International Journal of Mathematical, Engineering and Management Sciences
Vol. 4, No. 3, 531–541, 2019
https://dx.doi.org/10.33889/IJMEMS.2019.4.3-043

Reliability and Cost-Benefit Analysis for Two-Stage Intervened Decision-Making Systems with Interdependent Decision Units

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(Received March 15, 2019; Accepted March 29, 2019)

Abstract
This paper deals with a special type of voting systems, called two-stage intervened decision-making systems, in which the decision time of each decision unit will be a random variable and some supervising mechanism is included. A new decision rule is applied to such kind of systems, which makes the decision units become interdependent from each other. The reliability and cost-benefit models are developed. The optimization for the models is discussed and the optimal solution for a special case is also derived. A numerical example for model optimization is presented as well as some model comparison. Even though a specific application is used for model formulation and derivation throughout this paper, the modeling results can be easily modified and applied to many other systems.

Keywords- Intervened decision systems, Interdependent systems, Voting systems, System reliability, System cost-benefit model

1. Introduction

Decision-making is one of the most common and important activities in many situations of modern society including energy planning (Soroudi and Amraee, 2013), network resource reallocation (Zhou et al., 2015) and communication systems (Pham, 1997). Due to the limitation of human beings and modern technologies, it is impossible to guarantee that all the decisions will be correct all the time. In order to improve the quality and reliability of decisions, ideas from multiple sources can be put together and the final decision can be made based on some particular rule, which is the idea of voting systems. In the 1950s, J. von Neumann (von Neumann, 1956) put forward the idea that voting mechanism could be used to obtain highly reliable data from multiple unreliable versions, which is the initial idea of voting systems. Voting systems can be applied in many contexts, such as human organization (Nordmann and Pham, 1997; 1999), financial investment (Lin and Pham, 2017) and surveillance systems (Bai et al., 2009). Voting systems are fault tolerant with redundancy. However, if the number of units making incorrect decisions exceeds a certain limit, the final decisions of the voting systems will be incorrect. In addition, voting systems are usually used to make important decisions. Thus, the failure of such systems may lead to serious consequences. Therefore, it is important to evaluate the reliability of voting systems.

In the past years, many works have been done in the area of reliability for voting systems. The study of reliability evaluation of general voting systems can be found in Kim et al. (2002), Mine and Hatayama (1981) and Pham (1997). Weighted voting systems, which are special voting systems, are more realistic in many contexts and a lot of research has been done in this area. The problem formulation for weighted voting systems reliability evaluation was first proposed by Nordmann and Pham (1997). A simplified method for reliability evaluation has been proposed by Nordmann and Pham (1999) based on two restrictions. A method based on a universal generating function technique to evaluate the reliability of weighted voting systems with no restriction has been proposed by Levitin and Lisnianski (2001). A recursive method to calculate the reliability of weighted voting systems has been proposed by Xie and Pham (2005) and their work is also applied in time dependent systems with dynamic structures and threshold. The study of systems with different decision times for different voting units can be found in Levitin (2005). Instead of binary inputs, the evaluation of reliability for weighted voting systems with discrete states inputs and continuous state inputs can be found in Levitin (2002) and Long et al. (2008). Some recent work on reliability evaluation of weighted voting systems can be found in Liu et al. (2016; 2017).

In the works discussed above, the consideration of time issues in voting systems is limited. It is only considered in Xie and Pham (2005), and Levitin (2005), in which the time for each voting unit to make the decision is predetermined. However, in many real applications, especially the situations including human beings, the decision time for each unit can be random and unknown without continuous review. Based on this, Lin and Pham (2017) have proposed the reliability and cost models for two-stage weighted intervened decision systems in the application of human decision systems. To our knowledge, this is the first paper considering units with random decision times in the voting systems and introducing supervising mechanism to the system. In addition, an inspection mechanism in intervened decision systems is also considered by Lin and Pham (2019). However, the study for systems with decision units having random decision times is very limited, because except for different weights (Lin and Pham, 2017), all the decision units in Lin and Pham's models are assumed to be identical and independent from each other.

Based on the review of current works, the main objective of this paper is to develop the reliability and cost-benefit models for two-stage intervened decision systems with interdependent decision units. Specifically, the rest of the paper will be organized as follows: section 2 is the description of the system, including notations and assumptions, using the human decision system in financial investment as the application; in section 3, the reliability and cost-benefit models are developed, and model optimization is discussed; in section 4, a numerical example is presented as well as some discussion; section 5 is for conclusion.

2. System Description

Suppose that there is a financial investment company with many decision groups. The job of each decision group is to evaluate investment options and decide whether to do it or not. Suppose that there is a group with three identical decision agents. When an option (defined as input) comes to the group: either good (1) or not good (0), all the three agents will start working on it and submit the result (defined as output) to show that whether they should do the option (1) or not (0). If the input is consistent with the output of an agent, we can say that this agent has provided a correct result. At the end of the process, all results available will be put together and the final decision, which is whether to do (1) the option or not (0), will be made using the majority voting rule. It is assumed that all agents have equal weights, so the voting rule is 2-out-of-3 for this system. If the final output of the system after voting is consistent with the input, we can say that the decision from the system is correct. There is a due time to submit the results determined by the management level. The agents can submit their results any time before the due. In addition, a check point before the due time during the process is also determined by the management level. At the check point, the management level will review the results from all agents. If at least two consistent results (may be correct or incorrect) are available at the check point, the process can be terminated and the final decision can be made without waiting until the due time. Otherwise, the supervising mechanism will start to make sure that the remaining results can be submitted in time. The due time and check point are notified to the agents in advance. Figure 1 below shows how the decision process runs. Figure 2 shows the structure of system and notations in Table 1.

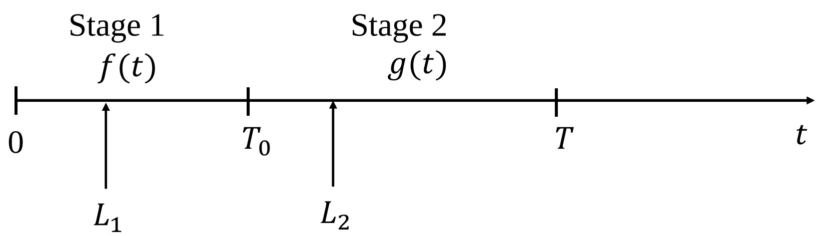

Figure 1. Two-stage intervened decision making process

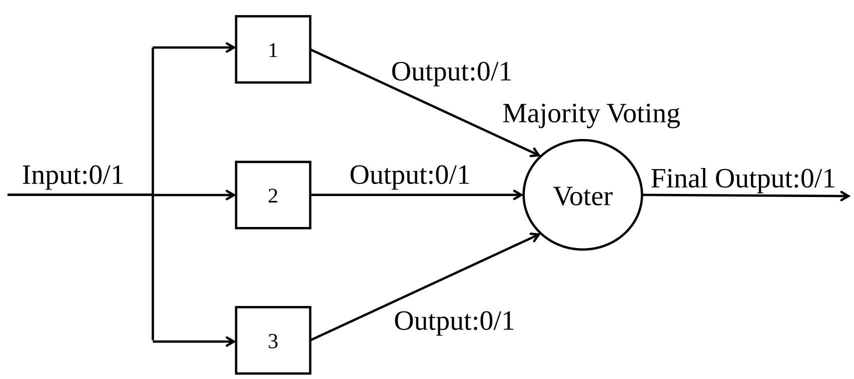

Figure 2. Triple-agent decision making system with binary input and output

International Journal of Mathematical, Engineering and Management Sciences
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Table 1. Notations

L_j	the decision time for each agent in stage j (j=1,2)
T_0	the check point
T	the ending time of the process
$f(t)$	the probability density function of decision time for each agent
g(t)	the conditional probability density function of decision time for each agent if the decision is not made by time T_0 but it is made in stage 2
$p(t)$	the probability for each agent to submit the correct result if the result is submitted at time t
$v(T_0)$	the probability that an agent provides the result in stage 2
$R_0(T_0, T)$	individual reliability function, which is defined as the probability that the agent will give a correct result
$R(T_0, T)$	the reliability function of the system, defined as the probability that the system will make a correct decision
q	the probability that the option is good (1)
r	the revenue generated by the option when it is good and it is implemented
c	the loss resulting from the option when it is not good but it is implemented
c_0	the fixed cost of the decision process
c_1	the operational cost per unit time during stage 1
c_2	the operational cost per unit time during stage 2
τ_1	the estimated loss of revenue per unit time in stage 1 if the option is good
τ_2	the estimated loss of revenue per unit time in stage 2 if the option is good
$u(T_0)$	the probability that the final decision is made at T_0
$W(T_0, T)$	the net profit of the decision system

2.1 Assumptions

(a) Each agent in the system must provide a result by time T. It means that for each agent, a result has to be submitted at stage 1 or if the agent fails to do so, the result has to be submitted in stage 2. This can be expressed as:

$$F(T_0) = \int_0^{T_0} f(t)dt \leq 1, \int_{T_0}^{T} g(t)dt = 1.$$

(b) The time dependent probability function to submit a correct result is a non-decreasing function over time since more information will be received over time and it should be always no greater than 1 by time T, that is $p\ (T) \leq 1$.

3. Reliability and Cost-Benefit Models

3.1 Reliability Model

According to the decision rule of the system described in section 2, each agent may not need to provide a result when the process ends. For each agent, whether it will provide a result by the end of the process depends on the decision behavior of the other two agents. Thus, the three agents are not independent from each other anymore and they become interdependent. To evaluate the reliability of such system, total probability equation will be used to calculate system reliability case by case.

The individual reliability function for each agent can be obtain using total probability equation:

$$R_0(T_0,T) = \Pr\{\text{Correct result in Stage 1}\} + \Pr\{\text{Correct result in Stage 2}\}$$
$$= \int_0^{T_0} f(t)p(t)\,dt + v(T_0)\int_{T_0}^{T} g(t)p(t)dt$$

where

$$v(T_0) = Pr\{\text{the agent will provide the result in stage 2}\}$$
$$= \big(1-F(T_0)\big)\left[1-\left(\int_0^{T_0} f(t)p(t)\,dt\right)^2 - \left(F(T_0)-\int_0^{T_0} f(t)p(t)\,dt\right)^2\right].$$

To obtain the reliability of the whole system, total probability equation should be used, which takes the sum of all possible cases, since the three agents are no longer independent from each other. The event space can be divided by the number of agents providing results in stage 1, so there are four possible cases to be considered as follows:

Case 1: all the three agents give results in stage 1.
Case 2: two of the three agents give results in stage 1 and one fails to give a result in stage 1.
Case 3: only one of the three agents gives a result in stage 1.
Case 4: none of the three agents gives result in stage 1.

Thus, the reliability of the whole system is given by

$$R(T_0,T) = \sum_{k=1}^{4}\left(Pr\{\text{Final decision is correct}|\text{Case } k\} * \Pr\{\text{Case } k\}\right).$$

Let us define the following two functions:

$$S_1(T_0) = \int_0^{T_0} f(t)p(t)\,dt, \qquad S_2(T_0,T) = \int_{T_0}^{T} g(t)p(t)d \tag{1}$$

Case 1:

$$Pr\{\text{Case 1}\} = F(T_0)^3$$

$$Pr\{\text{Final decision is correct}|\text{Case 1}\} = 3\left(\frac{S_1(T_0)}{F(T_0)}\right)^2 - 2\left(\frac{S_1(T_0)}{F(T_0)}\right)^3.$$

Case 2:

$$Pr\{\text{Case 2}\} = 3F(T_0)^2(1-F(T_0))$$
$$Pr\{\text{Final decision is correct}|\text{Case 2}\} =$$
$$Pr\{\text{Final decision is correct and two consistent results in stage 1 }|\text{Case 2}\}$$
$$+\,Pr\{\text{Final decision is correct and two inconsistent results in stage 1 }|\text{Case 2}\}$$
$$= \frac{S_1(T_0)^2}{F(T_0)^2} + 2\frac{S_1(T_0)}{F(T_0)}\left(1-\frac{S_1(T_0)}{F(T_0)}\right)S_2(T_0,T).$$

Case 3:
$Pr\{\text{Case } 3\} = 3F(T_0)(1-F(T_0))^2$
$Pr\{\text{Final decision is correct}|\text{Case } 3\} = \frac{S_1(T_0)}{F(T_0)}S_2(T_0,T)^2 + \left(1-\frac{S_1(T_0)}{F(T_0)}\right)S_2(T_0,T)^2 + 2\frac{S_1(T_0)}{F(T_0)}S_2(T_0,T)\left(1-S_2(T_0,T)\right).$

Case 4:
$Pr\{\text{Case } 4\} = (1-F(T_0))^3$
$Pr\{\text{Final decision is correct}|\text{Case } 4\} = 3S_2(T_0,T)^2 - 2S_2(T_0,T)^3$

Combine the results from the four cases, the system reliability function can be expressed as:

$$R(T_0,T) = 3\hat{R}_0(T_0,T)^2 - 2\hat{R}_0(T_0,T)^3 \neq 3R_0(T_0,T)^2 - 2R_0(T_0,T)^3 \tag{2}$$

where $\hat{R}_0(T_0,T) = S_1(T_0) + (1-F(T_0))S_2(T_0,T)$, which is the individual reliability function for the system with totally independent units according to Lin and Pham (2017).

3.2 Cost-Benefit Model

The net profit of the system consists of three parts: profit or loss from the option, system operational cost and loss of revenue. Since the last two portions depend on the time at which the final decision is made, the net profit of the system is a random variable. Therefore, its expected value should be considered. Each part in the model can be calculated as follows:

(i) Expected profit from the option:
$$E(\text{profit}) = rqR(T_0,T) - c(1-q)(1-R(T_0,T)) \tag{3}$$

(ii) Expected system operational cost:
The system operational cost is related to the final decision time point:

$$\mathrm{c}_{op} = \begin{cases} c_0 + c_1T_0 & \text{if at leaset two consistent results are submitted before } T_0 \\ c_0 + c_1T_0 + c_2(T-T_0) & \text{others} \end{cases}.$$

Thus, expected operational cost is:

$$\begin{aligned} E(\mathrm{c}_{op}) &= u(T_0)(c_0 + c_1T_0) + (1-u(T_0))\left(c_0 + c_1T_0 + c_2(T-T_0)\right) \\ &= c_0 + c_1T_0 + (1-u(T_0))c_2(T-T_0) \end{aligned} \tag{4}$$

where $u(T_0) = F(T_0)^3 + 3\left(1-F(T_0)\right)\left(\left(\int_0^{T_0} f(t)p(t)\,dt\right)^2 + \left(F(T_0) - \int_0^{T_0} f(t)p(t)\,dt\right)^2\right)$

(iii) Expected loss of revenue
The loss of revenue is also related to the final decision time point and it will happen only if the option is good:

$$\mathrm{c}_L = \begin{cases} q\tau_1T_0 & \text{if at leaset two consistent results are submitted before } T_0 \\ q(\tau_1T_0 + \tau_2(T-T_0)) & \text{others} \end{cases}.$$

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The expected loss of revenue is

$$E(c_L) = q\left(u(T_0)\tau_1 T_0 + \left(1-u(T_0)\right)\left(\tau_1 T_0 + \tau_2 (T-T_0)\right)\right)$$
$$= q\left(\tau_1 T_0 + \left(1-u(T_0)\right)\tau_2 (T-T_0)\right) \quad (5)$$

Therefore, taking the sum of equations (3)-(5), the expected net profit for the system with interdependent agents can be obtained as follows:

$$E\left(W(T_0,T)\right) = [rqR(T_0,T) - c(1-q)R] - E\left(c_{op}\right) - E(c_L)$$
$$= \left[rqR(T_0,T) - c(1-q)\left(1-R(T_0,T)\right)\right]$$
$$-\left[c_0 + c_1 T_0 + \left(1-u(T_0)\right)c_2(T-T_0)\right] - q\left[\tau_1 T_0 + \left(1-u(T_0)\right)\tau_2(T-T_0)\right] \quad (6)$$

3.3 Optimization for Cost-Benefit Model

Since the ultimate goal of the system is to make profit, we are interested in maximizing the expected net profit of the system in this section. To keep the system performing well, a required reliability level should be satisfied. Therefore, the goal is to find the optimal values of T_0 and T to maximize the expected net profit of the system given certain reliability level $\tilde{R}$ ($R(T_0,T) \geq \tilde{R}$).

To find T_0 and T that maximize the expected net profit, we can take the derivatives of $E(W(T_0,T))$ with respect to T_0 and T, respectively:

$$\frac{\partial E(W(T_0,T))}{\partial T_0} = \left(rq + c(1-q)\right)\left(6\hat{R}_0(T_0,T) - 6\hat{R}_0(T_0,T)^2\right)\frac{\partial \hat{R}_0(T_0,T)}{\partial T_0} - (c_1 + q\tau_1)$$
$$+ (c_2 + q\tau_2)[(1-u(T_0)) + (T-T_0)\frac{\partial u(T_0)}{\partial T_0}]$$

$$\frac{\partial E(W(T_0,T))}{\partial T} = \left(rq + c(1-q)\right)\left(6\hat{R}_0(T_0,T) - 6\hat{R}_0(T_0,T)^2\right)\frac{\partial \hat{R}_0(T_0,T)}{\partial T} - (c_2 + q\tau_2)(1$$
$$- u(T_0))$$

where

$$\frac{\partial R(T_0,T)}{\partial T} = \left(6\hat{R}_0(T_0,T) - 6\hat{R}_0(T_0,T)^2\right)\frac{\partial \hat{R}_0(T_0,T)}{\partial T_0}.$$

Since T_0 is no greater than T, the Lagrange Multiplier method should be used to solve this type of conditional extreme value problem, but the closed form solution is hard to find. Since the derivative for T is much simpler, the problem can become much easier if T_0 is given. In the human decision system for financial investment discussed in this paper, T_0 can be explained as the time point that there will be no loss of revenue before this, which can be determined first. Therefore, the goal is to find the optimal value of the due time T given the value of T_0 under some required reliability level $\tilde{R}$. A special case will be discussed in the next section and a numerical example will be presented in section 4.

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3.3.1 Special Case

Given that $f(t) = \lambda e^{-\lambda t}, g(t) = \frac{1}{T-T_0}, p(t) = a + bt$.

From section 3.1,

$$\hat{R}_0(T_0, T) = S_1(T_0) + \left(1 - F(T_0)\right)S_2(T_0, T)$$
$$= a\left(1 - e^{-\lambda T_0}\right) + b\left(\frac{1}{\lambda} - \frac{1}{\lambda}e^{-\lambda T_0} - T_0 e^{-\lambda T_0}\right) + \left[a + \frac{1}{2}b(T + T_0)\right]e^{-\lambda T_0}.$$

The derivative of $R(T_0, T)$ with respect to T is:

$$\frac{\partial R(T_0,T)}{\partial T} = \frac{\partial R(T_0,T)}{\partial \hat{R}_0(T_0,T)}\frac{\partial \hat{R}_0(T_0,T)}{\partial T} = \frac{1}{2}be^{-\lambda T_0}\left(6R_0(T_0,T) - 6R_0(T_0,T)^2\right) \geq 0$$

where $\frac{\partial \hat{R}_0(T_0,T)}{\partial T} = \frac{1}{2}be^{-\lambda T_0} > 0$.

The derivative of expected net profit $(E(W(T_0, T)))$ with respect to T is:

$$\frac{\partial E(W(T_0,T))}{\partial T} = \frac{1}{2}be^{-\lambda T_0}\left(rq + c(1-q)\right)\left(6\hat{R}_0(T_0,T) - 6\hat{R}_0(T_0,T)^2\right)$$
$$- (c_2 + q\tau_2)\left(1 - u(T_0)\right).$$

If $\frac{(c_2+q\tau_2)(1-u(T_0))}{\frac{1}{2}be^{-\lambda T_0}(rq+c(1-q))} \in (0, \frac{3}{2})$, $E(W(T_0, T))$ has two extreme points with respect to $\hat{R}_0(T_0, T)$:

$\hat{R}_0(T_0, T) = R_{01}$ and $\hat{R}_0(T_0, T) = R_{02}$. Since it is obvious that $\hat{R}_0(T_0, T)$ is increasing and uniquely determined by T, $E(W(T_0, T))$ has two extreme points with respect to T: $T = T_{01}$ and $T = T_{02}$. $E(W(T_0, T))$ is decreasing in $(0, T_{01})$, (T_{02}, ∞) and increasing in (T_{01}, T_{02}) with respect to T. Assume that $T \geq \hat{T} = T_0 + \frac{1}{\lambda}$, which is reasonable. Since $R(T_0, T) \geq \tilde{R}$, $T \geq \tilde{T}$ $\left(R\left(T_0, \tilde{T}\right) = \tilde{R}\right)$. If $max\{\hat{T}, \tilde{T}\} < \frac{1-a}{b}$, the problem will be feasible. Then, the optimal value of the due time (T^*) can be obtained as:

(a) $T^* = \begin{cases} max\{\hat{T}, \tilde{T}\} & \text{when } E\left(W(T_0, max\{\hat{T}, \tilde{T}\})\right) > E(W(T_0, T_{02})) \\ T_{02} & \text{when } E\left(W(T_0, max\{\hat{T}, \tilde{T}\})\right) \leq E(W(T_0, T_{02})) \end{cases}$, if $max\{\hat{T}, \tilde{T}\} < T_{02} < \frac{1-a}{b}$;

(b) $T^* = max\{\hat{T}, \tilde{T}\}$, if $T_{02} < max\{\hat{T}, \tilde{T}\}$;

(c) $T^* = \begin{cases} max\{\hat{T}, \tilde{T}\} & \text{when } E\left(W(T_0, max\{\hat{T}, \tilde{T}\})\right) > E\left(W(T_0, \frac{1-a}{b})\right) \\ \frac{1-a}{b} & \text{when } E\left(W(T_0, max\{\hat{T}, \tilde{T}\})\right) \leq E\left(W(T_0, \frac{1-a}{b})\right) \end{cases}$, if $T_{02} > \frac{1-a}{b}$.

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4. Numerical Examples and Discussion

Consider the case in section 3.3.1, given $\lambda = 0.4, a = 0.6, b = 0.02, T_0 = 3, \tilde{R} = 0.7,\ r = \text{c} = \$5{,}000{,}000,\ c_0 = \$50{,}000,\ c_1 = \$800,\ c_2 = \$1{,}000,\ \tau_1 = 0,\ \tau_2 = \$50{,}000, q = 0.6.$ The optimal value of T to maximize the expected net profit of the system will be determined.

From section 3.2, $u(T_0) = u(3) = 0.5755$. Also, $\hat{T} = 3 + 2.5 = 5.5$ from section 3.3.1.

Use the procedure in section 3.3.1, $R_{01} = 0.1770, R_{02} = 0.8230$.

Since $T_{02} = 65.42 > \frac{1-a}{b} = 20$, it can be shown that $T_{01} < max\{\hat{T}, \tilde{T}\} < T < \frac{1-a}{b} < T_{02}$ in this example and the expected net profit is an increasing function with respect to T in the feasible interval according to section 3.3.1. Therefore, $\hat{T}^* = \frac{1-a}{b} = 20$ and $\hat{R}_0(3{,}20) = 0.6861, R(3{,}20) = 0.7663$.

In conclusion, based on the given parameters, if the decision is not made at $T_0 = 3$, an extension of 17 days should be given . The maximum expected net profit is \$1,555,450. The behavior of the expected net profit function in term of T for the interdependent system can be found in Figure 3.

Under the same input parameters, consider the system with independent agents discussed in Lin and Pham (2017), the optimal solution is $T^* = 10$, with corresponding system reliability and expected net profit 0.7264 and \$1,436,848. The behavior of expected net profit function (Lin and Pham, 2017) can also be found in Figure 3.

From Figure 3, it can be identified that the behavior of the expected net profit function for the interdependent system is significant different from the independent system in Lin and Pham (2017), given the same input parameters. In addition, the optimal expected profit is much higher for interdependent system. Note that if $T = 10$, which is the optimal value for the independent system, the expected net profit is \$1,487,675 for the interdependent system, which is also higher than the optimal expected profit for the independent system. Based on these, the advantage of interdependent system in this paper is significant.

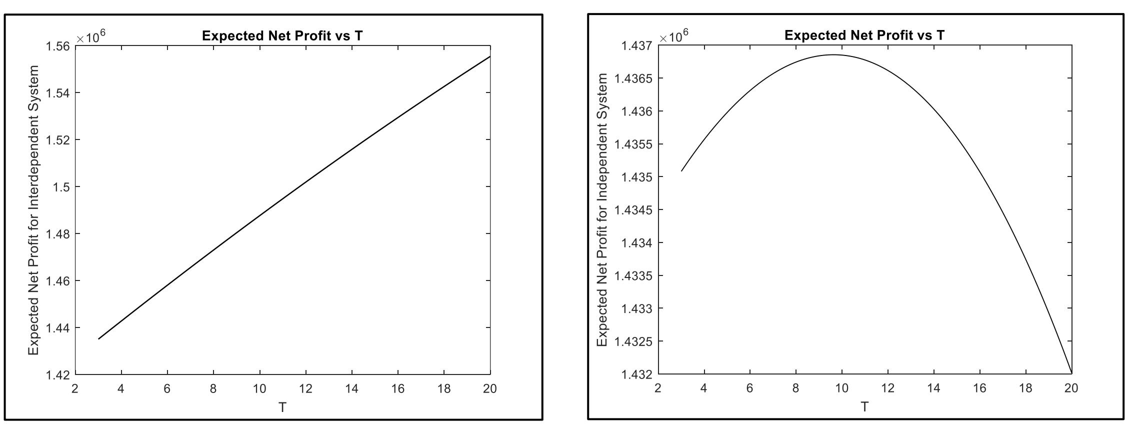

Figure 3. Expected Net Profit versus T for Interdependent (Left) System and Independent (Right) System

5. Conclusion

In this paper, the reliability and cost-benefit models for two-stage intervened decision making systems with interdependent units have been developed. Even though the financial investment system is used for model development in this paper, the modeling results can be generalized and applied to other situations as well, such as product releasing and recall, loan application, and academic papers review. There is still much work worth to be done for intervened decision making systems. For example, the systems will multiple units and/or non-identical units can be considered; the models for systems with multiple stages can be developed; the system with individual incentive can also be considered.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments
The authors would like to appreciate the effort from editors and reviewers. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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