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Some New Correlation Coefficient Measures Based on Fermatean Fuzzy Sets using Decision Making Approach in Pattern Analysis and Supplier Selection

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Abstract
Fermatean fuzzy set (FFS) is an effective tool to depict expert reasoning information in the decision-making process than fuzzy sets (FS), intuitionistic fuzzy sets (IFS), and Pythagorean fuzzy sets (PFS). Keeping in mind the importance of correlation coefficient and application in medical diagnosis, decision making and pattern recognition, several studies on correlation coefficient measures have been proposed in the literature. As there does not exist any study concerning correlation coefficient measures for FFS, in this communication, we propose novel entropy-correlation measures for Fermatean fuzzy sets and applied it decision making problems of pattern analysis and multi-criteria decision making for supplier selection. With the help of proposed correlation coefficient, we establish some weighted measures for FFS. Using numerical computations, we determine the efficacy of the suggested measures over other measures. The aim of this study is to propose a novel and efficient methodology for evaluation of supplier's selection with uncertain information. Finally, we establish the comparative study of our developed measures over the existing correlation coefficient measures. The analysis showed that the suggested methodology is reliable, flexible, and consistent with the existing techniques.

Keywords- Pythagorean fuzzy sets, Correlation coefficient, Entropy, Pattern recognition and multi-criteria decision-making.

1. Introduction

Every day humans are encircled by various situations demanding to make an optimal decision. Sometimes decision making is easy, and it is possible to make decisions using crisp numbers (Cao et al., 2020) but many a times humans must make tough choices and situations involves lots of randomness. Zadeh (1965) realized this problem and introduced fuzzy set theory as an extension to Cantor's classical set theory. Generalized orthopair fuzzy sets (OFS) described as q-rung OFS is a useful technique of conveying imprecise knowledge. In q-rung OFS sum of q^{th} power of membership degree and non-membership degree is restricted by unity. Fermatean fuzzy sets is an extension of intuitionistic fuzzy sets ($with\, q = 1$) and Pythagorean fuzzy sets ($with\, q = 2$) with $q = 3$ are more effective in dealing with several problems involving ambiguity and uncertainty. Many distance and similarity measures for intuitionistic fuzzy sets and Pythagorean fuzzy sets have been familiarized by several researchers and applied to several multi criteria decision-making (MCDM) fields such as medical analysis, clustering, image segmentation and pattern analysis etc. To enhance the capability of fuzzy sets Atanassov (1986, 1989) proposed intuitionistic fuzzy sets in which the degree of non-membership was also included such that if '$\alpha(v_j)$' is membership degree (MD) and '$\beta(v_j)$' is non-membership degree (NMD) then $\alpha(v_j) + \beta(v_j) \leq 1$ where $j = 1,2, \dots, n$. Since it's not necessary that the sum of membership and non-membership is always 1, the complement to 1 is degree of uncertainty called hesitancy. IFS has been applied to variety of real-life problems by many researchers in domains like pattern recognition (Dengfeng and Chuntian, 2002; Song et al., 2014; Garg 2016; Zeng et al., 2022), cluster analysis (Xu et al., 2008), medical diagnosis (De et al., 2001; Davvaz and Sadrabadi, 2016; Umar and Saraswat, 2022) and MCDM (Xu, 2007; Garg and Kumar, 2018).

Later, it was realized that there exist some values of MD and NMD such that their sum exceeds 1. For example, suppose a person must make a choice about an alternative and for him the degree of belongingness is 0.6 and degree of non-belongingness is 0.5 but, 0.6 + 0.5 > 1. To address such problems Yager (2013, 2014) suggested the concept of PFS. PFS states that if '$\alpha(v_j)$' is MD and '$\beta(v_j)$' is NMD than the sum of square of MD and NMD is less than or equal to unity. Mathematically, $0 \leq \alpha^2(v_j) + \beta^2(v_j) \leq 1$.

1.1 Research Gap and Motivation

The study of FFS has recently garnered attention due to its extensive application in uncertain environments. To deal with real-world problems, it can simply be used with MCDM approaches. To handle decision-making challenges, a variety of correlation coefficient methodologies have been proposed and implemented for IFS, PFS, etc. Though the existing correlation coefficient measures are informative, they have certain drawbacks in terms of accuracy and consistency with the FFS notion that need to be addressed to produce more accurate results. The work focuses on:

- Re-examining certain existing FFS correlation coefficient measures,
- Proposing an improved FFS correlation coefficient technique,
- Correlation coefficient measures satisfies the useful properties of proven theorems for validation,
- Aptness of the developed models in recognition of patterns and decision-making problem, and
- Compares the novel correlation coefficient technique in a Fermatean fuzzy environment.

The purpose of this article is to introduce a new correlation coefficient using FFS with applications in pattern analysis and supplier selection. The article is designed as follows: Section 2 includes the literature review in support to the present problem. Section 3 is dedicated to the basic definitions of FS, IFS, PFS and FFS to serve as a mathematical base for better understanding of the article. Section 4 introduces the proposed measures, proved various axiomatic properties of correlation coefficient, and provides numerical validation to these measures. Section 5 offers the real-life applications of these measures in pattern recognition and supplier selection and compares the proposed methods with existing measures to show the legitimacy of the suggested work. Section 6 summarises the work with brief conclusion and scope for future perspective in this area.

2. Literature Review

PFS has been a hot topic for research with its applications being applied in many diverse domains like pattern recognition (Yager, 2016; Wei and Wei, 2018; Ejegwa and Awolola, 2019), information measures (Peng et al., 2017), supplier selection (Villa Silva et al., 2019) and decision making (Zhou and Chen, 2019; Taruna et al., 2021; Bhatia et al., 2022) etc.

One more application of PFS grabbed a lot of attention from the researcher was the concept of correlation coefficient. Correlation analysis can be simply stated as a number which defines the interdependency between two variables. In other words, it tells extent to which one variable is related to other. The value of correlation coefficient lies between -1 and 1, where -1 means that variables have a strong negative relationship, 1 means they share a strong positive relationship and 0 means they are not related to each other. To understand it better suppose analysts wants to determine the impact on an asset by change in external factors a correlation of 0.1 implies that there is a low degree of positive correlation which is unimportant whereas correlation of 0.8 implies a positive and strong impact. Earlier correlation coefficient was done for crisp sets only but later it extended its roots to fuzzy set theory (Dumitrescu, 1978). Even though it was quite challenging but still researchers managed to study its properties and applied it in various dimensions like social sciences, science and technology, engineering, and economics (Yu, 1993; Hong 2006). Though the range of correlation is [-1,1] the research done by many users shows its values between 0 and 1 only. Chiang and Lin (1992) got the range of correlation between -1 and 1 but for that they treated membership values as crisp sets and due to this the fuzziness was missing in it. It was Gerstenkorn and Manko (1991) who introduced the concept of correlation in IFS. Till then many researchers have used the concept of correlation coefficient using IFS and applied it to various disciplines like decision making (Hung and Wu, 2002; Szmidt and Kacprzyk, 2010; Thao et al., 2019; Ejegwa et al., 2020b; Ejegwa, 2021; Zulqarnain et al., 2021). The concept was also applied in PFS introduced by Garg (2016) with uses in pattern recognition and medical analysis. The idea was also applied in MCDM problems by Ejegwa et al. (2020), Ejegwa (2021), Thao (2019), Ejegwa and Awolola (2021), Ejegwa et al. (2022).

In 2017, the concept of orthopair fuzzy sets was discussed by Yager and after that it was taken a step further and Fermatean Fuzzy sets (FFS) were introduced by Senapati and Yager (2019). The concept of FFS is a general case of Orthopair fuzzy sets. Ortho pair fuzzy sets are fuzzy sets where the sum of q^{th} power of membership and non- membership is less than equal to 1 i.e., $0 \le \alpha^q(\mathcal{v}_j) + \beta^q(\mathcal{v}) \le 1$ where '$\alpha(\mathcal{v}_j)$' MD and '$\beta(\mathcal{v}_j)$' is NMD. When q =1 it reduces to IFS, q =2 it is considered as PFS and when q=3 they are called Fermatean fuzzy sets i.e., $0 \le \alpha^3(\mathcal{v}_j) + \beta^3(\mathcal{v}_j) \le 1$. The concept of FFS is helpful where the theories of both IFS and PFS both fail. For instance, 0.8 + 0.7 > 1 and $0.8^2 + 0.7^2 > 1$ but $0.8^3 + 0.7^3 = 0.855 < 1$. The concept was applied in MCDM techniques like TOPSIS (Senapati and Yager, 2019) and VIKOR (Akram et al., 2022) and in decision making (Aydin, 2021; Sahoo, 2022; Sindhu et al. ,2022). The applications of correlation coefficients for FFS is new and very less work has been done on it. Kirisci (2022) has proposed correlation coefficient measures using FFS and using medical diagnosis to show the applicability of the concept.

3. Some Basic Definitions

This section covers basic definitions of IFS, PFS, FFS, and their properties in detail. These concepts will prove beneficial in the subsequent segments.

Let $V = \{\mathcal{v}_1, \mathcal{v}_2, \dots, \mathcal{v}_n\}$ be the universal set and FFS (V) be the set of all FFSs of V.

Definition 1. (Atanassov, 1986) An element Ƒ in the universe of discourse V is defined as an IFS, if

$$Ƒ = \{\langle \mathcal{v}_j, \alpha_{Ƒ}(\mathcal{v}_j), \beta_{Ƒ}(\mathcal{v}_j)\rangle | \mathcal{v}_j \in V\} \tag{1}$$

where, $\alpha_{Ƒ}(\mathcal{v}_j): Ƒ \to [0,1]$, $\beta_{Ƒ}(\mathcal{v}_j): Ƒ \to [0,1]$are the MG and NMG of $\mathcal{v}_j \in V$

$$0 \le \alpha_{Ƒ}(\mathcal{v}_j) + \beta_{Ƒ}(\mathcal{v}_j) \le 1, \forall\ \mathcal{v}_j \in V. \tag{2}$$

For each IFS Ƒ in V,

$$\gamma_{Ƒ}(\mathcal{v}) = 1 - \alpha_{Ƒ}(\mathcal{v}_j) - \beta_{Ƒ}(\mathcal{v}_j), \forall\ \mathcal{v}_j \in V \tag{3}$$

Then $\gamma_{Ƒ}(\mathcal{v}_j)$ is the HG of $\mathcal{v}_j$to V.

Definition 2. (Yager, 2013) An element Ғ in the universal set V is given by

$$Ғ = \{\langle \mathcal{v}_j, \alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j)\rangle | \mathcal{v}_j \in V\}$$

where, $\alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j): Ғ \rightarrow [0,1]$ are the MG and NMG of $\mathcal{v}_j \in$ V with the condition

$$0 \leq \alpha_Ғ^2(\mathcal{v}_j) + \beta_Ғ^2(\mathcal{v}_j) \leq 1, \forall\, \mathcal{v}_j \in V \quad (4)$$

The degree of hesitation of $\mathcal{v}_j$ to V is given by

$$\gamma_Ғ(\mathcal{v}_j) = \sqrt{1 - \alpha_Ғ^2(\mathcal{v}_j) - \beta_Ғ^2(\mathcal{v}_j)} \quad (5)$$

Definition 3. (Yager, 2017) Let a set Ғ is characterised by two functions on V that measures the MD - $\alpha_Ғ(\mathcal{v}_j)$ and NMD - $\beta_Ғ(\mathcal{v}_j)$ to Ғ for every element of the universal set $\mathcal{v}_j \in V$. Ғ is a q-rung OFS ($q \geq 1$) where $\alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j) \in [0,1]$ and validate the property

$$0 \leq \alpha_Ғ^q(\mathcal{v}_j) + \beta_Ғ^q(\mathcal{v}_j) \leq 1, \forall\, \mathcal{v}_j \in V \quad (6)$$

Definition 4. (Senapati and Yager, 2019) An FFS Ғ in V is specified as

$$Ғ = \{\langle \mathcal{v}_j, \alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j)\rangle | \mathcal{v}_j \in V\}$$

where $\alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j) \in [0,1]$ are the MG and NMG of $\mathcal{v}_j \in V$ subject to the condition

$$0 \leq \alpha_Ғ^3(\mathcal{v}_j) + \beta_Ғ^3(\mathcal{v}_j) \leq 1, \forall\, \mathcal{v}_j \in V \quad (7)$$

The degree of hesitation of $\mathcal{v}_j$ to V is given by

$$\gamma_Ғ(\mathcal{v}_j) = \sqrt[3]{1 - \alpha_Ғ^3(\mathcal{v}_j) - \beta_Ғ^3(\mathcal{v}_j)} \quad (8)$$

If $q = 1$, then q-rung OFS reduces to IFS, If $q = 2$, then q-rung OFS reduces to PFS; while $q = 3$, then q-rung OFS reduces to FFS.

Definition 5. (Senapati and Yager, 2019) Given that Ғ and Ġ are FFS in V, then

(a) $Ғ \subseteq Ġ$ iff
$\alpha_Ғ(\mathcal{v}_j) \leq \alpha_Ġ(\mathcal{v}_j),\ \beta_Ғ(\mathcal{v}_j) \geq \beta_Ġ(\mathcal{v}_j)\ \forall\, \mathcal{v}_j \in V.$

(b) $(Ғ)^c = \{\langle \mathcal{v}_j, \beta_Ғ(\mathcal{v}_j), \alpha_Ғ(\mathcal{v}_j)\rangle | \mathcal{v}_j \in V\}$

(c) $Ғ \cup Ġ = \left\{\left\langle \begin{matrix} \mathcal{v}_j, max[(\alpha_Ғ(\mathcal{v}_j), \alpha_Ġ(\mathcal{v}_j)], \\ min[\beta_Ғ(\mathcal{v}_j) \geq \beta_Ġ(\mathcal{v}_j)]\ | \mathcal{v}_j \in V \end{matrix} \right\rangle\right\}$

(d) $Ғ \cap Ġ = \left\{\left\langle \begin{matrix} \mathcal{v}_j, min[(\alpha_Ғ(\mathcal{v}_j), \alpha_Ġ(\mathcal{v}_j)], \\ max[\beta_Ғ(\mathcal{v}_j) \geq \beta_Ġ(\mathcal{v}_j)]\ | \mathcal{v}_j \in V \end{matrix} \right\rangle\right\}$

Definition 6. (Aydin, 2021). Let $Ғ = \{\langle \mathcal{v}_j, \alpha_Ғ(\mathcal{v}_j), \beta_Ғ(\mathcal{v}_j)\rangle | \mathcal{v}_j \in V\}$ be a FFS on V and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ where $\sum_{i=1}^n \omega_i = 1$. The Fermatean fuzzy weighted average operator E of Ғ is

$$E(Ғ) = (\bar{\alpha}_Ғ, \bar{\beta}_Ғ) = \left(\sum_{j=1}^n \omega_i\, \alpha_Ғ(\mathcal{v}_j),\ \sum_{j=1}^n \omega_i\, \beta_Ғ(\mathcal{v}_j)\right) \quad (9)$$

Definition 7. (Senapati and Yager, 2019). Let $\text{Ғ} = \{\langle \vartheta_j, \alpha_{\text{Ғ}}(\vartheta_j), \beta_{\text{Ғ}}(\vartheta_j)\rangle | \vartheta_j \in V\}$ be an FFS on V. The score function $\mathcal{S}(\text{Ғ})$ and accuracy function $\mathcal{A}(\text{Ғ})$ is given by

$$\mathcal{S}(\text{Ғ}) = \alpha_{\text{Ғ}}^3 - \beta_{\text{Ғ}}^3 \tag{10}$$

$$\mathcal{A}(\text{Ғ}) = \alpha_{\text{Ғ}}^3 + \beta_{\text{Ғ}}^3 \tag{11}$$

Definition 8. (Kirişci, 2022) For any two FFSs $\text{Ғ}, \dot{\text{G}} \in \text{FFS}\ (V)$, the correlation coefficients is defined as

$$\rho_K(\text{Ғ}, \dot{\text{G}}) = \frac{Cov.(\text{Ғ}, \dot{\text{G}})}{\sqrt{Var.\text{Ғ}}.\sqrt{Var.\dot{\text{G}}}} \tag{12}$$

$$Cov.(\text{Ғ}, \dot{\text{G}}) = \sum_{j=1}^{n}\left[\left(\alpha_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\alpha_{\mathcal{G}}(\vartheta_j)\right)^3 + \left(\beta_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\beta_{\mathcal{G}}(\vartheta_j)\right)^3 + \left(\gamma_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\gamma_{\mathcal{G}}(\vartheta_j)\right)^3\right]$$

$$Var.\text{Ғ} = \sum_{j=1}^{n}\left\{\left(\alpha_{\text{Ғ}}(\vartheta_j)\right)^6 + \left(\beta_{\text{Ғ}}(\vartheta_j)\right)^6 + \left(\gamma_{\text{Ғ}}(\vartheta_j)\right)^6\right\}^{2/3}$$

$$Var.\dot{\text{G}} = \sum_{j=1}^{n}\left\{\left(\alpha_{\dot{\text{G}}}(\vartheta_j)\right)^6 + \left(\beta_{\dot{\text{G}}}(\vartheta_j)\right)^6 + \left(\gamma_{\dot{\text{G}}}(\vartheta_j)\right)^6\right\}^{2/3}$$

and weighted measure is defined as

$$Cov.(\text{Ғ}, \dot{\text{G}}) = \frac{1}{n}\sum_{i=1}^{n} w_i \frac{\left[\left(\alpha_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\alpha_{\mathcal{G}}(\vartheta_j)\right)^3 + \left(\beta_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\beta_{\mathcal{G}}(\vartheta_j)\right)^3 + \left(\gamma_{\text{Ғ}}(\vartheta_j)\right)^3.\left(\gamma_{\mathcal{G}}(\vartheta_j)\right)^3\right]}{\left\{\left(\alpha_{\text{Ғ}}(\vartheta_j)\right)^6 + \left(\beta_{\text{Ғ}}(\vartheta_j)\right)^6 + \left(\gamma_{\text{Ғ}}(\vartheta_j)\right)^6\right\}^{1/3}\left\{\left(\alpha_{\dot{\text{G}}}(\vartheta_j)\right)^6 + \left(\beta_{\dot{\text{G}}}(\vartheta_j)\right)^6 + \left(\gamma_{\dot{\text{G}}}(\vartheta_j)\right)^6\right\}^{1/3}}.$$

4. Proposed Methodology

Even though the concept of correlation coefficient has been investigated in a lot of research for IFS, PFS, it fails to capture the key features of FFS. As a result, the outcomes of current methodologies cannot be relied. The correlation coefficient for FFS is defined axiomatically in this section.

Definition 9. Let $\text{Ғ} = \{\langle \vartheta_j, \alpha_{\text{Ғ}}(\vartheta_j), \beta_{\text{Ғ}}(\vartheta_j)\rangle | \vartheta_j \in V\}$ and $\dot{\text{G}} = \{\langle \vartheta_j, \alpha_{\dot{\text{G}}}(\vartheta_j), \beta_{\dot{\text{G}}}(\vartheta_j)\rangle | \vartheta_j \in V\}$ be two FFS, where $\alpha_{\text{Ғ}}(\vartheta_j)$, $\beta_{\text{Ғ}}(\vartheta_j) \in [0,1]$ are the MG and NMG and $\alpha_{\text{Ғ}}^3(\vartheta_j) + \beta_{\text{Ғ}}^3(\vartheta_j) \leq 1$ for each $\vartheta_j \in V$, we define deviations of Ғ and Ġ as

$$Dev.(\text{Ғ}) = \left(\left(\alpha_{\text{Ғ}}(\vartheta_j)\right)^3 - \bar{\alpha}_{\text{Ғ}}^3\right) - \left(\left(\beta_{\text{Ғ}}(\vartheta_j)\right)^3 - \bar{\beta}_{\text{Ғ}}^3\right) - \left(\left(\gamma_{\text{Ғ}}(\vartheta_j)\right)^3 - \bar{\gamma}_{\text{Ғ}}^3\right) \tag{13}$$

$$Dev.(\dot{\text{G}}) = \left(\left(\alpha_{\dot{\text{G}}}(\vartheta_j)\right)^3 - \bar{\alpha}_{\dot{\text{G}}}^3\right) - \left(\left(\beta_{\dot{\text{G}}}(\vartheta_j)\right)^3 - \bar{\beta}_{\dot{\text{G}}}^3\right) - \left(\left(\gamma_{\dot{\text{G}}}(\vartheta_j)\right)^3 - \bar{\gamma}_{\dot{\text{G}}}^3\right) \tag{14}$$

$\forall\ j = 1,2, \dots, n.$

Their variances are defined by

$$Var.\text{Ғ} = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ғ})\}^2 \tag{15}$$

$$Var.\dot{\text{G}} = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\dot{\text{G}})\}^2 \tag{16}$$

and their covariances is defined by

$$Cov.(\text{Ғ}, \dot{\text{G}}) = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ғ})\}.\{Dev.(\dot{\text{G}})\} \tag{17}$$

Definition 10. For any two FFSs $\text{Ғ}, \dot{\text{G}} \in \text{FFS}\ (V)$, the correlation coefficients are given as

$$\rho(\text{Ғ}, \dot{\text{G}}) = \frac{Cov.(\text{Ғ}, \dot{\text{G}})}{\sqrt{Var.\text{Ғ}}.\sqrt{Var.\dot{\text{G}}}} \tag{18}$$

where the components $Cov.(\text{Ƒ},\dot{\text{G}}), Var.\text{Ƒ}$ and $Var.\dot{\text{G}}$ are defined in (15), (16) and (17).

Proposition 1. Suppose Ƒ and Ġ are two FFS in V, then
(a) $Cov.(\text{Ƒ},\dot{\text{G}}) = Cov.(\dot{\text{G}},\text{Ƒ})$.
(b) $Cov.(\text{Ƒ},\text{Ƒ}) = Dev.(\text{Ƒ})$, or $Cov.(\dot{\text{G}},\dot{\text{G}}) = Dev.(\dot{\text{G}})$.
(c) $|Cov.(\text{Ƒ},\dot{\text{G}})| \leq \sigma_{\text{Ƒ}} \times \sigma_{\dot{\text{G}}}$, where $\sigma_{\text{Ƒ}}$ and $\sigma_{\dot{\text{G}}}$ are the standard deviations of Ƒ and Ġ, respectively.

Proof. (a) $Cov.(\text{Ƒ},\dot{\text{G}}) = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ƒ})\}.\{Dev.(\dot{\text{G}})\}$,

$$Cov.(\dot{\text{G}},\text{Ƒ}) = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\dot{\text{G}})\}.\{Dev.(\text{Ƒ})\}$$

Since $\{Dev.(\text{Ƒ})\}.\{Dev.(\dot{\text{G}})\} = \{Dev.(\dot{\text{G}})\}.\{Dev.(\text{Ƒ})\} \Rightarrow Cov.(\text{Ƒ},\dot{\text{G}}) = Cov.(\dot{\text{G}},\text{Ƒ})$

The proof of part (b) self-evident and straightforward.

(c) To prove this part, we use Cauchy's – Schwarz inequality. It states that for all sequence of real numbers a_i and b_i, we have
$|\langle a,b\rangle|^2 \leq \| a \|^2 \| b \|^2$ or
$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \leq \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right) \quad (19)$$

$\forall\ a = \{a_1, a_2, \dots, a_n\}$ and $b = \{b_1, b_2, \dots, b_n\} \in \mathbb{R}^n$. We have
$Cov.^2(\text{Ƒ},\dot{\text{G}}) = \frac{1}{(n-1)^2}\{Dev.(\text{Ƒ}).Dev.(\dot{\text{G}})\}^2 \leq \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ƒ})\}^2 \times \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\dot{\text{G}})\}^2$
$= Var.\text{Ƒ} \times Var.\dot{\text{G}}$.

Therefore, $|Cov.(\text{Ƒ},\dot{\text{G}})| \leq \sigma_{\text{Ƒ}} \times \sigma_{\dot{\text{G}}}$. Hence proved.

Theorem 1. Suppose Ƒ and Ġ are two FFS in V, then
(a) $\rho(\text{Ƒ},\dot{\text{G}}) = \rho(\dot{\text{G}},\text{Ƒ})$
(b) If $\text{Ƒ} = \lambda\,\dot{\text{G}}$ for some $\neq 0$, then $\rho(\text{Ƒ},\dot{\text{G}}) = 1$.
(c) $-1 \leq \rho(\text{Ƒ},\dot{\text{G}}) \leq 1$

Proof. (a) $\rho(\text{Ƒ},\dot{\text{G}}) = \frac{Cov.(\text{Ƒ},\dot{\text{G}})}{\sqrt{Var.\text{Ƒ}}.\sqrt{Var.\dot{\text{G}}}} = \frac{Cov.(\dot{\text{G}},\text{Ƒ})}{\sqrt{Var.\dot{\text{G}}}.\sqrt{Var.\text{Ƒ}}} = \rho(\dot{\text{G}},\text{Ƒ})$

(b) $\text{Ƒ} = \lambda\,\dot{\text{G}} \Rightarrow \alpha_{\text{Ƒ}} = \lambda\alpha_{\dot{\text{G}}}$, $\beta_{\text{Ƒ}} = \lambda\beta_{\dot{\text{G}}}$ and $\gamma_{\text{Ƒ}} = \lambda\gamma_{\dot{\text{G}}}$. We have
$$Dev.(\text{Ƒ}) = \left(\left(\alpha_{\text{Ƒ}}(v_j)\right)^3 - \bar{\alpha}_{\text{Ƒ}}^3\right) - \left(\left(\beta_{\text{Ƒ}}(v_j)\right)^3 - \bar{\beta}_{\text{Ƒ}}^3\right) - \left(\left(\gamma_{\text{Ƒ}}(v_j)\right)^3 - \bar{\gamma}_{\text{Ƒ}}^3\right)$$
$$= \lambda^3\left(\left(\alpha_{\dot{\text{G}}}(v_j)\right)^3 - \lambda^3\bar{\alpha}_{\dot{\text{G}}}^3\right) - \lambda^3\left(\left(\beta_{\dot{\text{G}}}(v_j)\right)^3 - \lambda^3\bar{\beta}_{\dot{\text{G}}}^3\right) - \lambda^3\left(\left(\gamma_{\dot{\text{G}}}(v_j)\right)^3 - \lambda^3\bar{\gamma}_{\dot{\text{G}}}^3\right)$$
$$= \lambda^3\left\{\left(\left(\alpha_{\text{Ƒ}}(v_j)\right)^3 - \bar{\alpha}_{\text{Ƒ}}^3\right) - \left(\left(\beta_{\text{Ƒ}}(v_j)\right)^3 - \bar{\beta}_{\text{Ƒ}}^3\right) - \left(\left(\gamma_{\text{Ƒ}}(v_j)\right)^3 - \bar{\gamma}_{\text{Ƒ}}^3\right)\right\}$$
$$= \lambda^3 Dev.(\dot{\text{G}})$$
and $Cov.(\text{Ƒ},\dot{\text{G}}) = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ƒ})\}.\{Dev.(\dot{\text{G}})\}$
$= \frac{1}{n-1}\sum_{j=1}^{n}\lambda^3\{Dev.(\dot{\text{G}})\}.\{Dev.(\dot{\text{G}})\}$
$= \lambda^3 Var.\dot{\text{G}}$.

Therefore,

$Var.\text{Ғ} = \frac{1}{n-1}\sum_{j=1}^{n}\{Dev.(\text{Ғ})\}^2 = \frac{\lambda^6}{n-1}\sum_{j=1}^{n}\{Dev.(\dot{\text{G}})\}^2 = \lambda^6 Var.\dot{\text{G}}.$

If $\lambda \neq 0$ then

$\rho(\text{Ғ},\dot{\text{G}}) = \frac{Cov.(\text{Ғ},\dot{\text{G}})}{\sqrt{Var.\text{Ғ}}.\sqrt{Var.\dot{\text{G}}}} = \frac{\lambda^3 Var.\dot{\text{G}}}{\sqrt{\lambda^6 Var.\dot{\text{G}}}.\sqrt{Var.\dot{\text{G}}}} = \frac{\lambda^3 Var.\dot{\text{G}}}{\lambda^3 Var.\dot{\text{G}}} = 1.$

(c) From proposition 1 (c), we have

$|Cov.(\text{Ғ},\dot{\text{G}})| \leq \sqrt{Var.(\text{Ғ})}.\sqrt{Var.(\dot{\text{G}})}$

$\Rightarrow -\sqrt{Var.(\text{Ғ})}.\sqrt{Var.(\dot{\text{G}})} \leq Cov_{\text{i}}(\text{Ғ},\dot{\text{G}}) \leq \sqrt{Var.(\text{Ғ})}.\sqrt{Var.(\dot{\text{G}})}$

$\Rightarrow -1 \leq \frac{Cov_i(\text{Ғ},\dot{\text{G}})}{\sqrt{Var.(\text{Ғ})}.\sqrt{Var.(\dot{\text{G}})}} \leq 1$

$\Rightarrow -1 \leq \rho(\text{Ғ},\dot{\text{G}}) \leq 1$. Hence proved.

4.1 Numerical Examples for Validation

Example 1. Consider two FFSs in V as

$\text{Ғ} = \{\langle v_1, 0.35, 0.62\rangle, \langle v_2, 0.59, 0.38\rangle, \langle v_3, 0.44, 0.56\rangle\}$,

$\dot{\text{G}} = \{\langle v_1, 0.37, 0.65\rangle, \langle v_2, 0.54, 0.31\rangle, \langle v_3, 0.38, 0.52\rangle\}$.

Now,

$\bar{\alpha}_{\text{Ғ}} = 0.46$, $\bar{\beta}_{\text{Ғ}} = 0.52$, and $\bar{\gamma}_{\text{Ғ}} = 0.901454$ and

$\bar{\alpha}_{\dot{\text{G}}} = 0.43$, $\bar{\beta}_{\dot{\text{G}}} = 0.493$, and $\bar{\gamma}_{\dot{\text{G}}} = 0.913456$.

$\therefore Var.\text{Ғ} = \frac{1}{n-1}\sum_{j=1}^{n}\left\{\left(\left(\alpha_{\text{Ғ}}(v_j)\right)^3 - \bar{\alpha}_{\text{Ғ}}^3\right) - \left(\left(\beta_{\text{Ғ}}(v_j)\right)^3 - \bar{\beta}_{\text{Ғ}}^3\right) - \left(\left(\gamma_{\text{Ғ}}(v_j)\right)^3 - \bar{\gamma}_{\text{Ғ}}^3\right)\right\}^2$

$= \frac{1}{2}[\{((0.35))^3 - (0.46))^3) - ((0.62))^3 - (0.52))^3) - ((0.895781)^3 - (0.901454)^3)\}^2 +$
$\{((0.59))^3 - (0.46))^3) - ((0.38))^3 - (0.52))^3) - ((0.90440)^3 - (0.901454)^3)\}^2 +$
$\{((0.44))^3 - (0.46))^3) - ((0.56))^3 - (0.52))^3) - ((0.904178)^3 - (0.901454)^3)\}^2]$

$\Rightarrow \sigma_{\text{Ғ}} = \sqrt{\frac{1}{2}(0.019165491 + 0.034807811 + 0.002896752)} = 0.168626887.$

$Var.\dot{\text{G}} = \frac{1}{n-1}\sum_{j=1}^{n}\left\{\left(\left(\alpha_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\alpha}_{\dot{\text{G}}}^3\right) - \left(\left(\beta_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\beta}_{\dot{\text{G}}}^3\right) - \left(\left(\gamma_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\gamma}_{\dot{\text{G}}}^3\right)\right\}^2$

$= \frac{1}{2}[\{((0.37))^3 - (0.43))^3) - ((0.65))^3 - (0.493))^3) - ((0.877)^3 - (0.913456)^3)\}^2 + \{((0.54))^3 -$
$(0.43))^3) - ((0.31))^3 - (0.493))^3) - ((0.9332)^3 - (0.913456)^3)\}^2 + \{((0.38))^3 - (0.43))^3) -$
$((0.52))^3 - (0.493))^3) - ((0.9300)^3 - (0.913456)^3)\}^2]$

$\Rightarrow \sigma_{\dot{\text{G}}} = \sqrt{\frac{1}{2}(0.009205287 + 0.013848068 + 0.007657332)}$

$= 0.12391668$

and

$Cov.(\text{Ғ},\dot{\text{G}}) = \frac{1}{n-1}\sum_{j=1}^{n}\left\{\left(\left(\alpha_{\text{Ғ}}(v_j)\right)^3 - \bar{\alpha}_{\text{Ғ}}^3\right) - \left(\left(\beta_{\text{Ғ}}(v_j)\right)^3 - \bar{\beta}_{\text{Ғ}}^3\right) - \left(\left(\gamma_{\text{Ғ}}(v_j)\right)^3 - \bar{\gamma}_{\text{Ғ}}^3\right)\right\}.\left\{\left(\left(\alpha_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\alpha}_{\dot{\text{G}}}^3\right) - \left(\left(\beta_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\beta}_{\dot{\text{G}}}^3\right) - \left(\left(\gamma_{\dot{\text{G}}}(v_j)\right)^3 - \bar{\gamma}_{\dot{\text{G}}}^3\right)\right\}$

$= \frac{1}{2}[\{((0.35))^3 - (0.46))^3) - ((0.62))^3 - (0.52))^3) - ((0.895781)^3 - (0.901454)^3)\}.\{((0.37))^3 - (0.43))^3) - ((0.65))^3 - (0.493))^3) - ((0.877)^3 - (0.913456)^3)\} + \{((0.59))^3 - (0.46))^3) - ((0.38))^3 - (0.52))^3) - ((0.90440)^3 - (0.901454)^3)\}.\{((0.54))^3 - (0.43))^3) - ((0.31))^3 - (0.493))^3) - ((0.9332)^3 - (0.913456)^3)\} + \{((0.44))^3 - (0.46))^3) - ((0.56))^3 - (0.52))^3) - ((0.904178)^3 - (0.901454)^3)\}.\{((0.38))^3 - (0.43))^3) - ((0.52))^3 - (0.493))^3) - ((0.9300)^3 - (0.913456)^3)\}]$
$= \frac{1}{2}(0.013282463 + 0.021954975 + 0.004709713)$
$= 0.0199736.$

Hence, the correlation coefficient between Ғ and Ġ for FFS is given by

$$\rho(\text{Ғ}, \dot{G}) = \frac{Cov.(\text{Ғ},\dot{G})}{\sqrt{Var.\text{Ғ}}.\sqrt{Var.\dot{G}}} = \frac{0.0199736}{0.168626887\times 0.12391668} = 0.955871.$$

Now, we take two examples to legalize the performance rating of our suggested measure with the existing methods.

Example 2. Consider an example where Ғ and Ġ are two FFS in V which shows a positive correlation between them.

$$\text{Ғ} = \left\{\left\langle\frac{0.14, 0.17}{v_1}\right\rangle, \left\langle\frac{0.14, 0.17}{v_2}\right\rangle, \left\langle\frac{0.17, 0.65}{v_3}\right\rangle\right\},$$
$$\dot{G} = \left\{\left\langle\frac{0.49, 0.595}{v_1}\right\rangle, \left\langle\frac{0.49, 0.595}{v_2}\right\rangle, \left\langle\frac{0.595, 0.70}{v_3}\right\rangle\right\}.$$

Here we can see a linear relationship between Ғ and Ġ as

$$\dot{G} = 3.5\text{Ғ}.$$

Table 1 depicts the comparative study of correlation coefficient between Ғ and Ġ for various methods

Table 1. Comparative results of correlation coefficient.

Methods	$\boldsymbol{\rho}(\text{Ғ}, \dot{G})$
Gerstenkorn and Mańko (1991)	**1**
Xu et al. (2008)	0.73487
Garg (2016)	0.572807
Garg (2018)	0.596709
Thao (2019)	**1**
Ejegwa (2021)	0.734874
Ejegwa et al. (2002)	**0.903535**
Kirişci (2022)	0.883193
Our method	**0.997042≈ 1**

Example 3. Consider an example where Ғ and Ġ are two FFS in V which shows a negative correlation between them.

$$\text{Ғ} = \left\{\left\langle\frac{0.84, 0.07}{v_1}\right\rangle, \left\langle\frac{0.74, 0.17}{v_2}\right\rangle, \left\langle\frac{0.69, 0.20}{v_3}\right\rangle\right\},$$
$$\dot{G} = \left\{\left\langle\frac{0.57, 0.80}{v_1}\right\rangle, \left\langle\frac{0.67, 0.70}{v_2}\right\rangle, \left\langle\frac{0.70, 0.65}{v_3}\right\rangle\right\}.$$

Here we can see the relationship between MD and NMD of Ƒ and Ġ as
$\beta_{\dot{G}}(\upsilon_j) = \alpha_{F}(\upsilon_j) - 0.04$ and $\beta_{F}(\upsilon_j) = \alpha_{\dot{G}}(\upsilon_j) - 0.50$.

Table 2 depicts the comparative study of correlation coefficient between Ƒ and Ġ for various methods.

Table 2. Comparative results of correlation coefficient.

Methods	$\boldsymbol{\rho}(F, \dot{G})$
Gerstenkorn and Mańko (1991)	0.782176
Xu et al. (2008)	0.74334
Garg (2016)	0.564408
Garg (2018)	0.664948
Thao (2019)	**-0.99952**
Ejegwa (2021)	0.743345
Ejegwa et al. (2022)	0.893621
Kirişci (2022)	0.722624
Our method	**-0.99934≈ −1**

4.2 Discussions

To authenticate the obtained results, we intended to compare them with the findings of some existing methods. Since using FFS is comparatively a novel concept and the present analysis is one of the first study in the field, we could compare proposed with only with the approach introduced by Kirişci (2022) for FFS. However, comparison with other measures of IFS and PFS are also considered. In example 2, only Gerstenkorn and Mańko (1991) and Thao (2019) measures shows value equal to 1 and our proposed measure also shows value equivalent to 1 as suggested measure incorporates hesitancy grade. Other measures do not show high degree of positive correlation. In the example 3, Thao (2019) and our method shows negative values. All other methods show only positive value. These methods suffer from the fact that correlation coefficient values lie in the interval [0,1]. From the above two examples, it has been concluded that our suggested method of computing correlation coefficient gives more precise results (high degree of positive correlation in the first example and high degree of negative correlation in the second) as compared to other existing methods as shown in Table 1 and Table 2.

4.3 Weighted Measure

In many real-life applications, weights (ω) of an object play an important role to be considered. Each criterion in decision-making usually have distinct significance, hence should be assigned different weights. Here, we extend our proposed correlation measure $\rho(F, \dot{G})$ to $\rho_\omega(\mathcal{F}, \dot{G})$ in the subsequent definition.

Definition 11. If two FFSs $F, \dot{G} \in$ FFS (V), the weighted correlation coefficient is defined by

$$\rho_\omega(F, \dot{G}) = \frac{Cov_\omega\,(F, \dot{G})}{\sqrt{Var_\omega\, F}.\sqrt{Var_\omega\, \dot{G}}} \tag{20}$$

where, the components $Cov_\omega\,(F, \dot{G}), Var_\omega\, F$ and $Var_\omega\, \dot{G}$ are defined as

$$Cov_\omega(F, \dot{G}) = \frac{1}{n-1}\sum_{j=1}^{n} \omega_j \left\{\left(\left(\alpha_F(\upsilon_j)\right)^3 - \bar{\alpha}_F^3\right) - \left(\left(\beta_F(\upsilon_j)\right)^3 - \bar{\beta}_F^3\right) - \left(\left(\gamma_F(\upsilon_j)\right)^3 - \bar{\gamma}_F^3\right)\right\}.\left\{\left(\left(\alpha_{\dot{G}}(\upsilon_j)\right)^3 - \bar{\alpha}_{\dot{G}}^3\right) - \left(\left(\beta_{\dot{G}}(\upsilon_j)\right)^3 - \bar{\beta}_{\dot{G}}^3\right) - \left(\left(\gamma_{\dot{G}}(\upsilon_j)\right)^3 - \bar{\gamma}_{\dot{G}}^3\right)\right\} \tag{21}$$

$$Var_\omega\, F = \frac{1}{n-1}\sum_{j=1}^{n} \omega_j \left\{\left(\left(\alpha_F(\upsilon_j)\right)^3 - \bar{\alpha}_F^3\right) - \left(\left(\beta_F(\upsilon_j)\right)^3 - \bar{\beta}_F^3\right) - \left(\left(\gamma_F(\upsilon_j)\right)^3 - \bar{\gamma}_F^3\right)\right\}^2 \tag{22}$$

$$Var_{\omega}\ \dot{G} = \frac{1}{n-1}\sum_{j=1}^{n}\omega_j\left\{\left(\left(\alpha_{\dot{G}}(v_j)\right)^3 - \bar{\alpha}_{\dot{G}}^3\right) - \left(\left(\beta_{\dot{G}}(v_j)\right)^3 - \bar{\beta}_{\dot{G}}^3\right) - \left(\left(\gamma_{\dot{G}}(v_j)\right)^3 - \bar{\gamma}_{\dot{G}}^3\right)\right\}^2 \quad (23)$$

We consider ω as the weight vector of $V = \{v_1, v_2, \ldots, v_n\}$ with $\omega \geq 0$ and $\sum_{j=1}^{n}\omega_j = 1$. In the above, if we assume $\omega = 1$, (20) reduces to (18).

Proposition 2. Suppose Ғ and Ġ are two FFS in V, then
(a) $Cov_{\omega}(Ғ, \dot{G}) = Cov_{\omega}(\dot{G}, Ғ)$.
(b) $Cov_{\omega}(Ғ, Ғ) = Dev_{\omega}(Ғ)$, or $Cov_{\omega}(\dot{G}, \dot{G}) = Dev_{\omega}(\dot{G})$.
(c) $|Cov_{\omega}(Ғ, \dot{G})| \leq \sqrt{Var_{\omega}(Ғ)} \times \sqrt{Var_{\omega}(\dot{G})}$
Proof. The proofs are similar to as done in preposition 1.

Theorem 2. Suppose Ғ and Ġ are two FFS in V, then
(a) $\rho_{\omega}(Ғ, \dot{G}) = \rho_{\omega}(\dot{G}, Ғ)$
(b) If $Ғ = \lambda\ \dot{G}$ for some $\neq 0$, then $\rho_{\omega}(Ғ, \dot{G}) = 1$.
(c) $-1 \leq \rho_{\omega}(Ғ, \dot{G}) \leq 1$
Proof. The proofs are similar to as done in theorem 1

5. Model Demonstration: Decision Making

Recognizing a decision, gaining information, and calculating probable remedies are all aspects in the decision-making process. It is a systematic procedure for dealing with complex problems such as medical diagnosis, recognition of patterns, clustering and multi criteria decision making and so on. In this section, we use our proposed correlation coefficient for Fermatean fuzzy sets and the weighted suggested method to address decision-making problems involving pattern recognition and multi-criteria decision-making problems, as it is the most reliable correlation coefficient of Fermatean fuzzy sets technique. To strengthen the reliability and uniqueness our measures, comparative analysis is performed in both the applications.

5.1 Pattern Analysis

Example 4. Let there be three known patterns $Ғ_1, Ғ_2$ and $Ғ_3$ and Ġ be an unknown pattern represented by FFS in a finite universe of discourse $V = \{v_1, v_2, v_3\}$ as

$Ғ_1 = \left\{\left\langle\frac{1.0, 0.0}{v_1}\right\rangle, \left\langle\frac{0.8, 0.0}{v_2}\right\rangle, \left\langle\frac{0.7, 0.1}{v_3}\right\rangle\right\}$,

$Ғ_2 = \left\{\left\langle\frac{0.8, 0.1}{v_1}\right\rangle, \left\langle\frac{0.1, 0.0}{v_2}\right\rangle, \left\langle\frac{0.9, 0.1}{v_3}\right\rangle\right\}$,

$Ғ_3 = \left\{\left\langle\frac{0.6, 0.2}{v_1}\right\rangle, \left\langle\frac{0.8, 0.0}{v_2}\right\rangle, \left\langle\frac{0.1, 0.0}{v_3}\right\rangle\right\}$,

and $\dot{G} = \left\{\left\langle\frac{0.5, 0.3}{v_1}\right\rangle, \left\langle\frac{0.6, 0.2}{v_2}\right\rangle, \left\langle\frac{0.8, 0.1}{v_3}\right\rangle\right\}$ where $\omega = \{0.5, 0.3, 0.2\}$ is the set of weights of V with $\omega_j \geq 0$ and $\sum_{j=1}^{3}\omega_j = 1$.

Now our aim is to classify the pattern Ġ into $Ғ_1, Ғ_2$ or $Ғ_3$. For this, we apply proposed measure (18) for computing correlation coefficient between $Ғ_j (j = 1,2,3)$ and Ġ. Comparative analysis of the proposed measure with the existing measures is shown in Table 3 and Figure 1.

Table 3. Pattern recognition results for FFS.

Methods	$\boldsymbol{\rho}(\text{Ƒ}_1, \dot{G})$	$\boldsymbol{\rho}(\text{Ƒ}_2, \dot{G})$	$\boldsymbol{\rho}(\text{Ƒ}_3, \dot{G})$
Xu et al. (2008)	0.81103	**0.90745**	0.86756
Garg (2016)	0.672639	**0.85620**	0.73496
Garg (2018)	0.83190	*NA*	0.908984
Thao (2019)	-0.91531	**0.370597**	-0.60034
Ejegwa (2021)	0.811029	**0.907446**	0.867562
Ejegwa et al. (2022)	0.836576	**0.939847**	0.919121
Kirişçi (2022)	0.564781	**0.883148**	0.826193
Our method	-0.81524	**0.551349**	-0.64006

From the above results shown in Table 3, our proposed measure shows optimal allocation of pattern $\dot{G}$ belongs to the class Ƒ_2. Also, results are analogous to the existing measures reveals the reliability and uniqueness of the novel measures.

Computation of weighted correlation coefficient using (20) between $\text{Ƒ}_j (j = 1,2,3)$ and $\dot{G}$ with weight vector $\omega = \{0.3, 0.4, 0.3\}$ and their comparative analysis is shown in Table 4 and Figure 2.

Table 4. Pattern recognition results for FFS with weights.

Methods	$\boldsymbol{\rho}(\text{Ƒ}_1, \dot{G})$	$\boldsymbol{\rho}(\text{Ƒ}_2, \dot{G})$	$\boldsymbol{\rho}(\text{Ƒ}_3, \dot{G})$
Xu et al. (2008)	0.8244	**0.89977**	0.87528
Garg (2016)	0.68967	**0.855629**	0.746521
Garg (2018)	0.491556	***0.609842***	0.532076
Thao (2019)	-0.90435	**0.373492**	-0.56892
Ejegwa (2021)	0.824402	**0.899772**	0.875282
Ejegwa et al. (2022)	0.845793	**0.931647**	0.919796
Kirişçi (2022)	0.589995	**0.891502**	0.82775
Our method	-0.7635	**0.556412**	-0.62438

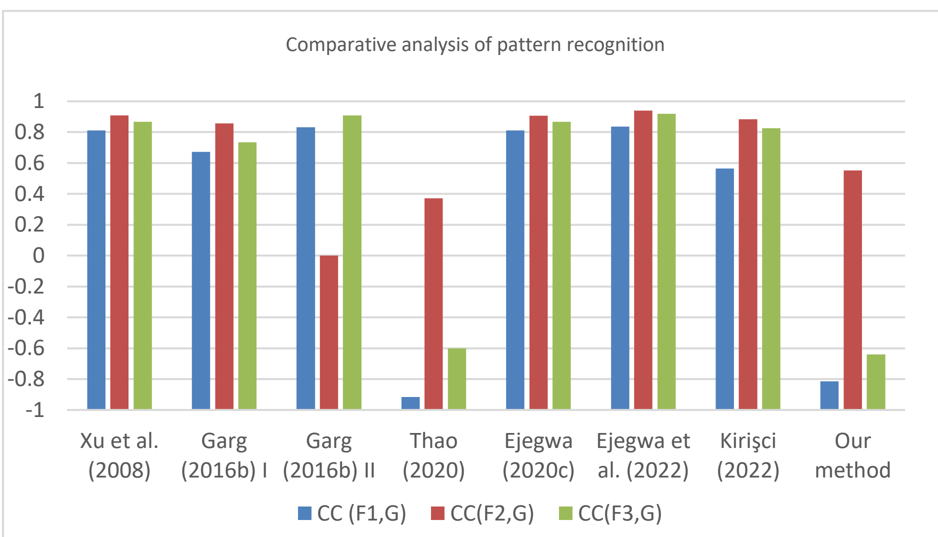

Figure 1. Comparative analysis of pattern recognition.

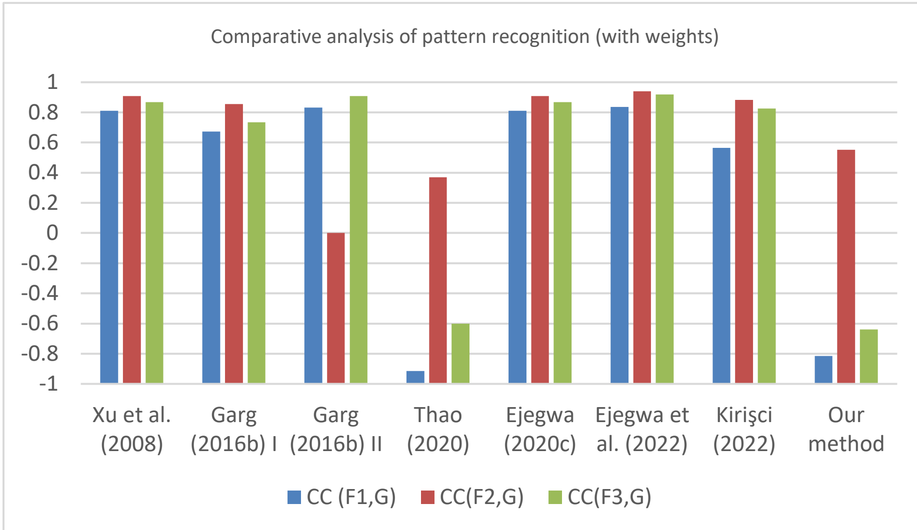

Figure 2. Comparative analysis of pattern recognition (weighted measures).

Figure 1 shows the graphical representation of comparative study of the suggested method and the prevailing correlation measures given by different authors like Xu et al. (2008), Garg (2016), Thao (2019), Ejegwa (2021) and Kirisci (2022) based on pattern recognition. Figure 2 shows the comparison done for the weighted measure. The graphical representation here adds one more dimension to the comparison and shows utility of the measures introduced.

5.2 Application to Supplier Selection

In MCDM, FFS is a preferable applied concept. We demonstrate that the designed correlation coefficient measures for FFS can be used to solve uncertainty and ambiguity in MCDM problems. For MCDM problem under FFS domain, assume that there are m options $\dot{S}_i(i = 1,2, \ldots m)$ and m criteria $\acute{Ç}_j(j = 1,2, \ldots n)$ with the vector weight of criteria $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^t$ with the condition $\omega_j \in [0,1]$ and $\sum_{j=1}^{n} \omega_j = 1$. The decision maker set weights of the criteria as $\omega = \{0.40, 0.20, 0.10, 0.30\}$.

We construct the decision matrix $\check{D} = \{\acute{Ç}_j(\dot{S}_i)\}_{m \times n}$ where $\acute{Ç}_j(\dot{S}_i) = (\alpha_{ij}, \beta_{ij})$ is a Fermatean score number for the alternative $\dot{S}_i(i = 1,2, \ldots m)$ with criteria $\acute{Ç}_j(j = 1,2, \ldots n)$.

5.2.1 Algorithm of the Proposed Method

Step 1. Formulate the Fermatean fuzzy decision matrix $\check{D} = \{\mathfrak{C}_j(\hat{A}_i)\}_{m \times n}$ as discussed above.

Step 2. Obtain the normalized decision matrix $\acute{Ç} = (\alpha_{ij}^*, \beta_{ij}^*)_{m \times n}$ where

$$(\alpha_{ij}^*, \beta_{ij}^*)_{m \times n} = \begin{cases} (\alpha_{ij}, \beta_{ij}), for\ benefit\ criterion\ \mathfrak{C}_j \\ (\beta_{ij}, \alpha_{ij}), for\ cost\ criterion\ \mathfrak{C}_j \end{cases} \quad (24)$$

Step 3. Compute positive ideal solution (PIS) $\mathfrak{I}^+ = \{r_1^+, r_2^+, \ldots, r_n^+\}$ and negative ideal solution $\mathfrak{I}^- = \{r_1^-, r_2^-, \ldots, r_n^-\}$ where,

$$\mathfrak{I}^{+} = \begin{cases} \max\langle \acute{Ç}_j(\dot{S}_i)\rangle; j = 1,2, \dots, n \\ if\ \acute{Ç}_j\ is\ a\ benefit\ criteria \\ \min\langle \acute{Ç}_j(\dot{S}_i)\rangle; j = 1,2, \dots, n \\ if\ \acute{Ç}_j\ is\ a\ cost\ criteria \end{cases} \quad (25)$$

and

$$\mathfrak{I}^{-} = \begin{cases} \min\langle \acute{Ç}_j(\dot{S}_i)\rangle; j = 1,2, \dots, n \\ if\ \acute{Ç}_j\ is\ a\ benefit\ criteria \\ \max\langle \acute{Ç}_j(\dot{S}_i)\rangle; j = 1,2, \dots, n \\ if\ \acute{Ç}_j\ is\ a\ cost\ criteria \end{cases} \quad (26)$$

Step 4. Compute the coefficient of correlation of each alternative $\dot{S}_i (i = 1,2, \dots, m)$ with PIS $\mathfrak{I}^{+}$ and NIS $\mathfrak{I}^{-}$, i.e., $\rho(\dot{S}_i , \mathfrak{I}^{+})$ and $\rho(\dot{S}_i , \mathfrak{I}^{-})$.

Step 5. Compute the closeness index $\mathfrak{E}(\dot{S}_i)$ of each alternative $\dot{S}_i (i = 1,2, \dots, m)$ using

$$\acute{Ç}(\dot{S}_i) = \frac{\rho(\dot{S}_i , \mathfrak{I}^{+})}{\rho(\dot{S}_i , \mathfrak{I}^{+}) + \rho(\dot{S}_i , \mathfrak{I}^{-})} \quad (27)$$

Step 6. Decide the optimal ranking of alternatives in decreasing order of their relative closeness coefficient. The alternative with the largest closeness coefficient $\acute{Ç}(\dot{S}_i)$ is the best/leading alternative.

The unexpected outbreak of COVID-19 has caused major supply disruption for numerous international organizations. Weakness of many organisations have been highlighted as a result of the pandemic, particularly those that rely on global supply networks and are highly depended on production centres and major marketplaces. The prospect of the COVID-19 pandemic spreading around the world has sparked concerns about the destruction and recovery of global supply chains. Managers now take supply disruption into account seriously due to the pandemic to maintain quality. Existing research, on the other hand, does not take this goal into account. To fill this research gap, this case study provides a model that attempts to handle the problem of sustainable supplier selection.

One of the most crucial decision obstacles for organizations looking to reduce supply chain expenses is the supplier selection process (Villa Silva et al., 2019). Because there is so much ambiguous information, supplier selection is commonly a MCDM problem. The proposed correlation coefficients for FFS is applied to address the hesitations inherent in associating options, conditions, and the opinion of decision creators to r.

For the past few years, the necessity of sustainable supply chain management has been recognized by organizations and researchers. In supply chain management, sustainable supplier selection and company performance are significant. This case study is about an automotive manufacturing organization.

Due to supply chain difficulties caused during COVID-19, the company encountered breakdown due to no viable suppliers and no effective plan in place to deal with the issue. In a challenging attempt to move forward from the current situation, the company is currently modifying its previous strategy of suppliers. The current application provides a systematic framework to assist the management of the organization in this direction to resolve this issue.

Example 5. Consider the situation that a manufacturing enterprise is experiencing with supplier selection to reduce costs during the finished goods manufacturing phase. The organization is examining five suppliers $\dot{S} = \{\dot{S}_1, \dot{S}_2, \dots, \dot{S}_5\}$ under the four criteria $\acute{Ç} = \{\acute{Ç}_1, \acute{Ç}_2, \dots, \acute{Ç}_4\}$ as price ($\acute{Ç}_1$), facility ($\acute{Ç}_2$), lead time ($\acute{Ç}_3$) and quality ($\acute{Ç}_4$).

Step 1. In accordance with the decision maker's evaluation/input, Fermatean fuzzy decision matrix is defined in Table 5.

Table 5. Decision matrix for FFS.

	$\acute{Ç}_1$	$\acute{Ç}_2$	$\acute{Ç}_3$	$\acute{Ç}_4$
$\dot{S}_1$	⟨0.17, 0.72⟩	⟨ 0.59, 0.31⟩	⟨ 0.84, 0.10⟩	⟨0.66, 0.23⟩
$\dot{S}_2$	⟨0.33, 0.62⟩	⟨ 0.60, 0.29⟩	⟨0.78, 0.11⟩	⟨0.53, 0.36⟩
$\dot{S}_3$	⟨ 0.11, 0.78⟩	⟨0.64, 0.25⟩	⟨0.76, 0.17⟩	⟨ 0.74, 0.15⟩
$\dot{S}_4$	⟨ 0.20, 0.70⟩	⟨ 0.57, 0.32⟩	⟨0.76, 0.12⟩	⟨ 0.64, 0.25⟩
$\dot{S}_5$	⟨0 .27, 0.56⟩	⟨ 0.46, 0.43⟩	⟨0.66, 0.23⟩	⟨0.52, 0.37⟩

Step 2. Establish normalized decision matrix (Table 6) using (24). Here, price ($\acute{Ç}_1$): the most minimum values are chosen (*cost criteria*), facility ($\acute{Ç}_2$): high valuations are chosen (*benefit criteria*), Lead time ($\acute{Ç}_3$): high valuations are chosen (*benefit criteria*), Quality ($\acute{Ç}_4$): high valuations are chosen (*benefit criteria*).

Table 6. Normalized decision matrix for FFS.

	$\acute{Ç}_1$	$\acute{Ç}_2$	$\acute{Ç}_3$	$\acute{Ç}_4$
$\dot{S}_1$	⟨0.72, 0.17⟩	⟨0.59, 0.31⟩	⟨0.84, 0.10⟩	⟨0.66, 0.23⟩
$\dot{S}_2$	⟨ 0.62, 0.33⟩	⟨0.60, 0.29⟩	⟨0.78, 0.11⟩	⟨0.53, 0.36⟩
$\dot{S}_3$	⟨0 .78, 0.11⟩	⟨0.64, 0.25⟩	⟨0.76, 0.17⟩	⟨0.74, 0.15⟩
$\dot{S}_4$	⟨ 0.70, 0.20⟩	⟨0.57, 0.32⟩	⟨0.76, 0.12⟩	⟨0.64, 0.25⟩
$\dot{S}_5$	⟨0.56, 0.27⟩	⟨0.46, 0.43⟩	⟨0.66, 0.23⟩	⟨0.52, 0.37⟩

Step 3. Determine PIS ($\mathfrak{I}^+$) and NIS ($\mathfrak{I}^-$) using (25) and (26) is shown in Table 7 as

Table 7. PIS and NIS for each criterion.

	$\acute{Ç}_1$	$\acute{Ç}_2$	$\acute{Ç}_3$	$\acute{Ç}_4$
$\mathfrak{I}^+$	⟨0.56, 0.33⟩	⟨0.64, 0.25⟩	⟨0.84, 0.10⟩	⟨0.74, 0.15⟩
$\mathfrak{I}^-$	⟨0.78, 0.11⟩	⟨0.46, 0.43⟩	⟨0.66, 0.23⟩	⟨0.52,0.37⟩

Step 4. With the help of step 3, we apply proposed correlation coefficient given in (18), we compute correlation coefficient of each alternative $\dot{S}_i (i = 1,2, \dots m)$ with PIS ($\mathfrak{I}^+$) and NIS ($\mathfrak{I}^-$) in Table 8.

Table 8. Correlation coefficient for each alternative.

	$\rho(\dot{S}_i, \mathfrak{I}^+)$	$\rho(\dot{S}_i, \mathfrak{I}^-)$
$\dot{S}_1$	0.697069	0.507374
$\dot{S}_2$	0.685812	0.304209
$\dot{S}_3$	0.177393	0.812385
$\dot{S}_4$	0.570362	0.656539
$\dot{S}_5$	0.721248	0.480256

Step 5. Obtain closeness coefficient $\acute{Ç}(\dot{S}_i)$ using (27) and shown in Table 9.

Step 6. Finally, establish the classification of alternatives as per the descending order of $\acute{Ç}(\dot{S}_i)$

$$\dot{S}_2 \succ \dot{S}_5 \succ \dot{S}_1 \succ \dot{S}_4 \succ \dot{S}_3.$$

Therefore, $\dot{S}_2$ is carefully chosen as the best supplier as the maximum among correlation coefficient is selected as the best option.

Table 9. Closeness coefficient for each alternative.

	$\acute{Ç}(\dot{S}_i)$	Ranking
$\dot{S}_1$	0.578748	3
$\dot{S}_2$	0.692725	1
$\dot{S}_3$	0.179225	5
$\dot{S}_4$	0.464880	4
$\dot{S}_5$	0.600288	2

In a similar way, we can apply our proposed weighted correlation coefficient measure for model validation. The decision maker set Assuming weights of the criteria as $\omega = \{0.40, 0.20, 0.10, 0.30\}$ to be set as per the decision maker's preference, we have calculated $\rho_\omega(\dot{S}_i, \mathfrak{I}^+)$ and $\rho_\omega(\dot{S}_i, \mathfrak{I}^-)$ in Table 10.

Table 10. Weighted correlation coefficient for each alternative.

	$\rho_\omega(\dot{S}_i, \mathfrak{I}^+)$	$\rho_\omega(\dot{S}_i, \mathfrak{I}^-)$
$\dot{S}_1$	0.411116	0.481467
$\dot{S}_2$	0.442479	0.210162
$\dot{S}_3$	-0.18119	0.823919
$\dot{S}_4$	0.188847	0.688759
$\dot{S}_5$	0.459438	0.437057

Table 11. Weighted closeness coefficient for each alternative.

	$\acute{Ç}(\dot{S}_i)$	Ranking
$\dot{S}_1$	0.460591	3
$\dot{S}_2$	0.677982	1
$\dot{S}_3$	-0.281910	5
$\dot{S}_4$	0.215184	4
$\dot{S}_5$	0.512483	2

From the above table, we can conclude that $\dot{S}_2$ is chosen as the best supplier. Based on the numerical computations in Table 9 and 11, $\dot{S}_2$ is selected (with or without weights) as the best supplier among others. The ranking outcomes shown in our proposed correlation coefficient measures for FFS rank the alternative in similar order and there is no contradiction in ranking the alternatives (weighted and without weights).

6. Conclusions

The correlation coefficient measures are applied to improve the decision results of the real-life applications. In this paper, novel correlation coefficient measures for FFS are proposed and proved some of their essential properties. The goal of the deliberation is to propose more reliable, efficient, and resourceful methods of computing correlation coefficient for FFS. The proposed measures in contrast to the pioneer work on correlation coefficient in IF, PF and FF environment such as Gerstenkorn and Mańko (1991), Garg (2016), Xu et al. (2008), Thao (2019), Ejegwa (2021), and Kirişci (2022) provides a better performing index with accuracy and efficiency in computation. The proposed correlation coefficient measures have by means of numerical examples been proven to be capable of handling situations where the present correlation coefficients fail. A weighted correlation coefficient measure for FFS has also been determined to cope with situations when the elements in a set are interdependent.

The salient features of the proposed correlation coefficient measures for FFS have been considered as

- The article extends the concept of correlation coefficient from IFS and PFS to FFS, removing the restriction that $\alpha_{\mathrm{F}}(\upsilon_j) + \beta_{\mathrm{F}}(\upsilon_j) \leq 1$ or $\alpha_{\mathrm{F}}^2(\upsilon_j) + \beta_{\mathrm{F}}^2(\upsilon_j) \leq 1$.
- Illustrations described in the paper clearly expresses that in many situations the existing measures do not provide productive results.
- The value of correlation coefficient always lies between $[-1,1]$. However, many existing measures fail to handle situation of values lying between $[-1,0]$.
- The proposed correlation coefficient has been demonstrated to be capable of handling situations where the present correlation coefficient in the FFS scenario fails as shown in Table 1 and Table 2.

Practical applications of the projected methods in pattern recognition and MCDM have been considered to discover the usefulness of the suggested coefficients. From the results, it has been revealed that the anticipated correlation coefficient measures for Fermatean fuzzy environment can certainly manage the real-life decision-making complications. One of the restrictions of this approach is that it is inadequate to deliberate the assessment information from individual decision maker when making the decision results. We look forward in developing some more generalizations of the proposed correlation coefficient measures for PFS and apply them to medical diagnosis, clustering analysis and MCDM through TOPSIS, COPRAS and VIKOR etc.

Conflict of Interest

Authors declare there is no conflict of Interest.

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