2+\tau)(1+\tau)(1+\lambda)(2\lambda(1+\tau)+2+\tau)^{-1} \quad (29)$$

which agrees with the result obtained by Rao and Rao (1970), Ramkissoon and Majumdar (1976).

5.1.2 Second Case

When $\tau = 0$ in Eq. (29),

$$F_\infty = -6\pi a U\mu \quad (30)$$

which is Stokes result (Happel and Brenner, 1965).

The normalised hydrodynamic drag force W is defined as the ratio of drag force acting on the particle in envelope to the drag experienced by particle in absence of envelope.

$$W = \frac{F}{F_\infty} \tag{31}$$

5.1.3 Third Case

If $\tau \to 0$ in Eq. (31),

$$W_{Hp} = -\frac{(2\alpha^2+1)\left(2(\alpha^2-10)\delta^{\frac{5}{3}}+3\alpha^2\right)}{3\alpha^2\left(2(\alpha^2-10)\delta^2-3(\alpha^2-2)\delta^{\frac{5}{3}}+3\alpha^2\delta^{\frac{1}{3}}-2\alpha^2-1\right)} \tag{32}$$

$$W_{Ku} = -\frac{5\,(2\alpha^2+1)}{2(\alpha^2-10)\delta^2-10(\alpha^2+2)\delta+18\alpha^2\delta^{\frac{1}{3}}-5(2\alpha^2+1)} \tag{33}$$

$$W_{Kv} = -\frac{2\,(2\alpha^2+1)((\alpha^2-10)\delta^{\frac{5}{3}}+4\alpha^2)}{\alpha^2(8(\alpha^2-10)\delta^2-9(\alpha^2-2)\delta^{\frac{5}{3}}-10(\alpha^2+2)\delta+27\alpha^2\delta^{\frac{1}{3}}-8(2\alpha^2+1))} \tag{34}$$

$$W_{Cu} = -\frac{2\,(2\alpha^2+1)((\alpha^2-10)\delta^{\frac{5}{3}}-\alpha^2)}{\alpha^2(4(\alpha^2-10)\delta^2-9(\alpha^2-2)\delta^{\frac{5}{3}}+10(\alpha^2+2)\delta-9\alpha^2\delta^{\frac{1}{3}}+2(2\alpha^2+1))} \tag{35}$$

Expressions (32)-(35) are normalized drag force exerted on a semipermable sphere in a cavity filled with Newtonian fluid.

5.1.4 Fourth Case

If $\alpha \to \infty$ in Eqs. (32), (33), (34), and (35) then

$$W_{Hp} = \frac{1+\frac{2}{3}\delta^{\frac{5}{3}}}{1-\delta^2+\frac{3}{2}\delta^{\frac{5}{3}}-\frac{3}{2}\delta^{\frac{1}{3}}} \tag{36}$$

$$W_{Ku} = \frac{1}{1-\frac{1}{5}\delta^2+\delta-\frac{9}{5}\delta^{\frac{1}{3}}} \tag{37}$$

$$W_{Kv} = \frac{\delta^{\frac{5}{3}}+4}{4-2\delta^2+\frac{9}{4}\delta^{\frac{5}{3}}+\frac{5}{2}\delta-\frac{27}{4}\delta^{\frac{1}{3}}} \tag{38}$$

$$W_{Cu} = \frac{1-\delta^{\frac{5}{3}}}{1+\delta^2-\frac{9}{4}\delta^{\frac{5}{3}}+\frac{5}{2}\delta-\frac{9}{4}\delta^{\frac{1}{3}}} \tag{39}$$

Eqs. (36)-(39) coincide with the expressions deduced in previous work of Saad (2012).

6. Results

To study the effect of micropolarity parameter τ for fixed value of permeability parameter $k_1 \left(=\frac{1}{\alpha}=\frac{k}{a^2}\right)$ for all the four models, the normalized drag force W versus volume fraction $\delta = \eta^3$ $(0 \leq \delta < 1)$ is presented in Figure 2 and 3. All figures are plotted for a fixed value of parameter $\frac{\gamma_0}{\mu\, a^2} = 0.3$. Figure 2 illustrate the variation of W with δ for semipermeable sphere filled with micropolar fluid in cell models. It indicates W monotonically increases with an increase in δ and W decreases as k_1 increases.

Figure 3 depicts the variation of W with k_1 for different values of the micropolarity parameter τ for fixed value of δ. For all the four models, when $k_1 \to 0$ and $\tau \to 0$, the problem reduces to motion of a solid sphere in a spherical container and it is observed in this case that W increases as τ increases. The drag force acting on a solid sphere in micropolar fluid is more than that on a solid sphere in Newtonian fluid. This behaviour agrees with the previous result obtained by Rao and Rao (1970). Also, the drag exerted on the semipermeable particle decreases as the permeability and micropolarity parameters increases. It is found that the drag on the semipermeable sphere in micropolar fluid is less than that on a semipermeable sphere in Newtonian fluid. Thus, the behaviour of W changes after a particular value of permeability.

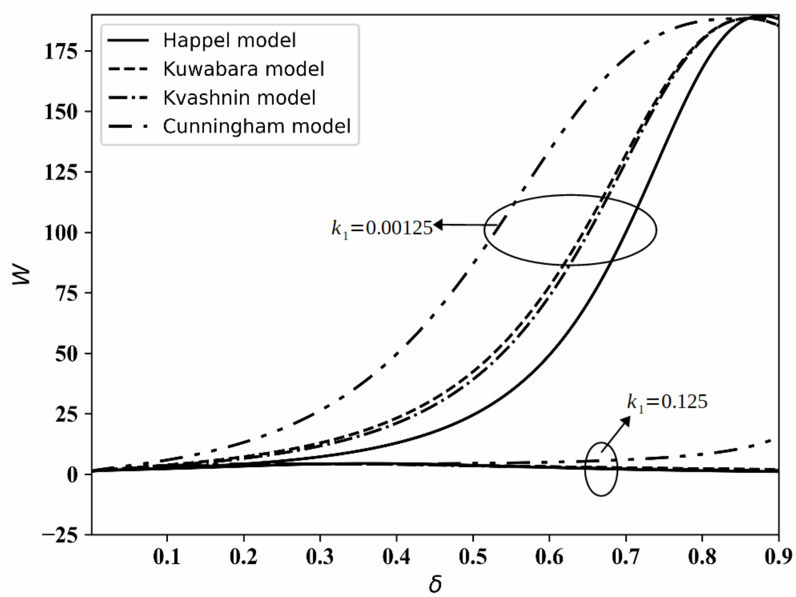

Figure 2. Variation of W with δ for different values of k_1 and $\tau = 0.1$

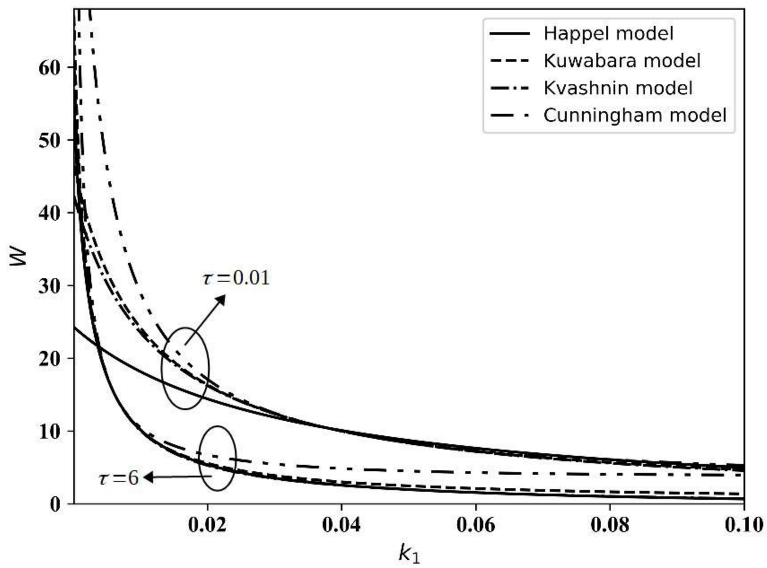

Figure 3. Variation of W with k_1 for different values of τ and fixed value of $\delta = 0.5$

7. Conclusions

An exact solution for micropolar fluid a past a semipermeable sphere in cell models is obtained. On the interface of semipermeable region and clear fluid region, normal velocity and pressure is continuous, vanishing of tangential velocity and no spin condition. Happel's, Kuwabara's, Kvashnin's, and Cunningham's boundary conditions and no spin condition at the cell surface are used. The closed form expression for the normalized drag force is obtained and its dependence on the various fluid parameters is studied. It is found that the drag on the semipermeable sphere in micropolar fluid is less than that on a semipermeable sphere in Newtonian fluid.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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