

Optimal Mass Design of 25 Bars Truss with Loading Conditions on Five Node Elements

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Abstract

The optimal design of a twenty-five bar space truss commonly involves multiple loading conditions acting on 4 node elements in the linear elastic model. In this paper, we describe the behavior of the truss system with our experimental loading conditions on five node elements subject to minimum displacement and stresses that are used to formulate the constrained nonlinear optimization problem. Numerical computations are developed with the objective of mass minimization and the best structural design is selected by applying the interior point method with the guidance of Matlab Optimization Toolbox. Our numerical results show the optimal values of cross-sectional areas, material densities, and internal forces which satisfy the minimum weight design. These results provide the appropriate mass to the experimental data and allow substantial changes in size, shape, and topology.

Keywords- Linear elastic model, Variational problem, Structural optimization, Interior point method, Trusses

1. Introduction

The twenty-five bars space truss problem is typically characterized by their large numbers of design variables and constraints within allowable limits. It is therefore difficult but not impossible to describe this truss system with all stresses and displacements for minimum weight, deflection and for the maximum fundamental natural frequency of vibration. This structure has been optimized by many workers including Rajeev and Krishnamoorthy (1992), Wu and Chow (1995a, 1995b), Adeli and Park (1996), Erbatur et al. (2000), Park and Sung (2002). In these studies, the loading conditions were acting on four node elements using the size and shape symmetry in certain study case. Other studies were made with the same loading conditions including Venkayya (1971), Schmit and Farshi (1974), Schmidt and Miura (1976), Adeli and Kamal (1986), Saka (1990), Lamberti (2008), Farshi and Ziazi (2010). To make the structure economical and save materials, the truss profiles were selected for optimal mass design using STADD.Pro in Bora (2016) and also in Jha and Paliwal (2017).

In this paper, we examine the case where the loading conditions are acting on five nodes elements in a linear elastic model in order to select the economical profile of the 25 bars space truss. For this, we find the displacement field from the variational problem and look for the design requirements and structural criteria that leads to the objective of mass minimization. Numerical

computations are developed about applied forces at five node elements to select the best structural design of minimal mass. By applying the interior point-method (Herskovits et al., 2000) with the guidance of Matlab optimization Toolbox (Coleman et al., 1999), the results provide the economical mass, which allows substantial changes in size, shape, and topology of the truss system.

The purpose is introduced in this section and the linear elastic model is formulated in section 2 with the minimization of the energy functional. Structural analysis is developed about the loading conditions on five nodes elements in order to state the objective of the mass minimization problem in section 3. The structural design of the minimum mass is selected in section 4 through numerical applications on two load cases and section 5 concludes the present study.

2. Model Formulation and Minimization of the Energy Functional

We consider that a twenty-five bars truss shown in Figure 1, occupies a domain Ω of N -dimensional design space and carries a load on the nodes $\{1\}$, $\{2\}$, $\{3\}$, $\{5\}$ and $\{6\}$ as given below in Figure 2.

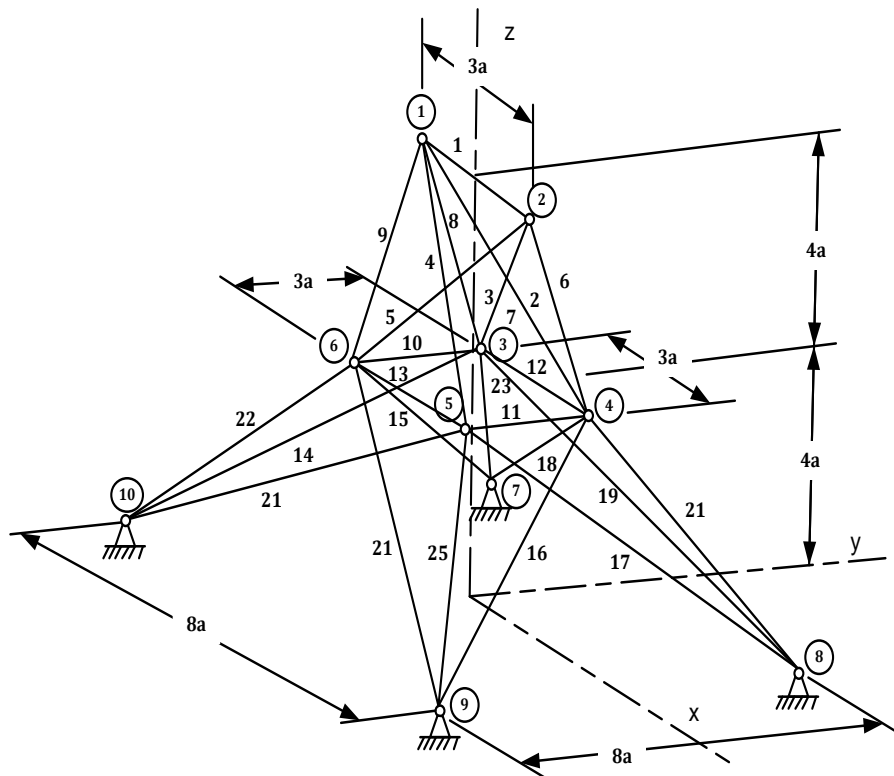


Figure 1. Twenty-five bars truss

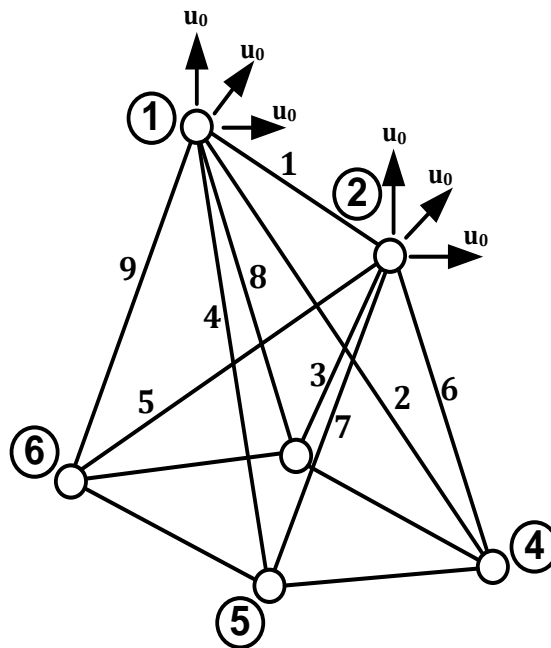


Figure 2. Applied loads at the Nodes {1}, {2}, {3}, {5} and {6}

The structural design of a twenty-five bars truss commonly involves a mathematical model in term of partial differential equations, a set of objective functions and a set of admissible constraints or stresses. In general, one denotes by the Hilbert space $\mathcal{L}^2(\Omega)$, the set of measurable and square integrable functions in Ω and $\mathcal{H}^1(\Omega) = \left\{ \omega \in \mathcal{L}^2(\Omega) \mid \forall i \in \{1, 2, \dots, N\}, \frac{\partial \omega}{\partial x_i} \in \mathcal{L}^2(\Omega) \right\}$, the Sobolev space. The boundary of Ω is denoted by $\partial\Omega$ and the loading forces $F \in \mathcal{L}^2(\Omega)$ are imposed on the boundary $\partial\Omega_F = \partial\{\{1\}, \{2\}, \{3\}, \{5\}, \{6\}\}$ by $g_i \in \mathcal{L}^2(\Omega)$ with $i \in \{1, 2, 3, 5, 6\}$. At the nodes $\{7\}, \{8\}, \{9\}$ and $\{10\}$, the truss is fixed in contact with the soil and we assume that all the displacements are equal to zero on the boundary denoted by $\partial\Omega_{u_0} = \partial\{\{7\}, \{8\}, \{9\}, \{10\}\}$. In Hilbert space of all displacement $\mathcal{H}^1(\Omega)$, we denote by u_{ad} , the admissible displacements at the nodes $\{1\}$ and $\{2\}$ and on the boundary $\partial\Omega_{u_1} = \partial\{\{1\}, \{2\}\}$. The loading conditions are not applied on the boundary $\partial\Omega_{F_0} = C_{\partial\Omega}^{\partial\Omega_F \cup \partial\Omega_u}$ and the overall boundary can be decomposed in such a manner that $\partial\Omega = \partial\Omega_F \cup \partial\Omega_{u_0} \cup \partial\Omega_{u_1} \cup \partial\Omega_{F_0}$. We consider that $f \in \mathcal{L}^2(\Omega)$ is the volume force of the truss structure and assume that the elastic properties of the cross sections are represented by a symmetric matrix function $A(x) = (a_{i,j}(x))_{1 \leq i,j \leq N} \in \mathbb{R}^{N \times N}$. According to Hooke law, the normal component of the stresses field $\sigma = \sum_{i=1}^N \left(\sum_{j=1}^N \frac{\partial \sigma_{ij}}{\partial x_j} \right) \in \mathcal{L}^2(\Omega)^N$ is equal to $A \nabla u$ and the elastic model equation is represented by

$$\operatorname{div} \sigma = f, \text{ in } \Omega \quad (1)$$

$$u = 0, \text{ on } \partial\Omega_{u_0} = \partial\{\{7\}, \{8\}, \{9\}, \{10\}\} \quad (2)$$

$$u = u_{\text{ad}}, \text{ on } \partial\Omega_{u_1} = \partial\{\{1\}, \{2\}\} \quad (3)$$

$$F = \sigma \cdot n = g_i, \text{ on } \partial\Omega_F = \partial\{\{1\}, \{2\}, \{3\}, \{5\}, \{6\}\} \quad (4)$$

$$F = \sigma \cdot n = 0, \text{ on } \partial\Omega_{F_0} \quad (5)$$

The solution space to model (1-5) is $\mathcal{H}_s(\Omega) = \{u \in \mathcal{H}^1(\Omega) \mid u = 0 \text{ on } \partial\Omega_{u_0} \text{ and } u = u_{\text{ad}} \text{ on } \partial\Omega_{u_1}\}$ and the variational problem can be stated as:

$$\int_{\Omega} A \nabla u \cdot \nabla v \, dx = \int_{\Omega} f \cdot v \, dx + \int_{\partial\Omega_F} g_i \cdot v \, ds, \quad (6)$$

for all $v \in \mathcal{H}_s(\Omega)$, with $i \in \{1, 2, 3, 5, 6\}$.

The Dirichlet conditions (2) and (3) are satisfied in the solution space $\mathcal{H}_s(\Omega)$ and the Neumann conditions (4) and (5) are imposed in the variational formulation. We assume that the unique solution of the variational problem (6) is also the unique solution of the boundary value problem (1-5). Its stability depends continuously on the surface force $g_i \in \mathcal{L}^2(\Omega)$ and the volume force $f \in \mathcal{L}^2(\Omega)$. This solution is obtained by minimizing the energy functional $J(v) = \frac{1}{2} q(v) - \left(\int_{\Omega} f \cdot v \, dx + \int_{\partial\Omega_F} g_i \cdot v \, ds \right)$ in the Hilbert space $\mathcal{H}_s(\Omega)$ as follows

$$u = \arg \min_{v \in \mathcal{H}_s(\Omega)} J(v) \quad (7)$$

where the bilinear form $\int_{\Omega} A \nabla u \cdot \nabla v \, dx \in \mathcal{H}_s(\Omega)$ can admit a quadratic form denoted by $q(v)$.

The problem (7) provides the minimum displacement $u \in \mathcal{H}_s(\Omega)$ which is used to attain the objective of the mass minimization.

2.1 Stress and Displacement Constraints of Our Approach

One denotes by $U_{\text{ad}} \subset \mathcal{H}_s(\Omega)$ and $\sigma_{\text{ad}} \subset \mathcal{H}_s(\Omega)$, the set of admissible displacements and stresses in tension or compression. Also, we assume that the coordinates at each node of the twenty-five bars truss are given in Table 1.

Table 1. Coordinate systems with respect to x, y, and z-axis

Nodes	x	Y	Z
1	-3/2	0	8
2	3/2	0	8
3	-3/2	3/2	4
4	3/2	3/2	4
5	3/2	-3/2	4
6	-3/2	-3/2	4
7	-4	4	0
8	4	4	0
9	4	-4	0
5	3/2	-3/2	4
10	-4	-4	0

As it is shown in Table 2, one denotes by $(l_i)_{1 \leq i \leq 25} > 0$, the i th length or line joining two node elements and by $(S_{i,j})_{1 \leq i,j \leq 25} \in \Omega$, the members of cross-sections. The formula used to compute the lengths are given by: $l_i = \sqrt{\Delta x_i^2 + \Delta y_i^2 + \Delta z_i^2}$.

Table 2. Length $(l_i)_{1 \leq i \leq 25}$ associated to the cross sections $(S_{i,j})_{1 \leq i,j \leq 25, i \neq j}$

Cross sections	Lengths
$S_{1,i}$	3
$S_{2,j}$	$\sqrt{109}/2$
$S_{3,j}$	$\sqrt{109}/2$
$S_{4,j}$	$\sqrt{109}/2$
$S_{5,j}$	$\sqrt{109}/2$
$S_{6,j}$	$\sqrt{73}/2$
$S_{8,j}$	$\sqrt{73}/2$
$S_{9,j}$	$\sqrt{73}/2$
$S_{10,i}$	3
$S_{11,j}$	3
$S_{12,i}$	3
$S_{13,j}$	3
$S_{14,j}$	$\sqrt{210}/2$
$S_{15,j}$	$\sqrt{210}/2$
$S_{16,j}$	$\sqrt{210}/2$
$S_{17,j}$	$\sqrt{210}/2$
$S_{18,j}$	$\sqrt{210}/2$
$S_{19,j}$	$\sqrt{210}/2$
$S_{20,j}$	$\sqrt{210}/2$
$S_{21,j}$	$\sqrt{210}/2$
$S_{22,j}$	$\sqrt{114}/2$
$S_{23,j}$	$\sqrt{114}/2$
$S_{24,j}$	$\sqrt{114}/2$
$S_{25,j}$	$\sqrt{114}/2$

We assume the permissible displacements on the node element 1 and 2 as it is presented in Table 3.

Table 3. Displacement limit u_0 at the nodes {1} and {2}

Nodes	x-direction	y-direction	z-direction
1	± 3.5	± 3.5	± 3.5
2	± 3.5	± 3.5	± 3.5

Each node has three degrees of freedom with respect to x, y and z-axis and these constraints of displacement on the nodes 1 and 2 are given by the equations:

$$|u_{1x}| \leq 3.5 = u_0, \quad |u_{1y}| \leq 3.5 = u_0, \quad |u_{1z}| \leq 3.5 = u_0 \quad (8)$$

$$|u_{2x}| \leq 3.5 = u_0, \quad |u_{2y}| \leq 3.5 = u_0, \quad |u_{2z}| \leq 3.5 = u_0 \quad (9)$$

The displacements imposed on the nodes 1 and 2 influence other nodes of the truss system. It appears an axial deformation at each member of the cross sections modeled by the displacements. We have the relation between axial deformations and stresses by $\sigma_0 = E\varepsilon$ where E is the modulus of elasticity and $\varepsilon = \frac{\Delta l}{l}$. Therefore we can consider the limitation of stresses presented in Table 4.

Table 4. Description of the elastic behavior

Cross sections	Stresses
$S_{1,j}$	$\frac{2}{3}Eu_0$
$S_{2,j}, S_{3,j}, S_{4,j}, S_{5,j}$	$\frac{10}{109}Eu_0$
$S_{6,j}, S_{7,j}, S_{8,j}, S_{9,j}$	$\frac{26}{73}Eu_0$

We denote by $h = \{h_{i,j} \in \mathcal{H}^1(\Omega) \mid 1 \leq i \leq 2 \text{ and } 1 \leq j \leq 6 \text{ and } i \neq j\}$, the set of internal forces that are distributed in Ω_F from the imposed displacements on the nodes 1 and 2. The behavior law leads to the following constraints:

$$|h_{1,2}| \leq \frac{2}{3}Eu_0, \quad |h_{1,4}| \leq \frac{10}{209}Eu_0, \quad |h_{2,3}| \leq \frac{10}{209}Eu_0 \quad (10)$$

$$|h_{1,5}| \leq \frac{10}{209}Eu_0, \quad |h_{2,6}| \leq \frac{10}{209}Eu_0, \quad |h_{1,6}| \leq \frac{10}{209}Eu_0 \quad (11)$$

$$|h_{1,3}| \leq \frac{26}{73}Eu_0, \quad |h_{2,4}| \leq \frac{26}{73}Eu_0, \quad |h_{2,5}| \leq \frac{26}{73}Eu_0 \quad (12)$$

The admissible constraint field σ_0 has to be in tension or in compression and those stresses are represented in Ω by the following equations:

$$|h_{1,2}| \leq \sigma_0 S_{1,2}, \quad |h_{1,3}| \leq \sigma_0 S_{1,3}, \quad |h_{1,4}| \leq \sigma_0 S_{1,4} \quad (13)$$

$$|h_{1,2}| \leq \sigma_0 S_{1,2}, \quad |h_{1,3}| \leq \sigma_0 S_{1,3}, \quad |h_{1,4}| \leq \sigma_0 S_{1,4} \quad (14)$$

$$|h_{1,4}| \leq \sigma_0 S_{1,4}, \quad |h_{1,5}| \leq \sigma_0 S_{1,5}, \quad |h_{1,6}| \leq \sigma_0 S_{1,6} \quad (15)$$

$$|h_{2,3}| \leq \sigma_0 S_{2,3}, \quad |h_{2,4}| \leq \sigma_0 S_{2,4}, \quad |h_{2,5}| \leq \sigma_0 S_{2,5} \quad (16)$$

$$|h_{2,6}| \leq \sigma_0 S_{2,6}, \quad |h_{3,4}| \leq \sigma_0 S_{3,4}, \quad |h_{3,6}| \leq \sigma_0 S_{3,6} \quad (17)$$

$$|h_{3,7}| \leq \sigma_0 S_{3,7}, \quad |h_{3,8}| \leq \sigma_0 S_{3,8}, \quad |h_{3,10}| \leq \sigma_0 S_{3,10} \quad (18)$$

$$|h_{4,5}| \leq \sigma_0 S_{4,5}, \quad |h_{4,8}| \leq \sigma_0 S_{4,8}, \quad |h_{5,6}| \leq \sigma_0 S_{5,6} \quad (19)$$

$$|h_{5,8}| \leq \sigma_0 S_{5,8}, \quad |h_{5,9}| \leq \sigma_0 S_{5,9}, \quad |h_{5,10}| \leq \sigma_0 S_{5,10} \quad (20)$$

$$|h_{6,7}| \leq \sigma_0 S_{6,7}, \quad |h_{6,9}| \leq \sigma_0 S_{6,9}, \quad |h_{6,10}| \leq \sigma_0 S_{6,10} \quad (21)$$

$$|h_{6,10}| \leq \sigma_0 S_{6,10} \quad (22)$$

2.2 External Stability Conditions of the Truss System

For stability, we denoted by $F = \{F_{i,j} \in \mathcal{H}^1(\Omega) \mid 1 \leq i \leq 6 \text{ and } 1 \leq j \leq 10\}$ the set of external forces $F_{i,j}$ which satisfies the equilibrium state of 25 bars space truss. The projections are represented in Ω on the node elements with respect to x, y and z axis and the equations describing this equilibrium are given as follows:

Projections on the node 1:

$$h_{1,2} + \frac{6}{\sqrt{109}}h_{2,3} + \frac{6}{\sqrt{109}}h_{1,5} = F_{1,x} \quad (23)$$

$$\frac{3}{\sqrt{73}}h_{1,3} + \frac{3}{\sqrt{109}}h_{1,4} - \frac{3}{\sqrt{109}}h_{1,5} - \frac{3}{73}h_{1,6} = F_{1,y} \quad (24)$$

$$-\frac{3}{\sqrt{73}}h_{1,3} - \frac{8}{\sqrt{109}}h_{1,4} - \frac{8}{\sqrt{109}}h_{1,5} - \frac{8}{73}h_{1,6} = F_{1,z} \quad (25)$$

Projections on the node 2:

$$h_{1,2} - \frac{6}{\sqrt{109}}h_{2,3} - \frac{6}{\sqrt{109}}h_{2,6} = F_{2,x} \quad (26)$$

$$\frac{3}{\sqrt{109}}h_{2,3} + \frac{3}{\sqrt{73}}h_{2,4} - \frac{3}{\sqrt{73}}h_{2,5} - \frac{3}{109}h_{2,6} = F_{2,y} \quad (27)$$

$$-\frac{8}{\sqrt{109}}h_{2,3} - \frac{8}{\sqrt{73}}h_{2,4} + \frac{8}{\sqrt{73}}h_{2,5} - \frac{8}{109}h_{2,6} = F_{2,z} \quad (28)$$

Projections on the node 3:

$$\frac{6}{\sqrt{109}}h_{2,3} + h_{3,4} - \frac{5}{\sqrt{114}}h_{3,7} + \frac{11}{\sqrt{210}}h_{3,8} = F_{3,x} \quad (29)$$

$$\frac{6}{\sqrt{73}}h_{1,3} + \frac{6}{\sqrt{109}}h_{2,3} + \frac{5}{\sqrt{114}}h_{3,7} + \frac{5}{\sqrt{210}}h_{3,8} = F_{3,y} \quad (30)$$

$$-\frac{8}{\sqrt{73}}h_{1,3} - \frac{8}{\sqrt{109}}h_{2,3} - \frac{8}{\sqrt{114}}h_{3,7} + \frac{8}{\sqrt{210}}h_{3,8} = F_{3,z} \quad (31)$$

Projections on the node 4:

$$-\frac{6}{\sqrt{109}}h_{1,4} - h_{3,4} - \frac{11}{\sqrt{210}}h_{4,7} + \frac{5}{\sqrt{114}}h_{4,8} = F_{4,x} \quad (32)$$

$$\frac{3}{\sqrt{109}}h_{1,4} + \frac{3}{\sqrt{73}}h_{2,4} - h_{4,5} + \frac{5}{\sqrt{210}}h_{4,7} + \frac{5}{\sqrt{114}}h_{4,8} = F_{4,y} \quad (33)$$

$$-\frac{8}{\sqrt{109}}h_{1,4} - \frac{8}{\sqrt{73}}h_{2,4} - \frac{8}{\sqrt{210}}h_{4,7} - \frac{4}{\sqrt{114}}h_{4,8} = F_{4,z} \quad (34)$$

Projections on the node 5:

$$-\frac{6}{\sqrt{109}}h_{1,5} - h_{5,6} + \frac{5}{\sqrt{210}}h_{5,8} + \frac{5}{\sqrt{114}}h_{5,9} - \frac{11}{\sqrt{210}}h_{5,10} = F_{5,x} \quad (35)$$

$$-\frac{3}{\sqrt{109}}h_{1,5} - \frac{3}{\sqrt{73}}h_{2,5} - h_{4,5} + \frac{11}{\sqrt{210}}h_{5,8} - \frac{5}{\sqrt{114}}h_{5,9} - \frac{5}{\sqrt{210}}h_{5,10} = F_{5,y} \quad (36)$$

$$-\frac{8}{\sqrt{109}}h_{1,5} - \frac{8}{\sqrt{73}}h_{2,4} - \frac{8}{\sqrt{210}}h_{5,8} - \frac{8}{\sqrt{114}}h_{5,9} - \frac{8}{\sqrt{210}}h_{5,10} = F_{5,z} \quad (37)$$

Projections on the node 6:

$$-\frac{6}{\sqrt{109}}h_{2,6} - h_{5,6} - \frac{5}{\sqrt{210}}h_{6,7} + \frac{11}{\sqrt{210}}h_{6,9} - \frac{5}{\sqrt{114}}h_{6,10} = F_{6,x} \quad (38)$$

$$-\frac{3}{73}h_{1,6} - \frac{3}{\sqrt{109}}h_{2,6} - h_{3,6} + \frac{11}{\sqrt{210}}h_{6,7} - \frac{5}{\sqrt{210}}h_{6,9} - \frac{5}{\sqrt{114}}h_{6,10} = F_{6,y} \quad (39)$$

$$-\frac{4}{73}h_{1,6} - \frac{8}{\sqrt{109}}h_{2,6} - \frac{8}{\sqrt{210}}h_{6,7} - \frac{8}{\sqrt{210}}h_{6,9} - \frac{8}{\sqrt{114}}h_{6,10} = F_{6,z} \quad (40)$$

3. Formulation of the Structural Optimization Problem

Size and shape optimization of truss structure is widely used despite large computation costs for massive structures. This optimization is often applied to obtain the optimum values of the cross section area $S_{i,j}$ and lengths l_i which lead to minimum weight design. Our approach imposes the loading conditions and movements of nodes which include the calculations of cross sections and internal forces $h_{i,j}$ with the objective of mass minimization.

3.1 Design Variables

The variables that are chosen to describe the design of truss system are represented in Tables 5, 6 and 7.

Table 5. Variables $(x_i)_{1 \leq i \leq 25}$ of the cross sections $(S_{i,j})_{1 \leq i,j \leq 25, i \neq j}$

Cross Sections	Unknown variables
$S_{1,j}$	x_1
$S_{2,j}$	x_2
$S_{3,j}$	x_3
$S_{4,j}$	x_4
$S_{5,j}$	x_5
$S_{6,j}$	x_6
$S_{7,j}$	x_7
$S_{8,j}$	x_8
$S_{9,j}$	x_9
$S_{10,j}$	x_{10}
$S_{11,j}$	x_{11}
$S_{12,j}$	x_{12}
$S_{13,j}$	x_{13}
$S_{14,j}$	x_{14}
$S_{15,j}$	x_{15}
$S_{16,j}$	x_{16}
$S_{17,j}$	x_{17}
$S_{18,j}$	x_{18}
$S_{19,j}$	x_{19}
$S_{20,j}$	x_{20}
$S_{21,j}$	x_{21}
$S_{22,j}$	x_{22}
$S_{23,j}$	x_{23}
$S_{24,j}$	x_{24}
$S_{25,j}$	x_{25}

Table 6. Variables $(x_i)_{25 \leq i \leq 50}$ of the Internal Forces $h_{i,j}$

Internal Forces	Unknown variables
$h_{1,2}$	x_{26}
$h_{1,3}$	x_{27}
$h_{1,4}$	x_{28}
$h_{1,5}$	x_{29}
$h_{1,6}$	x_{30}
$h_{2,3}$	x_{31}
$h_{2,4}$	x_{32}
$h_{2,5}$	x_{33}
$h_{2,6}$	x_{34}
$h_{3,4}$	x_{35}
$h_{3,6}$	x_{36}
$h_{3,7}$	x_{37}
$h_{3,8}$	x_{38}
$h_{3,10}$	x_{39}
$h_{4,5}$	x_{40}
$h_{4,7}$	x_{41}
$h_{4,8}$	x_{42}
$h_{4,9}$	x_{43}
$h_{5,6}$	x_{44}
$h_{5,8}$	x_{45}
$h_{5,9}$	x_{46}
$h_{5,10}$	x_{47}
$h_{6,7}$	x_{48}
$h_{6,9}$	x_{49}
$h_{6,10}$	x_{50}

Table 7. Variables $(x_i)_{25 \leq i \leq 50}$ of Material Density ρ_i

Material Density	Unknown variables s
ρ_1	x_{51}
ρ_2	x_{52}
ρ_3	x_{53}
ρ_4	x_{54}
ρ_5	x_{55}
ρ_6	x_{56}
ρ_7	x_{57}
ρ_8	x_{58}
ρ_9	x_{59}
ρ_{10}	x_{60}
ρ_{11}	x_{61}
ρ_{12}	x_{62}
ρ_{13}	x_{63}
ρ_{14}	x_{64}
ρ_{15}	x_{65}
ρ_{16}	x_{66}
ρ_{17}	x_{67}
ρ_{18}	x_{68}
ρ_{19}	x_{69}
ρ_{20}	x_{70}
ρ_{21}	x_{71}
ρ_{22}	x_{72}
ρ_{23}	x_{73}
ρ_{24}	x_{74}
ρ_{25}	x_{75}

3.2 Linear Design Constraints

The limitations of the truss system are behavior and side constraints in matrix form $A_{eq}x = b_{eq}$ and $A_{ineq}x \leq b_{ineq}$.

3.2.1 Equality Constraints

The first design requirement for $A_{eq} \in \mathbb{R}^{19 \times 75}$, $b_{eq} \in \mathbb{R}^{19 \times 1}$ and $x \in \mathbb{R}^{75 \times 1}$ are represented by the equality constraints expressed by:

$$x_{26} + \frac{6}{\sqrt{109}}x_{28} + \frac{6}{\sqrt{109}}x_{29} = F_{1,x} \quad (41)$$

$$\frac{3}{\sqrt{73}}x_{27} + \frac{3}{\sqrt{109}}x_{28} - \frac{3}{\sqrt{109}}x_{29} - \frac{3}{73}x_{30} = F_{1,y} \quad (42)$$

$$-\frac{8}{\sqrt{73}}x_{27} - \frac{8}{\sqrt{109}}x_{28} - \frac{8}{\sqrt{109}}x_{29} - \frac{8}{73}x_{30} = F_{1,z} \quad (43)$$

$$x_{26} - \frac{6}{\sqrt{109}}x_{31} - \frac{6}{\sqrt{109}}x_{34} = F_{2,x} \quad (44)$$

$$\frac{3}{\sqrt{109}}x_{31} + \frac{3}{\sqrt{73}}x_{32} - \frac{3}{\sqrt{73}}x_{33} - \frac{3}{109}x_{34} = F_{2,y} \quad (45)$$

$$-\frac{8}{\sqrt{109}}x_{31} - \frac{8}{\sqrt{73}}x_{32} + \frac{8}{\sqrt{73}}x_{33} - \frac{8}{109}x_{34} = F_{2,z} \quad (46)$$

$$\frac{6}{\sqrt{109}}x_{31} + x_{35} - \frac{5}{\sqrt{114}}x_{37} + \frac{11}{\sqrt{210}}x_{38} = F_{3,x} \quad (47)$$

$$\frac{6}{\sqrt{73}}x_{27} + \frac{6}{\sqrt{109}}x_{31} + \frac{5}{\sqrt{114}}x_{37} + \frac{5}{\sqrt{210}}x_{38} = F_{3,y} \quad (48)$$

$$-\frac{8}{\sqrt{73}}x_{27} - \frac{8}{\sqrt{109}}x_{31} - \frac{8}{\sqrt{114}}x_{37} + \frac{8}{\sqrt{210}}x_{38} = F_{3,z} \quad (49)$$

$$-\frac{6}{\sqrt{109}}x_{28} - x_{35} - \frac{11}{\sqrt{210}}x_{41} + \frac{5}{\sqrt{114}}x_{42} = F_{4,x} \quad (50)$$

$$\frac{3}{\sqrt{109}}x_{28} + \frac{3}{\sqrt{73}}x_{32} - x_{40} + \frac{5}{\sqrt{210}}x_{41} + \frac{5}{\sqrt{114}}x_{42} = F_{4,y} \quad (51)$$

$$-\frac{8}{\sqrt{109}}x_{28} - \frac{8}{\sqrt{73}}x_{32} - \frac{8}{\sqrt{210}}x_{41} - \frac{4}{\sqrt{114}}x_{42} = F_{4,z} \quad (52)$$

$$-\frac{6}{\sqrt{109}}x_{29} - x_{44} + \frac{5}{\sqrt{210}}x_{45} + \frac{5}{\sqrt{114}}x_{46} - \frac{11}{\sqrt{210}}x_{47} = F_{5,x} \quad (53)$$

$$-\frac{3}{\sqrt{109}}x_{29} - \frac{3}{\sqrt{73}}x_{33} - x_{40} + \frac{11}{\sqrt{210}}x_{45} - \frac{5}{\sqrt{114}}x_{46} - \frac{5}{\sqrt{210}}x_{47} = F_{5,y} \quad (54)$$

$$-\frac{8}{\sqrt{109}}x_{29} - \frac{8}{\sqrt{73}}x_{32} - \frac{8}{\sqrt{210}}x_{45} - \frac{8}{\sqrt{210}}x_{46} - \frac{8}{\sqrt{210}}x_{47} = F_{5,z} \quad (55)$$

$$-\frac{8}{\sqrt{109}}x_{29} - \frac{8}{\sqrt{73}}x_{32} - \frac{8}{\sqrt{210}}x_{45} - \frac{8}{\sqrt{210}}x_{46} - \frac{8}{\sqrt{210}}x_{47} = F_{5,z} \quad (56)$$

$$-\frac{6}{\sqrt{109}}x_{34} - x_{44} - \frac{5}{\sqrt{210}}x_{48} + \frac{11}{\sqrt{210}}x_{49} - \frac{5}{\sqrt{114}}x_{50} = F_{6,x} \quad (57)$$

$$-\frac{3}{\sqrt{73}}x_{30} - \frac{3}{\sqrt{109}}x_{34} - x_{36} + \frac{11}{\sqrt{210}}x_{48} - \frac{5}{\sqrt{210}}x_{49} - \frac{5}{\sqrt{114}}x_{50} = F_{6,y} \quad (58)$$

$$-\frac{4}{\sqrt{73}}x_{30} - \frac{8}{\sqrt{109}}x_{34} - \frac{8}{\sqrt{210}}x_{48} - \frac{8}{\sqrt{210}}x_{49} - \frac{8}{\sqrt{114}}x_{50} = F_{6,z} \quad (59)$$

An additional equality constraint on limited material density is given by:

$$\sum_{i=51}^{75} x_i = 1925.25 \quad (60)$$

3.2.2 Inequality Constraints

The second design requirement for $A_{ineq} \in IR^{118 \times 75}$, $b_{ineq} \in IR^{118}$ and $x \in IR^{75}$ is represented by the inequality constraints expressed by:

$$|x_{26}| \leq x_1\sigma_0, \quad |x_{27}| \leq x_8\sigma_0, \quad |x_{28}| \leq x_2\sigma_0 \quad (61)$$

$$|x_{29}| \leq x_4\sigma_0, \quad |x_{30}| \leq x_9\sigma_0, \quad |x_{31}| \leq x_3\sigma_0 \quad (62)$$

$$|x_{32}| \leq x_6\sigma_0, \quad |x_{33}| \leq x_7\sigma_0, \quad |x_{34}| \leq x_5\sigma_0 \quad (63)$$

$$|x_{35}| \leq x_{12}\sigma_0, \quad |x_{36}| \leq x_{10}\sigma_0, \quad |x_{37}| \leq x_{19}\sigma_0 \quad (64)$$

$$|x_{38}| \leq x_{23}\sigma_0, \quad |x_{39}| \leq x_{14}\sigma_0, \quad |x_{40}| \leq x_{11}\sigma_0 \quad (65)$$

$$|x_{41}| \leq x_{18}\sigma_0, \quad |x_{42}| \leq x_{24}\sigma_0, \quad |x_{43}| \leq x_{16}\sigma_0 \quad (66)$$

$$|x_{44}| \leq x_{13}\sigma_0, \quad |x_{45}| \leq x_{17}\sigma_0, \quad |x_{46}| \leq x_{25}\sigma_0 \quad (67)$$

$$|x_{47}| \leq x_{20}\sigma_0, \quad |x_{48}| \leq x_{15}\sigma_0, \quad |x_{49}| \leq x_{21}\sigma_0 \quad (68)$$

$$|x_{50}| \leq x_{22}\sigma_0, \quad |x_{26}| \leq \frac{2}{3}Eu_0, \quad |x_{27}| \leq \frac{26}{73}Eu_0 \quad (69)$$

$$|x_{28}| \leq \frac{10}{209}Eu_0, \quad |x_{29}| \leq \frac{10}{209}Eu_0, \quad |x_{30}| \leq \frac{26}{73}Eu_0 \quad (70)$$

$$|x_{31}| \leq \frac{10}{209}Eu_0, \quad |x_{32}| \leq \frac{26}{73}Eu_0, \quad |x_{33}| \leq \frac{26}{73}Eu_0 \quad (71)$$

$$|x_{34}| \leq \frac{10}{209}Eu_0, \quad 0 \leq (x_i)_{1 \leq i \leq 25}, \quad 0 \leq (x_i)_{51 \leq i \leq 75} \quad (72)$$

3.3 Nonlinear Objective Function

One represents the structural mass of the truss system by the nonlinear objective function of several variables stated as:

$$\begin{aligned} m(x) = \sum_{i=1}^n \rho_i S_i L_i = & + \frac{\sqrt{109}}{2} (x_{56}x_2 + x_{57}x_3 + x_{58}x_4 + x_{59}x_5) \\ & + \frac{\sqrt{73}}{2} (x_{60}x_6 + x_{61}x_7 + x_{62}x_8 + x_{63}x_9) \\ & + \frac{\sqrt{114}}{2} (x_{72}x_{22} + x_{73}x_{23} + x_{74}x_{24} + x_{75}x_{25}) \\ & + \frac{\sqrt{210}}{2} (x_{64}x_{14} + x_{65}x_{15} + x_{66}x_{16} + x_{67}x_{17}) \\ & + \frac{\sqrt{210}}{2} (x_{68}x_{18} + x_{69}x_{19} + x_{70}x_{20} + x_{71}x_{21}) \\ & + 3(x_{51}x_1 + x_{52}x_{10} + x_{53}x_{11} + x_{54}x_{12} + x_{55}x_{13}) \end{aligned} \quad (73)$$

where L_i , ρ_i , S_i and $n=25$ are respectively the length, material density, the cross-section area of the i^{th} element and the number of elements in the structure.

3.4 Optimization Statement

The structural optimization problem consists of finding the material density, stresses and cross section area which minimize the objective function $m(x)$ under equality (41-60) and inequality (61-72) constraints. This can be stated as:

$$\min_{x \in \mathbb{R}^{75}} m(x) \quad (74)$$

$$\text{subject to } \begin{cases} A_{\text{eq}}x = b_{\text{eq}} \\ A_{\text{ineq}}x \leq b_{\text{ineq}} \\ x, \text{ unrestricted} \end{cases} \quad (75)$$

The interior point method is applied through the solver of type `fmincom` to solve the nonlinear minimization problem (74-75) using Matlab Optimization Toolbox (Coleman et al., 1999).

4. Applications of the Structural Analysis

4.1 Additional Data

The loading cases imposed for this structural analysis is given by the next Table 8.

Table 8. Imposed loading conditions

Loading cases	Nodes	Imposed load on x-axis (kN)	Imposed load on y-axis (kN)	Imposed load on z-axis (kN)
1	1	100	100	-50
1	2	0	100	-50
1	3	50	0	0
2	5	0	200	-50
2	6	0	-200	50

We also have the following parameters:

- Coefficient $C_\alpha = 0.04$
- Partial coefficient of security $\gamma_{MO} = 1.1$
- Limit of elasticity $f_y = 235$ MPa
- Stress limit in compression or tension $\sigma_0 = \frac{f_y}{\gamma_{MO}} = 213.63$ MPa
- Minimum cross section area $S_{\min} = 1.0$ cm².

We define CSA, VIF, SN, and MD respectively as the cross-section area, values of internal forces, stress nature and material density.

4.2 Result of the Loading Case 1

Through the loading case 1, the minimal value of the structural mass $m_1 = 25679.755$ kg is found at the fifth iteration and the decisions parameters that provide this mass are presented in Table 9.

In Table 9, we find that the material density is the same at each cross section and it values 7.701 kg/m³. The internal force vanishes where the cross-section areas x_{10} , x_{14} and x_{18} are not equal to zero. This result shows that the internal forces are inactive at the cross section areas x_{10} , x_{14} and x_{18} . We observe that there are 13 elements in compression and only 9 in tension in the results of the loading case 1.

Table 9. Optimal values of cross-sectional area $(x_i)_{1 \leq i \leq 25}$ for the loading case 1

CSA (cm ²)	VIF (kN)	SN	MD (kg/m ³)
$x_1=2.2389$	$x_{26}=-47.830$	Compression	7.701
$x_2=10.5525$	$x_{27}=-15.393$	Compression	7.701
$x_3=2.7220$	$x_{28}=-225.440$	Tension	7.701
$x_4=1.4881$	$x_{29}=31.792$	Tension	7.701
$x_5=6.6177$	$x_{30}=-141.717$	Compression	7.701
$x_6=171.4113$	$x_{31}=58.151$	Tension	7.701
$x_7=177.10944$	$x_{32}=-41.534$	Compression	7.701
$x_8=170.4755$	$x_{33}=-163.045$	Compression	7.701
$x_9=176.1110$	$x_{34}=-141.379$	Compression	7.701
$x_{10}=2.3419$	$x_{35}=0.000$	-	7.701
$x_{11}=3.9908$	$x_{36}=50.031$	Tension	7.701
$x_{12}=1.0000$	$x_{37}=-44.258$	Compression	7.701
$x_{13}=1.000$	$x_{38}=-5.460$	Compression	7.701
$x_{14}=1.0000$	$x_{39}=0.000$	-	7.701
$x_{15}=2.9177$	$x_{40}=-85.259$	Compression	7.701
$x_{16}=1.0000$	$x_{41}=-92.355$	Compression	7.701
$x_{17}=1.6548$	$x_{42}=-221.207$	Compression	7.701
$x_{18}=4.3230$	$x_{43}=0.000$	-	7.701
$x_{19}=1.0000$	$x_{44}=13.489$	Tension	7.701
$x_{20}=1.0000$	$x_{45}=-35.353$	Compression	7.701
$x_{21}=1.7557$	$x_{46}=60.572$	Tension	7.701
$x_{22}=7.4694$	$x_{47}=-20.541$	Compression	7.701
$x_{23}=1.0000$	$x_{48}=62.332$	Tension	7.701
$x_{24}=10.3543$	$x_{49}=37.509$	Tension	7.701
$x_{25}=2.8353$	$x_{50}=159.573$	Tension	7.701

4.2 Result of the Loading Case 2

The minimal value of the structural mass $m_2 = 1963.908$ kg is found at the second iteration and the decisions parameters that provide this mass are presented in the Table 10. Through the loading case 2, we have also the material density equal to 7.701 kg/m³. The same observation was done in loading case 1 is made with loading case 2, namely, the internal forces are inactive at the cross section areas x_9 , x_{14} and x_{18} . These results show that the structural mass corresponding to the worst loading is case 1 with $m_1 = 25679.755$ kg compared to case 2 with lesser effects on the structure.

Table 10. Optimal values of cross-sectional area $(x_i)_{1 \leq i \leq 25}$ for the loading case 2

CSA (cm ²)	VIF (kN)	SN	MD (kg/m ³)
$x_1=1.0000$	$x_{26}=-5.081$	Compression	7.701
$x_2=1.0000$	$x_{27}=-1.208$	Compression	7.701
$x_3=1.0000$	$x_{28}=1.476$	Tension	7.701
$x_4=1.0000$	$x_{29}=7.365$	Tension	7.701
$x_5=1.0000$	$x_{30}=-6.027$	Compression	7.701
$x_6=1.0000$	$x_{31}=-8.840$	Compression	7.701
$x_7=1.0000$	$x_{32}=-2.326$	Compression	7.701
$x_8=1.0000$	$x_{33}=-9.560$	Compression	7.701
$x_9=1.0000$	$x_{34}=0.000$	-	7.701
$x_{10}=5.0905$	$x_{35}=9.428$	Tension	7.701
$x_{11}=1.0000$	$x_{36}=108.753$	Tension	7.701
$x_{12}=1.0000$	$x_{37}=11.432$	Tension	7.701
$x_{13}=1.8753$	$x_{38}=1.432$	Tension	7.701
$x_{14}=1.0000$	$x_{39}=0.000$	-	7.701
$x_{15}=5.2082$	$x_{40}=-1.470$	Compression	7.701
$x_{16}=1.0000$	$x_{41}=6.915$	Compression	7.701
$x_{17}=9.6594$	$x_{42}=-7.396$	Compression	7.701
$x_{18}=1.0000$	$x_{43}=0.000$	-	7.701
$x_{19}=1.0000$	$x_{44}=40.065$	Tension	7.701
$x_{20}=1.0000$	$x_{45}=206.360$	Tension	7.701
$x_{21}=1.0000$	$x_{46}=-79.786$	Compression	7.701
$x_{22}=1.0000$	$x_{47}=-13.778$	Compression	7.701
$x_{23}=1.0000$	$x_{48}=-111.226$	Compression	7.701
$x_{24}=1.0000$	$x_{49}=9.580$	Tension	7.701
$x_{25}=3.7346$	$x_{50}=11.955$	Tension	7.701

5. Conclusion

In this paper, the behavior of a twenty-five bars spatial truss has been described by a linear elastic model with imposed loading conditions acting on five node elements. The unique solution of the elastic model is found by minimizing the energy functional in the solution space of all the displacements and then used to formulate the mass minimization problem as a nonlinear minimization problem. For the two loading cases, the results of this structural optimization show the characteristics of the cross section areas, material densities and internal forces which satisfy the effectiveness of the minimum mass design. It appears that the minimum mass obtained with the loading case 2 is less than the mass obtained with loading case 1. We have found that the best structural design corresponding to our experimental data is the one which has an economical mass, uses less financial resources and allows substantial changes in size, shape, and topology.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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