International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 6, 1522–1533, 2019
https://dx.doi.org/10.33889/IJMEMS.2019.4.6-120

ANFIS based Machine Repair Model with Control Policies and Working Vacation

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(Received July 10, 2019; Accepted September 7, 2019)

Abstract
This study is concerned with the transient state analysis of M/M/1 machine repairable system consisting of M operating units. F-policy is quite useful to avoid the overloading of failed machines that arrive for repair in the system. The failed machines are repaired by a server that is susceptible to failure and follows the threshold recovery while being repaired. The server leaves for a vacation if there are no machines waiting in the system for the repair. Runge-Kutta method is implemented to solve the governing equations and evaluate the system's state probabilities. Cost function is also designed to determine the system's minimum cost. In addition, the numerical outcomes acquired by the Runge-Kutta method are compared with the results generated by adaptive neuro-fuzzy inference system (ANFIS).

Keywords- Machine-repair, Start-up time, Threshold recovery, Cost analysis, ANFIS.

1. Introduction

In this present era, the machining systems are of enormous benefit to people. Machining parts failure is quite common causing an adverse impact on reliability, efficiency and quality of the machining system. Modern engineering systems like computer systems, information systems, telecommunication systems, transportation systems etc. are designed to ensure successful operation. Moreover, in real life no server is perfect; incorporating the idea of unreliable server for performance modelling is useful in depicting the more flexible and realistic queueing situations. The significant contributions in varying situations to unreliable machining systems can be observed in the study by Jain and Bhagat (2013), Tan et al. (2013), Yang et al. (2015), Jain et al. (2017). The unreliable server can be repaired so as to serve the customers using various schemes like immediate repair, repair using different policies like threshold recovery. The latter concept of repair is usually applied so as to optimize time and cost involved in repair process. Threshold policy plays an important role to turn on the server when there are an adequate amount of customers in the system. Jain and Meena (2018) examined an unreliable machining system with mixed standby support and obtained the numerical results using Runge-Kutta method. They constructed a cost function to determine the repair rate and to minimize the total system cost. Further, Ezeagu et al. (2018) conducted the transient analysis of a queuing model with working breakdowns and obtained the numerical results by using a small k-threshold value.

The server can go on vacation in many real-life situations and is not available to provide service. More attention is paid to queueing models with vacation after the work of Levy and Yechiali (1975). In many queueing situations, the server may go for a vacation for some time. From a cost-effective perspective, it is vital to send a server to utilize the time to conduct a secondary job as quickly as it becomes idle. Servi and Finn (2002) introduced a different vacation policy i.e. working vacation policy, where the server operates at distinct service rates rather than stopping the service completely and the server returns to normal service if the service system accumulates a sufficient number of customers. Wang et al. (2009)studied a machine repair problem with finite capacity. Sharma and Kumar (2018) considered a machining system with multiple working vacation and analysed the model numerically using Runge-Kutta technique. Recently, Sethi and Bhagat (2019) examined a machine repair model with working vacation and used Runge-Kutta method to examine the performance indices.

Over the previous decade, by implementing the control policy, namely F-policy, an increasing consideration has been paid to control the arrivals in the queueing system. Optimal F-policy controls the main issue in controlling arrivals in a machining system and finds various applications in communication systems, transport, production systems. Gupta(1995)implemented the F-policy concept for the analysis of a finite capacity queueing system. The work of prominent experts (Wang and Yang, 2009; Jain and Bhagat, 2015; Jain and Meena, 2017) towards this path are valuable for the variation and expansion of the F- policy model. Recently, Sharma (2018) did the systematic analysis of queueing model with various threshold policies.

ANFIS, a hybrid soft computing development method, is a combination of neural network and fuzzy logic that can be used effectively to examine the performance indices of the machining system. Jang (1993) has performed the noticeable works on ANFIS. The method of ANFIS hybrid soft computing is well developed in distinct manufacturing environment sectors (Lin and Liu, 2001). Bhargava and Jain (2014) evaluated the use of the ANFIS to provide a comparative analysis of the outcomes acquired by an unreliable vacation queueing model using matrix method (MGM). Jain and Meena (2018) recently suggested an unreliable machining system comprising of mixed standby support and numerical outcomes are acquired using Runge-Kutta method. In addition, the numerical results obtained from Runge-Kutta method are compared with hybrid soft computing method based on ANFIS.

An unreliable machine repair model with finite capacity has been considered in the present study where the arrival of failed machines in the system is controlled using F-policy. Repair of the server is controlled using threshold recovery. The remaining paper is organized in different sections. The model description and assumptions for formulating the Markov model are discussed in Section 2. Section 3 deals with the transient state equations of the model. System characteristics have been established in Section 4 by using the probabilities of the transient state. In Section 5 cost function is formulated. Section 6 provides numerical analysis of the system and results are displayed in graphs and tables. To highlight the noble features of the work carried out, the concluding remarks have been given in Section 7.

2. Model Description and Assumptions

In this paper, we consider an unreliable machine repair problem (MRP), in which the repair process follows threshold recovery. Failed machines join the system from a single waiting line and only one machine is repaired at a time. Whenever the system is empty (i.e. no customers) the server operates with lower service rate rather than completely terminating the system entirely.

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2.1 Model Assumptions

The model is constructed by taking into account the following assumptions.

- Arrival Process: The failed machines arrive in the system following a Poisson process with arrival rate λ and only one failed machine can be repaired by the server at a time. The arrival is controlled using F-policy following an exponential distribution and failed machines are permitted in the system after a start-up time with rate ξ .
- Service Process: The service process follows an exponential distribution with mean service rate μ_b (μ_{b1}) when arrival allowed (not allowed), while the server serves with mean service rate μ_v during working vacation state.
- Working vacation: The server begins a working vacation of random length when the system is free and the vacation length is exponentially distributed with the parameter η .
- Breakdown Process: The server is assumed to be unreliable and may breakdown at any time with rate α_1 (α_2)following exponential distribution.
- Repair Process: The repair of the failed system is carried out according to threshold recovery with the following repair rates

$$\beta = \begin{cases} \beta_1, \text{repair rate when arrival is allowed} \\ \beta_2, \text{repair rate when arrival is not allowed.} \end{cases}$$

- Threshold Recovery: The repair mechanism follows the threshold recovery during both stages. Repair starts when a sufficient number of failed machines are accumulated in the system, say $q_1(q_2)$; $(q_2 < q_1)$ when arrival allowed (not allowed) in the system.

2.2 State of System

The following function *i*(t) is used to interpret the status of the server at time '*t*'

$$i(\mathrm{t}) = \begin{cases} 1, \text{the server is on working vacation} \\ 2, \text{the server is on busy period when arrival allowed} \\ 3, \text{the server break-down on busy period when arrival allowed} \\ 4, \text{the server is on busy period when arrival not allowed} \\ 5, \text{the server break-down on busy period when arrival not allowed} \end{cases}$$

2.3 Application

The model under investigation can be closely related to many real life situations. The present model can be related to working of service centre with restricted space for failed units (say M). Service centre representatives help customers to resolve issues with its products and services. The failed units join the repair service centre in accordance with FIFO discipline. The arrival of the failed units starts if the number of failed units waiting for service reduces to a level say 'F'. If the server does not find failed units in the system, it may go for working vacation. The server provides service at a slower rate during the vacation period rather than completely stopping the service. Upon returning from vacation if the server discovers no failed units waiting in the system, it can go on working vacation again. The repairman is unreliable and may breakdown (fall ill etc.) while repairing the faulty units which results in the loss of time and money. The repair process follows threshold policy i.e. the repair process starts only when the number of

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failed units reaches to a specified level say ‘q’. This concept of threshold recovery helps in the optimization of time and cost.

3. Governing Equations

Chapman-Kolmogorov equations are built using birth-death process to evaluate the probabilities affiliated with distinct states of the system. The transient state equations can be formulated as follows where $P'_{i,n}(t)=\frac{d}{dt}[P_{i,n}(t)]$ represents the probability at state *i* with n customers at time *t*. The transient state equations related to machine repair model are provided as:

i) for *i*=1, the server is on working vacation

$$P'_{1,0}(t)=\mu_b P_{2,1}(t)+\mu_v P_{1,1}(t)-M\lambda P_{1,0}(t) \tag{1}$$

$$\begin{aligned}P'_{1,n}(t)=&\mu_v P_{1,n+1}(t)+(M-n+1)\lambda P_{1,n-1}(t)\\&-[\mu_v+\eta+(M-n)\lambda]P_{1,n}(t);(1\le n\le M-1)\end{aligned} \tag{2}$$

$$P'_{1,M}(t)=\mu_v P_{1,M}(t)-\lambda P_{1,M-1}(t) \tag{3}$$

ii) for *i*=2, the server is on busy period when arrival allowed

$$P'_{2,1}(t)=\xi P_{4,1}(t)+\eta P_{1,1}(t)+\mu_b P_{2,2}(t)-(\alpha_1+(M-1)\lambda+\mu_b)P_{2,1}(t) \tag{4}$$

$$\begin{aligned}P'_{2,n}(t)=&\mu_b P_{2,n+1}(t)+\eta P_{1,n}(t)+(M-n+1)\lambda P_{2,n-1}(t)+\xi P_{4,n}(t)\\&+\beta_1 P_{3,n}(t)-(\alpha_1+(M-n)\lambda+\mu_b)P_{2,n}(t);(q_1\le n\le F)\end{aligned} \tag{5}$$

$$\begin{aligned}P'_{2,n}(t)=&(M-n+1)\lambda P_{2,n-1}(t)+\eta P_{1,n}(t)+\mu_b P_{2,n+1}(t)\\&-(\alpha_1+(M-n)\lambda+\mu_b)P_{2,n}(t);\ (F+1\le n\le M-2)\end{aligned} \tag{6}$$

$$P'_{2,M-1}(t)=2\lambda P_{2,M-2}(t)+\beta_1 P_{3,M-1}(t)+\eta P_{1,M-1}(t)-(\mu_b+\lambda+\alpha_1)P_{2,M-1}(t) \tag{7}$$

iii) for *i*=3, the server break-down on busy period when arrival allowed

$$P'_{3,1}(t)=\alpha_1 P_{2,1}(t)-(M-1)\lambda P_{3,1}(t) \tag{8}$$

$$\begin{aligned}P'_{3,n}(t)=&\alpha_1 P_{2,n}(t)+(M-n+1)\lambda P_{3,n-1}(t)-[(M-n)\lambda+\beta_1]P_{3,n}(t);\\&q_1\le n\le M-2\end{aligned} \tag{9}$$

$$P'_{3,M-1}(t)=2\lambda P_{3,M-2}(t)+\alpha_1 P_{2,M-1}(t)-\beta_1 P_{3,M-1}(t) \tag{10}$$

iv) for *i*=4, the server is on busy period when arrival not allowed

$$P'_{4,1}(t)=\mu b_1 P_{4,2}(t)-(\xi+\alpha_2)P_{4,1}(t) \tag{11}$$

$$P'_{4,n}(t)=\mu b_1 P_{4,n+1}(t)-(\mu b_1+\alpha_2+\xi)P_{4,n}(t);\ 2\le n\le q_2-1 \tag{12}$$

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$$P'_{4,n}(t) = \mu b_1 P_{4,n+1}(t) + \beta_2 P_{5,n}(t) - (\mu b_1 + \alpha_2) P_{4,n}(t);\ F+1 \le n \le M-1 \tag{13}$$

$$P'_{4,n}(t) = \mu b_1 P_{4,n+1}(t) + \beta_2 P_{5,n}(t) - (\mu b_1 + \alpha_2 + \xi) P_{4,n}(t);\ q_2 \le n \le F \tag{14}$$

$$P'_{4,M}(t) = \lambda P_{2,M-1}(t) + \beta_2 P_{5,n}(t) - (\mu b_1 + \alpha_2) P_{4,M}(t) \tag{15}$$

v) for i=5, the server break-down on busy period when arrival not allowed

$$P'_{5,1}(t) = \alpha_2 P_{4,1}(t) \tag{16}$$

$$P'_{5,n}(t) = \alpha_2 P_{4,n}(t); 2 \le n \le q_2 - 1 \tag{17}$$

$$P'_{5,n}(t) = \alpha_2 P_{4,n}(t) - \beta_2 P_{5,n}(t); q_2 \le n \le M \tag{18}$$

4. System Characteristics

The performance of a machine repair model can be evaluated in terms of performance indices that demonstrate system behaviour in distinct circumstances.

- Expected system length is

$$ENQ(t) = \sum_{n=1}^{M} n(P_{1,n}(t) + P_{3,n}(t)) + \sum_{n=1}^{M-1} nP_{2,n}(t)$$

(19)

- System throughput

$$TP(t) = \sum_{n=1}^{M} (\mu_v P_{1,n}(t) + \mu b_1 P_{4,n}(t)) + \sum_{n=1}^{M-1} \mu_b P_{2,n}(t) \tag{20}$$

- Probability of the server being busy

$$P_B(t) = \sum_{n=0}^{M} P_{1,n}(t) + \sum_{n=1}^{M-1} P_{2,n}(t) + \sum_{n=1}^{M} P_{5,n}(t) \tag{21}$$

- Probability that failed machines enter the system

$$P_F(t) = \sum_{n=0}^{F} P_{4,n}(t) \tag{22}$$

- Probability that system is blocked

$$P_O(t) = \sum_{n=1}^{M} P_{4,n}(t) \tag{23}$$

- Failure frequency

$$F_f(t) = \alpha_1 \sum_{n=1}^{M-1} P_{2,n}(t) + \alpha_2 \sum_{n=1}^{M} P_{4,n}(t) \tag{24}$$

- Expected system waiting time

$$W_S(t) = \frac{ENQ(t)}{\lambda} \quad (25)$$

5. Cost Analysis

Cost function has been developed in this section to examine the sensitivity of the machine repair model. The cost components involved in various operations are summarized as:

C_N : Holding cost in the system for each failed unit

C_B : Cost spent when the server is busy

C_V : Cost spent when the server is on working vacation

C_R : Cost spent when the server is under repair

C_M : fixed cost for system capacity

C_1, C_2, C_3: Cost for providing service to the customer with service rate μ_b, μ_v, μ_{b1} respectively

The cost components and performance measures are taken into account to develop the complete cost function

$$TC(t) = C_N ENQ(t) + C_B P_B(t) + C_V P_V(t) + C_M M + C_R P_R(t) + C_1\mu_b + C_2\mu_v + C_3\mu_{b1} \quad (26)$$

6. Numerical Analysis

The numerical results are provided to examine the machine repair model's sensitivity for the numerous performance indices. To provide the systems' numerical solution coding is done using the MATLAB software ode45 function. It is important to determine optimal policy for a queueing system that provides the minimum cost. The set of default parameters is taken as $\lambda = 0.5$, $\alpha_1 = 1$, $\alpha_2 = 1$, $\mu_v = 1.5$, $\mu_b = 2$, $\mu_{b1} = 1.5$, $\eta = 0.5$, $\beta_1 = 1.5$, $\beta_2 = 2$, $\xi = 0.5$. The values for various cost elements are taken as $C_N = 200$, $C_B = 150$, $C_V = 100$, $C_R = 100$, $C_M = 250$, $C_1 = 200$, $C_2 = 100$, $C_3 = 150$.The numerical outcomes are presented using graphs and tables to comprehend the system's behaviour. Gaussian membership function is considered for the input parameter λ, to calculate the ANFIS outcomes using the MATLAB Neuro-Fuzzy tool. We choose the five members which are shown in Figure 1 for fuzzification of λ. The members considered for λ are very low, low, average, high, and very high. The results acquired by the Runge-Kutta method are plotted using continuous lines in Figures 2-5, while the numerical findings acquired using ANFIS are shown by tick marks.

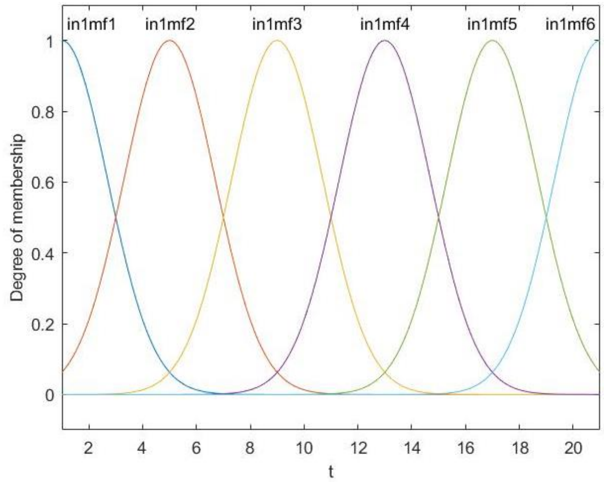

Figure 1. Membership function for input variable λ

Table 1 shows that the server's probability of being in busy state decreases and the system's throughput increases if the service rate increases. As the service rate increases the server's probability of being in blocked state reduces and the failed machines are allowed to enter the system at a fast rate as service is provided at fast rate.

Table 1. Effect on performance measures by varying M, F and λ

(M,F)	μ_b	$P_B(t)$	$P_F(t)$	$P_O(t)$	$TC(t)$
(10,8)	**1**	0.781	0.272	0.553	1.8555
	2	0.692	0.385	0.462	2.3751
	3	0.618	0.391	0.363	2.3396
(15,10)	**1**	0.856	0.251	0.582	1.9819
	2	0.791	0.357	0.486	2.5535
	3	0.748	0.348	0.321	2.9181
(15,8)	**1**	0.821	0.218	0.622	1.9621
	2	0.762	0.312	0.512	2.4247
	3	0.708	0.328	0.362	2.7328

As the server is prone to breakdown while providing service, the breakdown rate α_1 and α_2 have a significant impact on the machining system's efficiency. Figure 2 demonstrates a decrease in the queue length as the repair rate increases.

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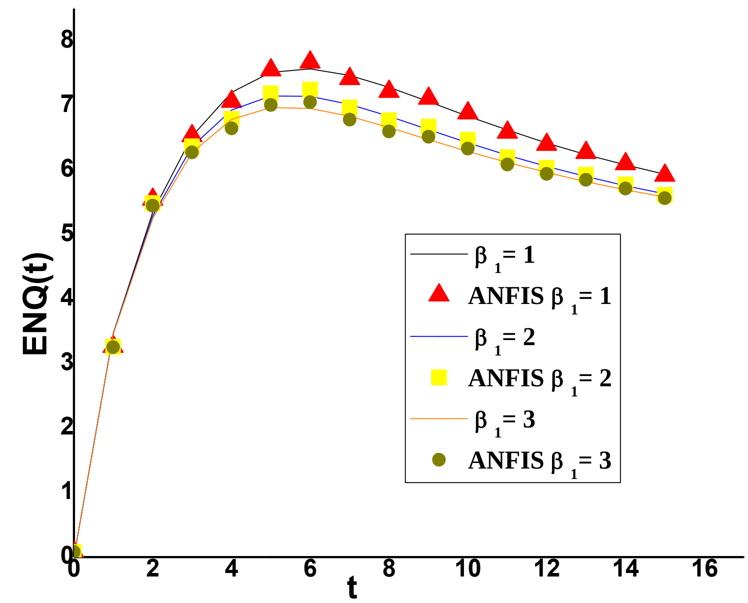

Figure 2. ENQ(t) at various values of β_1

It is noticed from Figure 3 that with the increase in service rate, the number of effective service TP(t) provided to the customers increases. There is a decrement in the waiting time with the increase in service rate as shown in Figure 4 and the obtained trends match the realistic situation also. It is clear that to increase the system's throughput and decrease the waiting time, the system's service rate should be improved.

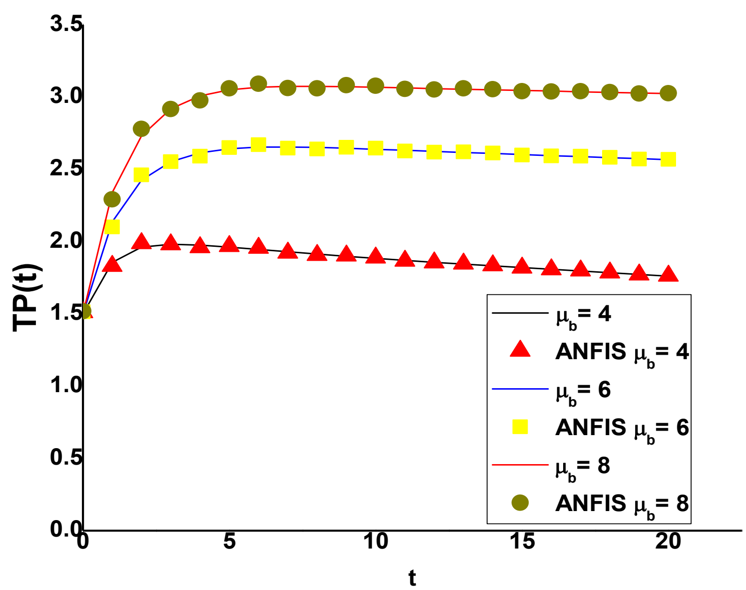

Figure 3. TP(t) vs t at various values of μ_b

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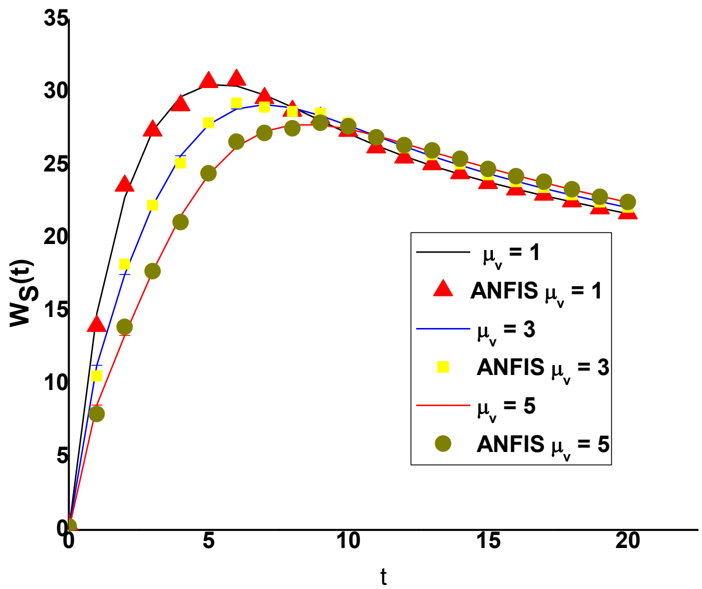

Figure 4. $W_S(t)$vs t at various values of μ_v

The cost function (26) is regarded and assessed in order to evaluate the optimal value by determining different cost elements. The efficiency of total cost of the system is assessed by the server's probability to be in various states, service rates and number of operating machines. From figures 5 and 6 it is noticed that the total cost of the system increases with the increase in service rate of the system. Figure 7 displays that as the operating machines increase in the system, the cost associated with the system increases.

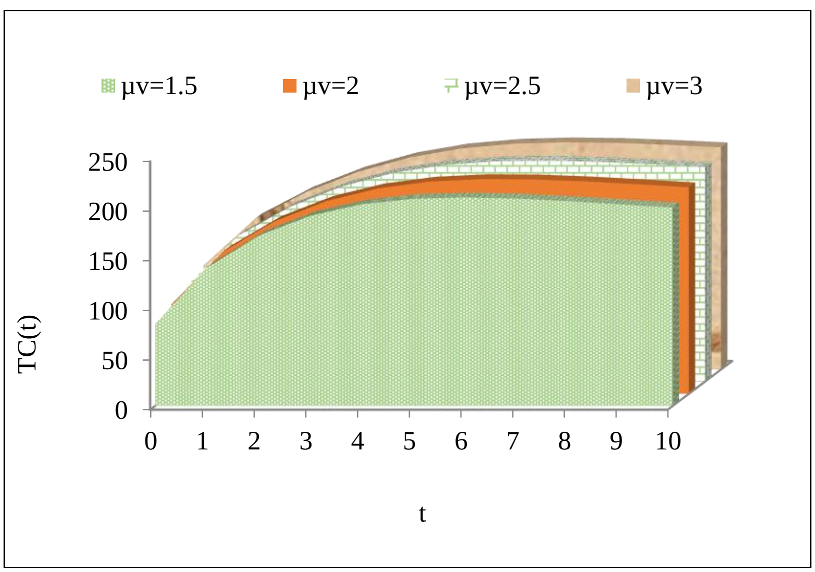

Figure 5. TC(t) vs t at various values of μ_v

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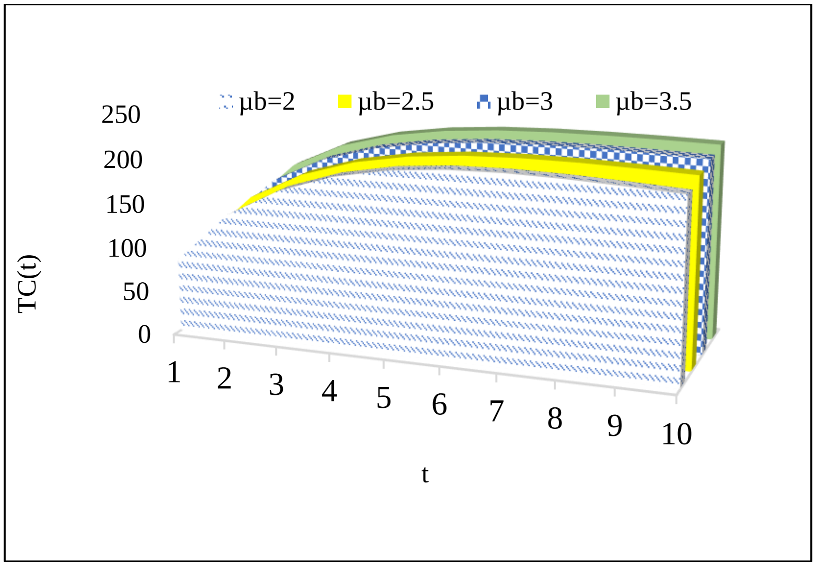

Figure 6. TC(t) vs t at various values of μ_b

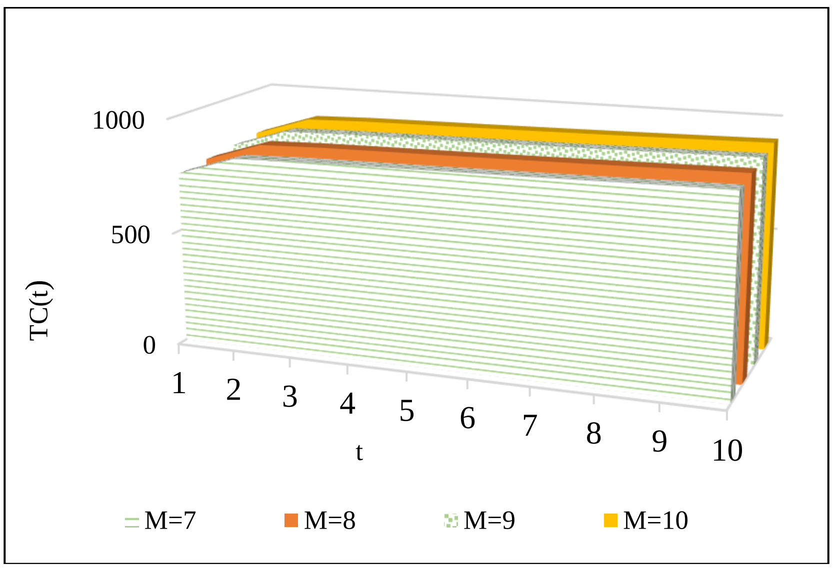

Figure 7. TC(t) vs t various values of M

7. Conclusion

In the current period of industrialisation, machining system has become part of our day to day life. Nowadays, human beings are completely dependent on the machines because of the automation of many systems. In this investigation, threshold-based repair facility for machining system with the concept of working vacation has been examined. The idea of working vacation was introduced to allow the repairman to provide service at a slow rate instead of totally stopping the service. In order to control the arrival in the system the concept of F-policy is used. The incorporation of threshold N-policy to start the repair makes the system cost effective.Using the fourth order Runge-Kutta technique, performance measures are determined and the numerical results acquired are compared with ANFIS. The model designed may provide profitable insight in industrial organizations, manufacturing systems, communication systems for effective and smooth operation of the machining system.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments

The authors would like to appreciate the effort from editors and reviewers. This study received no specific grant from government, commercial or non-profit funding organizations.

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