International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 2, 327–336, 2019
https://dx.doi.org/10.33889/IJMEMS.2019.4.2-026

Some Observations on 2-tuple Linguistic and Interval 2-tuple Linguistic Operators

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(Received October 3, 2018; Accepted December 22, 2018)

Abstract
Literature of aggregation operators defined in the domains of linguistic and interval-valued linguistic information is extensive and vast. But, some operators are not well-defined and do not benefit the researchers. In this paper, we have highlighted the flaws in defining the operators and demonstrated them via examples. Further, some remedial suggestions for the improvement of the definitions are given.

Keywords- Multiple criteria group decision making, 2-tuple linguistic variable, Interval 2-tuple linguistic variable, Aggregation operators.

1. Introduction
Multiple criteria group decision making (MCGDM) problems in linguistic framework emanate due to constraints, ambiguity and obscure knowledge of the experts. But, sometimes the subjectivity of the linguistic terms in the linguistic term set is not appropriate to cater the granularity of uncertainty in assessment on some criteria. In such scenario, interval 2-tuple linguistic variables, introduced by Zhang (2013), can be consolidated to model complicated real world decision making problems. The main aspect of MCGDM is the aggregation of the linguistic information to obtain a unified decision result. Over the last decades, researchers have proposed various aggregation operators for 2-tuple linguistic (Park et al., 2013; Wei, 2010) and interval 2-tuple linguistic variables (Zhang, 2012; Zhang, 2013). Although, some authors Liu et al. (2014a, 2014b, 2014c, 2014d), Wan (2013), You et al. (2015) have inadvertently made certain mistakes in defining the operators. In this paper, we highlight these conceptual mistakes and illustrate them via examples. Moreover, we have suggested some improvements in the definitions of the operators.

Now we briefly present the following notations for clarity.

Let $S = \{s_0, s_1, \dots, s_g\}$ be a totally ordered discrete linguistic term set with odd cardinality and $\beta \in [0, g]$ is a value representing the result of a symbolic aggregation operation. Herrera and Martinez (2000) proposed the generalized translation function $\Delta_{HM}: [0, g] \to S \times [-0.5, 0.5)$, used to obtain the 2-tuple linguistic variable equivalent to β. It is defined as $\Delta_{HM}(\beta) = (s_i, \alpha)$, $i =$ round (β) and $\alpha = \beta - i$, where round $(\cdot)$ is the usual rounding operation, s_i has the closest index label to β and α is the value of the symbolic translation. The function Δ_{HM} is a bijection and its inverse is given by $\Delta_{HM}^{-1}: S \times [-0.5, 0.5) \to [0, g]$ as $\Delta_{HM}^{-1}(s_i, \alpha) = i + \alpha = \beta$.

Instigated by this model, Tai and Chen (2000) proposed the generalized 2-tuple linguistic model in the following way.

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Let $S = \{s_0, s_1, \ldots, s_g\}$ be a linguistic term set as defined above and $\beta \in [0,1]$ represents the result of a symbolic aggregation. The equivalent 2-tuple linguistic variable can be obtained using the function $\Delta_{TC}: [0,1] \rightarrow S \times \left[\frac{-1}{2g}, \frac{1}{2g}\right)$, defined as $\Delta_{TC}(\beta) = (s_i, \alpha)$ where i = round (βg) and $\alpha = \beta - \frac{i}{g}$. The corresponding Δ_{TC}^{-1} is defined as:

$\Delta_{TC}^{-1}: S \times \left[\frac{-1}{2g}, \frac{1}{2g}\right) \rightarrow [0,1]$ as $\Delta_{TC}^{-1}(s_i, \alpha) = \frac{i}{g} + \alpha = \beta$.

For example, given a linguistic term set

$S = \{s_0: Very\ Poor\ (VP), s_1: Poor\ (P), s_2: Medium\ (M), s_3: Good\ (G), s_4: Very\ Good\ (VG)\}$, the symbolic translation α lies in $\left[\frac{-1}{8}, \frac{1}{8}\right) = [-0.125, 0.125)$. The generalized 2-tuple linguistic variables from set S can be given as $(G, 0.018), (P, -0.124)$ etc.

Zhang (2012) proposed interval 2-tuple linguistic representation model as an extension of 2-tuple linguistic model, given as follows.

An interval 2-tuple linguistic variable is composed of two 2-tuple linguistic information, denoted by $[(s_i, \alpha_i), (s_j, \alpha_j)]$ for $i \le j$, where α_i and α_j represent symbolic translation. In other words, if $\beta_1, \beta_2 \in [0,1]$, are such that $\beta_1 \le \beta_2$, then the interval 2-tuple variable that expresses the equivalent information to an interval $[\beta_1, \beta_2]$ is derivable by the function Δ_Z defined as follows.

$$\Delta_Z([\beta_1, \beta_2]) = \left[(s_i, \alpha_i), (s_j, \alpha_j)\right] \text{ with } \begin{cases} s_i & i = round(\beta_1 g) \\ \alpha_i = \beta_1 - \frac{i}{g} & \alpha_i \in \left[\frac{-1}{2g}, \frac{1}{2g}\right) \\ s_j & j = round(\beta_2 g) \\ \alpha_j = \beta_2 - \frac{j}{g} & \alpha_j \in \left[\frac{-1}{2g}, \frac{1}{2g}\right) \end{cases}.$$

2. Flaws in the Existing Operators' Definitions on 2-tuple Linguistic and Interval 2-tuple Linguistic Variables

In this section, we present the operators on 2-tuple linguistic and interval 2-tuple linguistic variables given in the works of Liu et al. (2014a, 2014b, 2014c, 2014d), Wan (2013) and You et al. (2015) and highlight the concerns related to them through examples.

(i) In the works of Wan (2013), the author has defined operation laws of 2-tuple linguistic variables as follows.

(a) $(s_k, \alpha_k) \oplus (s_l, \alpha_l) = \Delta_{HM}\left(\Delta_{HM}^{-1}(s_k, \alpha_k) + \Delta_{HM}^{-1}(s_l, \alpha_l)\right)$.

The above defined addition operation $\oplus$ for 2-tuple linguistic variables is not correct. The domain of Δ_{HM} operator is $[0, g]$ that could be contravened when $\Delta_{HM}^{-1}(s_k, \alpha_k) + \Delta_{HM}^{-1}(s_l, \alpha_l) > g$.

For instance, let the predefined linguistic term set be $S_1 = \{s_0, s_1, \dots, s_6\}$. Consider

$$\begin{aligned}(s_4, 0.2) \oplus (s_3, 0.4) &= \Delta_{HM}\left(\Delta_{HM}^{-1}(s_{4,}, 0.2) + \Delta_{HM}^{-1}(s_3, 0.4)\right)\\ &= \Delta_{HM}(4.2 + 3.4)\\ &= \Delta_{HM}(7.6)\end{aligned}$$

which is not defined, since $7.6 \notin [0,6]$.

(b) $(s_k, \alpha_k) \otimes (s_l, \alpha_l) = \Delta_{HM}\left(\Delta_{HM}^{-1}(s_k, \alpha_k).\Delta_{HM}^{-1}(s_l, \alpha_l)\right)$.

The operator $\otimes$ is also not correctly defined. For instance, consider same linguistic term set $S_1 = \{s_0, s_1, \dots, s_6\}$, then,

$$\begin{aligned}(s_{2,}, 0) \otimes (s_4, 0) &= \Delta_{HM}\left(\Delta_{HM}^{-1}(s_{2,}, 0).\Delta_{HM}^{-1}(s_4, 0)\right) = \Delta_{HM}(2 \times 4)\\ &= \Delta_{HM}(8),\end{aligned}$$

and, the same is not computable as the operand does not belong to the domain.

(c) $\lambda(s_k, \alpha_k) = \Delta_{HM}\left(\lambda \Delta_{HM}^{-1}(s_k, \alpha_k)\right);\ \lambda \geq 0$.

The given definition of scalar multiplication with scalar $\lambda \geq 0$ is ambiguous. Take $\lambda = 2$ and 2-tuple linguistic information as $(s_4, 0) \in S_1$. Then,

$$2(s_4, 0) = \Delta_{HM}(2\Delta_{HM}^{-1}(s_4, 0)) = \Delta_{HM}(8).$$

Again, $\Delta_{HM}(8)$ is not defined.

(d) $(s_k, \alpha_k)^\lambda = \Delta_{HM}\left((\Delta_{HM}^{-1}(s_k, \alpha_k))^\lambda\right);\ \lambda \geq 0$.

The power operation is also not well-defined. Consider $\lambda = 4$ and linguistic variable as $(s_2, 0)$. Then,

$$(s_2, 0)^4 = \Delta_{HM}((\Delta_{HM}^{-1}(s_2, 0))^4) = \Delta_{HM}(2^4) = \Delta_{HM}(16),$$
which cannot be computed.

(e) $(s_k, \alpha_k)^{(s_l, \alpha_l)} = \Delta_{HM}\left((\Delta_{HM}^{-1}(s_k, \alpha_k))^{\Delta_{HM}^{-1}(s_l, \alpha_l)}\right)$.

The operation defined for 2-tuple linguistic power of a 2-tuple linguistic variable, is also not correct for the same reasons as above. Consider two 2-tuple linguistic variables $(s_4, 0)$and $(s_2, 0)$. Then,

$$(s_4, 0)^{(s_2, 0)} = \Delta_{HM}\left((\Delta_{HM}^{-1}(s_4, 0))^{\Delta_{HM}^{-1}(s_2, 0)}\right) = \Delta_{HM}(4^2) = \Delta_{HM}(16),$$
leading to the same problem as encountered above.

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(ii) In Liu et al. (2014b), the authors defined the following operators for interval 2-tuple linguistic variables.

(a) The addition of two interval 2-tuple linguistic variables $\tilde{a} = [(s_i, \alpha_i), (s_j, \alpha_j)]$ and $\tilde{b} = [(s_k, \alpha_k), (s_l, \alpha_l)]$ is defined as follows.

$$\tilde{a} + \tilde{b} = [(s_i, \alpha_i), (s_j, \alpha_j)] + [(s_k, \alpha_k), (s_l, \alpha_l)]$$

$$= \Delta_Z\left[\Delta_{TC}^{-1}(s_i, \alpha_i) + \Delta_{TC}^{-1}(s_k, \alpha_k), \Delta_{TC}^{-1}(s_j, \alpha_j) + \Delta_{TC}^{-1}(s_l, \alpha_l)\right].$$

The addition operation is repeatedly defined and used in various works of Liu et al. (2014a, 2014c, 2014d).

The above definition of addition operation for interval-valued 2-tuple linguistic variables is not correct. For instance, let $S_2 = \{s_0, s_1, \dots, s_{10}\}$ be another predefined linguistic term set and two interval 2-tuple information are $\tilde{a} = [(s_3, 0), (s_5, 0)]$ and $\tilde{b} = [(s_5, 0), (s_7, 0)]$. Then,

$$\tilde{a} + \tilde{b} = \Delta_Z([\Delta_{TC}^{-1}(s_3, 0) + \Delta_{TC}^{-1}(s_5, 0), \Delta_{TC}^{-1}(s_5, 0) + \Delta_{TC}^{-1}(s_7, 0)])$$

$$= \Delta_Z([0.3 + 0.5, 0.5 + 0.7]) = \Delta_Z([0.8,1.2])$$

which cannot be evaluated for the resultant interval $[0.8,1.2] \not\subseteq [0,1]$.

(b) The Euclidean distance between two interval 2-tuple linguistic variables $\tilde{a} = [(s_i, \alpha_i), (s_j, \alpha_j)]$ and $\tilde{b} = [(s_k, \alpha_k), (s_l, \alpha_l)]$ is defined as,

$$D(\tilde{a}, \tilde{b}) = \Delta_{TC}\left(\sqrt{\left(\Delta_{TC}^{-1}(s_i, \alpha_i) - \Delta_{TC}^{-1}(s_k, \alpha_k)\right)^2 + \left(\Delta_{TC}^{-1}(s_j, \alpha_j) - \Delta_{TC}^{-1}(s_l, \alpha_l)\right)^2}\right).$$

The Euclidean distance between two interval 2-tuple variables is also ill-defined. Consider the same linguistic term set S_2 as above and interval 2-tuple linguistic variables $\tilde{a} = [(s_9, 0), (s_{10}, 0)]$ and $\tilde{b} = [(s_0, 0), (s_1, 0)]$. Then,

$$D(\tilde{a}, \tilde{b}) = \Delta_{TC}\left(\sqrt{\left(\Delta_{TC}^{-1}(s_9, 0) - \Delta_{TC}^{-1}(s_0, 0)\right)^2 + \left(\Delta_{TC}^{-1}(s_{10}, 0) - \Delta_{TC}^{-1}(s_1, 0)\right)^2}\right)$$
$$= \Delta_{TC}\left(\sqrt{(0.9 - 0)^2 + (1 - 0.1)^2}\right) = \Delta_{TC}(1.27).$$

Since, $1.27 \notin [0,1]$, the latter is not defined for linguistic 2-tuple Δ_{TC} function.

(c) The Euclidean distance between two finite sets of interval 2-tuple linguistic variables,

$\tilde{X}_1 = \{[(s_1, \alpha_1), (t_1, \varepsilon_1)], [(s_2, \alpha_2), (t_2, \varepsilon_2)], \dots, [(s_n, \alpha_n), (t_n, \varepsilon_n)]\}$ and
$\tilde{X}_2 = \{[(s_1', \alpha_1'), (t_1', \varepsilon_1')], [(s_2', \alpha_2'), (t_2', \varepsilon_2')], \dots, [(s_n', \alpha_n'), (t_n', \varepsilon_n')]\}$

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is defined as follows.

$$D(\tilde{X}_1,\tilde{X}_2) = \Delta_{TC}\left(\sqrt{\sum_{i=1}^{n}\left[\left(\Delta_{TC}^{-1}(s_i,\alpha_i)-\Delta_{TC}^{-1}(s_i{}',\alpha_i{}')\right)^2+\left(\Delta_{TC}^{-1}(t_i,\varepsilon_i)-\Delta_{TC}^{-1}(t_i{}',\varepsilon_i{}')\right)^2\right]}\right).$$

The distance operator between two sets of interval 2-tuple linguistic variables is also nebulous. Consider the linguistic term set $S_2 = \{s_0, s_1, \dots, s_{10}\}$ and take two sets $\tilde{X}_1$ and $\tilde{X}_2$, each having cardinality two, as $\tilde{X}_1 = \{[(s_3,0),(s_4,0)],[(s_6,0),(s_8,0)]\}$ and $\tilde{X}_2 = \{[(s_9,0),(s_{10},0)],[(s_2,0),(s_3,0)]\}$. Then, $D(\tilde{X}_1,\tilde{X}_2) = \Delta_{TC}(1.063)$, which is not defined for 1.063 is not in the domain of Δ_{TC}.

(iii) In the works of You et al. (2015), the authors defined distance operation between two finite sets of interval 2-tuples $\tilde{X}_1 = \{[(s_1,\alpha_1),(t_1,\varepsilon_1)],[(s_2,\alpha_2),(t_2,\varepsilon_2)],\dots,[(s_n,\alpha_n),(t_n,\varepsilon_n)]\}$ and

$\tilde{X}_2 = \{[(s_1{}',\alpha_1{}'),(t_1{}',\varepsilon_1{}')],[(s_2{}',\alpha_2{}'),(t_2{}',\varepsilon_2{}')],\dots,[(s_n{}',\alpha_n{}'),(t_n{}',\varepsilon_n{}')]\}$ as follows.

$$D(\tilde{X}_1,\tilde{X}_2) = \Delta_{TC}\sum_{i=1}^{n}\left[\frac{1}{2}\left(\left|\Delta_{TC}^{-1}(s_i,\alpha_i)-\Delta_{TC}^{-1}(s_i{}',\alpha_i{}')\right|+\left|\Delta_{TC}^{-1}(t_i,\varepsilon_i)-\Delta_{TC}^{-1}(t_i{}',\varepsilon_i{}')\right|\right)\right].$$

The aforementioned distance operation between two sets of interval 2-tuple linguistic is not accurately defined. Consider the above defined sets $\tilde{X}_1$ and $\tilde{X}_2$. Then, $D(\tilde{X}_1,\tilde{X}_2) = \Delta_{TC}(1.05)$, which is not defined as $1.05 \notin [0,1]$.

3. Application of the Defined Operations in MCGDM Problem

Besides the above defined incorrect operations, there are also some omissions in the example presented in Section 4 in the works of Liu et al. (2014b) to illustrate the proposed interval 2-tuple linguistic TOPSIS (ITL-TOPSIS) for multi-criteria decision making problems.

The problem considered is of robot selection where robots are desired to perform certain tasks efficiently. The specific example considered three shortlisted robots, A_1, A_2, A_3, to be evaluated under six predefined criteria C_1, C_2..., C_6 by committee of four decision makers $DM_1, DM_2, \dots, DM_4$ using the linguistic term sets as $A = \{a_0, a_1, \dots, a_4\}$, $B = \{b_0, b_1, \dots, b_6\}$, $C = \{c_0, c_1, \dots, c_8\}$ and $D = \{d_0, d_1, \dots, d_6\}$, as discussed in Liu et al. (2014b).

The assessment matrices of the alternatives under the 3 subjective criteria given by the experts using the above linguistic term sets, as mentioned in Liu et al. (2014b), are given in Table 1.

Table 1. Decision-makers' evaluations of the three robots under subjective criteria

Decision-makers	Alternatives	Criteria		
		C_1	C_2	C_3
DM_1	A_1	VG	F	P-F
	A_2	F-VG	F-G	G
	A_3	G	G-VG	VG
DM_2	A_1	MG-G	G	MG-G
	A_2	-	G-VG	VG
	A_3	VG	VG	G
DM_3	A_1	F	P-F	MP-MG
	A_2	G-VG	G	F
	A_3	MG	F-MG	G
DM_4	A_1	MP-F	G	-
	A_2	F	VG	MG
	A_3	G	MG-VG	G-VG

With this setting ITL-TOPSIS method is invoked to rank the alternatives. The linguistic entries in four respective alternatives-criteria assessment matrices of decision makers are such that the final computed values of the separation measures D_i^+(from positive ideal solution) and D_i^-(from negative ideal solution), $i = 1,2,3$ for three robots alternatives, are in [0,1].

But this is not always the case. When we consider the same example, but with minor changes in the four assessment matrices of the decision makers, then we realize that the values of $D_i^+$and D_i^-can be greater than 1.

Consider the assessment matrices given in Table 2 to evaluate three alternatives.

Table 2. Decision-makers' evaluations of the three robots under subjective criteria with minor changes. The entries with * represent changes in Table 1

Decision-makers	Alternatives	Criteria		
		C_1	C_2	C_3
DM_1	A_1	P-VG*	P-F*	P-F
	A_2	F-VG	F-G	G
	A_3	G	G-VG	VG
DM_2	A_1	MP-G*	P-G*	MG-G
	A_2	-	G-VG	VG
	A_3	VG	VG	G
DM_3	A_1	VP-F*	P-F	MP-MG
	A_2	G-VG	G	F
	A_3	MG	F-MG	G
DM_4	A_1	P-F*	VP-G*	-
	A_2	F-VG*	VG	MG
	A_3	G	MG-VG	G-VG

The ITL-TOPSIS method is applied on the above matrices, and after some computations, we have separation measures from positive ideal solution and negative ideal solution of three robots as mentioned in Table 3.

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https://dx.doi.org/10.33889/IJMEMS.2019.4.2-026

Table 3. Separation measures from Positive ideal solution and Negative ideal solution of three robots for evaluations given in Table 2 using Euclidean distance for interval 2-tuple linguistic variables

i	D_i^+	D_i^-
1	$\Delta_{TC}(1.039)^{**}$	$\Delta_{TC}(0.655)$
2	$\Delta_{TC}(0.548)$	$\Delta_{TC}(1.074)^{**}$
3	$\Delta_{TC}(0.348)$	$\Delta_{TC}(1.143)^{**}$

Since the domain of Δ_{TC} is [0,1], some of the values in Table 3 (marked as **) are not defined.

Note that this happens as a consequence of the wrongly conceptualized n-dimensional Euclidean distance formula for interval 2-tuples in Liu et al. (2014b).

4. Remedial Suggestions

In this section, we give some recommendations that make the definitions valid.

(i) The addition operation $\oplus$ for 2-tuple linguistic variable could not be defined yet properly but it may be replaced by arithmetic mean as defined in Singh et al. (2017) as follows.

Let $\left\{(s_{r_i}, \alpha_{r_i}), i = 1,2, \dots, q, r_i \in \{0,1, \dots, g\}\right\}$, be a set of 2-tuple linguistic information. Then,

$$AM\left((s_{r_i}, \alpha_{r_i}): i = 1,2, \dots, q\right) = (s_k, \alpha_k),$$

Where $$k = round\left(\frac{1}{q}\sum_{i=1}^{q} r_i + \frac{g}{q}\sum_{i=1}^{q} \alpha_{r_i}\right)$$

and $$\alpha_k = \frac{q}{g}\sum_{i=1}^{q} r_i + \frac{1}{q}\sum_{i=1}^{q} \alpha_{r_i}.$$

(ii) For multiplication $\otimes$, if we apply the Δ_{TC} and Δ_{TC}^{-1} definitions given by Tai and Chen (2009), then the $\otimes$ operator is well defined and given as.

$$(s_k, \alpha_k)\otimes(s_l, \alpha_l) = \Delta_{TC}\left(\Delta_{TC}^{-1}(s_k, \alpha_k).\Delta_{TC}^{-1}(s_l, \alpha_l)\right).$$

Now, the example given in $1(b)$ of Section 2 can be calculated as.

$$(s_{2,}, 0)\otimes(s_4, 0) = \Delta_{TC}\left(\Delta_{TC}^{-1}(s_{2,}, 0).\Delta_{TC}^{-1}(s_4, 0)\right) = \Delta_{TC}\left(\frac{2}{6}.\frac{4}{6}\right) = \Delta_{TC}(0.2222)$$
$$= (s_1, 0.0555).$$

(iii) The scalar multiplication is not well defined for any scalar$\lambda \in \mathbb{R}^+$. However, it is well defined for $0 \le \lambda \le 1$.

(iv) In two operators $1(d)$ and $1(e)$ defined in Section 2, if we apply the definitions of the Δ_{TC} and Δ_{TC}^{-1}, given by Tai and Chen (2009), the operators become well defined. For example, consider $\lambda = 4$ and the linguistic variable as $(s_2, 0)$ with granularity $= 6$ of linguistic term set. Then,

$$(s_2,0)^4 = \Delta_{TC}((\Delta_{TC}^{-1}(s_2,0))^4) = \Delta_{TC}(0.3333^4) = \Delta_{TC}(0.0123) = (s_0, 0.0123)$$
and $$(s_4,0)^{(s_2,0)} = \Delta_{TC}\left(\left(\Delta_{TC}^{-1}(s_4,0)\right)^{\Delta_{TC}^{-1}(s_2,0)}\right) = \Delta_{TC}\left(\left(\frac{4}{6}\right)^2\right) = \Delta_{TC}(0.4444) = (s_3, -0.0556).$$

(v) The distance operation for interval 2-tuple linguistic, defined in Singh et al. (2017), allows the operand to always lie inside the domain. Hence, the Euclidean distance between two interval 2-tuple linguistic may be replaced by the following.

$$D\left(\left[(s_i,\alpha_i),(s_j,\alpha_j)\right],\left[(s'_i,\alpha'_i),(s'_j,\alpha'_j)\right]\right) = (s_k,\alpha_k)$$

where
$$k = round\left(0.5\left(\left|(i+\alpha_i g)-(i'+\alpha'_i g)\right| + \left|(j+\alpha_j g)-(j'+\alpha'_j g)\right|\right)\right)$$
and
$$\alpha_k = \left(\frac{\left|(i+\alpha_i g)-(i'+\alpha'_i g)\right| + \left|(j+\alpha_j g)-(j'+\alpha'_j g)\right|}{2g}\right) - \frac{k}{g}.$$

(vi) Again, the Euclidean distance between two sets of interval 2-tuple linguistic makes the operator ill-defined. Although, You et al. (2015) proposed the distance operation for sets of interval 2-tuple linguistic using absolute difference but, here also, the domain of Δ_{TC} may be violated. A remedial course is that instead use the following separation measures for set of interval 2-tuple linguistic variables.

$$D_i^+ = \Delta_{TC}\left(\frac{\sum_{j=1}^{n}\left(\left|\Delta_{TC}^{-1}(s'_{ij},\alpha'_{ij}) - \Delta_{TC}^{-1}(r_j^+,\alpha_j^+)\right| + \left|\Delta_{TC}^{-1}(t'_{ij},\varepsilon'_{ij}) - \Delta_{TC}^{-1}(r_j^+,\alpha_j^+)\right|\right)}{2n}\right), i = 1,2,\dots,m,$$

$$D_i^- = \Delta_{TC}\left(\frac{\sum_{j=1}^{n}\left(\left|\Delta_{TC}^{-1}(s'_{ij},\alpha'_{ij}) - \Delta_{TC}^{-1}(r_j^-,\alpha_j^-)\right| + \left|\Delta_{TC}^{-1}(t'_{ij},\varepsilon'_{ij}) - \Delta_{TC}^{-1}(r_j^-,\alpha_j^-)\right|\right)}{2n}\right), i = 1,2,\dots,m.$$

where m is number of alternatives, n is number of criteria, $[(r_1^+,\alpha_1^+),\dots,(r_n^+,\alpha_n^+)]$ is a 2-tuple linguistic positive ideal and $[(r_1^-,\alpha_1^-),\dots,(r_n^-,\alpha_n^-)]$ is a 2-tuple linguistic negative ideal solutions.

Note that the weighted average factor $\frac{1}{2n}$ in above expressions ensure that the values within braces are in $[0,1]$ and hence, $\Delta_{TC}(.)$ is meaningful.

On using these measures, in the example taken from Section 4 of Liu et al. (2014b), the Table 3 is replaced by the Table 4.

Table 4. Separation measures from Positive ideal solution and Negative ideal solution of three robots for evaluations given in Table 2 using the suggested distance measure

i	D_i^+	D_i^-
1	$\Delta_{TC}(0.234)$	$\Delta_{TC}(0.138)$
2	$\Delta_{TC}(0.116)$	$\Delta_{TC}(0.255)$
3	$\Delta_{TC}(0.084)$	$\Delta_{TC}(0.288)$

All values are now well-defined. Thereafter, one can easily work out the remaining steps of ITL-TOPSIS procedure to conclude the problem.

Note 1. The addition operation for 2-tuple linguistic and interval 2-tuple linguistic variables are not well defined in literature till now. As we need to perform relative comparison of the alternatives in decision making problems, hence ad interim, we may adopt the arithmetic mean operation for 2-tuple and interval 2-tuple linguistic variables. Although, the researchers should explore in this direction for the amelioration of the literature.

5. Conclusion

In this paper, we have pointed out some flaws in the addition, multiplication and distance operators of the 2-tuple linguistic and interval-valued 2-tuple linguistic variables in the works of Liu et al. (2014b), Wan (2013) and You et al. (2015). The paper emphasizes on the existing flaws as some authors Liu et al. (2014a, 2014b, 2014c) have inadvertently adopted these operators in linguistic MCGDM problems. To improvise the literature on aggregation operators of linguistic variables, the remedial suggestions are also given to overcome the difficulties in defining the aforementioned operators. The defined remedies are also illustrated via examples to demonstrate the modifications in the calculations.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgement

The authors are thankful to the esteemed referees for their valuable suggestions for improving the paper. The authors thank the editor for being supportive and considerate.

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