Reliability Measures of Two Dissimilar Units Parallel System Using Gumbel-Hougaard Family Copula

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Abstract
The goal of the present study is to investigate the availability and the reliability of the system, which has two dissimilar units in the parallel network under copula. Other key parameters, such as mean time to failure (MTTF) and expected profit are also evaluated. Simultaneous malfunctioning of units, common cause failure and human fault are the causes of system breakdown. The present mathematical model is examined under the assumption that each failure rate is constant and is exponentially distributed. The system undergoes repair in the completely failed state as well as in degraded state. In the case of complete failure, the system is repaired by two repair facilities and that are tackled by utilizing Gumbel-Hougaard family copula. The present system has been studied by applying the concepts of probability theory, supplementary variable technique and Laplace transformation.

Keywords- Reliability, Availability, MTTF, Copula.

1. Introduction
In the present scenario of the competitive world, increasing complexity in components and systems has made it imperative for industries to produce highly reliable, user friendly, efficient and cost effective systems. Reliability is the probability that a unit or a machine will perform its specified task adequately for an assured period of time under the given set of conditions. Customer satisfaction is highly governed by the reliability, availability and the performance of the system. Thus, reliability analysis is an important phase in the planning, designing and manufacturing of any system. In recent time, it has emerged as one of the most challenging and demanding theory.

In spite of advanced automation techniques, manpower is involved in system operation and hence system may fail at any instant due to unexpected human activity. Lack of training, inadequate experience, mental stress, high noise levels, improper work layouts, inadequate tools and poorly written manuals are the chief causes of human error. Apart from human error, some other major causes of system failure are common cause failure, catastrophic failure, hardware failure, partial failure and unavailability of repairman. Common cause failure occurs when multiple units or components fail in the same manner due to a single cause. Temperature, humidity, pressure, equipment design deficiencies and maintenance errors are some of the reasons of common cause failures.

Ram (2013) has summarized the survey of various reliability approaches that are used in the fields of engineering and applied sciences. Complex systems are extensively discussed by a number of earlier authors like Dhillon and Yang (1992, 1993), Kontoleon and Kontoleon (1974); Gupta and Agarwal (1984) by considering various failure and repair disciplines. Recently, Saxena et al. (2019) analyzed the performance of the water treatment plant by evaluating its various reliability measures. Zheng et al. (2018) considered component importance analysis of a real-time computing system with warm standby redundancy and common cause failures. Kumar et al. (2017) studied multistate casting system with the help of supplementary variable technique and Laplace transform.

Parallel redundancy is an important commonly used method for enhancing system reliability. Li (2016) discussed the pros and cons of the active redundancy and standby redundancy. The author utilized Markov model technique to evaluate mean time between failures for active and standby redundant systems. Chung (1981) used the Laplace transform technique for analyzing a redundant system comprising of two dissimilar three-state active units and one standby unit. Author has assumed that the units of the system may go down due to two mutually exclusive failure modes or by common cause failure. Dhillon and Viswanath (1991) obtained reliability, steady state availability, time dependent availability and MTTF for three models, where each model corresponds to two distinct units parallel system with common cause failure. Dhillon and Anuda (1993) further considered the same parallel system with gamma distributed repair time. Authors employed the supplementary variable technique to provide the Laplace transform solution of state probability equations. Sridharan and Mohanavadivu (1997) studied three Markov models each representing the parallel system comprising of two non-identical units with different repair rules. In model 1, the system undergoes repair in both the partially failed state as well as in the completely failed state, whereas in model 2 it undergoes repair only in the case of partial failure. The considered system in model 3 is not at all repairable. Researchers concluded that the availability, reliability and the MTTF of parallel system shrinks with the increment in common cause failure rate and human error rate. Chopra and Ram (2017) further evaluated the reliability measures of the dissimilar units parallel system by incorporating waiting time.

Agnihotri and Satsangi (1996) analyzed two distinct parallel units system with priority based repair, inspection and post repair. Gupta et al. (1999) used regenerative point technique for studying the system in which the failure times of two dissimilar operating parallel units are correlated. Kumar et al. (2012) analyzed system having priority unit and non priority unit arranged in a parallel configuration. The priority unit is repairable and non priority unit is not repairable and is replaced after a random period of operation. Authors have supposed that the failure and repair times of the priority unit are correlated random variable having bivariate exponential distribution. Malik et al. (2010); Deswal and Malik (2015) also used regenerative point technique for analyzing parallel systems. EL-Sherbeny (2013) considered four types of failure: hardware failure, common cause failure, critical human error and non-critical human error to analyze the performance of two dissimilar units parallel system under preventive maintenance and two types of repair. They employed graphical evaluation and review technique to derive various reliability measures.

The concept of Copula is studied by many authors including Nelsen (2006). Copulas are defined as multivariate distribution functions with uniform margins over the interval [0, 1]. The Gumbel-Hougaard family copula is the bivariate distribution given by:

$$C_\theta(u_1, u_2) = e^{\left(-\left((-\log u_1)^\theta + (-\log u_2)^\theta\right)^{1/\theta}\right)}.$$

Here, θ lies in the interval $[1, \infty)$ and θ controls the dependency between u_1 and u_2. Gumbel-Hougaard copula belongs to the Archimedean family of copulas. Independence copula and comonotonicity copula are the special cases of Gumbel-Hougaard family copula for $\theta = 1$ and $\theta \to \infty$ respectively. Gumbel-Hougaard family copula is not symmetric and possesses higher dependence at right tails. The work presented in a series of papers of Ram and Singh (2008, 2010), Ram et al. (2013), Singh and Gulati (2014); Ram and Goyal (2018) is based on copula approach. In all these papers, authors have incorporated two types of repair facilities between neighbouring states. The two repair facilities are modelled by using Gumbel-Hougaard family copula.

Ram and Singh (2008) mentioned several reasons for using copula as a tool for modelling dependence. They applied copula to analyze the complex system which consists of two independent repairable subsystems namely 1-out-of-2:F and 1-out-of-*n*:F in series. Authors studied system under preemptive resume repair policy. They found significant improvement in the reliability of the system by incorporating copula. Ram et al. (2013) considered standby system where repair of main and standby unit follows general distribution, but repair in case of human error is handled with the aid of Gumbel-Hougaard family copula. Singh and Gulati (2014) further adopted copula to study a standby complex system under waiting time discipline. Authors modelled the need of fast repair in the case of complete failure and human error by using Gumbel-Hougaard family copula distribution. Ram and Goyal (2018) presented a novel concept for three state fault tolerant repairable system with two kinds of repair facilities. They have shown that the coverage factor and copula improves the performance of the system.

Redundant parallel systems are unanimously used in computers, power plants, navigation systems, fire stations, aircraft systems, communication systems and many other critical systems. Therefore, this study is carried out to compute the reliability measures of two distinct units parallel system subjected to two kinds of failure under two repair facilities. The two different types of repair are modelled by using Gumbel-Hougaard family copula. Supplementary variable technique and Laplace transformation are used in the present study.

2. Model Description
The system has two dissimilar units, 1 and 2 which are arranged in a parallel network. Simultaneous hardware failure of distinct units, common cause failure and human fault leads to total system failure. Normal, degraded and complete failure are the three states of the present redundant system. These states and corresponding transition diagram is presented in Table 1 and Figure 1 respectively. There are two repair facilities namely exponential and general between the completely failed states (S_3, S_4, S_5) and the normal state (S_0).

Table 1. States explanation

State	Description
S_0	Normal state: Both dissimilar units are functioning properly
S_1	Degraded state: Unit 1 has stopped working and is under repair, Unit 2 is functioning normally
S_2	Degraded state: Unit 2 has stopped working and is under repair, Unit 1 is operational
S_3	Completely failed state: System failure because of hardware breakdown of both dissimilar units
S_4	Completely failed state: System failure due to common cause failure
S_5	Completely failed state: System failure due to human error

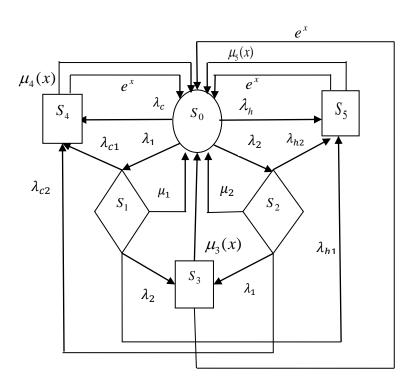Figure 1. State transition diagram

3. Notations and Assumptions
The notations used in this paper are as follows:

t	Time scale
s	Laplace transform variable
S_i $(i= 0, 1, 2, 3, 4, 5)$	Transition states
λ_1/λ_2	Hardware failure rate of unit 1/unit 2
$\lambda_c/\lambda_{c1}/\lambda_{c2}$	Common cause failure rate for state $S_0/S_1/S_2$
$\lambda_h/\lambda_{h1}/\lambda_{h2}$	The rate of human error for state $S_0/S_1/S_2$
μ_1/μ_2	Constant repair rate of unit 1/ unit 2
$\mu_3(x)/\mu_4(x)/\mu_5(x)$	Repair rate for the completely failed state $S_3/S_4/S_5$
$P_i(t)$ $(i= 0, 1, 2, 3, 4, 5)$	Probability of system being in state S_i at instant t
$\overline{P_i}(s)$ $(i= 0, 1, 2, 3, 4, 5)$	Laplace transformation of $P_i(t)$
$P_3(x,t)/P_4(x,t)/P_5(x,t)$	Probability density function of the system being in completely failed state $S_3/S_4/S_5$ at instant t with elapsed repair time x
$C_{\theta3}(u(x),\mu_3(x))$ /$C_{\theta4}(u(x),\mu_4(x))$ /$C_{\theta5}(u(x),\mu_5(x))$	Joint probability for repair rate from failed state $S_3/S_4/S_5$ to normal state S_0 according to Gumbel-Hougaard copula, where $u(x) = e^x$, $$C_{\theta3}(u(x),\mu_3(x)) = exp\left[x^\theta + \left(log(\mu_3(x))\right)^\theta\right]^{1/\theta}$$ $$C_{\theta4}(u(x),\mu_4(x)) = exp\left[x^\theta + \left(log(\mu_4(x))\right)^\theta\right]^{1/\theta}$$ $$C_{\theta5}(u(x),\mu_5(x)) = exp\left[x^\theta + \left(log(\mu_5(x))\right)^\theta\right]^{1/\theta}$$
$\overline{\chi_3}(s)/\overline{\chi_4}(s)/\overline{\chi_5}(s)$	Laplace transformation of probability density functions of $\mu_3(x)/\mu_4(x)/\mu_5(x)$

In this study, the following assumptions were made:
- The system has two distinct units which are connected in parallel network.
- Initially, the system is in normal state as both parallel units are functional.
- System complete failure occurs on account of common cause failure, human fault and hardware malfunctioning of both units.
- All failures rates are constant. Hardware failure, common cause failure and human failure are statistically independent and exponentially distributed.
- Common cause failure and human error can arise at any instant irrespective of the fact whether one unit is operational (S_1 / S_2) or both are operational (S_0).
- The system undergoes repair in the completely failed state as well as in degraded state.
- Unit 1 and unit 2 have constant repair rates.
- In complete failed states (S_3, S_4, S_5), the system is repaired by two repair facilities.
- Gumbel-Hougaard family copula is used for modelling the repairs of completely failed states (S_3, S_4, S_5).
- After repairing, the system behaves like a new one.

4. Analysis and Discussion
The set of differential equations corresponding to the present model are as follows:

$$\left(\frac{\partial}{\partial t} + \lambda_1 + \lambda_2 + \lambda_h + \lambda_c\right) P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t)$$
$$= \int_0^\infty C_{\theta 3}\big(u(x), \mu_3(x)\big) P_3(x,t)\, dx + \int_0^\infty C_{\theta 4}\big(u(x), \mu_4(x)\big) P_4(x,t)\, dx$$
$$+ \int_0^\infty C_{\theta 5}\big(u(x), \mu_5(x)\big) P_5(x,t)\, dx \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \lambda_2 + \lambda_{h1} + \lambda_{c1} + \mu_1\right) P_1(t) = \lambda_1 P_0(t) \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \lambda_1 + \lambda_{h2} + \lambda_{c2} + \mu_2\right) P_2(t) = \lambda_2 P_0(t) \tag{3}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta 3}\big(u(x), \mu_3(x)\big)\right) P_3(x,t) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta 4}\big(u(x), \mu_4(x)\big)\right) P_4(x,t) = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial t} + C_{\theta 5}\big(u(x), \mu_5(x)\big)\right) P_5(x,t) = 0 \tag{6}$$

At instant $t = 0$, the probability of the system being in state S_i ($i = 0, 1, 2, 3, 4, 5$) is given by following two equations.

$$P_0(0) = 1 \tag{7}$$

$$P_1(0) = P_2(0) = P_3(0) = P_4(0) = P_5(0) = 0 \tag{8}$$

The present model is governed by following boundary conditions:

$$P_3(0,t) = \lambda_2 P_1(t) + \lambda_1 P_2(t) \tag{9}$$

$$P_4(0,t) = \lambda_c P_0(t) + \lambda_{c1} P_1(t) + \lambda_{c2} P_2(t) \tag{10}$$

$$P_5(0,t) = \lambda_h P_0(t) + \lambda_{h1} P_1(t) + \lambda_{h2} P_2(t) \tag{11}$$

Using Equations (7-11), we obtain Laplace transform solution of Equations (1-6) as follows:

$$\overline{P_0}(s) = \frac{1}{G(s)} \tag{12}$$

$$\overline{P_1}(s) = \frac{\lambda_1}{(s + B_1)} \overline{P_0}(s) \tag{13}$$

$$\overline{P_2}(s) = \frac{\lambda_2}{(s + B_2)}\overline{P_0}(s) \tag{14}$$

$$\overline{P_3}(s) = \lambda_1\,\lambda_2\,\overline{P_0}(s)\left(\frac{1}{s + B_1} + \frac{1}{s + B_2}\right)\left(\frac{1 - \overline{\chi_3}(s)}{s}\right) \tag{15}$$

$$\overline{P_4}(s) = \overline{P_0}(s)\left(\lambda_c + \frac{\lambda_1\,\lambda_{c1}}{s + B_1} + \frac{\lambda_2\,\lambda_{c2}}{s + B_2}\right)\left(\frac{1 - \overline{\chi_4}(s)}{s}\right) \tag{16}$$

$$\overline{P_5}(s) = \overline{P_0}(s)\left(\lambda_h + \frac{\lambda_1\,\lambda_{h1}}{s + B_1} + \frac{\lambda_2\,\lambda_{h2}}{s + B_2}\right)\left(\frac{1 - \overline{\chi_5}(s)}{s}\right) \tag{17}$$

where,

$$G(s) = s + \lambda_c + \lambda_h + \lambda_1 + \lambda_2 - \frac{\lambda_1\,\mu_1}{s + B_1} - \frac{\lambda_2\,\mu_2}{s + B_2} - \lambda_1\,\lambda_2\left(\frac{1}{s + B_1} + \frac{1}{s + B_2}\right)\overline{\chi_3}(s)$$
$$- \left(\lambda_c + \frac{\lambda_{c1}\,\lambda_1}{s + B_1} + \frac{\lambda_{c2}\,\lambda_2}{s + B_2}\right)\overline{\chi_4}(s)$$
$$- \left(\lambda_h + \frac{\lambda_{h1}\,\lambda_1}{s + B_1} + \frac{\lambda_{h2}\,\lambda_2}{s + B_2}\right)\overline{\chi_5}(s) \tag{18}$$

$B_1 = \lambda_2 + \lambda_{c1} + \lambda_{h1} + \mu_1$ and $B_2 = \lambda_1 + \lambda_{c2} + \lambda_{h2} + \mu_2$.

Equation (19) and Equation (20) give Laplace transformation of the probabilities of the system being in up and down state at any moment t.

$$\overline{P_{up}}(s) = \overline{P_0}(s) + \overline{P_1}(s) + \overline{P_2}(s) = \frac{1}{G(s)}\left[1 + \frac{\lambda_1}{(s + B_1)} + \frac{\lambda_2}{(s + B_2)}\right] \tag{19}$$

$$\overline{P_{down}}(s) = \overline{P_3}(s) + \overline{P_4}(s) + \overline{P_5}(s) \tag{20}$$

4.1 Availability
4.1.1 System Availability in Comprehensive State

System availability at any instant t is obtained by setting $\overline{\chi_3}(s) = \frac{C_{\theta 3}(u(x), \mu_3(x))}{s + C_{\theta 3}(u(x), \mu_3(x))}$, $\overline{\chi_4}(s) = \frac{C_{\theta 4}(u(x), \mu_4(x))}{s + C_{\theta 4}(u(x), \mu_4(x))}$ and $\overline{\chi_5}(s) = \frac{C_{\theta 5}(u(x), \mu_5(x))}{s + C_{\theta 5}(u(x), \mu_5(x))}$ and assuming the values of parameters. In Equation (19), we assumed $\lambda_1 = 0.50$, $\lambda_2 = 0.40$, $\lambda_h = 0.30$, $\lambda_{h1} = 0.20$, $\lambda_{h2} = 0.10$, $\lambda_c = 0.25$, $\lambda_{c1} = 0.15$, $\lambda_{c2} = 0.10$, $\mu_1 = 1$, $\mu_2 = 1$, $\mu_3 = \mu_4 = \mu_5 = 1$, $x = 1$ and $\theta = 1$ and then applied inverse Laplace transformation. The obtained expression is

$$A(t) = 0.183563466447622\,e^{(-2.948513192969554\,t)}\cos(0.240846721380981\,t)$$
$$- 0.037060362890448\,e^{(-2.948513192969554\,t)}\sin(0.240846721380981\,t)$$
$$- 0.000095958716709\,e^{(-1.721255442519939\,t)}$$
$$+ 0.816532492269086 \tag{21}$$

4.1.2 System Availability without Common Cause Failure

We have computed system availability in this case by considering $\lambda_1 = 0.50, \lambda_2 = 0.40, \lambda_h = 0.30, \lambda_{h1} = 0.20, \lambda_{h2} = 0.10, \lambda_c = 0, \lambda_{c1} = 0, \lambda_{c2} = 0, \mu_1 = 1, \mu_2 = 1, \mu_3 = \mu_4 = \mu_5 = 1, x = 1$ and $\theta = 1$ in Equation (19) and after that evaluating inverse Laplace transformation. The obtained system availability at time t is

$$
\begin{aligned}
A(t) = {} & 0.130506468190394 \, e^{(-2.759140914229522t)} \cos(0.450384265458169t) \\
& + 0.133409935834662 \, e^{(-2.759140914229522t)} \sin(0.450384265458169t) \\
& - 0.000000000000002 \, e^{(-1.59999999999999 \, t)} \\
& + 0.869493531809607
\end{aligned}
\tag{22}
$$

4.1.3 System Availability without Human Error

In this case, system availability is obtained by substituting $\lambda_1 = 0.50, \lambda_2 = 0.40, \lambda_h = 0, \lambda_{h1} = 0, \lambda_{h2} = 0, \lambda_c = 0.25, \lambda_{c1} = 0.15, \lambda_{c2} = 0.10, \mu_1 = 1, \mu_2 = 1, \mu_3 = \mu_4 = \mu_5 = 1, x = 1$ and $\theta = 1$ in Equation (19) and then taking inverse Laplace transformation. We get availability as

$$
\begin{aligned}
A(t) = {} & 0.119100727810722 \, e^{(-2.720712511315509t)} \cos(0.478452459176194t) \\
& + 0.154424764456622 \, e^{(-2.720712511315509t)} \sin(0.478452459176194t) \\
& - 0.000097619485088 \, e^{(-1.576856805828027t)} \\
& + 0.880996891674365
\end{aligned}
\tag{23}
$$

Table 2 reveals that initially, there is a sharp decrease in the system availability but in long run it stabilizes to 0.81, 0.87 and 0.88 in the comprehensive state, and in the absence of common cause failure and human error respectively. Figure 2 depicts that system availability is maximum when it is free from human error and is minimum in the comprehensive state. It can also be seen from Figure 2 that system availability in the absence of common cause failure is slightly less as compared to its availability when there is no human error.

4.2 Reliability

System reliability is evaluated by equating each repair rate in Equation (19) to zero and then taking inverse Laplace transformation. We have computed system reliability in the following different cases:

4.2.1 System Reliability in Comprehensive State

System reliability at any time t is computed by equating each repair rate as zero and considering parameters $\lambda_1 = 0.50, \lambda_2 = 0.40, \lambda_h = 0.30, \lambda_{h1} = 0.20, \lambda_{h2} = 0.10, \lambda_c = 0.25, \lambda_{c1} = 0.15$ and $\lambda_{c2} = 0.10$ in Equation (19) and then employing inverse Laplace transformation. The attained system reliability is

$$
R(t) = -0.247619047619045 e^{-1.45t} + 0.714285714285736 \, e^{-0.75\,t} + 0.53 e^{-0.70t}
\tag{24}
$$

4.2.2 System Reliability without Common Cause Failure

The expression for system reliability in case of absence of common cause failure is obtained by considering all repairs in Equation (19) as zero and assuming parameters as $\lambda_1 = 0.50$, $\lambda_2 = 0.40$, $\lambda_h = 0.30$, $\lambda_{h1} = 0.20$, $\lambda_{h2} = 0.10$, $\lambda_c = 0$, $\lambda_{c1} = 0$ and $\lambda_{c2} = 0$ and then applying inverse Laplace transformation. The evaluated expression is

$$R(t) = -0.50e^{-1.2t} + 1.50e^{-0.60t} + 0.000000000000002e^{-0.60t} \tag{25}$$

4.2.3 System Reliability without Human Error

Equating each repair rate in Equation (19) to zero and substituting $\lambda_1 = 0.50$, $\lambda_2 = 0.40$, $\lambda_h = 0$, $\lambda_{h1} = 0$, $\lambda_{h2} = 0$, $\lambda_c = 0.25$, $\lambda_{c1} = 0.15$ and $\lambda_{c2} = 0.10$ followed by inverse Laplace transformation, the attained expression of system reliability is

$$R(t) = -0.560606060606044e^{-1.15t} + 0.727272727272615e^{-0.60t} + 0.83e^{-0.55t} \tag{26}$$

Information pertaining to the variation of system reliability over the time when each failure rate has some specific value is shown in Table 2. As depicted in Table 2 and Figure 3, the reliability of the present system shrinks with an increase in time. System reliability is maximum in absence of any chances of human failure. The considered system has the lowest reliability in the comprehensive state. Figure 3 exhibits that in the absence of common cause failure reliability of the assumed system is slightly less as compared to its reliability in absence of human error.

4.3 MTTF

MTTF is computed with the help of Laplace transform (Equation 19), by putting each repair rate as zero and evaluating the limit, i.e.,

$$MTTF = \lim_{s \to 0} \overline{P_{up}}(s).$$

MTTF of the present system is given by

$$MTTF = \frac{1}{(\lambda_1 + \lambda_2 + \lambda_c + \lambda_h)}\left(1 + \frac{\lambda_1}{B_1} + \frac{\lambda_2}{B_2}\right) \tag{27}$$

The present study examines the impact of each failure rate on MTTF by altering that failure rate as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 in Equation (27). The constant values of failure rates are assumed as $\lambda_1 = 0.50$, $\lambda_2 = 0.40$, $\lambda_h = 0.30$, $\lambda_{h1} = 0.20$, $\lambda_{h2} = 0.10$, $\lambda_c = 0.25$, $\lambda_{c1} = 0.15$ and $\lambda_{c2} = 0.10$ in Equation (27). Table 3 and Figure 4 reveal that, in general, system MTTF decreases as the common cause failure rate, human failure rate and hardware failure rates of both dissimilar parallel units increases. In interval (0.1, 0.2), decrease in MTTF with respect to hardware failure rate of unit 1 is high as compared to its decrease with respect to common cause failure rate, human failure rate and hardware failure rate of unit 2. Moreover, Figure 4 exhibits that corresponding to the interval (0.7, 0.9), the consequence of the increase of hardware failure rates of unit 1 and unit 2 on system MTTF is the same. The impact of an increase in human failure rate and common cause failure rate on system MTTF is similar.

4.4 Cost Analysis

Expected profit over the time t is evaluated with the help of the following equation

$$E_p(t) = K_1 \int_0^t P_{up}(t)\, dt - t\, K_2 \tag{28}$$

where K_1 and K_2 refer to revenue cost and system service cost per unit time, respectively. In a comprehensive state, by using Equation (21) in Equation (28), we obtained the following expression for the system expected profit

$$
\begin{aligned}
E_p(t) = K_1 \big(& -0.060823745500503\, e^{-2.948513192969554t} \cos 0.240846721380981t \\
& + 0.17537504224045 e^{-2.948513192969554t} \sin 0.240846721380981t \\
& + 0.060823745500503\, e^{-2.948513192969554\, t} \\
& + 0.000055749259720 e^{-1.721255442519939t} + 0.816532492269086\, t \\
& - 0.000055749259720 \big) - tK_2
\end{aligned}
\tag{29}
$$

We evaluated the expected profit by assuming revenue cost as 1 and changing service cost as 0.10, 0.30, 0.40 and 0.70 in Equation (29). Figure 5 indicates that the increasing service cost decreases the expected profit of the present redundant system.

Table 2. Availability and Reliability of system in Comprehensive State

Time	Availability in Comprehensive State	Reliability in Comprehensive State
0	1	1
1	0.825396197	0.544166105
2	0.816929218	0.277272269
3	0.816548245	0.137399283
4	0.816532955	0.067244545
5	0.816532487	0.032727808
6	0.816532489	0.015891389
7	0.816532492	0.007710062
8	0.816532492	0.003740462
9	0.816532492	0.001815173
10	0.816532492	0.000881272
11	0.816532492	0.000428092
12	0.816532492	0.000208072
13	0.816532492	0.000101193
14	0.816532492	0.000049243
15	0.816532492	0.000023977

Table 3. MTTF as function of failure rates in the Comprehensive State

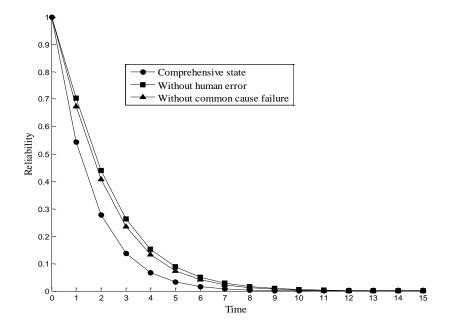| λ_1	λ_2	λ_h	λ_c	λ_{h1}	λ_{h2}	λ_{c1}	λ_{c2}
2.35	1.96	1.79	1.72	1.61	1.54	1.58	1.54
1.97	1.76	1.66	1.60	1.54	1.49	1.51	1.49
1.76	1.63	1.54	1.49	1.49	1.46	1.47	1.46
1.63	1.54	1.44	1.40	1.45	1.43	1.43	1.43
1.54	1.49	1.36	1.32	1.41	1.40	1.40	1.40
1.48	1.44	1.28	1.24	1.38	1.38	1.37	1.38
1.44	1.41	1.21	1.18	1.36	1.36	1.35	1.36
1.41	1.39	1.15	1.12	1.34	1.35	1.33	1.35
1.39	1.37	1.091	1.07	1.32	1.33	1.31	1.33

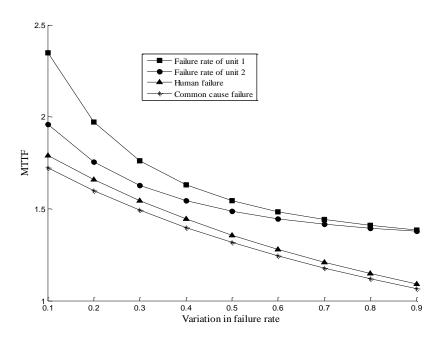Figure 2. Time vs Availability

Figure 3. Time vs Reliability

Figure 4. MTTF as function of failure rates in the Comprehensive State
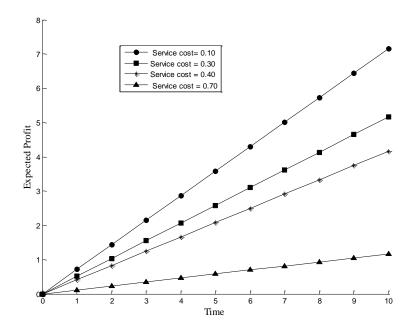
Figure 5. Expected profit as a function of time in Comprehensive State

5. Conclusion

The proposed model has incorporated copula to analyze the performance of two dissimilar units parallel system by considering common cause failure, human fault and hardware failure of units. The use of non-identical units in the parallel network decreases system cost and hence considered the system is more economical as compared to systems having identical units. Such dissimilar units parallel systems are widely found in power generating stations, aircrafts, medical equipments and industrial setups. The present model is significant because it has two repair facilities in completely failed states. The present system has maximum availability and reliability when there is no human error. Moreover, results reveal that the system has minimum availability and reliability in the comprehensive state. This study shows that at any moment, system availability is more than its reliability. Graph of system MTTF in comprehensive state illustrates that with an increase in values of each failure rate MTTF decreases. Through cost analysis, it is found that the rising service cost decreases the expected profit. So, the present system can be made more reliable by preventing the occurrence of the human error and common cause failure.

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