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Phased-Mission Reliability and Importance Measure Analysis for Linear and Circular UAV Swarms

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Abstract
The phased-mission reliability of unmanned aerial vehicle (UAV) swarm refers to its capability to successfully complete the missions of each phase under specified conditions for a specified period. In order to study the reliability of phased-mission in UAV swarm, this paper firstly studies the reliability of a single UAV under fault coverage. Then, considering the mission characteristics of UAV swarm, the consecutive *k*-out-of-*n* system is studied to model and predict the reliability of UAV swarm phase mission. Some importance measures are introduced to analyze the influence of UAV in different positions on the reliability of the whole system. Finally, numerical examples of linear and circular UAV swarms are given to demonstrate and verify the correctness of the model. The reliability modeling established in this paper can predict the phased-mission reliability of UAV swarm scientifically.

Keywords- Reliability analysis, UAV swarm, Phased-mission, Consecutive *k*-out-of-*n*: F system, Importance measure.

1. Introduction

1.1 Background

In many real-world situations, a task needs to be divided into some phases. These phases must be accomplished in sequence (Xing and Dugan, 2002; Xing and Amari, 2008). Systems used in such missions are referred to as phased-mission systems. The unmanned aerial vehicle (UAV) swarm system is a typical phased-mission system. It includes several phases such as take-off, cruise, mission execution and return. The mission execution may include multiple areas. Different phases may have different areas, so its requirement is different. The system is dynamic. The system configuration, success criteria, and component behavior may vary from phase to phase (Huang et al., 2019; Wang et al., 2020). In every phase, each component has an overall effect on the probability of system failure (Wang et al., 2015). What's more, each phase probably has a certain dependency, so the statistical dependencies across phases must be considered (Wang et al., 2012). In certain conditions, the faults of each component are irreparable, and faults are accumulating. Eventually, the component will break down and can't work anymore (Levitin et al., 2020; Peng et al., 2019; Wu et al., 2022).

The working mode of UAV is adaptive way. When the UAV swarm receives the mission target, the UAV swarm can respond to various situations spontaneously. In order to achieve self-organization, each UAV must be able to communicate with each other. The flight distance between each two UAV can't exceed the specified length. When a UAV fails, its adjacent UAVs can perceive the abnormal situation and adjust to the position. Thus, the whole UAV swarm can be driven to change and realize adaptation. Due to the limited range of each UAV, in the actual mission scenario, the UAVs can all work normally before the start, and some UAVs may fail during the mission execution.

The common mode of UAVs is a swarm. It is similar to the distributed system of bee colonies. When some components are damaged, the whole system can automatically adjust itself and construct distributed network system. Thus, it can ensure the successful completion of the mission. With the help of distributed cooperative control strategy, the UAV swarm can maintain the overall behavior. Thus, it will have a strong and relatively stable capability. This reflects a redundant idea in quantity. The UAV system can still complete the mission when some UAVs are in failure, so the UAV swarm can be regarded as a *k*-out-of-*n* system.

1.2 Relevant Work

K-out-of-*n* is a common system structure and its reliability has been widely studied. There are two types of systems: *k*-out-of-*n*: G system and *k*-out-of-*n*: F system (Yu et al., 2017). The *k*-out-of-*n*: G system is a system consisting of *n* components, which functions if at least *k* components can work (Su et al., 2020; Pham, 2010). The *k*-out-of-*n*: F system is a system consisting of *n* components, which fails if and only if there are at least *k* failed components. If the system has *k* consecutive components work or fail, this system can be called a consecutive *k*-out-of-*n*: G or F system. The alignment of components affects the reliability of the system (Endharta et al., 2018; Wang et al., 2018; Yam et al., 2003).

For the analysis of the reliability of phased-mission, Dui et al. (2021a; 2022a) thought that the mission reliability and structural optimization of UAV swarms can be studied based on importance measures. Then, for the reliability of two-phase weighted-*k*-out-of-*n*, Chaube and Singh (2016) studied the fuzzy reliability of two-stage weighted-*k*-out-of-*n* model with components in common based on the minimal cuts and minimal paths. Meanwhile, Rushdi (2019) proposed to use the Symmetric switching functions (SSFs) to analyze the reliability analysis of a binary *k*-out-of-*n* system. Then for the fault detection of this system, Li (2016) proposed a dormant *k*-out-of-*n* systems redundancy calculation to detect a dormant fault. Müller and Domínguez-García (2012) proposed that fault coverage gave a measure of the likelihood that a system will be able to recover after a fault occurrence. What's more, importance measures can be used for the recovery measures after fault analysis (Dui et al., 2021b). Dui et al. (2022b) and Bai et al. (2021) thought that in the analysis of the system, a data-driven evaluation of its maintenance priority can be carried out. Then, component maintenance priority can be used for analysis of polymorphic systems, continuous systems, and incoherent systems (Dui et al., 2021c).

In terms of algorithms, in recent research, the simple recursive algorithm (Wang et al., 2016) and multi-cut enumeration are useful for small-scale components (Mohammadi et al., 2018), but it's inefficient in large-scale conditions. Because the number of combinations increases exponentially, Yan and Liu (2022) proposed to use of signal momentum contrast for unsupervised representation learning to automatically extract fault features from different diagnostic objects. Mo et al. (2015) used multi-valued decision diagrams to calculate the reliability of multi-state *k*-out-of-*n* systems. It can offer lower computational complexity than the recursive algorithms and apply to larger practical cases. But the calculation process is a little tedious. Xing et al. (2012) put forward the recursive formula based on the Markov method and conditional probability. Computation time and memory requirements are linear in terms of system size, so mission reliability can be obtained quickly.

1.3 Our Work

In this paper, we describe the consecutive *k*-out-of-*n* UAV swarm system with phased-mission as follows. It is a UAV swarm system consisting of *n* UAVs, which fails if and only if at least *k* components fail in succession. In this system, *n* indicates the number of UAVs in the swarm, and *k* indicates the number of UAVs allowed to fail. The mission includes several phases, such as take-off, mission execution and returns flight. In the full mission, each phase of the mission is continuous and does not overlap. Regardless of the path planning between missions, the order of each mission has been determined. The range and duration of each mission are known and can be estimated. What's more, it is not affected by the system state. To be regarded as the overall mission success, UAV swarm operations must be completed successfully at all phases of the mission. That is, when the UAV swarm fails at a certain phase of the mission, the whole mission is regarded as a failure. When a UAV fails, the queue can be automatically adjusted. When the UAV is in a swarm, the whole system can be self-organized and adaptive. During the execution of the mission, when a UAV fails, the fault can't be repaired, and the fault state will remain for the remaining mission time.

The communication function fault of a single UAV is considered separately from the detection function fault. The failure mode of the identical type of the UAV is the same, and the UAV failure is independent at each mission phase. There is a certain relationship between different mission phases. According to above description, the fault probability is analyzed firstly. Then we analyze the phase reliability and system reliability according to the characteristics of the UAV swarm. To determine the key components in the mission process, the Birnbaum importance measure (BM) and integrated importance measure (IIM) are introduced. Next, the reliability of phased-mission in linear UAV and circular UAV swarms are analyzed.

The rest of this paper is organized as follows. Section 2 analyzes phased-mission reliability and importance measures. Section 3 presents the phased-mission reliability analysis of linear UAV swarm. Section 4 presents the phased-mission reliability analysis of circular UAV swarm. Section 5 uses a numerical example of a two-phase mission to verify the effectiveness of the proposed method. Section 6 summarizes the full text and subsequent prospects.

Notations

t	Mission time
N	Number of mission phases
φ_j	Duration of phase *j*
n	Number of UAVs
k_j	Number of failed UAVs in phase *j*
c_j	Fault coverage factor of single UAV in phase *j*
d_j	Probability of a single UAV being hit to fail in phase *j*
γ_j	Acceleration factors in phase *j*
p_j	Reliability of phase *j*
q_j	Unreliability of phase *j*
f_j	Probability of a single UAV's first failure
L_u	Accumulated failure probability in M phases
P_u	Probability of n UAVs achieving the mission
$F_j(t)$	Cumulative distribution function of its lifetime
$Q_j(t)$	Cumulative failure probability at the end of phase *j*
$P_j(t)$	Reliability of UAV in phase *j*
$I(BM)_i^t$	BM of component i in time t

$I(IIM)_i^t$	IIM of component i in time t
$r_{L/C,j}(p, k_j, n)$	Reliability of the UAV swarm
$H_{j,i}$	Probability of the UAV swarm system

2. Phased-Mission Reliability and Importance Measures Analysis

UAV mission includes several phases, such as take-off, mission execution and returns flight. The phased-mission system of the UAV swarm needs to be carried out continuously. The overall performance will decrease with the incremental number of missions in each phase. It is modeled according to the geometric process. With the incremental number of mission phases, the number of failed UAVs in the swarm will increase. The reliability of system will not meet the requirements of the mission. The cumulative distribution function of its life can be expressed as.

$$F_j(t) = F(\ \gamma_j t). \quad (1)$$

$$Q_j = F(\ \gamma_1\varphi_1\ + \cdots +\ \gamma_j\varphi_j). \quad (2)$$

$$P_j = 1 - Q_j. \quad (3)$$

where, γ_j represents the acceleration factor in phase j, φ_j represents the duration of phase j, Q_j and P_j are set to the cumulative failure probability and reliability of a single UAV at the end of phase j. Before the mission: Q_0=0, P_0=1. Let f_j be the probability of a single UAV's first failure due to its own life loss during the phase j,

$$f_j = Q_j - Q_{j-1}. \quad (4)$$

In this paper, the communication function and the detection function of a UAV are considered separately. As long as the communication function of UAV fails, the ground base station can't detect the position of the UAV and other normal UAVs can't perceive the failed UAV. Thus, the UAV can be regarded as non-existent on the map. However, there will also be cases where the communication function of the UAV is intact, but the detection function fails. In this situation, because the adjacent UAVs around the failed UAV can perceive its existence, the failed UAV can still complete the mission normally.

But in fact, due to the failure of the detection function, the UAV cannot complete the delivery mission, resulting in the failure of the UAV swarm in this mission phase. Thus, the whole phased-mission will fail. In this case, even if the number of normally working UAVs meets the requirements, the whole mission can't be successfully completed. Thus, the fault coverage factor c_j is introduced. It means the probability of failure of detection function while the communication function is in good condition. It is assumed that each UAV has the same probability of failure in the same mission phase. Let d_j be the probability of a single UAV for failure in phase j. When each UAV's communication and detection failures can be monitored, the probability of failure in phase j is

$$f_{jc} = f_j c_j. \quad (5)$$

When the communication function of a single UAV works in sync with the detection function or fails ($c_j = 1, f_{jc} = f_j$), which can be regarded as a perfect fault coverage.

Due to the single UAV may have the normal communication function and failed detection function (the fault is not completely covered). The probability that the UAV first fails in phase j with uncovered fault, denoted by f_{ju}, can be calculated as

$$f_{ju} = 1 - f_{jc}. \quad (6)$$

During the whole mission period, due to the existence of normal communication function and failed detection function, some faults can't be found. A single UAV failure probability can be accumulated in M phases,

$$L_u = \sum_{j=1}^{M} f_{ju}. \tag{7}$$

When the fault can be fully monitored, the UAV detection function fault can be found at the end of the phase j, the cumulative probability of failure is

$$Q_{jc} = \frac{\sum_{i=1}^{j} f_{ic}}{1-L_u}. \tag{8}$$

When the fault is perfectly covered, $Q_{jc} = Q_j$, the reliability of UAV in phase j is

$$P_{jc} = \left(1 - Q_{jc}\right) * (1 - d_j). \tag{9}$$

If the swarm fails the mission in phase j, the swarm must complete the (i−1) phases successfully. The unreliability of phase j can be calculated as

$$q_j = \frac{f_{jc}}{1-Q_{(j-1)c}}. \tag{10}$$

At phase j, the conditional reliability of the UAV needs to be satisfied so that it is working at beginning of the phase and no uncovered failure happens. The probability of being hit is also added. The reliability is denoted by p_j, can be calculated as

$$p_j = \left(1 - \frac{f_{jc}}{1-Q_{(j-1)c}}\right) * \left(1 - d_j\right) = \left(\frac{1-Q_{jc}}{1-Q_{(j-1)c}}\right) * \left(1 - d_j\right) = \left(\frac{P_{jc}}{P_{(j-1)c}}\right) * (1 - d_j), \tag{11}$$

$$q_j = 1 - p_j. \tag{12}$$

After evaluating the reliability of UAV, in order to clarify the key components in the swarm, BM and IIM are introduced. BM considers the sensitivity of UAV unreliability to changes in component failure probability, and the BM of component i in time t is defined in binary systems,

$$I(BM)_i^t = Pr\{\phi\left(X(t)\right) = 1|X_i = 1\} - Pr\{\phi\left(X(t)\right) = 1|X_i = 0\}. \tag{13}$$

Dui et al. (2015) have proposed and extended the IIM of component state, which assesses how the transition of the component state affects system performance. The IIM of component i in a binary system is defined as

$$\begin{aligned} I(IIM)_i^t &= Pr\{X_i(t) = 1\} \cdot \lambda_i(t) \cdot \begin{cases} Pr\{\phi\left(X(t)\right) = 1|X_i(t) = 1\} \\ -Pr\{\phi\left(X(t)\right) = 1|X_i(t) = 0\} \end{cases} \\ &= Pr\{X_i(t) = 1\} \cdot \lambda_i(t) \cdot I(BM)_i^t, \end{aligned} \tag{14}$$

where $\lambda_i(t)$ is the failure rate of component i at time t.

Then, among the UAV system, the BM and IIM of the UAV i is

$$I(BM)_i^t = \Pr\{R\left(m_i(t)\right) = 1\big|m_i(t) = 1\} - \Pr\{R\left(m_i(t)\right) = 1\big|m_i(t) = 0\}. \tag{15}$$

$$I(IIM)_i^t = \Pr\{m_i(t) = 1\} \cdot q_i(t) \cdot I(BM)_i^t. \tag{16}$$

According to above equations, we can derive the *I(BM)* and *I(IIM)*. They are used to identify weak components of the system and signify the roles of components of the system. BM considers the sensitivity of UAV unreliability to the change of component failure probability. IIM is used to evaluate the impact of component state transition on system performance. What's more, BM focuses on the influence of structure, while IIM lays more emphasis on the influence of state change. But the trends they show are similar.

3. Phased-Mission Reliability Analysis of Linear UAV Swarm

The UAV swarm is regarded as a k-out-of-n system, k is the total number of failed UAVs. But in practice, when the UAV swarm executes the mission, in order to successfully complete the mission, it is necessary that up to phase j, UAVs can fail at the end of the mission. To achieve self-organization, each UAV needs to be able to communicate with each other. To improve the reliability of the whole communication system, the UAV swarm will form a one-dimensional linear spatial distributed system, and each UAV can communicate with the nearest k UAVs. As a result, the number of faulted UAVs can't continuously exceed number k. Otherwise, it will cause the communication interruption of the swarm, resulting in the swarm can't achieve self-organization.

For a mission, there are UAVs in the swarm that can work properly before the mission starts. The UAV swarm needs to perform mission in a specified area, and each UAV is considered a mobile sensor with a limited detection range. Up to phase j , UAVs can fail at the end of the mission to complete the task. To improve the effectiveness of the whole mission, the mission range of UAVs can be covered repeatedly, and with $(k-1)$ UAVs failing continuously, the swarm can complete the mission. The linear UAV swarm is shown in Figure 1.

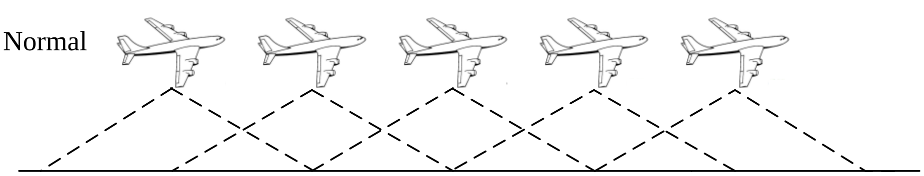

Figure 1. The linear UAV swarm.

As shown in Figure 1, the UAVs are arranged linearly and all work normally at the beginning of the mission. As UAVs perform multi-phase missions, the target linear distance may be different. The distance between two adjacent UAVs also changes, so the k value may be different in a different mission.

Through the above analysis, there should be further requirements for the number of failed UAVs in practice. The continuous failure should be considered to set the model by using the continuous k-out-of-n system. In this continuous k-out-of-n system, k is the number of continuously damaged UAVs. In a certain mission, when there are k continuous failed UAVs, the whole aircraft swarm will be unable to complete the self-organization, resulting in the overall mission failure.

It is assumed that the failure mode of identical UAVs is the same, and the UAV's failure is independent in each mission phase. Thus, if all UAVs have the same type in a certain mission, the UAV swarm is independent and identically distributed. At one phase, if i components fail continuously, there will be $(n-i)$ components that can work normally. These $(n-i)$ components have $(n-i+1)$ positions to place the i continuously failed components and keep the system working normally, $i < k$. N (i, $k-1$, $n-i+1$) represents the number of ways to put i same balls into $(n-i+1)$ different units and make the number of balls per unit not exceed $(k-1)$. Then

$$r_L(p,k,n) = \sum_{i=0}^{n} N(i,k-1,n-i+1)p^{n-i}q^i. \tag{17}$$

The total number of damaged UAVs is also an important factor affecting the reliability of the system. To satisfy the condition of the continuous k-out-of-n system, the value of l is limited. When continuous $(k-1)$ damaged UAVs and 1 normal UAV alternate, l has the maximum number. If the value of l continues to increase, the total system will fail. To realize the self-organizing and self-adaptation among UAV swarms, the maximum value l can reach

$$G = \begin{cases} \frac{n}{k}(k-1), & \text{n can be k divided} \\ n - \left[\frac{n}{k}\right], & \text{n can't be k divided} \end{cases}. \tag{18}$$

In particular, in order to ensure the successful completion of the mission, all target ranges must be able to be detected. Thus, at least x UAVs need to be normal, and the system can damage up to $n-x$ UAVs. l can neither exceed the G, nor exceed the $n-x$. $l = min\{G, x\}$. Reliability of the mission in phase j can be expressed as

$$r_{L,j}(p, k_j, n) = \sum_{i=0}^{\min(n-x)} N(i, k_j - 1, n - i + 1) p^{n-i} q^i. \tag{19}$$

According to the knowledge of combinatorial mathematics, $N(i, k-1, j)$ is the coefficient before x^j in polynomial $(1 + x + x^2 + \cdots + x^{k-1})$,

$$\begin{aligned}(1 + x + x^2 + \cdots + x^{k-1})^j = \frac{(1-x^k)^j}{(1-x)^j} &= \sum_{\lambda=0}^{j} (-1)^\lambda \binom{j}{\lambda} x^{k\lambda} \sum_{\mu=0}^{\infty} \binom{j+\mu-1}{\mu} x^\mu \\ &= \sum_{\mu=0}^{\infty} \sum_{\lambda=0}^{j} (-1)^\lambda \binom{j}{\lambda} \binom{j+\mu-1}{\mu} x^{\mu+k\lambda} \\ &= \sum_{i=k\lambda}^{\infty} \sum_{\lambda=0}^{i} (-1)^\lambda \binom{j}{\lambda} \binom{j+i-k\lambda-1}{i-k\lambda} x^i,\end{aligned} \tag{20}$$

$$N(i, k-1, n-i+1) = \sum_{\lambda=0}^{n-j+1} (-1)^\lambda \binom{n-i+1}{\lambda} \binom{n-k\lambda}{i-k\lambda} x^i, \tag{21}$$

$$r_L(p, k, n,) = \sum_{i=0}^{l} \sum_{\lambda=0}^{n-j+1} \ldots \sum_{\lambda=0}^{n+1} \sum_{i=0}^{l-\lambda+1} = \sum_{\lambda=0}^{n+1} \frac{(-1)^\lambda}{\lambda!} \sum_{i=\lambda k}^{l-\lambda+1} \binom{n-\lambda k}{i-\lambda k} P(n-i+1, \lambda) p^{n-i} q^i, \tag{22}$$

$$t(pt+q)^{n-\lambda k} = \frac{1}{q^{\lambda k}} \sum_{q^{\lambda k}}^{1} \binom{n-\lambda k}{i-\lambda k} P^{n-i} q^i t^{n-i+1}. \tag{23}$$

Take the differential λ times on both sides of the upper formula and make $t = 1$. Then

$$\sum_{i=\lambda k}^{l-\lambda+1} \binom{n-\lambda k}{i-\lambda k} = p^\lambda q^{\lambda k} P(l-\lambda k, \lambda) + \lambda p^{\lambda-1} q^{\lambda k} P(l-\lambda k, \lambda-1), \tag{24}$$

$$r_{L,j}(p, k_j, n,) = \sum_{\lambda=0}^{n+1} \binom{n-\lambda k_j}{\lambda} (-1)^\lambda (pq^{k_j})^\lambda - q^{k_j} \sum_{\lambda=0}^{n} \binom{n-\lambda k_j - k_j}{\lambda} (-1)^\lambda (pq^{k_j})^\lambda. \tag{25}$$

It's mentioned in the hypothesis that the failure of the UAV swarm is not repairable when performing phased-mission. If a UAV fails, the failure state will remain until the end of all phases. The number of normal UAVs in phase j is that the initial total number of UAVs minus the total number of failed UAVs in phase (j-1). At the beginning of the mission j, the number of UAVs that can complete the mission successfully is

$$n_j = n - \sum_{j=1}^{j-1} l_j. \tag{26}$$

There is dependency among these mission phases, and task j is successfully completed as the ($j-1$) task is successfully completed,

$$r_{L,j}(k_j,n_j) = r_{L,j-1}(k_{j-1},n_{j-1}) * \sum_{j=1}^{L_j} p^{n_j-i} q^i. \tag{27}$$

The reliability of a phased-mission can be obtained by accumulating the reliability of these N phases. When writing the algorithm, considering that the whole state of a single UAV only has two: normal and failed, the normal state of the UAV is recorded as 1 and the failed state is recorded as 0. In phase j, the reliability of the UAV swarm can be expressed as

$$r_{L,j}(p,k_j,n) = \sum_{j=1}^{L_j} \sum_{L_i k_j} \prod_{\mu=1}^{n} \binom{1}{m_\mu} p^{n-i} q^i, \tag{28}$$

where, Σ_{L_i,k_j} denotes the sum of the states of n UAVs, number UAVs from 1 to n, m_μ is the state of UAV $\mu, m_\mu \in \{0,1\}, (\mu = 1,2,\dots,n)$.

At the end of phase j, in order to ensure the success of the mission,

$$\begin{cases} m_1 + m_2 + \cdots + m_n > n - l_j \\ m_s + m_{s+1} + \cdots + m_{s+k-1} > n - k_j' \end{cases} \tag{29}$$

(s=1, 2, ..., n−k+1).

During phase j, the probability of the UAV swarm system is

$$H_{j,i} = \Pr\{l_j = i;\ m_1 + m_2 + \cdots + m_n > n - l_j, m_s + m_{s+1} + \cdots + m_{s+k-1} > n - k_j\} \quad (s = 1,2,\dots,n-k+1). \tag{30}$$

The reliability of the entire mission system with N phases can be calculated as

$$R_L(k,n) = \prod_{j=1}^{N} r_{L,j}(p,k_j,n). \tag{31}$$

Using the Markov property of a phase, the state of this phase is equal to the state of the previous phase multiplied by the state probability of this phase,

$$H_{j,i} = \sum_{a=0}^{m_{j-1}-1} H_{(j-1)a} * Pr\{l_j = i \mid l_{j-1} = a\}, \tag{32}$$

$$R_L = \sum_{i=0}^{m_M-1} H_{M,i}. \tag{33}$$

According to the above formulas, we can get the phase mission reliability and the overall mission reliability of the system. This is important for our further analysis.

4. Phased-Mission Reliability Analysis of in Circular UAV Swarm

Compared with linearly arranged UAV swarm, circularly arranged UAV swarm are connected head and tail. If there are i consecutive failed components, there will be (n−i) components working normally, and there will still be (n−i+1) positions at this time. The circular UAV swarm is shown in Figure 2.

As shown in Figure 2, the UAVs are arranged circularly and all work normally at the beginning of the mission. But unlike the linear UAV swarm, the head and tail are connected. The sum of the failure parts at the beginning and end positions can't exceed (k−1), and the number of continuously failed components placed at the rest positions can also not exceed (k−1). The mode number write as y_i. Take any x components from the i consecutive failed components, and then respectively put in the beginning and end positions, the number of ways is $\binom{2+x-1}{x} = 1 + x$, and the (i−x) failed components are placed in the remaining $(n - i - 1)$ position.

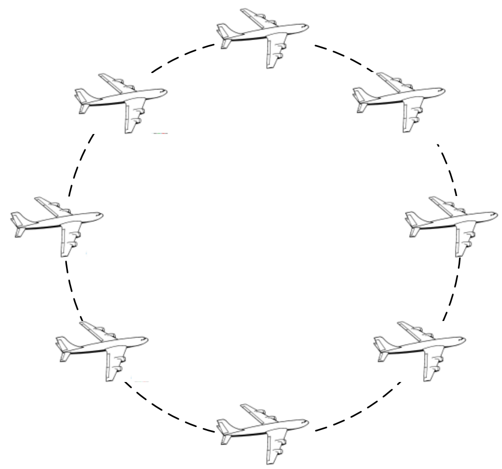

Figure 2. The circular UAV swarm.

$$a_i = \sum_{x=0}^{k-1}(1+x)N(i-x, k-1, n-i-1) = \omega_1(i) + \omega_2(i), \tag{34}$$

$$\omega_1(i) = \sum_{x=0}^{k-1} N(i-x, k-1, n-i-1) = N(i, k-1, n-i), \tag{35}$$

$$\omega_2(i) = (\sum_{x=0}^{k-1} xt^{tx})(\sum_{i=0}^{n-1} N(i, k-1, n-i-1)t^i = t\left(\frac{1-t^k}{1-t}\right)^{n-i-1}. \tag{36}$$

Launch $\omega_1(i)$, $\omega_2(i)$, and get

$$a_i = \sum_{\lambda=0}^{n-i}\binom{n-i}{\lambda}\binom{n-k\lambda}{i-k\lambda}(-1)^\lambda - k\sum_{\lambda=0}^{n-i-1}\binom{n-i-1}{\lambda}\binom{n-k\lambda-k-1}{i-k\lambda-k}(-1)^\lambda. \tag{37}$$

$$r_C(p,k,n,) = \sum_{\lambda=0}^{n}\sum_{i=0}^{n-\lambda}\frac{(-1)^\lambda}{\lambda!}\binom{n-k\lambda}{i-k\lambda}P(n-i,\lambda)p^{n-i}q^i - k\sum_{\lambda=0}^{n-1}\sum_{i=0}^{n-\lambda-1}\frac{(-1)^\lambda}{\lambda!}\binom{n-k\lambda-k-1}{i-k\lambda-k}P(n-i-1,\lambda)p^{n-i}q^i - q^n = \sum_{\lambda=0}^{n}\sum_{i=\lambda k}^{n-\lambda}\frac{(-1)^\lambda}{\lambda!}\binom{n-k\lambda}{i-k\lambda}P(n-i,\lambda)p^{n-i}q^i - k\frac{p}{q}\sum_{\lambda=0}^{n-1}\sum_{i=\lambda k+k+1}^{n-\lambda}\frac{(-1)^\lambda}{\lambda!}\binom{n-(k\lambda+k+1)}{i-(k\lambda+k+1)}P(n-i,\lambda)p^{n-i}q^i - q^n. \tag{38}$$

$$r_{C,j}(p,k_j,n) = \sum_{\lambda=0}^{n-i}\binom{n-\lambda k_j}{\lambda}(-1)^\lambda(pq^{k_j})^\lambda - k_j\sum_{\lambda=0}^{n-1}\binom{n-k_j\lambda-k_j-1}{\lambda}(-1)^\lambda(pq^{k_j})^{\lambda+1} - q^n. \tag{39}$$

When components are arranged in the alternating mode in which ($k-$ 1) components are damaged and a component is normally, if the value of l continues to increase, the model will fail. In order to realize the self-organization and adaptation among the swarms, the maximum value of l can be achieved

$$G = \begin{cases} \frac{n}{k}(k-1), & n \text{ can be divided by } k \\ n - \left[\frac{n}{k}\right](k-2) - 1, & n \text{ can't be divided by } k \end{cases}. \tag{40}$$

To ensure the successful completion of the mission, all target ranges must be detectable, the mission condition is that at least x UAVs are normal, then the system can damage up to (n−x) UAVs, so l can neither exceed the G, nor exceed the n−x.

$$l = min\{G, x\}, \tag{41}$$

$$r_{C,j}(p, k_j, n) = \sum_{i=0}^{\min(G,n-x)} N(i, k_j - 1, n - i + 1) p^{n-i} q^i. \tag{42}$$

Consider dependency between phases

$$r_{C,j}(k_j, n_j) = r_{C,j-1}(k_{j-1}, n_{j-1}) * \sum_{i=1}^{l_j} p^{n_j - i} q^i. \tag{43}$$

$$R_C(k, n) = \prod_{j=1}^{N} r_{C,j}(p, k_j, n). \tag{44}$$

When designing the algorithm, similar to the previous part, the UAVs are numbered sequentially from 1 to n, and component numbered as n is after the component numbered as 1. The m_μ indicates the state of UAV μ, $m_\mu \in \{0,1\}, (\mu = 1,2, \dots, n)$. The state of continuously damaged k UAVs can be divided into the following two situations.

The serial numbers contain 1 or n,

$$\{m_s + m_{s+1} + \cdots + m_{s+k-1} = 0, (s = 1,2,3, \dots, n - k + 1)\}. \tag{45}$$

The serial numbers contain 1 and n,

$$\{m_s + m_{s+1} + \cdots + m_n + m_1 + m_2 + \cdots m_{k+s-n-1} = 0, (s = n - k + 1, \dots, n)\}. \tag{46}$$

To ensure the success of the mission, the total number of damaged components at the end of task j does not exceed M, and a number of continuously damaged components can't exceed k

$$m_1 + m_2 + \cdots + m_n > n - l_j;$$
$$\{m_s + m_{s+1} + \cdots + m_{s+k-1} > n - k_j, (s = 1,2, \dots, n - k + 1)\} \cup \{m_s + m_{s+1} + \cdots + m_n + m_1 + m_2 + \cdots + m_{k+s-n-1} > n - k_j, \ (s = n - k + 1, \dots, n). \tag{47}$$

The reliability of mission j is

$$r_{C,j}(p, k_j, n) = \sum_{i=0}^{n-1} \sum_{C_i, \ k_j} \prod_{\mu=1}^{n} \binom{1}{m_\mu} p^{n-i} q^i, \tag{48}$$

where Σ_{C_i,k_j} epresents the sum of the states of n UAVs. Numbering the UAV from 1 to n, m_μ is the state of the μ-th UAV, $m_\mu \in \{0,1\}, (\mu = 1,2, \dots, n)$,

$$m_1 + m_2 + \cdots + m_n > n - l_j;$$
$$\{m_s + m_{s+1} + \cdots + m_{s+k-1} > n - k_j, (s = 1,2, \dots, n - k + 1)\} \cup \{m_s + m_{s+1} + \cdots + m_n + m_1 + m_2 + \cdots + m_{k+s-n-1} > n - k_j, (s = n - k + 1, \dots, n). \tag{49}$$

The reliability of the entire mission system is

$$R_C(k, n) = \prod_{j=1}^{N} r_{C,j}(p, k_j, n). \tag{50}$$

5. Numerical Example

In this part, we take a two-phase mission as an example to verify the model. In the mission, there are two main phases. At the beginning of the mission, there are 7 UAVs which are all operational. In the first phase, the 7 UAVs are arranged linearly. Except for the UAVs located at the head and tail, other UAVs are adjacent to two UAVs. But during the execution, two UAVs are failing. Thus, in the second phase, only 5 UAVs work and are arranged in a circle. They are connected end to end, and each UAV is adjacent to two UAVs. This is shown in Figure 3.

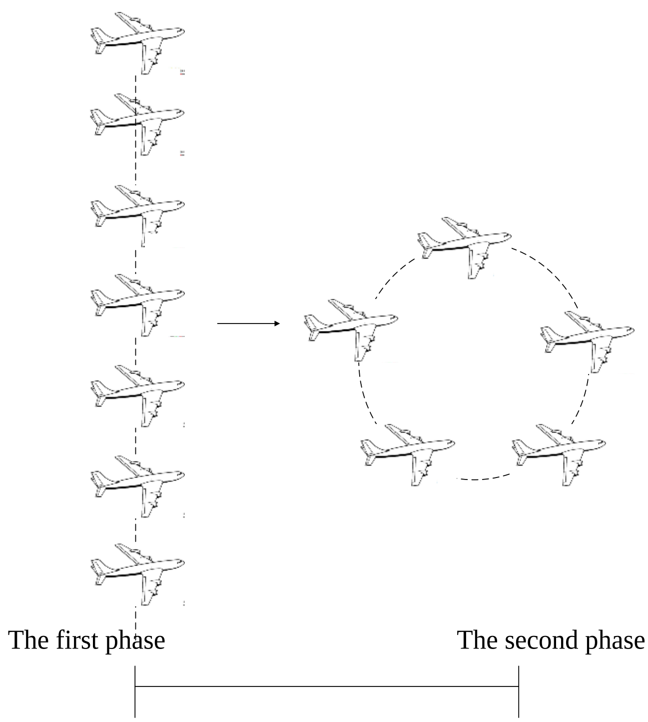

Figure 3. A numerical example with two-phase.

To study the mission reliability, we start with the failure probability of a single UAV. Then, phase reliability is analyzed by introducing the fault coverage factor. When the UAV detection function fault can be found at the end of the phase 2, the cumulative probability of failure can be calculated by Equation (8), which is

$$Q_{jc} = \frac{\sum_{i=1}^{2} f_{ic}}{1 - L_u}.$$

When the fault is completely covered, according to Equation (11), the reliability of the phase is

$$p_j = \left(1 - \frac{f_{jc}}{1 - Q_{(j-1)c}}\right) * \left(1 - d_j\right) = \left(\frac{1 - Q_{jc}}{1 - Q_{(j-1)c}}\right) * \left(1 - d_j\right) = \left(\frac{P_{jc}}{P_{(j-1)c}}\right) * \left(1 - d_j\right),$$

where $j = 1$ at first phase, $j = 2$ at second phase.

At first phase, it is a linear UAV swarm. According to the characteristics of the UAV swarm, 7 UAVs in the swarm can work normally before the mission starts. In the consecutive 4-out-7 system, when a certain mission is performed, there are 4 UAVs that fail continuously, and the entire fleet will not be able to complete self-organization, failing the overall mission. The reliability of this phase is obtained by Equation (28).

$$r_{L,j}(p,k_j,7)=\sum_{j=1}^{L_j}\sum_{L_i,k_j}\prod\nolimits_{\mu=1}^{7}\binom{1}{m_\mu}p^3q^4,$$

where, Σ_{L_i,k_j} denotes the sum of the states of 7 UAVs, m_μ is the state of μ-th UAV , $m_\mu \in \{0,1\}, (\mu = 1,2,\dots,7)$.

At second phase, it is a circular UAV swarm. When the UAV swarm is arranged in a ring, if 3 components consecutively fail, 2 components work normally, and there are still 4 positions at this time. However, unlike the linear formation, the head and tail are connected, and the sum of the failed components at the start and end positions cannot exceed 2. The reliability of the mission is obtained according to Equation (48).

$$r_{C,j}(p,k_j,5)=\sum_{i=0}^{4}\sum_{C_i,k_j}\prod_{\mu=1}^{5}\binom{1}{m_\mu}p^2\,q^3,$$

where, Σ_{C_i,k_j} presents the sum of the states of 5 UAVs, m_μ is the state of the μ-th UAV, $m_\mu \in \{0,1\}, (\mu = 1,2,\dots,5)$.

After evaluating the reliability of UAV swarm, in order to clarify the key components in the swarm, according to analysis, the reliability and importance of single UAV should be obtained. Therefore, we mainly analyze the BM and IIM of each UAV in the first and second phases, to analyze the difference in their importance in linear and circular arrangements.

5.1 Linear UAV Swarm for First Phase

Through the above analysis, the mission reliability of the UAV swarm at different phases can be obtained. But to determine the most important position in the phase mission, it needs to analyze the importance measure. According to Equations (15) and (16), it can be obtained that the BM and IIM of the 7 UAVs and the influence of the state of its UAV $i(i = 1,2 \dots,7)$ UAV on the entire fleet, to determine the machine that needs the most attention. In the linear consecutive 4-out-of-7 system, the BM and IIM of component i in time t are

$$I(BM)_i^t=\Pr\{R(m_i(t))=1|m_i(t)=1\}-\Pr\{R(m_i(t))=1|m_i(t)=0\}.$$

$$I(IIM)_i^t=\Pr\{m_i(t)=1\}*q_i(t)*I(BM)_i^t=p_i(t)*q_i(t)*\frac{R_{4/i-1}^t*R'^t_{4/7-i}-R_{4/7}^t}{1-p_i(t)}.$$

With the number, i as the boundary, $(i-1)$ indicates the number of UAVs before the UAV numbered i, and $(7-i)$ indicates the number of UAVs behind the UAV numbered i.

$R_{4/i-1}^t$is the reliability of consecutive 4-out-of-(i−1) at the time t.

$R'^t_{4/7-i}$ is the reliability of consecutive 4-out-of-(7−i) at the time t.

Based on the above analysis, we plot the trends of the BM and IIM of the 7 UAVs in the first phase of the mission over time, as shown in Figure 4 and Figure 5.

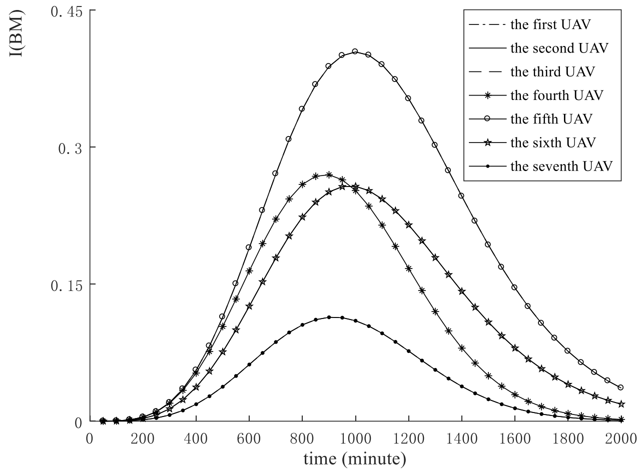

Figure 4. BM in linear UAV swarm.

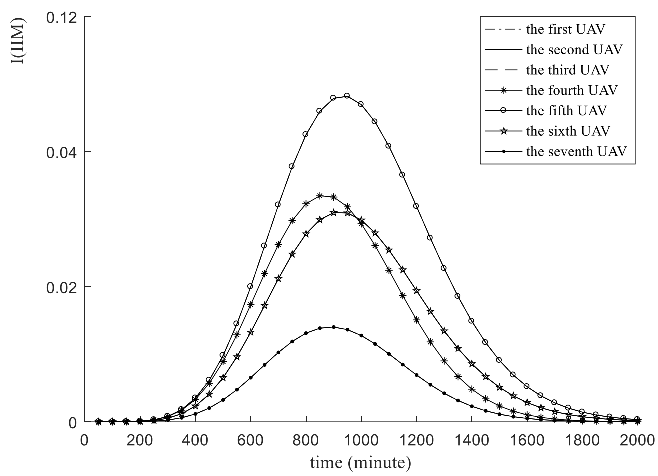

Figure 5. IIM in leaner UAV swarm.

In Figures 4 and 5, it can be seen that the changing trends of BM and IIM of 7 UAVs in line formation are first increasing and then decreasing with time adding. The importance of the 5th UAV is the largest, so the 5th UAV has the highest impact on the reliability of the system mission. The state of the 5th UAV should be emphasized.

Because the reliability of this phased-mission is accumulated by the reliability of each phase, it can be analyzed from a certain phase when analyzing the influence of each UAV on the reliability of the system. According to the characteristics of the formula, in linear formation, the UAV numbered j is the same as the UAV numbered $(n-j+1)$. The IIM is the BM multiples the failure rate of component i at the moment t. Therefore, the images of two importance are not different very in shape. The above images verified the analysis. The pictures of importance between the UAV numbered 1 and 7, the UAV numbered 2 and 6, and the UAV numbered 3 and 5 coincide. From the point of view of their location, they are in the symmetric position of the swarm, so the impacts on the swarm structure are similar.

5.2 Circular UAV Swarm for Second Phase

In this phase, since 2 UAVs failed in the previous phase, the number of UAVs in this phase is 5. Compared with the linearly arranged UAVs, the UAVs connected in a circular arrangement are different, which also need to be determined according to the importance analysis. According to Equations (15) and (16), the influence of the state of the UAV $i(i = 1,2 \ldots,5)$ UAV on the entire fleet can be obtained, because the circle structure can be regarded as a linear structure with the head and tail connected. There are 4 UAVs in front of UAV i and 4 UAVs behind i. On the basis of the previous analysis, the BM formula of component i at time t is,

$$I(BM)_i^t = \Pr\{R(m_i(t)) = 1 \big| m_i(t) = 1\} - \Pr\{R(m_i(t)) = 1 \big| m_i(t) = 0\}.$$

The IIM of component i at time t is defined as:

$$(IIM)_i^t = \Pr\{m_i(t) = 1\} * q_i(t) * I(BM)_i^t = p_i(t) * q_i(t) * \frac{R_{3/(i-1)}^t * R'^t_{3/(5-i)} - R_{3/5}^t}{1 - p_i(t)}$$

With the number, i as the boundary, (i−1) indicates the number of UAVs before the UAV numbered i, and (5−i) indicates the number of UAVs behind the UAV numbered i.

$R_{3/i-1}^t$is the reliability of consecutive 3-out-of-(i−1) at the time t.

$R'^t_{3/5-i}$ is the reliability of consecutive 3-out-of-(5−i) at the time t.

Based on the above analysis, we plotted the trends of the BM and IIM of the seven UAVs in the first phase of the mission over time, as shown in Figure 6 and Figure 7.

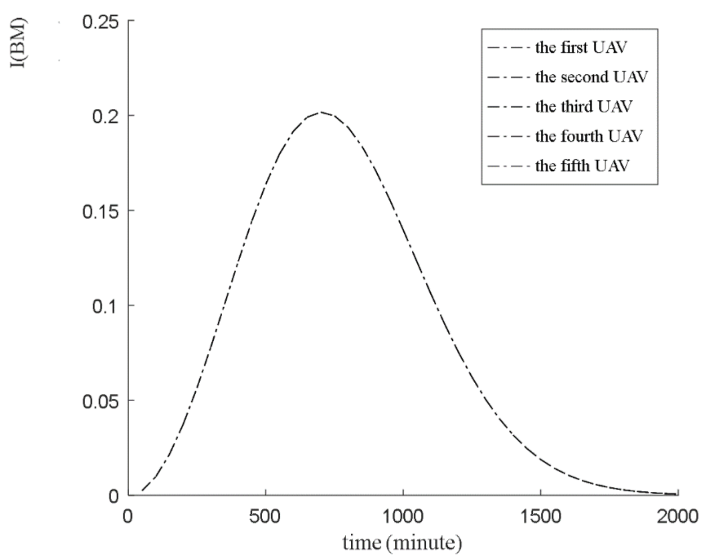

Figure 6. BM in circular UAV swarm.

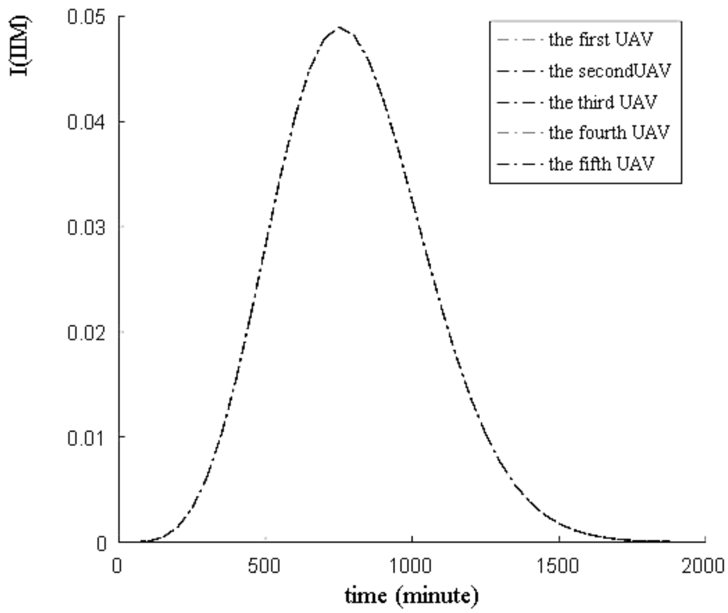

Figure 7. IIM in circular UAV swarm.

From Figures 6 and 7, it can be seen that the images of the BM and the IIM of each UAV in the circle formation are coincident. It indicates that the importance of each UAV is same. This is because the ring formation can be seen as a linear formation with its head and tail connected. There are $(n-1)$ UAVs in front of the UAV i and $(n-1)$ UAVs behind the i. Thus, at the same time, the importance of any UAV in the above equations calculated the same results. The images are also coincidental. Impact of each UAV in the system is the same. It can be seen from the image that the changing trend of the importance degree of each UAV increases first and then decreases as time increases.

6. Conclusions

Taking the UAV swarm system as object, and taking the characters of the UAV swarm as the background, this paper discusses the reliability of phased-mission in linear UAV and circular UAV swarms. The main work of this paper is as follows.

By disassembling the fault caused of UAV, it is divided into detection fault, communication fault, the influence of its service life, and the failure caused by being hit. Considering the actual situation, the fault coverage factor is introduced in the model, and then get the reliability of a single UAV in each phase. The importance of parameters is analyzed, which lays a certain foundation for later research.

The actual situation and mission requirements of UAV swarm operation are deeply considered. Based on the k-out-of-n system, the reliability of the UAV swarm mission is modeled and predicted. Then combined with a phased-mission, a recursive formula of linear formation in performing a phased-mission is proposed. Based on the linear UAV formation, the reliability formula of the UAV in ring performs phased-mission is studied.

The reliability models of the linear and circular UAV swarms are more realistic. It can help to find out the key UAV in the actual mission execution process. What's more, they actually represent two basic types of structures. At the same time, on this basis, it is also more convenient to expand UAV swarms of other types of structures. We can also expand other more complex structures of other systems.

In the future, we can continue to study the reliability of two-dimensional rectangular formation, irregular shape formation, and even three-dimensional formation. For the consideration of simplifying models, the UAVs are regarded as mutually independent, but in practical missions, the failure of a certain UAV may affect its surrounding UAVs, and increase their load burden, then affect the overall reliability. The coupling between UAVs can be further considered in the future, and the cascade failure problem of the UAV swarm can also be further studied. The reliability of the UAV swarm can be extended to a wide range, and then we can explore the impact of obstacles encountered in UAV swarm flight on reliability.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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References

Bai, G., Wang, H., Zheng, X., Dui, H., & Xie, M. (2021). Improved resilience measure for component recovery priority in power grids. *Frontiers of Engineering Management*, *8*(4), 545-556.

Chaube, S., & Singh, S.B. (2016). Fuzzy reliability of two-stage weighted-k-out-of-n systems with common components. *International Journal of Mathematical*, *Engineering and Management Sciences*, *1*(1), 41-51. https://dx.doi.org/10.33889/IJMEMS.2016.1.1-005.

Dui, H., Si, S., Zuo, M.J., & Sun, S. (2015). Semi-Markov process-based integrated importance measure for multi-state systems. *IEEE Transactions on Reliability*, *64*(2), 754-765.

Dui, H., Tian, T., Zhao, J., & Wu, S. (2022a). Comparing with the joint importance under consideration of consecutive-k-out-of-n system structure changes. *Reliability Engineering & System Safety*, *219*, 108255.

Dui, H., Wu, S., & Zhao, J. (2021c). Some extensions of the component maintenance priority. *Reliability Engineering & System Safety*, *214*, 107729.

Dui, H., Xu, Z., Chen, L., Xing, L., & Liu, B. (2022b). Data-driven maintenance priority and resilience evaluation of performance loss in a main coolant system. *Mathematics*, *10*(4), 563.

Dui, H., Zhang, C., Bai, G., & Chen, L. (2021a). Mission reliability modeling of UAV swarm and its structure optimization based on importance measure. *Reliability Engineering & System Safety*, *215*, 107879.

Dui, H., Zheng, X., & Wu, S. (2021b). Resilience analysis of maritime transportation systems based on importance measures. *Reliability Engineering & System Safety*, *209*, 107461.

Endharta, A.J., Yun, W.Y., & Ko, Y.M. (2018). Reliability evaluation of circular k-out-of-n: G balanced systems through minimal path sets. *Reliability Engineering & System Safety*, *180*, 226-236.

Huang, X., Aslett, L.J., & Coolen, F.P. (2019). Reliability analysis of general phased mission systems with a new survival signature. *Reliability Engineering & System Safety*, *189*, 416-422.

Levitin, G., Finkelstein, M., & Xiang, Y. (2020). Optimal multi-attempt missions with cumulative effect. *Reliability Engineering & System Safety*, *203*, 107091.

Li, J. (2016). Reliability calculation for dormant k-out-of-n systems with periodic maintenance. *International Journal of Mathematical, Engineering and Management Sciences*, *1*(2), 68-76.

Mo, Y., Xing, L., Amari, S.V., & Dugan, J.B. (2015). Efficient analysis of multi-state k-out-of-n systems. *Reliability Engineering & System Safety*, *133*, 95-105.

Mohammadi, F., Sáenz-de-Cabezón, E., & Wynn, H.P. (2018). Efficient multicut enumeration of k-out-of-n: F and consecutive k-out-of-n: F systems. *Pattern Recognition Letters*, *102*, 82-88.

Müller, M.A., & Domínguez-García, A.D. (2012). Fault coverage modeling in nonlinear dynamical systems. *Automatica*, *48*(7), 1372-1379.

Peng, R., Wu, D., Xiao, H., Xing, L., & Gao, K. (2019). Redundancy versus protection for a non-reparable phased-mission system subject to external impacts. *Reliability Engineering & System Safety*, *191*, 106556.

Pham, H. (2010). On the estimation of reliability of k-out-of-n systems. *International Journal of Systems Assurance Engineering and Management*, *1*(1), 32-35.

Rushdi, A.M.A. (2019). Utilization of symmetric switching functions in the symbolic reliability analysis of multi-state k-out-of-n systems. *International Journal of Mathematical, Engineering and Management Sciences*, *4*(2), 306-326.

Su, P., Wang, G., & Duan, F. (2020). Reliability evaluation of a k-out-of-n (G)-subsystem based multi-state system with common bus performance sharing. *Reliability Engineering & System Safety*, *198*, 106884.

Wang, C., Xing, L., & Levitin, G. (2012). Competing failure analysis in phased-mission systems with functional dependence in one of phases. *Reliability Engineering & System Safety*, *108*, 90-99.

Wang, C., Xing, L., & Levitin, G. (2015). Probabilistic common cause failures in phased-mission systems. *Reliability Engineering & System Safety*, *144*, 53-60.

Wang, C., Xing, L., Amari, S.V., & Tang, B. (2020). Efficient reliability analysis of dynamic k-out-of-n heterogeneous phased-mission systems. *Reliability Engineering & System Safety*, *193*, 106586.

Wang, G., Peng, R., & Xing, L. (2018). Reliability evaluation of unrepairable k-out-of-n: G systems with phased-mission requirements based on record values. *Reliability Engineering & System Safety*, *178*, 191-197.

Wang, R., Wang, X., Wang, L., & Chen, X. (2016). Efficient computational method for the non-probabilistic reliability of linear structural systems. *Acta Mechanica Solida Sinica*, *29*(3), 284-299.

Wu, C., Zhao, X., Wang, S., & Song, Y. (2022). Reliability analysis of consecutive-k-out-of-r-from-n subsystems: F balanced systems with load sharing. *Reliability Engineering & System Safety*, *228*, 108776.

Xing, L., & Amari, S.V. (2008). Reliability of phased-mission systems. In: Misra, K.B. (ed) *Handbook on Performability Engineering* (pp. 349-68). Springer, London.

Xing, L., & Dugan, J.B. (2002). Analysis of generalized phased-mission system reliability, performance, and sensitivity. *IEEE Transactions on Reliability*, *51*(2), 199-211.

Xing, L., Amari, S.V., & Wang, C. (2012). Reliability of k-out-of-n systems with phased-mission requirements and imperfect fault coverage. *Reliability Engineering & System Safety*, *103*, 45-50.

Yam, R.C., Zuo, M.J., & Zhang, Y.L. (2003). A method for evaluation of reliability indices for repairable circular consecutive-k-out-of-n: F systems. *Reliability Engineering & System Safety*, *79*(1), 1-9.

Yan, Z., & Liu, H. (2022). SMoCo: A powerful and efficient method based on self-supervised learning for fault diagnosis of aero-engine bearing under limited data. *Mathematics*, *10*(15), 2796.

Yu, H., Yang, J., Lin, J., & Zhao, Y. (2017). Reliability evaluation of non-repairable phased-mission common bus systems with common cause failures. *Computers & Industrial Engineering*, *111*, 445-457.

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