

Mathematical Modeling for Hospitalization due to Temperature Variations

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Abstract

Sudden changes in temperature occur due to ecological disturbances which results into global warming, volcanic eruption, depletion of ozone layer etc. This has become a crucial environmental and social issue all over the world. Changes in temperature influences transmission pattern for several diseases which leads to increase in the death rate. This research focusses on hospitalization due to temperature variations. Here, class of exposed individuals are divided into two types: asymptomatic individuals who do not show any symptoms and symptomatic individuals which shows symptoms. The indoor or outdoor medications are the remedial steps to get cured. Severe dehydration, blood pressure fluctuations etc. need hospitalization. The rate at which the individual gets hospitalized is scrutinized using *SEIRS* - model. The system of non-linear ordinary differential equations is formulated for the given model and the reproduction number is then computed using next generation matrix method which connotes the recovery rate of an individual. Stability and numerical simulations are carried out.

Keywords: Mathematical model, Basic reproduction number, Stability, Simulation.

1. Introduction

Temperature is a measure of the warmth or coldness of an object or substance with reference to some standard value. In the proposed model, temperature refers to ambient temperature. Change in temperature can occur at any location for any period. On this basis, it is classified as in Figure 1.

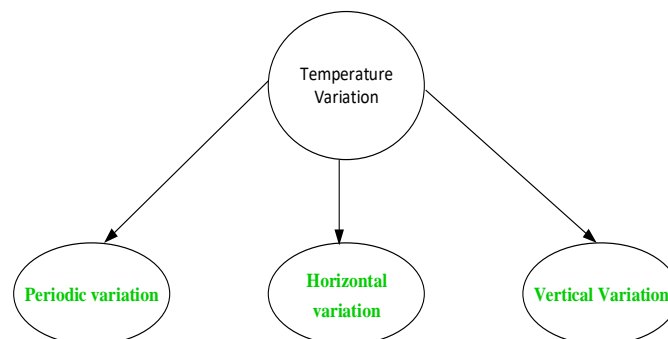


Figure 1. Classification of temperature variation

Periodic temperature variation is the continuous change of temperature during a day, week, month, year or any period, whereas a horizontal temperature variation is the change of solar energy with

respect to equatorial region and latitude. Vertical temperature variation is divided according to four spheres namely troposphere, stratosphere, mesosphere and thermosphere. This model is based on periodic temperature variations. During the last few years, the association between extreme temperatures and health issues has been frequently investigated. Unstable changes in temperature disturbs the weather we experience and our basic necessities resulting in increased health issues. The sternness of these health risks (hospitalization rate) depends on factors such as age, gender, individual's strength to sustain temperature change and monetary status. It will also vary based on location, sensitivity and adapting capability of an individual. Health effects may include gastrointestinal illness like vomiting, diarrhoea, effects on the body's nervous and respiratory systems, liver and kidney damage, etc. Some health issues may show some symptoms, others may not. Those individuals who show symptoms such as dehydration, low blood pressure, respiratory problems, heat stroke etc. are known as symptomatic individuals and those who do not show any symptoms such as any type of infection are asymptomatic individuals. Proper intime treatment is required to control these health issues. If the condition becomes critical and cannot be controlled by taking indoor medications, they get hospitalized. This research is focused on transmission rate of any health issue due to temperature variations on an individual from susceptible stage to hospitalised stage.

A lot of survey is done about effects of climate change on human health such as in book "Global climate change and health: an old story writ large" (McMichael, 2003) which contains information about every issue related to climate change and human health: Risks and responses. It is also reviewed (Wu et al., 2016) about the empirical evidence and human adaptation due to climate change on human infectious diseases. One of the study (Paaijmans et al., 2010) showed the influence of climate on malaria transmission which depends on daily temperature variation. The impact of rainfall and temperature variation on diarrheal prevalence in sub-saharan Africa was studied (Bandyopadhyay et al., 2012). A dynamic model of air pollution, health and population growth was examined using system dynamics: a study on Tehran-Iran (Shahgholian and Hajihosseini, 2009). Direct and indirect effect of climate change was analysed on human health (McMichael et al., 1997). Analysis was done for greater temperature variation within a day associated with increased emergency hospital admissions for asthma (Qiu et al., 2015). The temperature variation and emergency hospital admissions were reported for stroke in Brisbane, Australia, 1996–2005 (Wang et al., 2009). The impact was studied Zell et al. (2008) for global warming on viral diseases. Also, the basics of SEIR model were discussed and applied it for vector borne diseases to analyse steady state conditions. Also, after finding threshold parameter R_0 it was shown that the disease will spread only if its value exceeds 1 (Shah and Gupta, 2013, 2014). A mathematical model was formulated for water pollutants with stability analysis (Shah et al., 2018). A preliminary mathematical model was also computed for the transmission of dengue, chikungunya and zika (Isea and Lonngren, 2016). Similarly, a system of ordinary differential equations was designed to simulate the transmission of infection between humans and *Aedes albopictus* (Yakob and Clements, 2013).

The mathematical model for hospitalization rate is shown in Section 2 followed by stability at equilibrium points in Section 3. Numerical simulation is then computed in Section 4, concluding the model finally in Section 5.

2. Mathematical Model

The hospitalization rate due to temperature variations is studied in this mathematical model. It is based on *SEIRS*-model. This model consists of five compartments, namely, susceptible individual

(S_T) who are likely to be influenced by temperature variations which are then exposed (E) to temperature variations. These individuals are then classified into two: those who do not symptoms i.e. asymptomatic (A) and who show symptoms which are symptomatic (S) individuals. Symptomatic individuals after taking some medications may become asymptomatic and vice-versa. If the rate of asymptomatic and symptomatic individuals cannot be controlled by taking indoor medications, they get hospitalized (H). After getting cured, they may again get susceptible.

Table 1. Notations and parametric values

Notation	Description	Parametric values
B	New recruitment rate	0.6
α_1	The rate at which susceptible individuals gets exposed	0.5
α_2	The rate at which exposed individuals are asymptomatic	0.01
α_3	The rate at which exposed individuals start showing symptoms	0.02
α_4	The rate at which asymptomatic individuals becomes symptomatic	0.12
α_5	The rate at which symptomatic individuals after taking proper medications becomes asymptomatic	0.05
α_6	The rate at which asymptomatic individuals gets hospitalized	0.02
α_7	The rate at which symptomatic individuals is hospitalized	0.05
α_8	The rate at which hospitalized individuals after getting cured again becomes susceptible	0.08
μ	Mortality rate	0.03

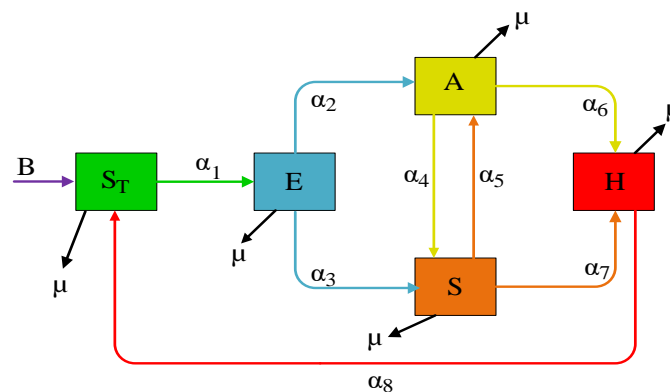


Figure 2. Transmission diagram for hospitalization

Using parametric values, the mathematical model shown in Figure 2 is formulated. The system of non-linear differential equations is as follows:

$$\begin{aligned}
 \frac{dS_T}{dt} &= B - \alpha_1 S_T E + \alpha_8 H - \mu S_T \\
 \frac{dE}{dt} &= \alpha_1 S_T E - \alpha_2 EA - \alpha_3 ES - \mu E \\
 \frac{dA}{dt} &= \alpha_2 EA + \alpha_5 S - \alpha_4 A - \alpha_6 A - \mu A \\
 \frac{dS}{dt} &= \alpha_3 ES + \alpha_4 A - \alpha_5 S - \alpha_7 S - \mu S \\
 \frac{dH}{dt} &= \alpha_6 A + \alpha_7 S - \alpha_8 H - \mu H
 \end{aligned} \tag{1}$$

where, $S_T + E + A + S + H \leq N$. Also, $S_T > 0; E, A, S, H \geq 0$.

Adding the above differential equations of system (1), we have

$$\frac{d}{dt}(S_T + E + A + S + H) = B - \mu(S_T + E + A + S + H) \geq 0; \quad \text{which implies that}$$

$$\limsup_{t \rightarrow \infty} (S_T + E + A + S + H) \leq \frac{B}{\mu}.$$

Therefore, the feasible region of the model is $\Lambda = \left\{ (S_T, E, A, S, H) \in R^5 : S_T + E + A + S + H \leq \frac{B}{\mu} \right\}$.

Now, the disease-free equilibrium point of the transmission of model is $Y_0 = \left(\frac{B}{\mu}, 0, 0, 0, 0 \right)$.

Unaffected equilibrium point is $Y_1 = \left(\frac{\mu}{\alpha_1}, \frac{B\alpha_1 - \mu^2}{\alpha_1\mu}, 0, 0, 0 \right)$ and endemic point is

$$Y^* = (S_T^*, E^*, A^*, S^*, H^*)$$

$$\text{where } S_T^* = \frac{(r\alpha_3 - \alpha_5 - \alpha_7 - \mu)(B\alpha_2(\alpha_8 + \mu) - \alpha_6\alpha_8\mu) - \alpha_4(B\alpha_3(\alpha_8 + \mu) + \alpha_7\alpha_8\mu)}{\mu \left[\alpha_1 \{ (\alpha_6 + \alpha_8 + \mu)(\alpha_3 r - \alpha_5) - \alpha_4(\alpha_7 + \alpha_8 + \mu) - \alpha_6(\alpha_7 + \mu) \} + (\alpha_8 + \mu)(\alpha_2(\alpha_3 r - \alpha_5) - (\alpha_7 + \mu)(\alpha_1 + \alpha_2) - \alpha_3\alpha_4) \right]},$$

$$E^* = r,$$

$$A^* = \frac{(\alpha_2(r\alpha_3 - \alpha_5 - \alpha_7 - \mu) + \alpha_3\alpha_5)(\alpha_1(B - r\mu) - \mu^2)(\alpha_8 + \mu)}{\mu \left[r\alpha_2\alpha_3(\alpha_1(\alpha_6 - \alpha_7) + (\alpha_8 + \mu)(\alpha_2 - \alpha_3)) - \alpha_1\alpha_2((\alpha_6 + \alpha_8 + \mu)(\alpha_5 + \alpha_7 + \mu) + \alpha_4(\alpha_7 + \alpha_8 + \mu)) + \alpha_1\alpha_3((\alpha_7 + \alpha_8 + \mu)(\alpha_4 + \alpha_6 + \mu) + \alpha_5(\alpha_6 + \alpha_8 + \mu)) - (\alpha_8 + \mu)(\alpha_2^2(\alpha_5 + \alpha_7 + \mu) + \alpha_2\alpha_3(\alpha_4 - \alpha_5) - \alpha_3^2(\alpha_4 + \alpha_6 + \mu)) \right]},$$

$$S^* = \frac{\alpha_4(\alpha_8 + \mu)(r\alpha_1\mu - B\alpha_1 + \mu^2)}{\mu \left[\alpha_1 \{ (\alpha_6 + \alpha_8 + \mu)(\alpha_3 r - \alpha_5) - \alpha_4(\alpha_7 + \alpha_8 + \mu) - \alpha_6(\alpha_7 + \mu) \} + (\alpha_8 + \mu)(\alpha_2(r\alpha_3 - \alpha_5) - (\alpha_1 + \alpha_2)(\alpha_7 + \mu) - \alpha_3\alpha_4) \right]},$$

$$H^* = \frac{B\alpha_1\alpha_2(\alpha_6(r\alpha_3 - \alpha_5 - \alpha_7 - \mu) - \alpha_4\alpha_7) + r\mu(\alpha_1(\alpha_2\alpha_4\alpha_7 - \alpha_3\alpha_6(\alpha_4 + \alpha_6 + \mu)) - \alpha_2\alpha_3\alpha_6\mu) + \alpha_6\mu(\alpha_1(\alpha_6 + \mu)(\alpha_5 + \alpha_7 + \mu) + (\alpha_7 + \mu)(\alpha_1\alpha_4 + \alpha_2\mu)) + \alpha_2\mu^2(\alpha_4\alpha_7 + \alpha_5\alpha_6)}{\alpha_2\mu \left[\alpha_1 \{ (\alpha_6 + \alpha_8 + \mu)(\alpha_3 r - \alpha_5) - \alpha_4(\alpha_7 + \alpha_8 + \mu) - \alpha_6(\alpha_7 + \mu) \} + (\alpha_8 + \mu)(r\alpha_3(\alpha_3 - \alpha_4) - (\alpha_7 + \mu)(\alpha_1 + \alpha_2) - \alpha_2\alpha_5) \right]},$$

and

$$r = \frac{1}{2\alpha_2\alpha_3} \left[\alpha_2(\alpha_5 + \alpha_7 + \mu) + \alpha_3(\alpha_4 + \alpha_6 + \mu) + \left\{ \alpha_2^2(\alpha_5(\alpha_5 + 2\alpha_7 + 2\mu) + (\alpha_7 + \mu)^2) + 2\alpha_2\alpha_3(\alpha_4(\alpha_5 - \alpha_7 - \mu) - (\alpha_6 + \mu)(\alpha_5 + \alpha_7 + \mu)) + \alpha_3^2(\alpha_4(\alpha_4 + 2\alpha_6 + 2\mu) + (\alpha_6 + \mu)^2) \right\}^{\frac{1}{2}} \right].$$

Now, we calculate the threshold called basic reproduction number R_0 using next generation matrix method (Diekmann et al., 2009).

Let $X' = (S_T, E, A, S, H)'$ and $X' = \frac{dX}{dt} = F(X) - V(X)$, where $F(X)$ denotes the rate of appearance of new individual in the compartment and $V(X)$ denotes the rate of transfer of health issue which are given by

$$F(X) = \begin{bmatrix} \alpha_1 S_T E \\ \alpha_2 EA \\ \alpha_3 ES \\ 0 \\ 0 \end{bmatrix} \text{ and } V(X) = \begin{bmatrix} E(\alpha_2 A + \alpha_3 S + \mu) \\ -\alpha_5 S + A(\alpha_4 + \alpha_6 + \mu) \\ -\alpha_4 A + S(\alpha_5 + \alpha_7 + \mu) \\ -\alpha_6 A - \alpha_7 S + H(\alpha_8 + \mu) \\ -B + \alpha_1 S_T E - \alpha_8 H + \mu S_T \end{bmatrix}.$$

Now, $DF(X_0) = \begin{bmatrix} f & 0 \\ 0 & 0 \end{bmatrix}$ and $DV(X_0) = \begin{bmatrix} v & 0 \\ J_1 & J_2 \end{bmatrix}$, where f and v are 5×5 matrices defined as

$$f = \left[\frac{\partial F_i(X_0)}{\partial X_j} \right] \text{ and } v = \left[\frac{\partial V_i(X_0)}{\partial X_j} \right]$$

Finding f and v , we get

$$f = \begin{bmatrix} \alpha_1 S_T & 0 & 0 & 0 & \alpha_1 E \\ \alpha_2 A & \alpha_2 E & 0 & 0 & 0 \\ \alpha_3 S & 0 & \alpha_3 E & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} \alpha_2 A + \alpha_3 S + \mu & \alpha_2 E & \alpha_3 E & 0 & 0 \\ 0 & \alpha_4 + \alpha_6 + \mu & -\alpha_5 & 0 & 0 \\ 0 & -\alpha_4 & \alpha_5 + \alpha_7 + \mu & 0 & 0 \\ 0 & -\alpha_6 & -\alpha_7 & \alpha_8 + \mu & 0 \\ \alpha_1 S_T & 0 & 0 & -\alpha_8 & \alpha_1 E + \mu \end{bmatrix}.$$

Here, v is non-singular matrix.

For this model, the basic reproduction number R_0 is not permissible at Y_0 . So, we find R_0 at $Y_1 = \left(\frac{\mu}{\alpha_1}, \frac{B\alpha_1 - \mu^2}{\alpha_1 \mu}, 0, 0, 0 \right)$ which is obtained as the spectral radius of matrix fv^{-1} .

$$R_0 = \frac{\left(\alpha_2 \mu^3 + (\alpha_2 \{ \alpha_5 + \alpha_7 + \alpha_8 \} + \alpha_3 \alpha_4) \mu^2 - \alpha_8 (\alpha_1 \alpha_6 - \alpha_2 \{ \alpha_5 - \alpha_7 \} - \alpha_3 \alpha_4) \mu \right) (\mu^2 - B\alpha_1) - \alpha_8 \alpha_1 (\{ \alpha_4 + \alpha_6 \} \alpha_7 + \alpha_5 \alpha_6)}{\alpha_1^2 (\mu^2 + (\alpha_4 + \alpha_5 + \alpha_6 + \alpha_7) \mu + (\alpha_4 + \alpha_6) \alpha_7 + \alpha_5 \alpha_6) B (\alpha_8 + \mu)} \quad (2)$$

3. Stability

In this section, for all equilibrium points local as well as global stability is discussed.

3.1. Local Stability

Theorem 3.1.1 (Stability at Y_0): If $\alpha_1 B < \mu^2$ then $Y_0 = \left(\frac{B}{\mu}, 0, 0, 0, 0 \right)$ is locally asymptotically stable.

Proof: The Jacobian matrix J_0 at Y_0 is given by

$$J_0 = \begin{bmatrix} -\mu & -\frac{\alpha_1 B}{\mu} & 0 & 0 & 0 \\ 0 & \frac{\alpha_1 B}{\mu} - \mu & 0 & 0 & 0 \\ 0 & 0 & -(\alpha_4 + \alpha_6 + \mu) & \alpha_5 & 0 \\ 0 & 0 & \alpha_4 & -(\alpha_5 + \alpha_7 + \mu) & 0 \\ 0 & 0 & \alpha_6 & \alpha_7 & -(\alpha_8 + \mu) \end{bmatrix}.$$

Above Jacobian J_0 has eigenvalues,

$$\lambda_1 = -\mu < 0, \quad \lambda_2 = -(\alpha_8 + \mu) < 0, \\ \lambda_3 = - \left(\frac{(\alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + 2\mu)}{2} - \sqrt{\frac{(\alpha_4 + \alpha_5)^2 + (\alpha_6 - \alpha_7)^2 + 2(\alpha_4 - \alpha_5)(\alpha_6 - \alpha_7)}{2}} \right) < 0,$$

$$\lambda_4 = - \left(\frac{(\alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + 2\mu)}{2} + \frac{\sqrt{(\alpha_4 + \alpha_5)^2 + (\alpha_6 - \alpha_7)^2 + 2(\alpha_4 - \alpha_5)(\alpha_6 - \alpha_7)}}{2} \right) < 0,$$

$$\lambda_5 = \frac{\alpha_1 B - \mu^2}{\mu} < 0 \text{ if } \alpha_1 B < \mu^2.$$

As all eigenvalues of above matrix are negative. Therefore, equilibrium point of system is stable.

Theorem 3.1.2 (Stability at Y_1): If $x_2 x_3 < \alpha_4 \alpha_5$ where $x_1 = \frac{B\alpha_1 - \mu^2}{\mu}$, $x_2 = \frac{\alpha_2 x_1}{\alpha_1} - \alpha_4 - \alpha_6 - \mu$ and $x_3 = \frac{\alpha_3 x_1}{\alpha_1} - \alpha_5 - \alpha_7 - \mu$ then $Y_1 = \left(\frac{\mu}{\alpha_1}, \frac{B\alpha_1 - \mu^2}{\alpha_1 \mu}, 0, 0, 0 \right)$ is locally asymptotically stable.

Proof: For $Y_1 = \left(\frac{\mu}{\alpha_1}, \frac{B\alpha_1 - \mu^2}{\alpha_1 \mu}, 0, 0, 0 \right)$, let $\frac{B\alpha_1 - \mu^2}{\mu} = x_1$. The Jacobian matrix J_1 is given by

$$J_1 = \begin{bmatrix} -(\mu + x_1) & -\mu & 0 & 0 & 0 \\ x_1 & 0 & -\frac{\alpha_2 x_1}{\alpha_1} & -\frac{\alpha_3 x_1}{\alpha_1} & 0 \\ 0 & 0 & \frac{\alpha_2 x_1}{\alpha_1} - \alpha_4 - \alpha_6 - \mu & \alpha_5 & 0 \\ 0 & 0 & \alpha_4 & \frac{\alpha_3 x_1}{\alpha_1} - \alpha_5 - \alpha_7 - \mu & 0 \\ 0 & 0 & \alpha_6 & \alpha_7 & -(\alpha_8 + \mu) \end{bmatrix}.$$

Above Jacobian J_1 has eigenvalues

$$\lambda_1 = -x_1 < 0, \lambda_2 = -\mu < 0, \lambda_3 = -(\alpha_8 + \mu) < 0$$

$$\lambda_4 = \frac{x_2 + x_3}{2} + \frac{\sqrt{4\alpha_4 \alpha_5 + (x_2 - x_3)^2}}{2} < 0 \text{ if } x_2 x_3 < \alpha_4 \alpha_5, \lambda_5 = \frac{x_2 + x_3}{2} - \frac{\sqrt{4\alpha_4 \alpha_5 + (x_2 - x_3)^2}}{2} < 0.$$

Here, all the eigen values are negative. So, Y_1 is locally asymptotically stable.

Theorem 3.1.3 (Stability at Y^*): Endemic point Y^* is locally asymptotically stable if the following conditions hold:

- i) $x_6 x_7 > \alpha_4 \alpha_5$, ii) $\alpha_2 A^* + \alpha_3 S^* + \mu > \alpha_1 S_7$, iii) $\alpha_4 + \alpha_6 + \mu > \alpha_2 E^*$, iv) $\alpha_5 + \alpha_7 + \mu > \alpha_3 E^*$

Proof: We determine the local stability behavior of endemic equilibrium point Y^* by using the Jacobian matrix J^* , which is given by

$$J^* = \begin{bmatrix} -(\alpha_1 E^* + \mu) & -\alpha_1 S_T^* & 0 & 0 & 0 \\ \alpha_1 E^* & \alpha_1 S_T^* - \alpha_2 A^* - \alpha_3 S^* - \mu & -\alpha_2 E^* & -\alpha_3 E^* & 0 \\ 0 & \alpha_2 A^* & \alpha_2 E^* - \alpha_4 - \alpha_6 - \mu & \alpha_5 & 0 \\ 0 & \alpha_3 S^* & \alpha_4 & \alpha_3 E^* - \alpha_5 - \alpha_7 - \mu & 0 \\ 0 & 0 & \alpha_6 & \alpha_7 & -(\alpha_8 + \mu) \end{bmatrix}.$$

Let,

$$x_4 = \alpha_1 E^* + \mu, \quad x_5 = \alpha_2 A^* + \alpha_3 S^* - \alpha_1 S_T^* + \mu, \quad x_6 = \alpha_4 + \alpha_6 - \alpha_2 E^* + \mu, \quad x_7 = \alpha_5 + \alpha_7 - \alpha_3 E^* + \mu, \\ x_8 = \alpha_8 + \mu.$$

The corresponding characteristic equation of Jacobian matrix is

$$\lambda^5 + a_1 \lambda^4 + a_2 \lambda^3 + a_3 \lambda^2 + a_4 \lambda + a_5 = 0 \text{ where}$$

$$a_1 = x_4 + x_5 + x_6 + x_7 + x_8$$

$$a_2 = E^* (\alpha_2^2 A^* + \alpha_3^2 S^* + \alpha_1^2 S_T^*) + (x_6 x_7 - \alpha_4 \alpha_5) + (x_4 + x_5)(x_6 + x_7 + x_8) + x_8(x_6 + x_7) + x_4 x_5$$

$$a_3 = E^* (\alpha_2^2 A^* (x_4 + x_7 + x_8) + \alpha_2 \alpha_3 (A^* \alpha_4 + S^* \alpha_5) + \alpha_3^2 S^* (x_4 + x_6 + x_8)) \\ + (x_6 + x_7 + x_8)(E^* S_T^* \alpha_1^2 + x_4 x_5) + (x_6 x_7 - \alpha_4 \alpha_5)(x_4 + x_5 + x_8) + (x_4 + x_5)(x_6 + x_7) x_8$$

$$a_4 = A^* E^* \alpha_2 (\alpha_2 x_4 (x_7 + x_8) + x_7 (\alpha_2 x_8 + \alpha_3 x_4) + \alpha_3 (x_7 x_8 + \alpha_5 (x_4 + x_8))) \\ + E^* S^* \alpha_3^2 (x_4 x_8 + x_6 (x_4 + x_8)) + (x_6 x_7 - \alpha_4 \alpha_5)(E^* S_T^* \alpha_1^2 + x_4 x_8 + x_5 (x_4 + x_8)) \\ + (x_6 + x_7) x_8 (E^* S_T^* \alpha_1^2 + x_4 x_5)$$

$$a_5 = x_8 (E^* x_4 (A^* \alpha_2 (\alpha_2 x_7 + \alpha_3 \alpha_4) + S^* \alpha_3 (\alpha_2 \alpha_5 + \alpha_3 x_6)) + (x_6 x_7 - \alpha_4 \alpha_5)(E^* S_T^* \alpha_1^2 + x_4 x_5)).$$

The equilibrium point is locally asymptotically stable (Routh, 1877), if following conditions are hold:

$$\text{i) } x_6 x_7 > \alpha_4 \alpha_5, \quad \text{ii) } \alpha_2 A^* + \alpha_3 S^* + \mu > \alpha_1 S_T^*, \quad \text{iii) } \alpha_4 + \alpha_6 + \mu > \alpha_2 E^*, \quad \text{iv) } \alpha_5 + \alpha_7 + \mu > \alpha_3 E^*.$$

3.2 Global Stability

Here, we discuss the global stability behavior of the equilibrium Y_0 , Y_1 and Y^* by Lyapunov's function.

Theorem 3.2.1 (Stability at Y_0): The disease-free equilibrium point Y_0 is globally asymptotically stable.

Proof: Consider the Lyapunov's function $L_1(t) = E(t) + A(t) + S(t) + H(t)$ then $L_1'(t) = E'(t) + A'(t) + S'(t) + H'(t) = E(\alpha_1 S_T - \mu) - \alpha_8 H - \mu(A + S + H)$, we get $\frac{dL}{dt} < 0$ if $\alpha_1 B < \mu^2$ whereas $\frac{dL}{dt} = 0$ only if $E, A, S, H = 0$.
 Therefore, Y_0 is globally asymptotically stable (LaSalle, 1976).

Theorem 3.2.2 (Stability at Y_1): The unaffected equilibrium point Y_1 is globally asymptotically stable.

Proof: Consider the Lyapunov function $L_2(t) = A(t) + S(t) + H(t)$, then $L_2'(t) = A'(t) + S'(t) + H'(t) = A(\alpha_2 E - \mu) + S(\alpha_3 E - \mu) - H(\alpha_8 + \mu)$, we get $\frac{dL}{dt} < 0$ if $B\alpha_1\alpha_2 < \mu^2(\alpha_1 + \alpha_2)$ and $B\alpha_1\alpha_3 < \mu^2(\alpha_1 + \alpha_3)$ whereas $\frac{dL}{dt} = 0$ only if $A, S, H = 0$.

Therefore, Y_1 is globally asymptotically stable (LaSalle, 1976).

Theorem 3.2.3 (Stability at Y^*): The endemic equilibrium point Y^* is globally asymptotically stable.

Proof: Consider the Lyapunov function,

$$L(t) = \frac{1}{2} \left[(S_T - S_T^*) + (E - E^*) + (A - A^*) + (S - S^*) + (H - H^*) \right]^2, \text{ then}$$

$$L'(t) = \left[(S_T - S_T^*) + (E - E^*) + (A - A^*) + (S - S^*) + (H - H^*) \right] (S_T' + E' + A' + S' + H')$$

$$= -\mu \left[(S_T - S_T^*) + (E - E^*) + (A - A^*) + (S - S^*) + (H - H^*) \right]^2 \leq 0.$$

Here, we denote $B = \mu(S_T^* + E^* + A^* + S^* + H^*)$. Therefore, Y^* is globally stable. Next, we validate the model by numerical simulation.

4. Numerical Simulation

In Figure 3, transmission pattern of humans are observed which are affected due to temperature variations. One can see that if any individual experience sudden temperature change while traveling from one place to another, they gets exposed immediately. Some individuals do not show symptoms for two days and after approximately nine days they begins indicating symptoms. It can also be observed that after approximately eight days six individuals again gets susceptible out of which two individual gets exposed.

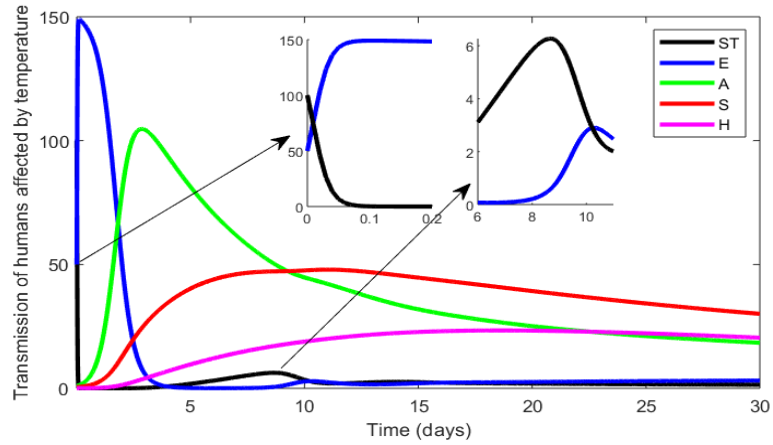


Figure 3. Transmission of humans affected by temperature variations

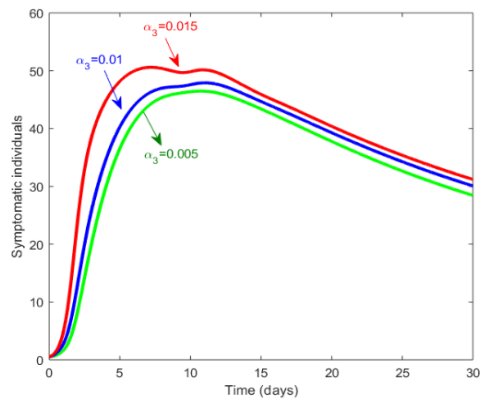


Figure 4. Effect of the rate at which exposed individuals moves to symptomatic individuals

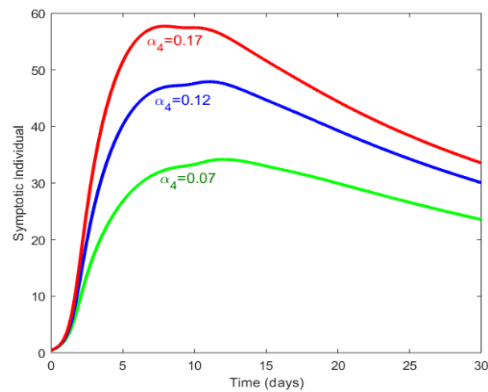


Figure 5. Effect of the rate at which asymptomatic individuals moves to symptomatic individuals

From Figure 4 and 5, one can observe that increase in rate results in increase of symptomatic individuals. Figure 4 shows that increase in α_3 by 1% results in increase of symptomatic individuals by 8%. Figure 5 implies that 64% increase of symptomatic individuals will be observed if there is an increase in rate of α_4 by 10%. It also connotes that during exposed or asymptomatic stage if proper precautions or medication are not taken then increase in symptomatic individuals will be observed.

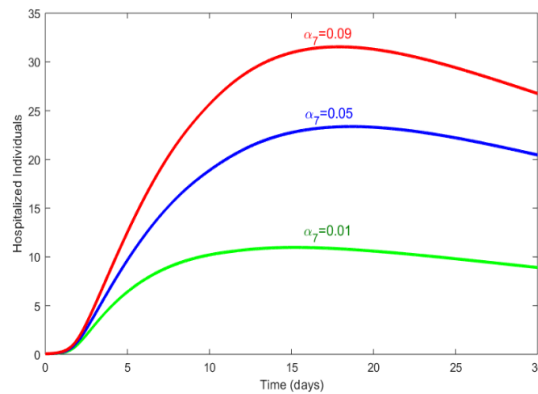


Figure 6. Effect of the rate at which symptomatic individuals moves to hospitalization

Figure 6 shows that 8% increase in rate of symptomatic individuals results in increase of hospitalized rate from 8 individuals to 26 individuals. So, increase in the rate of symptomatic individuals is directly proportional to hospitalization.

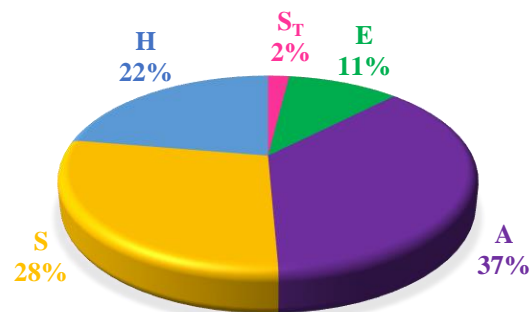


Figure 7. Percentage of all compartments

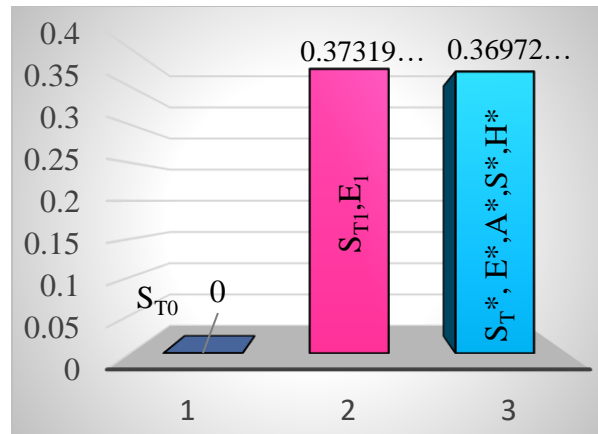


Figure 8. Graph representation of reproduction number

Figure 7 shows that 2% individuals are susceptible out of which 11% gets exposed. From that 11% exposed individuals, 37% are asymptomatic individuals and 28% are symptomatic out of which 22% gets hospitalized. Figure 8 shows that at every equilibrium point the value of reproduction number is less than 1. Therefore, the mathematical model is stable for the given parametric values.

5. Conclusion

Using system of nonlinear ordinary differential equations, a mathematical model is formulated to study the hospitalization rate due to impact of temperature variations on human health. System is locally and globally asymptotically stable at all equilibrium points. A basic reproduction number has been evaluated at an equilibrium point and is equal to 0.3732 which shows that approx. 37% of the individuals in the society gets hospitalized due to temperature variations. To validate the model, numerical simulations has been carried out which interprets that with the increase in rate of exposed and asymptomatic compartments, increase of susceptible individuals are observed which further results in increased hospitalization rate and the mathematical model is stable at every equilibrium point. Hospitalization rate can be controlled by taking proper precautions at the early stages. As we know better health helps to face physical, mental and social challenges. So, it is essential to become hale and hearty human as it makes people live longer and more productive resulting in decrease in hospitalization rate. Hence, every individual in the society should take proper care of their body and respond immediately if symptoms are observed by taking proper medications.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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