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An Efficient Portfolio with Several Objectives and Varying Parameters

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Abstract
We investigate a portfolio selection model with several objective functions, whose coefficients are uncertain and vary between some bounds. A preferable efficient portfolio of the model is obtained, which provides the range within which the portfolio return and the moments would vary. An optimal portfolio for the forecasted returns of stocks is found with actual market data from the Bombay Stock Exchange, India.

Keywords- Nonlinear interval programming, Multi-objective optimization, Portfolio selection, Forecasting, Efficient portfolio.

1. Introduction

Theory of portfolio optimization has grown in popularity in investment management and now forms an important tool for portfolio managers to assist in asset allocation. But uncertainty in the market often leads to uncertainty with respect to the returns of the individual securities, which in turn translates into uncertainty with respect to the different moments of the return and thereby, increases the complexity of the portfolio optimization model. Portfolio optimization in its mathematical form was initiated by Markowitz (1952) with the classical Mean-Variance portfolio selection model, which is widely considered as the foundation of modern portfolio theory. However, over the years a number of alternative models have been proposed (Konno and Yamazaki, 1991; Elton et al., 2009; Longerstaey and Spencer, 1996; Rockafellar and Uryasev, 2002 and references therein are some recent developments). One of the key concerns among the refined models, that has been proposed subsequently, is the uncertainty in price movements (King, 1993).

In order to analyze the uncertainty in the financial market, the portfolio optimization theory has been increasingly focused on multivariate data analysis and modified in several directions using various mathematical tools. Due to uncertainty in the financial market, the return of the asset is not fixed and usually expected return is estimated from historical data. This expected return of the assets affects the risk and performances, which further controls the selection process for an efficient portfolio. Recently many researchers have tried to select efficient portfolios by solving portfolio optimization models, which deal the parameters like return, risk etc. using probability theory (Owen and Rabinovitch, 1983; Hong et al., 1987; Laloux et al., 2000; Korn and Korn, 2001) and fuzzy set theory (Korn and Korn, 2001; Abiyev and Menekay, 2007; Li et al., 2010). These methods have certain limitations while calculating suitable probability distribution functions and membership functions respectively, which also depend upon decision maker's choice. Selection of suitable distribution function and membership function can be avoided if we consider the lower and upper level of the return from historical data. In that case the return of an asset will lie in the closed interval which can cover all types of market uncertainties. But if the return is considered as a closed interval then the risk and performance of the portfolio have to be expressed in terms of intervals, which may be treated as interval valued functions in the mathematical sense. For example if x_1 and x_2 are the proportion of the total fund, invested in two assets A and B; lower and upper bounds of the return of A are considered as a^L, a^U and lower and upper bounds of the return B are considered as b^L, b^U respectively, then the total return of the portfolio is $[a^L, a^U]x_1 + [b^L, b^U]x_2$. Variance of the portfolio will be $\left[{\sigma_1^2}^L, {\sigma_1^2}^U\right]x_1^2 + 2[\sigma_{12}^L, \sigma_{12}^U]x_1x_2 + \left[{\sigma_2^2}^L, {\sigma_2^2}^U\right]x_2^2$, where ${\sigma_1^2}^L({\sigma_1^2}^U)$ is the lower (upper) bound of variance of expected return of asset A, ${\sigma_2^2}^L({\sigma_2^2}^U)$ is the lower (upper) bound of variance of expected return of asset B and σ_{12} is the covariance between assets A and B. In these situations the portfolio optimization model can not be handled using general optimization technique and hence becomes a challenging factor. There exist few contributions in the literature to deal such type of portfolio optimization models. Lin et al. (2012) studied portfolio selection model with interval values based on fuzzy probability distribution functions. Jong (2012) introduced the concept of satisfaction index to consider interval portfolio selection with uncertain objective functions, but it was not extended to address uncertainty in constraints. Li et al. (2010) proposed an interval semi-absolute deviation model for portfolio selection, which can be transformed into a class of linear programming problem. This paper discussed a class of linear programming problems with interval uncertainties in both the objective functions and constraints.

These uncertainties originate due to the real-world conditions existing in the financial markets which render investors incapable of predicting the exact rate of future returns. In order to obtain an optimal portfolio for a future period, this uncertainty thus needs to be necessarily accounted for. So if instead, an investor considers the lower bound and the upper bound of the future price movements of the portfolio's assets and employ these to derive the optimal portfolio, it would provide a broader and a more prudent estimate of the portfolio's performance. This therefore motivates the use of interval analysis in the portfolio optimization theory. An uncertainty in the returns of the portfolio's assets is also translated in that of the higher moments such as variance, skewness and kurtosis, which are subsequently reflected as interval uncertainty in the non-linear objective functions.

The development of the paper is organized into several sections as follows: Section 2 provides the framework and formulation of the proposed portfolio optimization model. Section 3 discusses the methodology used to forecast the parameters and subsequently derives the preferable efficient portfolio for the portfolio optimization problem. Section 4 provides the result analysis, its subsequent validation through the Bombay Stock Exchange (BSE), India data and also provides some concluding remarks. All figures are tables are provided in Appendix.

2. Model Formulation

Balancing reward against risk is the base of a general mean-variance portfolio optimization problem. The reward is measured by the portfolio expected return and risk is measured by the portfolio variance. In the most basic form, portfolio optimization model determines the proportion of the total investment of n number of assets of a portfolio. In general, rate of expected returns of the assets of a portfolio are estimated from previous data. Due to the presence of uncertainty in market an investor can not estimate the exact rate of expected return. As we discussed in Section 1, if the investor finds the lower bound r_i^L and upper bound r_i^U of the return of the assets from previous data for a fixed time period then the expected rate of return of i^{th} asset lies in the interval $[r_i^L, r_i^U]$ and can cover all uncertainties. In a portfolio optimization problem, an investor wants to maximize the expected return of the portfolio with minimum risk. Some more realistic factors also affect the portfolio selection like skewness and kurtosis. In this situation, an investor needs to maximize the expected return as well as skewness of the expected return and to minimize the variance and kurtosis. Since variance, covariance, skewness and kurtosis depend upon the rate of expected returns, so they are also in the form of intervals.

The following definitions and notations are used in the models:

i: Asset index.

t: Time period.

n: Total number of assets in the portfolio.

$r_i^L (r_i^U)$: Lower (Upper) bound of the returns of asset i.

${\sigma_i^2}^L ({\sigma_i^2}^U)$: Lower (Upper) bound of variance of expected returns of i^{th} asset.

σ_{ij} : The covariance between i^{th} and j^{th} assets returns.

$\sigma_{ij}^L (\sigma_{ij}^U)$: Lower (Upper) bound of covariance of return between i^{th} and j^{th} assets,

i.e., $\sigma_{ij}^L \leq \sigma_{ij} \leq \sigma_{ij}^U$, this implies $\sigma_{ij} \in [\sigma_{ij}^L, \sigma_{ij}^U]$.

$s_{iij}^L (s_{iij}^U)$: Lower (Upper) bound of co-skewness of expected returns of i^{th} and j^{th} assets.

$k_{iiij}^L (k_{iiij}^U)$: Lower (Upper) bound of co-kurtosis of expected returns of i^{th} and j^{th} assets.

x_i: The proportion of the total funds invested on i^{th} assets.

In a portfolio optimization problem, an investor wants to maximize the expected return of the portfolio with minimum risk. Some more realistic factors also affect the portfolio selection like skewness and kurtosis. In an attempt to better approximate investor preferences, and because most asset returns exhibit strong deviations from normality, optimal portfolio selection techniques should involve higher-order co-moments like skewness and kurtosis of asset return distribution as inputs, in addition to the covariance matrix (Martellini and Ziemann, 2009). In this situation, an investor needs to maximize the expected return as well as skewness of the expected return and to minimize the variance and kurtosis. Since variance, covariance, skewness and kurtosis depend upon the rate of expected returns, so they are also in the form of intervals.

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Suppose, the expected return, variance, skewness and kurtosis of portfolio are denoted by $\hat{R}(x), \hat{\sigma}^2(x), \hat{S}(x)$ and $\hat{K}(x)$ respectively, which can be defined as follows.

$$\hat{R}(x) = \sum_{i=1}^{n} [r_i^L, r_i^U] x_i \tag{1}$$

$$\hat{\sigma}^2(x) = \sum_{i=1}^{n} x_i^2 [\sigma_i^{2^L}, \sigma_i^{2^U}] + \sum_{i=1}^{n} \sum_{j=1}^{n} [\sigma_{ij}^L, \sigma_{ij}^U] x_i x_j, (i \neq j) \tag{2}$$

$$\hat{S}(x) = \sum_{i=1}^{n} \left[s_i^{3^L}, s_i^{3^U}\right] x_i^3 + 3 \sum_{i=1}^{n} \left(\sum_{j=1}^{n} [s_{iij}^L, s_{iij}^U] x_i^2 x_j + \sum_{j=1}^{n} [s_{ijj}^L, s_{ijj}^U] x_i x_j^2 \right), (i \neq j) \tag{3}$$

$$\hat{K}(x) = \sum_{i=1}^{n} \left[k_i^{4^L}, k_i^{4^U}\right] x_i^4 + 4 \sum_{i=1}^{n} \left(\sum_{j=1}^{n} [k_{iiij}^L, k_{iiij}^U] x_i^3 x_j + \sum_{j=1}^{n} [k_{ijjj}^L, k_{ijjj}^U] x_i x_j^3 \right) + 6 \sum_{i=1}^{n} \sum_{j=1}^{n} [k_{iijj}^L, k_{iijj}^U] x_i^2 x_j^2, (i \neq j). \tag{4}$$

If $\hat{\hat{R}}_i = [\bar{r}_i^L, \bar{r}_i^U]$ denotes the range of expected return of asset i in time period t, then

$$[s_{iij}^L, s_{iij}^U] = \frac{1}{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\hat{R}_i - \hat{\hat{R}}_i\right)^2 \left(\hat{R}_j - \hat{\hat{R}}_j\right)$$

$$[s_{ijj}^L, s_{ijj}^U] = \frac{1}{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\hat{R}_j - \hat{\hat{R}}_j\right)^2 \left(\hat{R}_i - \hat{\hat{R}}_i\right)$$

$$[k_{iiij}^L, k_{iiij}^U] = \frac{1}{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\hat{R}_i - \hat{\hat{R}}_j\right)^3 (\hat{R}_j - \hat{\hat{R}}_i)$$

$$[k_{ijjj}^L, k_{ijjj}^U] = \frac{1}{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\hat{R}_j - \hat{\hat{R}}_i\right)^3 (\hat{R}_i - \hat{\hat{R}}_j)$$

$$[k_{iijj}^L, k_{iijj}^U] = \frac{1}{t} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\hat{R}_i - \hat{\hat{R}}_j\right)^2 \left(\hat{R}_j - \hat{\hat{R}}_j\right)^2.$$

The arithmetic operations in these expressions are sum, difference and product of interval arithmetic operations. In classical method, an arithmetic operation $* \in \{+, -, \cdot, /\}$ in a set of intervals is defined as follows. For two intervals:

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$\hat{A}=[a^L,a^U]$ and $\hat{B}=[b^L,b^U]$, $\hat{A}\boxed{*}\hat{B}=\{a*b: a\in\hat{A}, b\in\hat{B}\}$. For $\hat{A}\boxed{\setminus}B$, $0\notin\hat{B}$. $\boxed{-}=[-a^U,-a^L]$.

So, the objective of the model is to minimize the functions $-\hat{R}'(x)$, $\hat{\sigma}'^2(x)$, $-\hat{S}'(x)$ and $\hat{K}'(x)$. In the most basic form, portfolio optimization model determines the proportion of the total investment x_i of i^{th} asset of a portfolio $x=(x_1,x_2,\ldots,x_n)$, where $\sum_{i=1}^{n}x_i=1$. In case short selling is not allowed, $x_i\geq 0, \forall\, i$.

Hence, the portfolio model can be represented mathematically as

(**P**)
$$\min\{-\hat{R}(x),\hat{\sigma}^2(x),\hat{S}(x),\hat{K}(x)\}$$
$$\text{Subject to } \sum_{i=1}^{n}x_i=1, x_i\geq 0.$$

This is an interval nonlinear multi-objective programming problem. Solution methodology to solve a general interval multi-objective programming problem is described in Jana and Panda (2014). We say the solution of the portfolio optimization problem **(P)** as a preferable efficient portfolio.

3. Finding Preferable Efficient Portfolio for Model (P)

In this section, we have discussed a methodology to find preferable efficient portfolio for the model (**P**) following the steps.

3.1. Collection of Data

As previously stated, the uncertainty in the stock prices presents itself as uncertainty in the higher moments when historical values of the returns are used, which leads to an implicit inaccuracy in the final optimal solution. In order to better account for this uncertainty in the returns for the stock prices, we consider the range of fluctuations of these returns instead of their actual values. In an exchange, the daily low and daily high prices respectively form the lower and upper bounds for the prices of the respective stocks for the day are as follows:

$$H_t=\frac{HP_t-CP_{t-1}}{CP_{t-1}},\quad L_t=\frac{LP_t-CP_{t-1}}{CP_{t-1}},$$

where

H_t: Upper bound for returns on day t,
L_t: Lower bound for returns on day t,
CP_{t-1}: Closing price for previous day $(t-1)$,
HP_t: Daily high price on day t,
LP_t: Daily low price on day t.

We consider the daily low and high price of return data for the 15 stocks of BSE-India from Yahoo for the period of Feb '09-Feb '14 and forecast the upper and lower bound for each scrip for March '14. Detail of these fifteen stocks is provided in Table 1. Subsequently we determine the lower and upper bounds for each of the parameters of the objective function (Expressions (1)-(4)) using the moments of forecasted return bounds for March '14. Finally, we analyze the performance of the optimal portfolio for March 2014.

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The softwares **R** and **MATLAB** are used to derive the preferable efficient solution for the portfolio optimization model (**P**).

Table 1. Code of fifteen stocks

Stock	Stock 1	Stock 2	Stock 3	Stock 4	Stock 5
Code	Axis	Bank of Baroda	Cairn	Hindalco	HDFC
Stock	Stock 6	Stock 7	Stock 8	Stock 9	Stock 10
Code	Jindal	JP Associates	JSW Steel Ltd.	Maruti	RCom
Stock	Stock 11	Stock 12	Stock 13	Stock 14	Stock 15
Code	SBI	Tata Motors	Tata Steel	United Spirits Lt	Yes bank

3.2. Forecasting

In order to forecast the upper and lower bound for each scrip for March '14, the statistical computing software **R** is used with the packages `*fractal*' and `*rugarch*'. To forecast the data, we require the specifications of the model that can be used to fit the series. So, start with the tests for stationarity and lag length to identify the model and the corresponding order which best explains the given series.

3.2.1. Test for Stationarity

To test the stationarity, we use the Augmented Dickey Fuller (ADF) test to check the presence of a unit root with the tool `*adf.test'*• in R under the package `*tseries*'. Table 2 lists the ADF statistics for historical upper bound series for returns of all the scrips. As can be inferred from the table of critical values, the ADF statistics for each of the daily high return found, is outside the critical values corresponding to the conventional significance levels. Similar results are obtained for the lower bound series for returns suggesting their non-stationarity.

Table 2. Augmented Dickey Fuller Test statistics for daily high and low returns

Scrip	Upper bound series	Lower bound series
Axis	-10.8993	-11.0706
Bank of Baroda	-7.8598	-8.0448
Cairn	-7.8933	-9.3538
Hindalco	-8.1577	-9.3641
HDFC	-6.9063	-8.4997
Jindal	-7.9997	-9.1153
JP Associates	-7.2178	-9.2099
JSW Steel Ltd.	-6.7258	-8.0934
Maruti	-8.1134	-9.4397
RCom	-7.2167	-9.6262
SBI	-8.1307	-8.9645
Tata Motors	-8.3630	-9.6856
Tata Steel	-6.8896	-9.0783
United Spirits Ltd.	-10.6754	-10.6450
Yes Bank	-6.9669	-7.4365

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The returns series appears not to have a trend component. This can be seen in Fig. 1 through the time series plot for the Axis bank stock for the period from Feb '09- Feb '14. This is further confirmed using the Augmented Dickey Fuller test, which reveals the absence of a trend, but the presence of a unit root.

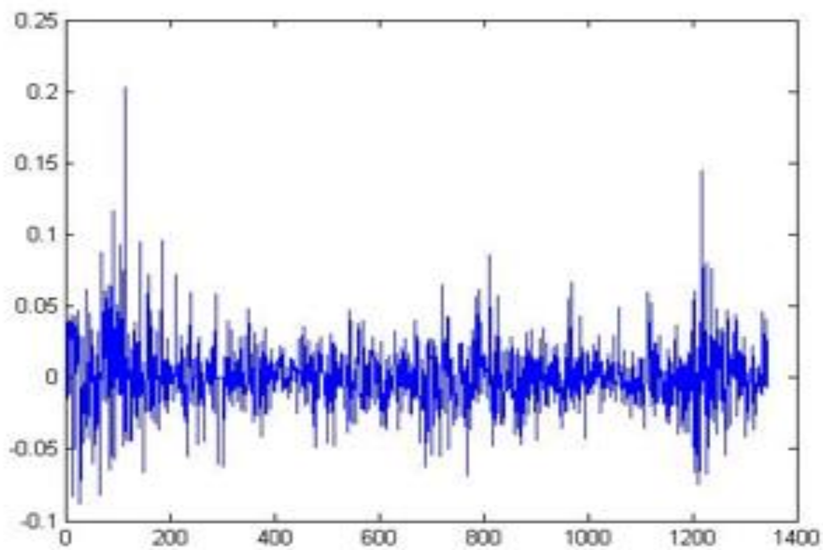

Fig. 1. Time series plot for the Axis Bank stock from Feb'09-Feb '14

Next, proceed to differencing the returns series so as to eliminate the unit root. First differencing the returns followed by the Augmented Dickey Fuller test on the first differences, indeed confirms that the returns are difference stationary (DS).

3.2.2. Identification of the Best Fit Model

All the series are first order DS, so we proceed to identify the orders p, q for the model **ARIMA** $(p, 1, q)$, that would provide the best fit for our series. For this purpose, we form the initial estimate using the autocorrelation (ACF) and partial autocorrelation (PACF)) plots. The likely candidates for the values of p and q are then used to fit different **ARMA** (p, q) models for the first differences. For instance, Fig. 2 provides the ACF and PACF plots for the Hindalco daily high returns. The plot reveals that the correlation decays relatively slowly, but in any case this needs to be studied no further than order 2 for both AR and MA components.

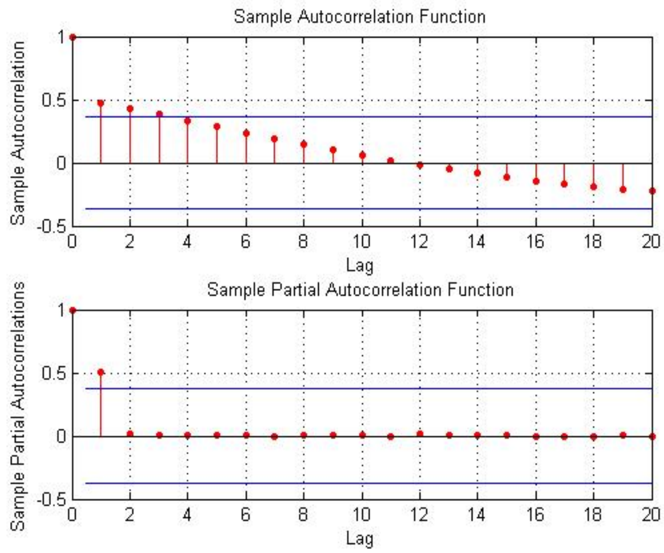

Fig. 2. ACF and PACF plots for Hindalco high returns series

Once we have the estimates of the **AR** and **MA** process orders for the first differences of the returns series, we proceed to estimate the **ARMA** (p, q) for the same. For this, we use the `*arma*'• function under the package `*tseries*' in **R**. It fits an **ARMA** model to a univariate time series by conditional least squares. Depending on the lag specification of the model (given using the initial estimates obtained from the ACF-PACF plots), the estimated residuals are then used for computing a least squares estimator of the full **ARMA** model. Fig. 3 displays the output by `*arma*'• for the Hindalco daily high returns discussed above. The output reveals that the **AR(**1) and **MA**(1) coefficients are highly significant. This suggests **ARMA** (1,1) for the first differences of the Hindalco daily high returns series. This in turn implies **ARIMA** (1,1,1) model for Hindalco daily high returns data.

```
Model:
ARMA(2,1)

Residuals:
      Min        1Q    Median        3Q       Max
-0.115861 -0.013358 -0.003881  0.009128  0.164517

Coefficient(s):
            Estimate  Std. Error  t value Pr(>|t|)
ar1        0.9213345   0.0747635   12.323   <2e-16 ***
ar2        0.0550265   0.0349453    1.575    0.115
ma1       -0.9293034   0.0825031  -11.264   <2e-16 ***
intercept  0.0004982   0.0010351    0.481    0.630
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Fit:
sigma^2 estimated as 0.0004683,  Conditional Sum-of-Squares = 0.64,  AIC = -6568.41
```

Fig. 3. R output by function *arma* for Hindalco daily returns (BSE)

The entire process is replicated for all the 15 scrips, both for the daily high returns and daily low returns data to obtain the **ARIMA** (p, d, q) model, which would be used further for forecasting future returns in each series.

The next important estimation is the appropriate **ARCH/ GARCH** process to model the variance of the errors obtained through estimation by the **ARIMA** (p, d, q) model, identified for each returns series.

The **GARCH** estimation is done using the Akaike Information Criterion (AIC), Bayesian information criterion (BIC) and log likelihood corresponding to the different combinations (a, b). The order (a, b) providing the least **AIC** and **BIC** values and the maximum log likelihood values corresponds to the best fit model. Since most of the time series rarely go any further than a and/or $b = 2$, we try to find the above coefficients corresponding to all possible combinations of $a - b$.

Table 3 displays the values of the three parameters for the Hindalco daily high returns series. It is evident that the $(a, b) = (1,2)$ yields highest value of log likelihood and the minimum for AIC and BIC. We thus consider **GARCH** (1,2) to be the model to best fit the Hindalco daily high series. The process is similarly repeated for other scrips, for both the daily high and daily low series.

Table 3. AIC, BIC, Loglikelihood coefficients for the Hindalco daily high series

Coefficients	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH(2,2)
Loglikelihood	3376.3190	3382.463	3376.51	3382.463
AIC	-4.9520	-4.9596	-4.9508	-4.9581
BIC	-4.9463	-4.9404	-4.9317	-4.9531

Next, we proceed to use these to forecast the returns for the month of March 2014.

3.2.3. Forecasting for March 2014

As mentioned earlier, we employ the package `*rugarch*' in R and its functions- `*Ugarchspec*'• , `*Ugarchfit*'• and `*Ugarchforecast*' particularly. `*Ugarchspec*'• is used to provide a specification for the model to be used for the series and using which subsequent forecasting would be done. We input the appropriate specifications: the order of **ARIMA** (p, d, q) along with the **GARCH** (a, b) estimated previously.

`*Ugarchfit*'• is then subsequently used to fit the model. Once we have the required fitted model, the expected returns (daily low and daily high individually) for March '14, can be forecasted using the function `*Ugarchforecast*'• , which provides the lower and upper bound respectively for the expected returns.

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The forecasted returns (both daily high and daily low) for the month of March '14 are provided in Table 4 and 5. This data will be used to compute the values of moments to be the input in the portfolio optimization problem using the **Matlab**.

Table 4. The forecasted returns (daily high) for the month of March 14

	Axis	Bank of Baroda	Cairn	Hindalco	IDFC	Jindal	JP	JSW	Maruti	Rcom	SBI	Tata Motors	Tata Steel	United Spirits Ltd	Yes Bank
Day 1	0.00516	0.01992	0.008593	0.01399	0.01504	0.01722	0.02004	0.01774	0.01255	0.01822	0.01757	0.01872	0.01224	0.02823	0.01505
Day 2	-0.000699	0.0189	0.008819	0.01545	0.0152	0.01735	0.02059	0.01778	0.01354	0.01874	0.01722	0.01687	0.01268	0.03108	0.0152
Day 3	0.00208	0.01879	0.008871	0.01546	0.01526	0.01736	0.02064	0.01788	0.01363	0.01905	0.01715	0.01676	0.0128	0.03146	0.01527
Day 4	0.0000841	0.01876	0.008915	0.01549	0.01533	0.01737	0.0207	0.01799	0.01365	0.01935	0.01712	0.01677	0.01294	0.03174	0.01533
Day 5	0.00145	0.01876	0.008957	0.01553	0.01539	0.01738	0.02076	0.01809	0.01367	0.01963	0.0171	0.01679	0.01308	0.03174	0.0154
Day 6	0.000508	0.01874	0.009	0.01556	0.01545	0.01739	0.02082	0.01818	0.01369	0.01989	0.01708	0.01681	0.01321	0.03177	0.01546
Day 7	0.00116	0.01873	0.009042	0.01559	0.01551	0.01739	0.02087	0.01828	0.01371	0.02014	0.01706	0.01683	0.01333	0.03176	0.01552
Day 8	0.000711	0.01871	0.009084	0.01563	0.01557	0.0174	0.02092	0.01837	0.01373	0.02037	0.01704	0.01685	0.01346	0.03177	0.01558
Day 9	0.00102	0.0187	0.009125	0.01566	0.01563	0.01741	0.02097	0.01846	0.01375	0.0206	0.01702	0.01687	0.01358	0.03177	0.01564
Day 10	0.000807	0.01869	0.009166	0.01569	0.01569	0.01742	0.02102	0.01855	0.01377	0.02081	0.01700	0.01689	0.01369	0.03177	0.0157
Day 11	0.000954	0.01868	0.009207	0.01572	0.01574	0.01743	0.02107	0.01864	0.01379	0.021	0.01698	0.01691	0.01381	0.03177	0.01575
Day 12	0.000852	0.01867	0.009247	0.01576	0.0158	0.01744	0.02112	0.01873	0.01381	0.02119	0.01696	0.01693	0.01392	0.03177	0.01581
Day 13	0.000922	0.01866	0.009287	0.01579	0.01585	0.01744	0.02116	0.01881	0.01383	0.02137	0.01694	0.01695	0.01402	0.03177	0.01586
Day 14	0.000874	0.01865	0.009327	0.01582	0.01591	0.01745	0.02121	0.01889	0.01385	0.02153	0.01692	0.01697	0.01412	0.03177	0.01592
Day 15	0.000907	0.01864	0.009367	0.01585	0.01596	0.01746	0.02125	0.01897	0.01387	0.02169	0.0169	0.01698	0.01422	0.03177	0.01597
Day 16	0.000884	0.01863	0.009406	0.01588	0.01601	0.01746	0.02129	0.01905	0.01389	0.02184	0.01688	0.017	0.01432	0.03177	0.01602
Day 17	0.0009	0.01862	0.009445	0.01592	0.01606	0.01747	0.02133	0.01912	0.0139	0.02198	0.01686	0.01702	0.01441	0.03177	0.01607
Day 18	0.000889	0.01861	0.009483	0.01595	0.01611	0.01748	0.02137	0.0192	0.01392	0.02211	0.01685	0.01704	0.0145	0.03177	0.01612
Day 19	0.000897	0.0186	0.009521	0.01598	0.01616	0.01748	0.02141	0.01927	0.01394	0.02224	0.01683	0.01706	0.01459	0.03177	0.01617
Day 20	0.000892	0.01859	0.009559	0.01601	0.01621	0.01749	0.02144	0.01934	0.01396	0.02236	0.01681	0.01708	0.01468	0.03177	0.01622
Day 21	0.000895	0.01858	0.009597	0.01604	0.01626	0.0175	0.02148	0.01941	0.01398	0.02247	0.01679	0.0171	0.01476	0.03177	0.01627
Day 22	0.000893	0.01857	0.009634	0.01607	0.0163	0.0175	0.02151	0.01947	0.01399	0.02257	0.01678	0.01711	0.01484	0.03177	0.01632
Day 23	0.000895	0.01856	0.009671	0.0161	0.01635	0.01751	0.02154	0.01954	0.01401	0.02267	0.01676	0.01713	0.01492	0.03177	0.01636
Day 24	0.000893	0.01856	0.009708	0.01613	0.01639	0.01751	0.02158	0.0196	0.01403	0.02277	0.01674	0.01715	0.01499	0.03177	0.01641
Day 25	0.000894	0.01855	0.009744	0.01616	0.01644	0.01752	0.02161	0.01967	0.01404	0.02285	0.01673	0.01717	0.01506	0.03177	0.01645
Day 26	0.000894	0.01854	0.00978	0.01619	0.01648	0.01753	0.02164	0.01973	0.01406	0.02294	0.01671	0.01718	0.01514	0.03177	0.0165
Day 27	0.000894	0.01853	0.009816	0.01622	0.01652	0.01753	0.02167	0.01979	0.01408	0.02302	0.01669	0.0172	0.0152	0.03177	0.01654
Day 28	0.000894	0.01852	0.009852	0.01625	0.01656	0.01754	0.0217	0.01984	0.01409	0.02309	0.01668	0.01722	0.01527	0.03177	0.01658
Day 29	0.000894	0.01851	0.009887	0.01628	0.01661	0.01754	0.02172	0.0199	0.01411	0.02316	0.01666	0.01724	0.01533	0.03177	0.01662
Day 30	0.000894	0.0185	0.009922	0.01631	0.01665	0.01755	0.02175	0.01996	0.01413	0.02323	0.01664	0.01725	0.0154	0.03177	0.01666

Table 5. The forecasted returns (daily low) for the month of March 14

	Axis	Bank of Baroda	Cairn	Hindalco	IDFC	Jindal	JP	JSW	Maruti	Rcom	SBI	Tata Motors	Tata Steel	United Spirits Ltd	Yes Bank
Day 1	0.000580	-0.01258	-0.006738	-0.01469	-0.01497	-0.01396	-0.02159	-0.01558	-0.01557	-0.02022	-0.004419	-0.01859	-0.01685	-0.02031	-0.01183
Day 2	0.000394	-0.01337	-0.007031	-0.01559	-0.01532	-0.01443	-0.02205	-0.01589	-0.01202	-0.01976	-0.005714	-0.02609	-0.01648	-0.02269	-0.01269
Day 3	0.000404	-0.01359	-0.007047	-0.01587	-0.01546	-0.01449	-0.02211	-0.01599	-0.0117	-0.0198	-0.006009	-0.02484	-0.01645	-0.0233	-0.01283
Day 4	0.000396	-0.01375	-0.007043	-0.01609	-0.01558	-0.01451	-0.02214	-0.01608	-0.01167	-0.01985	-0.006207	-0.0212	-0.01644	-0.0233	-0.01291
Day 5	0.000397	-0.0139	-0.007039	-0.01629	-0.01569	-0.01451	-0.02216	-0.01618	-0.01167	-0.01989	-0.006393	-0.01776	-0.01643	-0.02334	-0.01298
Day 6	0.000397	-0.01403	-0.007034	-0.01648	-0.0158	-0.01452	-0.02218	-0.01626	-0.01167	-0.01994	-0.006572	-0.01528	-0.01642	-0.02333	-0.01305
Day 7	0.000397	-0.01416	-0.007029	-0.01664	-0.01591	-0.01453	-0.0222	-0.01635	-0.01166	-0.01998	-0.006747	-0.01376	-0.01641	-0.02334	-0.01312
Day 8	0.000397	-0.01428	-0.007024	-0.01679	-0.01601	-0.01453	-0.02222	-0.01643	-0.01166	-0.02001	-0.006916	-0.01294	-0.0164	-0.02333	-0.01319
Day 9	0.000397	-0.0144	-0.007019	-0.01692	-0.01611	-0.01454	-0.02224	-0.01651	-0.01166	-0.02004	-0.00708	-0.01254	-0.01639	-0.02333	-0.01326
Day 10	0.000397	-0.01451	-0.007014	-0.01704	-0.0162	-0.01455	-0.02226	-0.01659	-0.01166	-0.02007	-0.00724	-0.01239	-0.01639	-0.02333	-0.01332
Day 11	0.000397	-0.01461	-0.00701	-0.01714	-0.01629	-0.01455	-0.02227	-0.01666	-0.01166	-0.0201	-0.00740	-0.01235	-0.01638	-0.02333	-0.01338
Day 12	0.000397	-0.01471	-0.007005	-0.01724	-0.01638	-0.01456	-0.02229	-0.01673	-0.01166	-0.02013	-0.00755	-0.01236	-0.01637	-0.02333	-0.01344
Day 13	0.000397	-0.0148	-0.007	-0.01733	-0.01647	-0.01456	-0.02231	-0.0168	-0.01166	-0.02015	-0.00770	-0.01238	-0.01636	-0.02333	-0.0135
Day 14	0.000397	-0.01489	-0.006995	-0.0174	-0.01655	-0.01457	-0.02232	-0.01686	-0.01166	-0.02017	-0.00784	-0.01239	-0.01636	-0.02333	-0.01356
Day 15	0.000397	-0.01497	-0.00699	-0.01747	-0.01662	-0.01457	-0.02234	-0.01693	-0.01166	-0.02019	-0.00798	-0.01241	-0.01635	-0.02333	-0.01361
Day 16	0.000397	-0.01504	-0.006986	-0.01753	-0.0167	-0.01458	-0.02235	-0.01699	-0.01166	-0.02021	-0.00812	-0.01242	-0.01634	-0.02333	-0.01367
Day 17	0.000397	-0.01512	-0.006981	-0.01759	-0.01677	-0.01458	-0.02237	-0.01705	-0.01166	-0.02022	-0.00825	-0.01242	-0.01634	-0.02333	-0.01372
Day 18	0.000397	-0.01519	-0.006976	-0.01764	-0.01684	-0.01459	-0.02238	-0.0171	-0.01166	-0.02024	-0.00838	-0.01242	-0.01633	-0.02333	-0.01377
Day 19	0.000397	-0.01525	-0.006971	-0.01769	-0.01691	-0.01459	-0.02239	-0.01715	-0.01166	-0.02025	-0.00851	-0.01242	-0.01633	-0.02333	-0.01382
Day 20	0.000397	-0.01531	-0.006966	-0.01773	-0.01697	-0.0146	-0.02241	-0.01721	-0.01166	-0.02026	-0.00863	-0.01242	-0.01632	-0.02333	-0.01387
Day 21	0.000397	-0.01537	-0.006961	-0.01776	-0.01703	-0.0146	-0.02242	-0.01726	-0.01165	-0.02027	-0.0087	-0.01242	-0.01631	-0.02333	-0.01391
Day 22	0.000397	-0.01537	-0.006957	-0.0178	-0.01709	-0.01461	-0.02244	-0.0173	-0.01165	-0.02028	-0.00886	-0.01242	-0.01631	-0.02333	-0.01396
Day 23	0.000397	-0.01548	-0.006952	-0.01783	-0.01715	-0.01461	-0.02245	-0.01735	-0.01165	-0.02029	-0.00897	-0.01242	-0.01631	-0.02333	-0.014
Day 24	0.000397	-0.01552	-0.006947	-0.01785	-0.0172	-0.01461	-0.02247	-0.0174	-0.01165	-0.0203	-0.00908	-0.01242	-0.0163	-0.02333	-0.01404
Day 25	0.000397	-0.01557	-0.006942	-0.01788	-0.01725	-0.01462	-0.02248	-0.01744	-0.01165	-0.02031	-0.00919	-0.01242	-0.0163	-0.02333	-0.01408
Day 26	0.000397	-0.01561	-0.006937	-0.0179	-0.01731	-0.01462	-0.02250	-0.01748	-0.01165	-0.02031	-0.00929	-0.01242	-0.01629	-0.02333	-0.01412
Day 27	0.000397	-0.01566	-0.006932	-0.01792	-0.01735	-0.01462	-0.02251	-0.01752	-0.01165	-0.02032	-0.00940	-0.01242	-0.01629	-0.02333	-0.01416
Day 28	0.000397	-0.01569	-0.006928	-0.01793	-0.0174	-0.01463	-0.02253	-0.01756	-0.01165	-0.02033	-0.00949	-0.01242	-0.01628	-0.02333	-0.0142
Day 29	0.000397	-0.01573	-0.006923	-0.01795	-0.01745	-0.01463	-0.02254	-0.01759	-0.01165	-0.02033	-0.00959	-0.01242	-0.01628	-0.02333	-0.01424
Day 30	0.000397	-0.01577	-0.006918	-0.01796	-0.01749	-0.01463	-0.02256	-0.01763	-0.01165	-0.02034	-0.00968	-0.01242	-0.01628	-0.02333	-0.01427

3.3. Solving (P) for Preferable Efficient Portfolio

In Jana and Panda (2014), the theory of nonlinear vector optimization models with interval uncertainty in both objective function and constraints is studied. Since the portfolio selection model of this paper is a nonlinear multi objective optimization problem whose objective functions are nonlinear interval valued functions so the methodology by Jana and Panda (2014) can be comfortably implemented for this model.

Recall the portfolio selection model **(P)**. One may observe that uncertainty is associated with **(P)** in the form of intervals in four objective functions. Any portfolio x satisfying $\sum x_i = 1$ can be a compromise solution of **(P)** if it minimizes four objective functions simultaneously. Since every objective function is an interval valued mapping, so interval vectors have to be compared corresponding to every feasible portfolio x. To compare interval valued objective functions in **(P)**, we accept $\preccurlyeq_{LU}$ and $\preccurlyeq_{LU}^{n}$ partial orderings which are as follows.

For two intervals $\hat{A} = [a^L, a^U]$ and $\hat{B} = [b^L, b^U]$, $\hat{A} \preccurlyeq_{LU} \hat{B}$ iff $a^L \leq b^L$ and $a^U \leq b^U$.

For two interval vectors $\hat{A}_v = \left(\hat{A}_1, \hat{A}_2, \dots, \hat{A}_n\right)^T$ and $\hat{B}_v = \left(\hat{B}_1, \hat{B}_2, \dots, \hat{B}_n\right)^T$, $\hat{A}_v \prec_{LU}^{n} \hat{B}_v$ iff $\hat{A}_i \preccurlyeq_{LU} \hat{B}, \forall\, i = 1,2, \dots, n$.

In the light of the definition of the solution of a general vector optimization problem Jana and Panda, 2014, x^* is the efficient solution of (**P**) with respect to $\preccurlyeq_{LU}^{4}$ partial ordering if there does not exists any feasible solution x of (**P**) such that $\hat{f}(x) \preccurlyeq_{LU}^{4} \hat{f}(x^*)$ with $\hat{f}(x) \neq \hat{f}(x^*)$. Here $\hat{f}(x) = \{-\hat{R}(x), \hat{\sigma}^2(x), -\hat{S}(x), \hat{K}(x)\}$.

To solve (**P**), we will assign a target/goal to every interval valued objective functions $-\hat{R}(x), \hat{\sigma}^2(x), -\hat{S}(x), \hat{K}(x)$. These goals may be provided by the decision maker, otherwise we can assign these goals using the following procedure.

Determination of goal to the objective functions:
Consider the following single objective problems corresponding to the feasible set $S = \{x: \sum_{i=1}^{n} x_i = 1, x_i \geq 0\}$ as,

$$(P_1^L): \min_{x \in S} -R^U(x)\,;\ \ (P_1^U): \min_{x \in S} -R^L(x)$$
$$(P_2^L): \min_{x \in S} \sigma^{2L}(x)\,;\ \ (P_2^U): \min_{x \in S} \sigma^{2U}(x)$$
$$(P_3^L): \min_{x \in S} -S^U(x)\,;\ \ (P_3^U): \min_{x \in S} -S^L(x)$$
$$(P_4^L): \min_{x \in S} K^L(x)\,;\ \ (P_4^U): \min_{x \in S} K^U(x)\,.$$

Denote the solution of the problems

$(P_1^L), (P_1^U), (P_2^L), (P_2^U), (P_3^L), (P_3^U), (P_4^L)$ and $(P_4^U) as\ x_1^L, x_1^U, x_2^L, x_2^U, x_3^L, x_3^U, x_4^L$ and x_2^U respectively, and $S_{ideal} = \{x_i^L, x_i^U, i \in \Lambda_4\}$.

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Let

$$l_1^L = \min_{x \in S} -R^U(x), u_1^L = \max_{x \in S} -R^U(x), l_1^U = \min_{x \in S} -R^L(x), u_1^U = \max_{x \in S} -R^L(x).$$

l_1^L, u_1^L and l_1^U, u_1^U can be treated as the goals of $-R^U(x)$ and $-R^L(x)$ respectively.

For every $x \in S$, deviation of $-R^U(x)$ from the goals l_1^L and u_1^L and of $-R^L(x)$ from the goals l_1^U and u_1^U may be more or less acceptable for the decision maker. This implies that $-R^U(x)$ and $-R^L(x)$ are associated with certain degree of flexibility from their goals. For every $x \in S$, the degree of flexibility of $-R^U(x)$ is higher if deviation of $-R^U(x)$ from l_1^L is less and the degree of flexibility is less if deviation of $-R^U(x)$ from u_1^L is more. Similar interpretation can be made for the upper bound function $-R^L(x)$. Hence the degree of flexibility of $-R^U(x)$ and $-R^L(x)$ can be measured through some functions η_1^L and η_1^U from $\mathbb{R}$ to [0,1] respectively. Mathematically we may write this function as

$$\eta_1^L\left(-R^U(x)\right) = \begin{cases} 1, & -R^U(x) \le l_1^L \\ \dfrac{u_1^L + R^U(x)}{u_1^L - l_1^L}, & l_1^L \le -R^U(x) \le u_1^L \\ 0, & -R^U(x) \ge u_1^L \end{cases}$$

$$\eta_1^U\left(-R^L(x)\right) = \begin{cases} 1, & R^L(x) \le l_1^U \\ \frac{u_1^U + R^L(x)}{u_1^U - l_1^U}, & l_1^L \le -R^L(x) \le u_1^U. \\ 0, & R^L(x) \ge u_1^U \end{cases}$$

Similarly, the degree of flexibility of the other objective functions is calculated. Suppose goals of the other objective functions are given as follows.

For $\delta \in \{L, U\}$, goals of $\sigma^{2\,\delta}(x)$ are l_2^δ and u_2^δ, goals of $-S^\delta(x)$ are l_3^δ and u_3^δ and goals of $K^\delta(x)$ are l_4^δ and u_4^δ.

Then the degree of flexibility of the lower and upper bound of the objective functions are given by the functions $\eta_2^L(\sigma^{2L}(x))$; $\eta_2^U(\sigma^{2U}(x))$, $\eta_3^L(-S^U(x))$; $\eta_3^U(-S^L(x))$ and $\eta_4^L(K^L(x))$; $\eta_4^U(K^U(x))$, which are mathematically defined by

$$\eta_2^L\left(\sigma^{2L}(x)\right) = \begin{cases} 1, & \sigma^{2L}(x) \le l_2^L \\ \dfrac{u_2^L - \sigma^{2L}(x)}{u_2^L - l_2^L}, & l_2^L \le \sigma^{2L}(x) \le u_2^L\,; \\ 0, & \sigma^{2L}(x) \ge u_2^L \end{cases}$$

$$\eta_2^U\left(\sigma^{2U}(x)\right) = \begin{cases} 1, & \sigma^{2U}(x) \le l_2^U \\ \dfrac{u_2^L - \sigma^{2U}(x)}{u_2^L - l_2^L}, & l_2^U \le \sigma^{2U}(x) \le u_2^U\,; \\ 0, & \sigma^{2U}(x) \ge u_2^U \end{cases}$$

$$\eta_3^L\left(-S^U(x)\right) = \begin{cases} 1, & -S^U(x) \le l_3^U \\ \dfrac{u_3^L + S^U(x)}{u_3^L - l_3^L}, & l_3^U \le -S^U(x) \le u_3^U\,; \\ 0, & -S^U(x) \ge u_3^U \end{cases}$$

$$\eta_3^U\left(-S^L(x)\right) = \begin{cases} 1, & -S^L(x) \le l_3^U \\ \dfrac{u_3^L + S^L(x)}{u_3^L - l_3^L}, & l_3^U \le -S^L(x) \le u_3^U\,; \\ 0, & -S^L(x) \ge u_3^U \end{cases}$$

$$\eta_4^L\left(K^L(x)\right) = \begin{cases} 1, & K^L(x) \le l_4^L \\ \dfrac{u_4^L - K^L(x)}{u_4^L - l_4^L}, & l_4^L \le K^L(x) \le u_4^L\,; \\ 0, & K^L(x) \ge u_4^L \end{cases}$$

$$\eta_4^U\left(K^U(x)\right) = \begin{cases} 1, & K^U(x) \le l_4^U \\ \dfrac{u_4^U - K^U(x)}{u_4^U - l_4^U}, & l_4^U \le K^U(x) \le u_4^U. \\ 0, & K^U(x) \ge u_4^U \end{cases}$$

That is, the objective functions are characterized by their degree of flexibility. So a decision x in this uncertain environment is the selection of activities that simultaneously satisfies all the objective functions, which is

$$\min_{\{i,l\}\in\Lambda_4} \left\{\eta_i^L(.); \eta_l^U(.); x \in S\right\}. \tag{7}$$

This problem can be rewritten in expanded form as

$$\begin{gathered} \max \theta \\ \text{subject to } \theta \le \eta_1^L\left(-R^U(x)\right), \theta \le \eta_1^U\left(-R^U(x)\right) \\ \theta \le \eta_2^L\left(\sigma^{2L}(x)\right), \theta \le \eta_2^U\left(\sigma^{2U}(x)\right) \\ \theta \le \eta_3^L\left(-S^U(x)\right), \theta \le \eta_3^U\left(-S^L(x)\right) \\ \theta \le \eta_4^L\left(K^L(x)\right), \theta \le \eta_4^U\left(K^U(x)\right) \\ x \in S, 0 \le \theta \le 1. \end{gathered}$$

This is a general nonlinear programming problem, which is free from interval uncertainty and can be solved using nonlinear programming techniques. By Theorem 4.1 of Jana and Panda (2014), if (θ^{opt}, x^{opt}) be the solution of the problem (**P**), then it can be shown that x^{opt} is preferable efficient solution of (**P**). We say this solution as an efficient portfolio.

4. Analysis and Validity of the Result

Following the discussion of Subsection 3.2.1, Augmented Dickey Fuller test statistics for the daily high and low returns of 15 stocks is provided in Table 2. Implementation of the transformation of the portfolio model (**P**) with these 15 stocks to the model (**P'**) is done following

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the procedure of Subsection 3.3. The solution of (**P**)' provides the efficient portfolio, which is provided in Table 6.

Table 6. Preferable efficient solution

Scrip	Percentage of portfolio investment
Axis	5.47
Bank of Baroda	4.38
Cairn	9.34
Hindalco	4.56
HDFC	6.34
Jindal	6.25
JP Associates	6.95
JSW Steel Ltd.	5.75
Maruti	7.34
RCom	6.73
SBI	7.87
Tata Motors	5.23
Tata Steel	8.94
United Spirits Ltd.	5.46
Yes Bank	9.39

As per the theory developed in this paper, investment will be made according to this portfolio for the month of Mar '14, by an investor, which uses returns data for the period of Feb '09- Feb '14. The corresponding expected a range of the returns of the portfolio and the subsequent moments, as predicted by the modified (**P**) model are calculated and provided in Table 7. So, if an investor stays invested in the efficient portfolio as in Table 6 for March '14, he would tend to maximize his mean returns and skewness while minimizing the risk (variance) and kurtosis, subject to the investor expectations and STT constraints given in (**P**).

Accordingly, the investor would also be able to derive a lower and an upper bound for each of the four moments- mean, variance, skewness and kurtosis corresponding to the expected returns of the optimal portfolio, as provided in Table 7.

Table 7. Expected range of different moments for the optimal portfolio predicted by the model (yearly)

Moments	Lower bound	Upper bound
Mean	0.1948	0.2734
Variance	0.0634	0.0703
Skewness	-4.63 e-05	-3.92 e-05
Kurtosis	7.32 e-06	8.02 e-06

In order to check the validity of the results of Table 7 one may look at the actual performance of the optimal portfolio predicted by the model at the end of March '14. We obtain the prices for the 15 scrips from the Bombay Stock Exchange for 1^{st} March '14- 31^{st} March '14 and derive their actual returns for the period. We also compute the different moments corresponding to these realized returns of the optimal portfolio, to check the performance of the model against the values predicted above. Results of the same are compiled in Table 8.

Table 8. Performance of the optimal portfolio for March 2014

Moments	Value (yearly)
Mean	0.0256
Variance	0.0678
Skewness	-4.02 e-05
Kurtosis	7.33 e-06

It is evident from Table 7 and Table 8 that the four moments corresponding to the returns of the optimal portfolio lie within the range predicted by the model. Thus, through the aforementioned methodology, we are not only able to predict the portfolio, optimizing the returns, but also efficiently predict the range within which the performance parameters of the proposed portfolio would vary.

5. Concluding Remarks

In this present work, we consider a multi-objective portfolio selection model in which the objective functions are expected return, variance, skewness and kurtosis of the portfolio of 15 stocks. The model is applied to find an efficient portfolio of the model. We consider the daily low and high price of return data for the 15 stocks of BSE-India from Yahoo for the period of Feb '09-Feb '14 and forecast the upper and lower bound for each scrip for March '14. Subsequently we determine the lower and upper bounds for each of the parameters of the objective function (variance, skewness and kurtosis) using the moments of forecasted return bounds for March '14. A methodology to find an efficient portfolio of the model of 15 stocks is illustrated. In this paper, we discuss a methodology to derive an efficient portfolio.

Finally, we analyze the performance of the optimal portfolio for March 14. Comparing the predicted range of the four basic moments by the interval vector model and their realized values for March '14, it can be concluded that the methodology is able to estimate the expected range of all these moments efficiently.

Thus an investor who invests in this portfolio is not only able to optimize the four basic moments corresponding to the expected returns of the portfolio but can also be able to estimate the range within which these will vary for the future period under consideration. This methodology is also applicable to more number of stocks. Further the results obtained in this paper can readily be extended to find different portfolio performance measures like Sharpe ratio, Treynor ratios under capital market specific constraints.

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