International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 6, 1362–1372, 2019
https://dx.doi.org/10.33889/IJMEMS.2019.4.6-107

Effects of Double Stratification on MHD Flow and Heat Transfer of Nanofluid along a Permeable Vertical Plate

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(Received December 12, 2018; Accepted August 17, 2019)

Abstract
This numerical work deals with the problem of Magnetohydrodynamics (MHD) flow of a doubly stratified Buongiorno's nanofluid along a permeable vertical flat plate. The resultant dimensionless system of coupled non-linear differential equations is solved numerically by utilizing Pseudo Spectral Collocation Method with Local Linearization Technique. For diverse values of the flow influenced parameters, the fluid flow, heat and mass transfer characteristics are explored and shown graphically. The presence of both ε_1 and ε_2 parameters strongly influences the convective heat and mass transport in the nanofluid medium.

Keywords- MHD, Double stratification, Spectral collocation method.

1. Introduction

Most of the researchers have analyzed the natural and mixed convective flows with heat transfer over various geometries by assuming the fluid as Newtonian. But, nowadays, the problem of convective flows with heat transfer of non-Newtonian fluids has been one of the active areas in the computational fluid dynamics and one of such fluids is Nanofluid. A suspension of nano-sized fibres or solid particles in conventional fluids (ethylene glycol, water, oil, etc.) is called nanofluid and it is recommended by Choi and Eastman (1995). Both the experimental and theoretical studies have been carried out by several researchers to get enhanced heat transfer rate and higher energy efficiency in various thermal exchange systems for numerous industrial applications. Buongiorno (2006) explored seven slip mechanisms such as fluid drainage, thermophoresis, inertia, Magnus effect, Brownian diffusion, diffusiophoresis, and gravity settling. With this experimentation, he found that the Brownian diffusion and thermophoresis are more significant effects to investigate the nanofluid flows. A detailed review and applications of nanofluids have been presented by Das et al. (2007), Das and Choi (2009) and Kakac and Pramuanjaroenkij (2009).

The combined effects of thermal and solutal stratification are one of the important aspects in heat and mass transport problems due to their significant applications in various engineering applications. Generally, the stratification of fluids occurs due to the presence of various fluids with distinct densities, variations in the temperature, concentration differences, etc. The natural convective flow past a vertically heated plate surrounded by a thermally stratified medium has been investigated by Chen and Eichhorn (1976). Later, Kulkarni et al. (1987) attained the similarity solution for free convective flow of a thermally stratified fluid along a heated vertical surface. The combined effects of cross diffusion and buoyancy forces on the free convective flow along a vertical surface embedded in a thermally stratified medium have been discussed by Angirasa and Srinivasan (1992). The combined convection boundary layer flow over a vertical surface in a stable stratified medium has been studied by Ishak et al. (2008). Mukhopadhyay et al. (2012) considered the problem of convective flow and heat transfer along a porous stretching sheet under the influence of thermal stratification. Moreover, Srinivasacharya and RamReddy (2010, 2011a, 2011b) explored the thermal and solutal stratification effects on natural and combined convective flows of a micropolar fluid along a vertical flat plate. The non-similarity solutions for free and combined convective flows of a nanofluid over a vertical surface embedded in a doubly stratified porous media have been obtained by Srinivasacharya and Surender (2014, 2015). The numerical study of a natural convection flow of a nanofluid saturated non-Darcy type porous media under the influence of double stratification has been presented by Srinivasacharya et al. (2015). Very recently, Hayat et al. (2018) discussed the MHD flow of a nanofluid in the presence of double stratification and slip conditions.

The present numerical study aims to examine the impact of double stratification on the MHD flow of a nanofluid along a permeable vertical flat plate. Pseudo-spectral collocation method along with local linearization technique, given in Motsa (2013), Motsa and Animasaun (2015), is employed to solve the present problem numerically. The effects of significant physical parameters on the nanofluid flow, local heat and mass transfer characteristics have been discussed and shown graphically.

2. Convective Transport Model

The 2-D natural convection and steady state flow over a permeable plate embedded in a nanofluid is considered. The 2-D coordinate system is chosen in which the x-axis is along the vertical plate and y-axis is normal to it. The surface of vertical plate is kept at uniform temperature T_w and uniform solid volume fraction C_w, whereas those at the ambient medium are $T_\infty(x) = T_{\infty,0} + Ax$ and $C(x) = C_{\infty,0} + Bx$, where $T_{\infty,0}$ and $C_{\infty,0}$ are ambient temperature and solid volume fraction, A and B are constants. Further, the uniform strength of B_0 opposite to the nanofluid flow direction is also considered. Following the Boussinesq approximations and standard boundary-layer assumptions, the governing equations, based on the Buongiorno's nanofluid model, are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho_{f_{\infty,0}}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \rho_{f_{\infty,0}}\left(1-\phi_{\infty,0}\right) g\beta_T (T - T_{\infty,0}) - \sigma B_0^{\,2} u - (\rho_p - \rho_{f\infty,0}) g(\phi - \phi_{\infty,0}) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\frac{\partial T}{\partial y}\frac{\partial \phi}{\partial y}+\frac{D_T}{T_{\infty,0}}\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

$$u\frac{\partial \phi}{\partial x}+v\frac{\partial \phi}{\partial y}=D_B\frac{\partial^2 \phi}{\partial y^2}+\frac{D_T}{T_{\infty,0}}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

and the related B.C. are

$$\begin{gathered} u(0)=0,\ v(0)=v_w,\ T(0)=T_w,\ \phi(0)=\phi_w \\ u(\infty)=0,\ T(\infty)=T_\infty(x),\ \phi(\infty)=\phi_\infty(x) \end{gathered} \tag{5}$$

Next, the stream function $\psi(x,y)$ with $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$ is utilized to satisfies Eq. (1) automatically. To simplify the present problem, we use the local Rayleigh number defined by

$$Ra_x=\frac{(1-\phi_{\infty,0})g\beta_T(T_w-T_{\infty,0})x^3}{\nu\alpha_m} \tag{6}$$

and also we introduce the following similarity transformations

$$\eta=\frac{y}{x}Ra_x^{1/4},\ \psi=\alpha_m Ra_x^{1/4}f(\eta),\ \theta(\eta)=\frac{T-T_\infty(x)}{T_w-T_{\infty,0}},\ S(\eta)=\frac{\phi-\phi_\infty(x)}{\phi_w-\phi_{\infty,0}} \tag{7}$$

Using the above similarity transformations, the equations (1)-(5) take the following form

$$f'''+\frac{1}{\Pr}\left(\frac{3}{4}ff''-\frac{1}{2}(f')^2\right)+\theta-N_r\phi-M\ f'=0$$

(8)

$$\theta''+\frac{3}{4}f\theta'+Nb\,\phi'\theta'+Nt\left(\theta'\right)^2-\varepsilon_1 f'=0 \tag{9}$$

$$\frac{1}{Le}\phi''+\frac{3}{4}f\phi'+\frac{1}{Le}\frac{Nt}{Nb}\theta''-\varepsilon_2 f'=0 \tag{10}$$

and the boundary conditions reduce to

$$\begin{gathered} f'(0)=0,\ f(0)=fw,\ \theta(0)=1-\varepsilon_1,\phi(0)=1-\varepsilon_2\ \text{ at } \eta=0 \\ f'(\infty)=0,\ \theta(\infty)=0,\ S(\infty)=0 \text{ as } \eta\to\infty \end{gathered} \tag{11}$$

where prime shows the differentiation with respect to η. Next, $M=\sigma B_0^2/\mu Ra_x^{1/2}$, $Nr=[(\rho_p-\rho_{f_{\infty,0}})(\phi_w-\phi_{\infty,0})]/[\rho_{f_{\infty,0}}\beta_T(1-\phi_{\infty,0})(T_w-T_{\infty,0})]$, $Nt=D_T\tau(T_w-T_{\infty,0})/\alpha_m T_{\infty,0}$, $f_w=-4v_w x/3\alpha_m Ra_x^{1/4}$, $\Pr=\nu/\alpha_m$, $Nb=[D_B\tau(\phi_w-\phi_{\infty,0})]/\alpha_m$, $Le=\alpha_m/D_B$, $\varepsilon_2=Bx/\phi_w-\phi_{\infty,0}$, $\varepsilon_1=Ax/T_w-T_{\infty,0}$ defines the magnetic parameter, nanoparticle buoyancy ratio, thermophoresis parameter, suction/injection parameter, Prandtl number, Brownian motion parameter, Lewis number, solutal and thermal stratification parameters, respectively.

If $\varepsilon_1 = \varepsilon_2 = 0$, the present problem reduces to the problem of free convective flow of a un-stratified nanofluid along a vertical plate with uniform wall conditions, which studied by Kuznetsov and Nield (2010).

The local Nusselt number Nu_x and the local nanoparticle Sherwood number NSh_x, respectively, are given by $Nu_x = \dfrac{x\,q_w}{k(T_w - T_{\infty,0})}$ and $NSh_x = \dfrac{x\,q_m}{D_B(C_w - C_{\infty,0})}$ where $q_w = -k\left.\dfrac{\partial T}{\partial y}\right|_{y=0}$ and $q_m = -D_B\left.\dfrac{\partial C}{\partial y}\right|_{y=0}$. The non-dimensional local Nusselt number and local nanoparticle Sherwood number are as follows

$$\frac{Nu_x}{Ra_x^{1/4}} = -\theta'(0) \text{ and } \frac{NSh_x}{Ra_x^{1/4}} = -\phi'(0) \tag{12}$$

3. Results and Discussion

The non-dimensional momentum, energy and solid volume fraction equations (8)-(10) along with B.C. (11) are coupled and non-linear differential equations for which analytical solution cannot be achieved. In this numerical work, the governing equations (8)-(10) are solved by employing the pseudo-spectral collocation method along with local linearization procedure, developed by Motsa (2013), Motsa and Animasaun (2015).

The numerical results for non-dimensional velocity, temperature, solid volume fraction, local heat transfer rate and local nanoparticle mass transfer rates have been computed and shown graphically. To explore the effects of various physical parameters such as the magnetic field, suction/injection, thermal stratification and solutal stratification on the nanofluid flow, heat and mass transfer characteristics, the numerical computations are performed for $Nr = 0.3$, $Pr = 1$, $Nt = 0.1$, $Le = 10$, $Nb = 0.1$. Further, the obtained results are plotted for velocity, temperature, solid volume fraction profiles and, heat and mass transfer rates, respectively. To authorize the MATLAB code, the present results are correlated with the existing results reported by Kuznetsov and Nield (2010), Ibrahim and Shanker (2012) and they are found to be in very good agreement as exhibited in Table 1.

Figures 1-3 plotted to show the non-dimensional velocity (f'), temperature (θ) and solid volume fraction (ϕ) for various inputs of the magnetic parameter (M) and $f_w = 0.2$, $\varepsilon_1 = \varepsilon_2 = 0.1$. It is seen from Figure 1 that the dimensional velocity enhances up to some maximum value and thereafter it diminishes gradually and finally it follows the velocity boundary condition at infinity. But, the velocity decreases in the momentum boundary layer region with the increase of magnetic field parameter. As the magnetic field parameter rises, the dimensionless temperature rises in the presence of both thermal and solutal stratifications as shown in Figure 2. But, the dimensional solid volume fraction profile enhances with the enhancement in the magnetic parameter. It is also seen that the solid volume fraction gets negative values in the presence of strong magnetic field and this occurs due to the presence of solutal stratification.

Table 1. Comparison of local Nusselt number when $Nt = Nb = Nr = 10^{-5}$, M=0, Le = 10, $\varepsilon_1 = \varepsilon_2 = f_w = 0$ for various Pr values

Pr	Ibrahim and Shanker (2012)	Kuznetsov and Nield (2010)	Present Results
1.0	0.401	0.401	0.401052
10.0	0.463	0.463	0.463371
100.0	0.485	0.481	0.481128
1000.0	0.491	0.484	0.483691

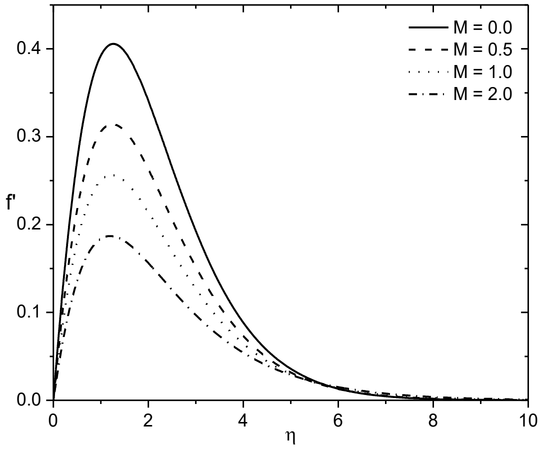

Figure 1. Effet of M on the Velocity

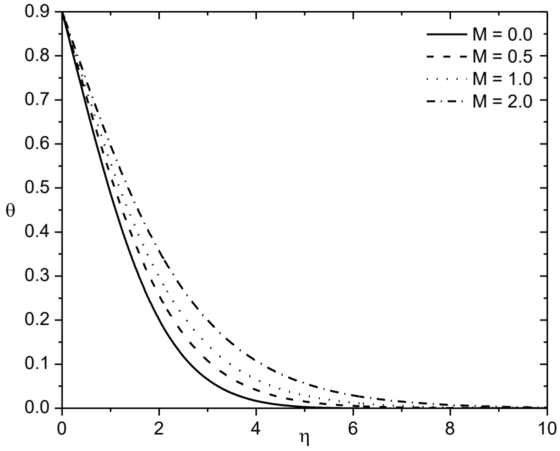

Figure 2. Effet of M on the Temperature

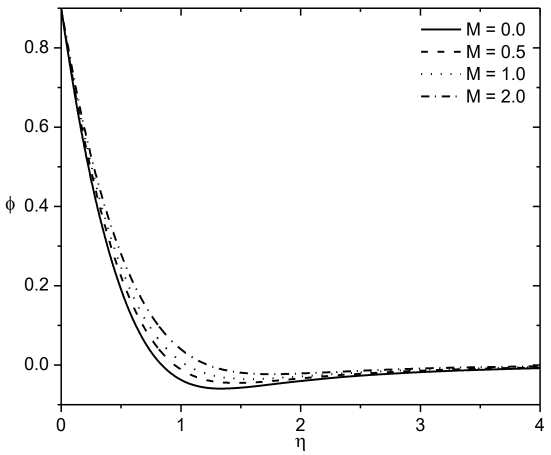

Figure 3. Effet of M on the Solid volume fraction

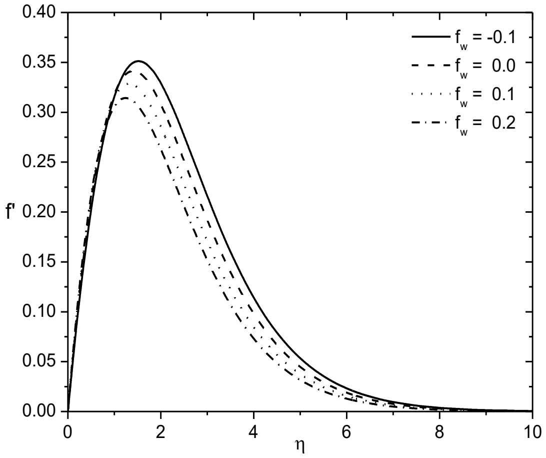

Figure 4. Effet of f_w on the Velocity

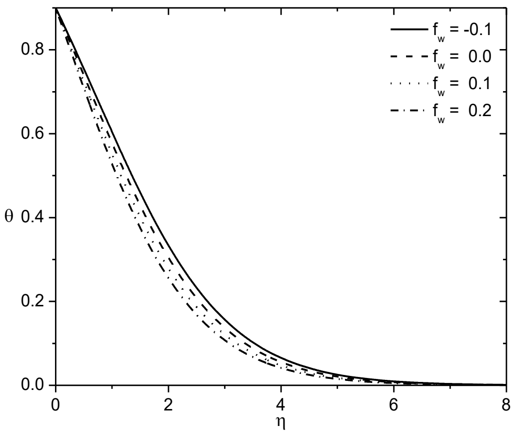

Figure 5. Effet of f_w on the Temperature

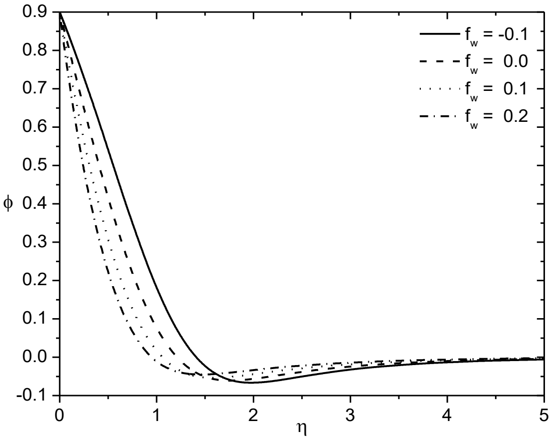

Figure 6. Effet of f_w on the Solid volume fraction

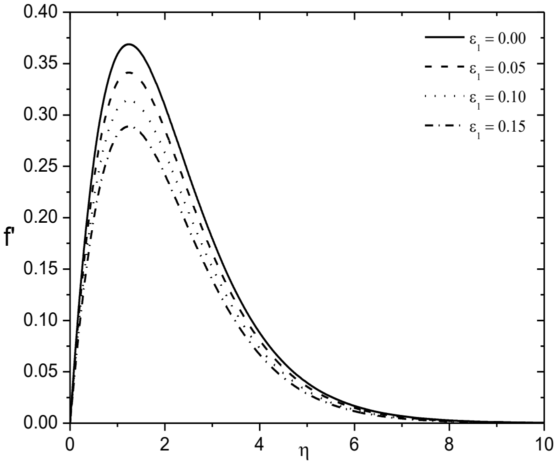

Figure 7. Effet of ε_1 on the Velocity

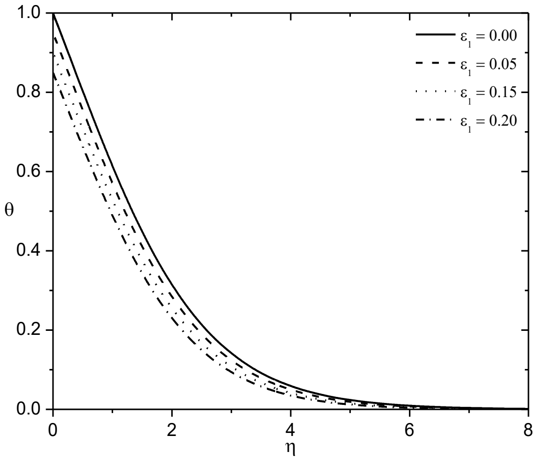

Figure 8. Effet of ε_1 on the Temperature

For fixed values *M = 0.5*, $\varepsilon_1 = \varepsilon_2 = 0.1$, the influence of suction/injection parameter (f_w) on the non-dimensional velocity (f'), temperature (θ) and solid volume fraction (ϕ) profiles is shown in Figures 4-6. Here, the suction/injection parameter f_w determines the fluid injection case for $f_w < 0$, fluid suction case for $f_w > 0$ and impermeable surface case for $f_w = 0$. Figure 4 depicts that the fluid velocity increases with the increase of suction/injection parameter. Moreover, the velocity of the fluid near to the plate is more for fluid suction case when compared to the fluid injection and impermeable surface cases. But, the dimensionless temperature and solid volume fraction decreases with the increase of suction/injection parameter as shown in Figures 5 and 6, respectively.

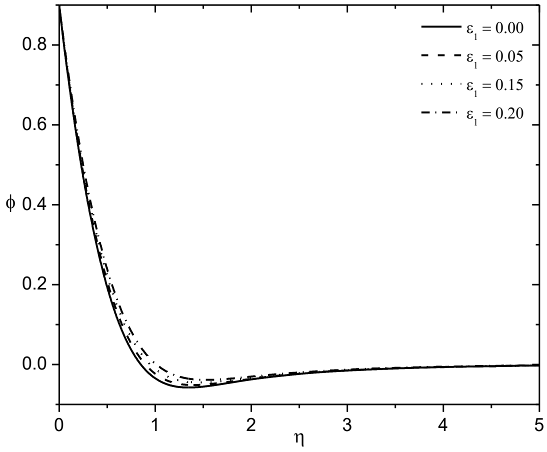

Figure 9. Effet of ε_1 on the Solid volume fraction

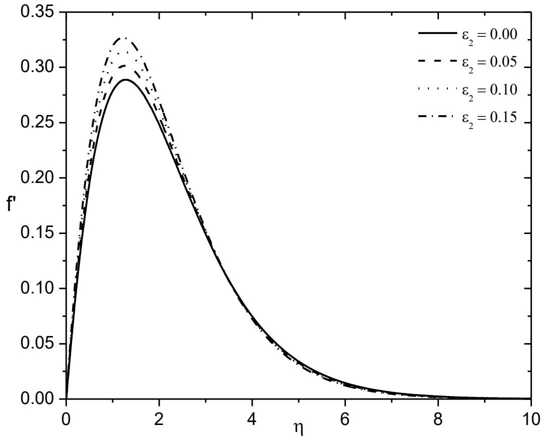

Figure 10. Effet of ε_2 on the Velocity

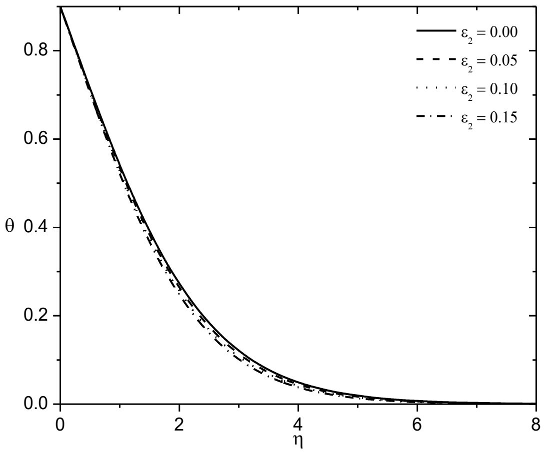

Figure 11. Effet of ε_2 on the Temperature

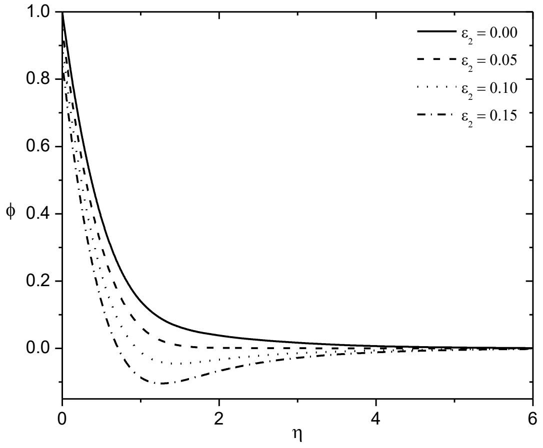

Figure 12. Effet of ε_2 on the Solid volume fraction

Figures 7-9 demonstrate the influence of thermal stratification (ε_1) on the non-dimensional velocity (f'), temperature (θ) and solid volume fraction (ϕ) profiles for $f_w = 0.2$, $M = 0.5$, $\varepsilon_2 = 0.1$. The dimensionless velocity diminishes with the rise of thermal stratification as presented in Figure 7. It is also seen from the Figure 8 that the temperature profile gradually decreases from its maximum temperature $\theta(0) = 1$. at the vertical plate as the thermal stratification increases whereas the opposite behaviour is captured for the non-dimensional solid volume fraction. These observations are due to the decrease of $T_w - T_{\infty,0}$ and $C_w - C_{\infty,0}$, respectively. For various values of the solutal stratification parameter, the non-dimensional velocity (f'), temperature (θ) and solid volume fraction (ϕ) are plotted in Figures 10-12. An increase in the solutal stratification parameter causes to increase the dimensionless velocity as shown in Figure 10. But, the dimensionless temperature and solid volume fraction profiles decrease as depicted in Figures 11 and 12.

Figures 13-14 depict the influence of magnetic parameter on the local heat transfer rate $Nu_x / Ra_x^{1/4}$ and local nanoparticle mass transfer rate $NSh_x / Ra_x^{1/4}$ versus the thermophoresis parameter (Nt). It is noticed from the Figures 13-14 that the local Nusselt and local nanoparticle Sherwood number reduce with the increase of magnetic field parameter. Also, the same trend is noticed for both $Nu_x / Ra_x^{1/4}$ and $NSh_x / Ra_x^{1/4}$ with the increase of Nt. Further, the influence of thermophoresis parameter on the local heat transfer rate $Nu_x / Ra_x^{1/4}$ and local nanoparticle mass transfer rate $NSh_x / Ra_x^{1/4}$ under various inputs of the suction/injection parameter is shown in Figures 15-16. As the suction/injection parameter rises, the local Nusselt and nanoparticle Sherwood numbers rises whereas an opposite behaviour is noticed in the case of thermophoresis parameter. Moreover, the Nusselt and nanoparticle Sherwood number are high for fluid suction case when compared to those of fluid injection and impermeable surface cases.

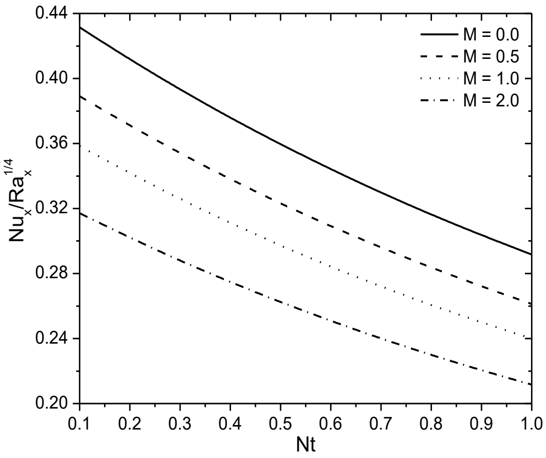

Figure 13. Effet of *M* on the Local Nusselt number

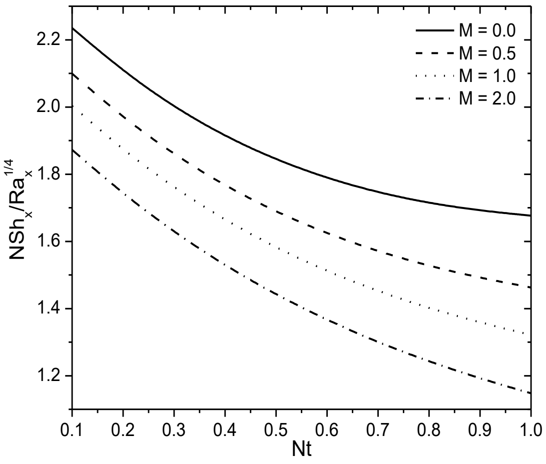

Figure 14. Effet of *M* on the Local nanoparticle Sherwood number

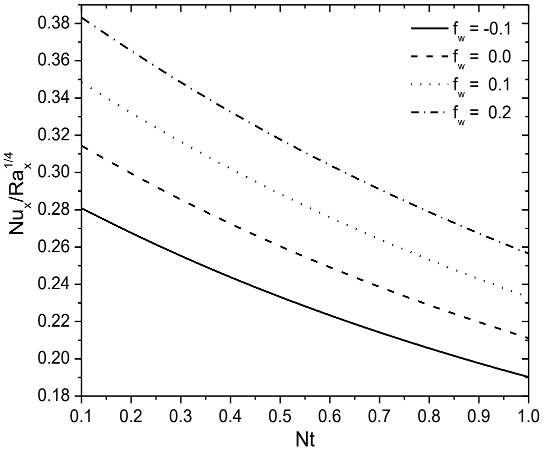

Figure 15. Effet of f_w on the Local Nusselt number

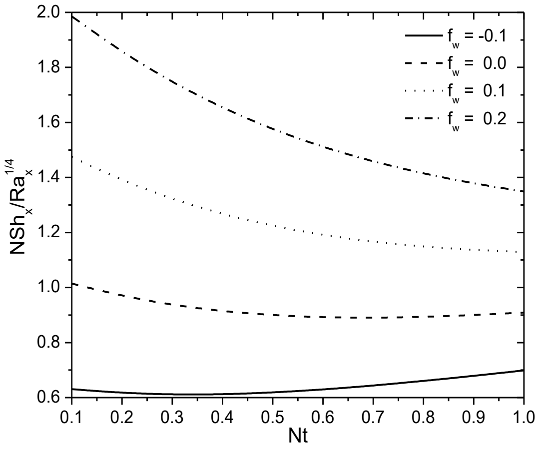

Figure 16. Effet of f_w on the Local nanoparticle Sherwood number

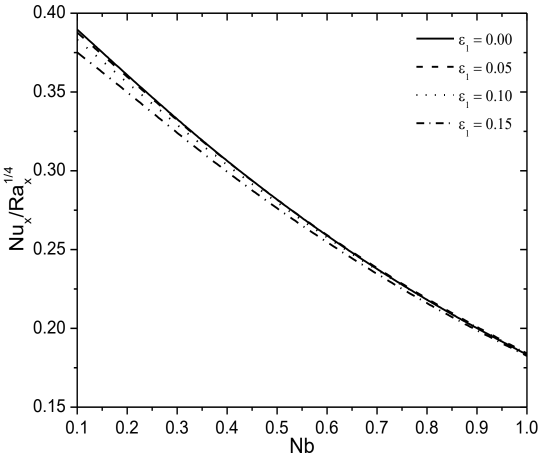

Figure 17. Effet of ε_1 on the Local Nusselt number

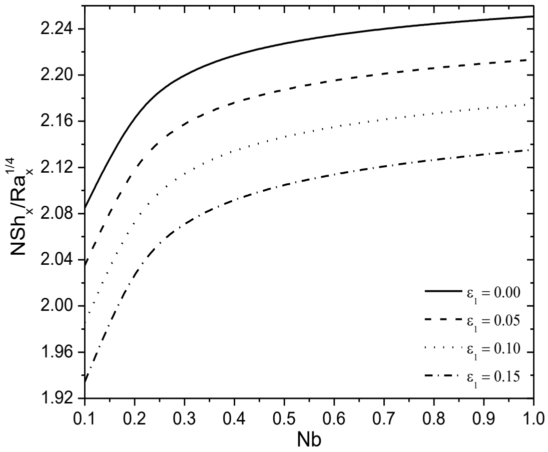

Figure 18. Effet of ε_1 on the Local nanoparticle Sherwood number

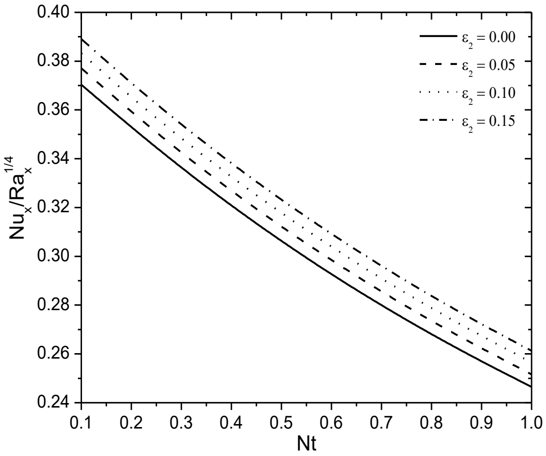

Figure 19. Effet of ε_2 on the Local Nusselt number

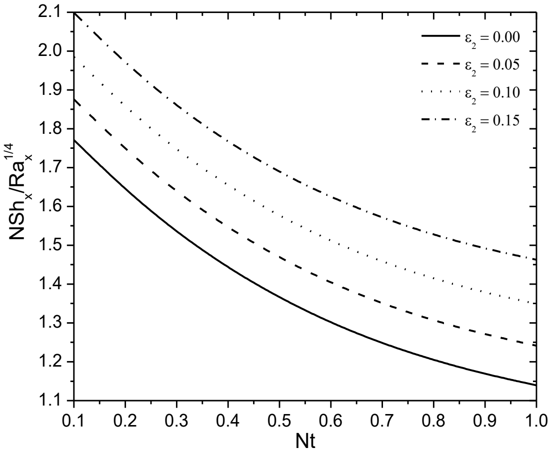

Figure 20. Effet of ε_2 on the Local nanoparticle Sherwood number

The impact of thermal stratification and solutal stratification on the local heat transfer rate $Nu_x / Ra_x^{1/4}$ and local nanoparticle mass transfer rate $NSh_x / Ra_x^{1/4}$ are presented in Figures 17-18 and Figures 19-20, respectively. The local heat transfer rate and local nanoparticle mass transfer rate decreases with increase of thermal stratification parameter as shown in Figures 17-18. The higher values of Nusselt and nanoparticle Sherwood numbers are found for un-thermally stratified nanofluid when compared to those for thermally stratified nanofluid. Further, the heat transfer rate diminishes whereas the nanoparticle mass transfer rate enhances with the enhancement of thermophoresis parameter. Moreover, the local Nusselt and local nanoparticle Sherwood number increases with the increase of solutal stratification parameter as shown in Figures 19-20. It is also seen that the heat and nanoparticle mass transfer rate are high in the presence of solutal stratification effect when compared those in the absence of solutal stratification effect.

4. Conclusions

This numerical work deals with the effect of double stratification on the MHD flow of a nanofluid along a vertical flat plate under the presence of suction/injection. An appropriate set of similarity variables are acquired to reduce the governing equations into a dimensionless form and solved numerically by employing pseudo-spectral collocation method along with local linearization technique. The effects of various physical parameters on nanofluid flow, heat and mass transfer attributes are explored. The main results in this investigation are listed as follows:

- With the presence and increasing values of MHD parameter strongly increases the temperature and solid volume fraction profiles and, decreases the local Nusselt and nanoparticle Sherwood numbers.
- An increase in the suction/injection parameter increases the velocity, local Nusselt and nanoparticle Sherwood numbers.
- The presence of ε_1 parameter decreases the velocity, temperature, local Nusselt and nanoparticle Sherwood numbers.
- Increasing values of ε_2 parameter leads to increase the velocity, local Nusselt and nanoparticle Sherwood numbers.
- The presence of both ε_1 and ε_2 parameters strongly influences the convective heat and mass transport in the nanofluid medium.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgement

The authors would like to express their sincere thanks to the referee and for their valuable suggestions towards the improvement of the paper.

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