International Journal of Mathematical, Engineering and Management Sciences
Vol. 11, No. 1, 353-375, 2026
https://doi.org/10.33889/IJMEMS.2026.11.1.015

An Integrated Inventory Model for Imperfect Production with Environmental Costs and Carbon Taxation

Saurabh Bahuguna
Department of Mathematics,
Graphic Era Hill University, Dehradun, 248002, Uttarakhand, India.
E-mail: bahugunasauri1996@gmail.com

Shilpy Tayal
Department of Mathematics,
Graphic Era Hill University, Dehradun, 248002, Uttarakhand, India.
Corresponding author: agarwal_shilpy83@yahoo.com

(Received on May 15, 2025; Revised on June 18, 2025 & July 21, 2025 & August 25, 2025 & October 3, 2025; Accepted on October 6, 2025)

Abstract
This study introduces a joint inventory framework that links the vendor and the buyer, while addressing the production of both good-quality and defective units. During inspection, the defective items are separated to ensure that only flawless products are delivered to the buyer in several shipments. Instead of being discarded, the rejected units are directed to a secondary market where they are accepted at a lower price, which helps in minimizing waste. The model also integrates environmental aspects by considering carbon emissions generated in manufacturing, transportation, and storage activities. To promote greener practices, a carbon tax is incorporated, aligned with real-world pricing mechanisms and environmental regulations. This tax motivates firms to control emissions and adopt sustainable operations. The paper further develops a mathematical model to identify the optimal delivery schedule and cycle length under the effect of carbon taxation. A numerical example is presented to demonstrate the applicability of the model. The sensitivity analysis shows that while rising demand boosts profits, higher production speeds may decrease them, and increased carbon costs lower vendor earnings. These findings highlight the importance of strategic demand management, balanced production planning, and the adoption of eco-efficient technologies.

Keywords- Supply chain management, Production, Imperfect items, Carbon emission, Primary and secondary market, Carbon taxes.

1. Introduction

The Economic Production Quantity (EPQ) model is a refined approach to inventory management that determines the most economical batch size for production. Its purpose is to balance the trade-off between production levels and inventory-related costs, allowing firms to lower overall expenses while improving profitability. Unlike basic models that assume a constant production rate, the EPQ framework reflects the realities of actual operations, where production speed may fluctuate. Issues such as machine breakdowns, variations in workforce productivity, and scheduled maintenance can all affect the pace of output. By taking these uncertainties into account, the EPQ model provides solutions that are both realistic and efficient, helping businesses avoid overproduction or shortages while making better use of resources and meeting customer needs.

Supply Chain Management (SCM) refers to the coordination of interconnected businesses that work together to move products and services from their raw material stage to the final consumer. It encompasses all key activities, including procurement of materials, production, inventory storage, transportation, and the timely delivery of finished goods. The primary aim of SCM is to create a smooth and efficient flow across the entire chain, reducing costs while increasing overall value for customers. This efficiency is achieved through better management of material movement, information sharing, and financial resources throughout production and distribution.

Carbon emissions have become an important concern in supply chain management because they harm the environment and increase global warming. These emissions mostly come from transportation, manufacturing, energy use, and logistics. Earlier, supply chains mainly focused on cost, efficiency, and service. But now, with more awareness and stricter rules, companies are also paying attention to their carbon footprint. This has led to "green" supply chains that try to balance profit with environmental care. High carbon emissions cause many problems. By using methods such as carbon taxes, emission limits, or eco-friendly targets, businesses can reduce their impact, build a better image, and meet global sustainability standards.

In retail and supply chain management, the secondary market is important for promoting sustainability. It deals with selling excess, returned, overstocked, or slightly imperfect products through channels other than the main retail network. Factory outlets, a key part of this market, let manufacturers sell out-of-season or slightly flawed items at lower prices. This approach helps recover value, reduce waste, and preserve the brand's reputation by ensuring quality and pricing remain under control, while maintaining customer trust.

This study brings together some important ideas such as EPQ, SCM, environmental sustainability, and secondary markets into a single inventory model. The model focuses on both cost and environmental goals, aiming to improve production and delivery decisions while reducing waste and carbon emissions. This makes it very useful for modern supply chains that prioritize sustainability.

1.1 Research Questions

To address the growing challenges in modern supply chains, several key research questions need to be explored.

a) How can a cooperative vendor -buyer inventory model be optimized to effectively manage both perfect and imperfect items while minimizing the total cost of the system?

b) What are the effects of carbon emission costs incurred during production, transportation, and storage on the operational structure and overall efficiency of an integrated inventory system?

c) How does incorporating carbon emission considerations into inventory decision-making contribute to the sustainability of supply chain operations?

d) What are the optimal number of deliveries and cycle times that minimize the total system cost in a carbon-taxed cooperative inventory model involving imperfect items?

1.2 Flow of Study

This paper is structured into nine sections. Section 1 outlines the research background and motivation. Section 2 presents a comprehensive review of the relevant literature. Section 3 outline the problem description, the key assumptions considered, and the notations used to represent variables and parameters throughout the model. In Section 4, the mathematical formulation of the proposed model is introduced, followed by the solution methodology in Section 5. Section 6 illustrates the model's application through a numerical example, while Section 7 analyzes the results using sensitivity analysis. Section 8 discusses the key insights and managerial implications. Finally, Section 9 concludes the study and suggests directions for future research.

2. Literature Review

In recent years, integrated inventory models have gained significant attention for their ability to coordinate vendor–buyer decisions while addressing practical challenges such as defective items and environmental impacts. These joint inventory systems have progressively evolved to include quality inspection processes and sustainability considerations, with recent research incorporating both product imperfections and carbon emission costs into supply chain coordination.

2.1 Imperfect items

Most manufacturing systems face challenges in producing entirely perfect products, resulting in the production of imperfect items simultaneously. These defective products form a portion of total output, characterizing the process as an imperfect production system. Sana (2010) proposed a model to determine the optimal quality and production rate in imperfect manufacturing, with the goal of maximizing total integrated profit. Similarly, Hsu and Hsu (2012) developed an Economic Production Quantity (EPQ) model specifically for products containing imperfect items, integrating the overall production and inventory system. Hsu and Hsu (2016) extended EPQ models by including backordered shortages in imperfect production processes, enhancing the realism of inventory decision-making. AlDurgam et al. (2017) and Mou et al. (2017) formulated models that account for stochastic demand and imperfect production processes, allowing more realistic decision making in uncertain environments. These approaches generally assume that a certain portion of each batch is defective and include strategies for inspection, screening, or reworking. Ben-Daya et al. (2019) studied a centralized system involving multiple buyers and imperfect production, highlighting the difficulties in coordinating activities between several buyers and vendors. Similarly, Saxena et al. (2017) developed a green supply chain model for remanufacturing that considers both product defects and sustainability factors. Lin (2021) analyzed an EPQ model with backlogging and imperfect rework under uncertain demand, assuming that a certain portion of each batch is defective. In general, these models use approaches like inspection, screening, or rework to effectively handle imperfect items. Jauhari et al. (2021) incorporated methods for managing defective products, such as offering discounts, remanufacturing or disposal, while aiming to reduce total costs and environmental impacts. Ruidas et al. (2021) presented a model illustrating an inventory system that accommodates imperfect production. Improvements in the production process could potentially reduce the proportion of defective items. In this process, both perfect and imperfect units are produced, perfect items are sold in the market, while imperfect ones are either repaired at a cost or sold at a discounted price. Tshinangi et al. (2025) focused on a two-echelon supply chain in which incoming raw materials may contain defective items.

While earlier studies on imperfect production primarily emphasized cost or profit optimization under quality uncertainty (Sana, 2010; Hsu and Hsu, 2012; AlDurgam et al., 2017; Jauhari et al., 2021), the present model differs by integrating imperfect items with carbon taxation, thereby connecting product quality management with environmental sustainability.

2.2 Carbon Emission

In recent years, growing environmental concerns and stricter regulations have driven extensive research on incorporating carbon emissions into inventory and supply chain management models. Traditional approaches, which primarily focused on cost minimization, are now being adapted to address both economic performance and environmental sustainability. Early work by Hovelaque and Bironneau (2015) introduced demand functions influenced by carbon emissions, demonstrating how production and transportation emissions can affect supply and demand decisions. Sarkar et al. (2016) developed sustainable models by examining carbon dioxide emissions associated with both fixed and variable transportation activities, while Mishra et al. (2020) identified major emission sources, including shipping, inventory replenishment, and order processing. Saxena et al. (2018) explored tactical planning in tyre remanufacturing, considering the effects of carbon tax policies on production and inventory management. Expanding this framework, Datta et al. (2020) integrated carbon emission taxes and environmental regulations into traditional inventory models. Medina-Santana and Cárdenas-Barrón (2020) proposed a sustainable inventory model that introduces a novel carbon emissions function, capturing multiple pollution sources often overlooked in conventional methods. Mishra (2022) focused on emissions arising from the manufacturing, storage, and disposal of deteriorating items.

Shee and Chakrabarti (2024) introduced carbon credit systems and price-dependent emissions, highlighting market based mechanisms influencing operational decisions. Yadav et al. (2025) analyzed a two-warehouse inventory system for deteriorating items, integrating carbon emissions from inventory holding, while Utama et al. (2025) proposed a sustainable integrated production inventory model for the agri-food sector, considering stochastic demand and carbon emissions from electricity, fuel, and storage activities. Suvetha et al. (2025) developed a power pattern demand based model for deteriorating and non-deteriorating items, incorporating carbon taxes for storage, transportation, and deterioration costs. Complementing these efforts, Ceylan and Aydemir (2025) presented a sustainable EPQ model addressing imperfect production, shortages, and variable defect rates while explicitly evaluating carbon emissions.

Together, these studies highlight the increasing importance of integrating environmental regulations and emission related costs into inventory decision making. While previous research often addressed carbon emissions separately or focused only on perishable products (Hovelaque and Bironneau, 2015; Mishra et al., 2020).

The present model considers both carbon emissions and imperfect items within a single-vendor, single-buyer system, providing a more realistic representation of production where quality losses and environmental impacts coexist.

2.3 Carbon Tax

Carbon taxes, which charge fees for activities that produce carbon, are key tools for reducing greenhouse gas (GHG) emissions. These taxes influence decisions across the supply chain, such as choosing vendors, managing inventory, planning transportation, and scheduling production. Fahimnia et al. (2015) found that carbon taxes can encourage manufacturers to invest in renewable energy. Bazan et al. (2015) showed that including carbon costs in the Economic Order Quantity (EOQ) model leads companies to reduce order sizes when taxes are high. Ghadimi et al. (2019) highlighted that a vendor's carbon footprint is an important factor in supplier selection.

By making carbon emissions costly, these taxes encourage businesses and individuals to lower their environmental impact. Datta et al. (2020) proposed combining carbon taxes with cap-and-trade policies. Jauhari et al. (2021) examined taxes on emissions from production, storage, and transportation. Wang et al. (2021) showed how companies can adjust supply chains to reduce transport-related emissions. Zhang et al. (2024) suggested using blockchain technology to monitor carbon emissions, and Rani et al. (2025) studied a two warehouse inventory system that accounts for emissions from storing inventory.

Most carbon emissions come from using fossil fuels in machinery, transport, and other energy intensive operations. Many countries have implemented carbon pricing systems, such as taxes or cap-and-trade programs (e.g., the EU Emissions Trading System, India's carbon credit market). Including these policies in inventory models helps companies understand the financial impact of emissions and make informed, sustainable decisions.

While earlier research often examined either imperfect production or carbon emissions separately, this study integrates both in a single model for a single-vendor, single-buyer system with multiple deliveries. By considering product quality alongside carbon costs, the model helps companies balance profit, product quality, and environmental responsibility.

2.4 Research Gap and Contribution

Although several studies have addressed production and inventory systems with imperfect items, and others have explored carbon emissions and taxation in supply chain models, there is a lack of integrated approaches that consider both issues together. Previous research has often focused on improving profitability or reducing costs in the presence of defective items (e.g., through inspection, rework, or secondary sales), while separate efforts have aimed at reducing environmental impact through carbon emission modeling or carbon taxes. A summary of the discussed literature has been shown with the help of **Table 1**.

Few models simultaneously incorporate:

- The production and handling of imperfect items.
- The financial and operational impact of carbon emissions, particularly under carbon tax policies, and strategies such as selling imperfect goods at a reduced price or remanufacturing them to minimize waste.

Table 1. Author's contribution table.

Authors	Production rate	Demand rate	Holding cost	Carbon emission
Sana (2010)	Deterministic	Constant	Yes	No
Hsu and Hsu (2012)	Constant	Constant	Yes	No
Alfares (2014)	Finite	Stock dependent	Yes	No
Fahimnia et al. (2015)	Constant	Constant	Yes	Yes
Bazan et al. (2015)	Constant	Constant	Yes	Yes
Hovelaque and Bironneau (2015)	Constant	Emission dependent	Yes	Yes
Hsu and Hsu (2016)	Constant	Constant	Yes	No
Sarkar et al. (2016)	Constant	Constant	Yes	Yes
AlDurgam et al. (2017)	Variable	Stochastic	No	No
Mou et al. (2017)	Variable	Stochastic	No	No
Saxena et al. (2017)	Variable	Price dependent	Yes	No
Saxena et al. (2018)	Constant	Constant	Yes	Yes
Ghadimi et al. (2019)	variable	Stochastic	Yes	Yes
Giri and Masanta (2019)	Constant	Constant	Yes	No
Ben-Daya et al. (2019)	Constant	Constant	Yes	No
Datta et al. (2020)	Constant	Price sensitive	Yes	Yes
Medina-Santana and Cárdenas-Barrón (2020)	Constant	Constant	Yes	Yes
Mishra et al. (2020)	Deterministic	Deterministic	Yes	Yes
Lin (2021)	Variable	Variable	Yes	No
Ruidas et al. (2021)	Variable	Price dependent	Yes	Yes
Jauhari et al. (2021)	Variable	Stochastic	Yes	Yes
Wang et al. (2021)	Variable	Price dependent	Yes	Yes
Mishra et al. (2021)	Variable	Stochastic	Yes	Yes
Alfares and Ghaithan (2022)	Variable	Linear	Yes	No
Mishra (2022)	Variable	Price dependent	Yes	Yes
Shee and Chakrabarti (2024)	Stock & demand dependent	Price dependent	Yes	Yes
Zhang et al. (2024)	Variable	Constant	Yes	Yes
Rani et al. (2025)	Variable	Price-advertising dependent	Yes	Yes
Utama et al. (2025)	Stochastic	Stochastic	Yes	Yes
Suvetha et al. (2025)	Deterministic	Power-pattern	Yes	Yes
Yadav et al. (2025)	Constant	Price & promotion	Yes	Yes
Ceylan and Aydemir (2025)	Deterministic	Constant	Yes	Yes
Tshinangi et al. (2025)	Variable	Stock dependent	Yes	Yes
Present paper	Constant	Constant	Yes	Yes

Furthermore, studies combining these elements within a single-vendor, single-buyer framework are rare, especially those that account for real world complexities such as demand uncertainty and emission related costs from production, storage, and transportation. This creates a significant opportunity for research to develop comprehensive models that bridge this gap and support sustainable, cost effective decision making.

The proposed model considers constant production and demand rates while including both inventory holding costs and carbon emissions. This approach promotes sustainability without making the system overly complex, keeping it practical and efficient. Unlike many existing models that either ignore environmental effects or deal with highly uncertain demand, this model offers a balanced solution. It is easy to implement while addressing real-world concerns such as cost control and environmental responsibility. By using fixed parameters, the model remains analytically manageable and is suitable for industries that aim for steady, eco-friendly inventory management.

3. Description of the Supply Chain Production Inventory Model

The proposed model is structured to provide a clear understanding of the vendor–buyer inventory system. The following subsections outline the problem description, the key assumptions considered, and the notations used to represent variables and parameters throughout the model.

3.1 Problem Description

In this model, the vendor produces items that are inspected and classified into two groups: perfect and defective units. The perfect products are sent to the buyer in N separate shipments, while the defective items are redirected to a secondary market. The model also takes into account carbon emissions arising from production, storage, and transportation activities.

3.2 Assumptions

These following are the assumptions made throughout the study of this model.

i) Carbon emissions are produced at every stage of a product's lifecycle, from manufacturing to final sale. Hence, implementing a carbon tax is essential to help reduce environmental pollution.

ii) The production process generates a small proportion of imperfect products, which are sold at a discounted price in the secondary market, while perfect items are sold in the primary market. In this paper, α represents the rate of perfect items, with $\alpha \ll 1$, capturing the proportion of high quality output in the system.

iii) The model considered in this study is a single-vendor, single-buyer system with multiple deliveries (Saxena et al., 2017).

iv) The model assumes an infinite time horizon to focus on long-term decisions. Lead time is considered negligible, means items are replenished instantly. This simplifies the analysis without affecting the overall validity of the model (Mishra et al. 2021).

v) The production and demand rates are assumed to be constant over time, meaning items are produced and consumed at a steady, unchanging rate. This simplifies inventory planning and ensures a stable, predictable system, as is common in inventory and supply chain models. Here, D denotes the constant demand rate and P the constant production rate, with the condition $P > D$ to maintain a feasible inventory system.

vi) In various sectors, including retail, pharmaceuticals, and essential manufacturing, stockouts can lead to lost sales, damaged reputations, and reduced future business opportunities. Therefore, we implement a model that avoids shortages.

vii) After production, all items undergo 100% reliable inspection. Based on quality, they are classified as either perfect or imperfect. Perfect items are sent to the primary market, while imperfect ones are directed to the secondary market. This ensures accurate quality control, effective product management, and reduced waste.

3.3 Notations

Notations for Buyer and Vendor

- **For Buyer**

D constant demand rate
O cost of ordering per order ($/order)
h_b holding cost for buyer ($/unit/unit time)
h_{cb} carbon emission cost for keeping good items in storage at the buyer's place ($/unit)
C_3 selling price per unit for the perfect items($/unit)
N number of shipments
t_b time between consecutive deliveries
q units sent in each delivery
α percentage of perfect items

- **For Vendor**

T cycle time
C vendor's purchasing cost ($/unit)
C_m vendor's production cost ($/unit)
C_1 price at which the vendor sells perfect items and the buyer purchases them ($/unit)
h_m vendor's holding cost ($/unit/unit time)
h_{cm} carbon emission expense incurred by the vendor for storing recent production
P constant production rate
C_i inspection cost for produced items($/unit)
C_2 selling price of the imperfect items($/unit)
C_{ef} carbon cost per setup for moving materials between factory sections
C_{ev} emission cost per unit due to manufacturing operations

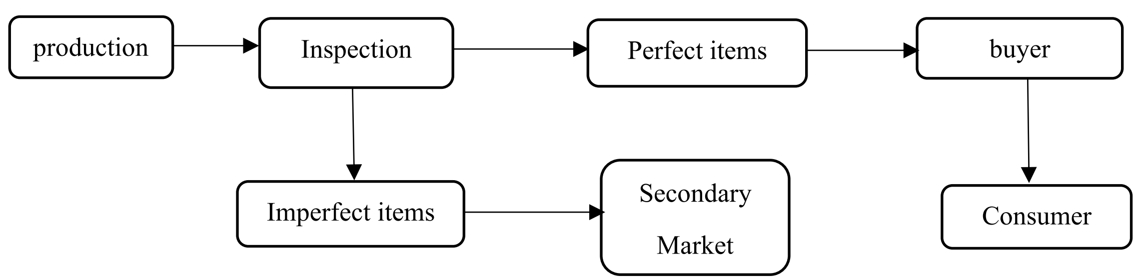

Figure 1. Flow chart of open loop supply chain.

Figure 1 illustrates an open-loop supply chain process where products move linearly from production to end consumers. After production, all items undergo inspection. Perfect items are sent to buyer and then to consumers, while imperfect items exist into a secondary market for resale or alternative use. This model does not include any return flow from consumers, focusing solely on forward logistics and quality-based item segregation.

4. Mathematical Formulation

The model starts with the production system, leading to a rise in the vendor's inventory due to production and demand over time t_1. After production, the inventory level decreases as a result of demand, reaching zero at the time T. After inspection, buyers acquire perfect items in N shipments of delivery size q until their inventory level reaches zero at time t_b. Subsequently a next delivery size q is dispatched to the vendor. **Figure 2** illustrates the changes in the inventory levels of the buyer and the vendor. Vendor produces a total of Nq units to fulfill buyer demand.

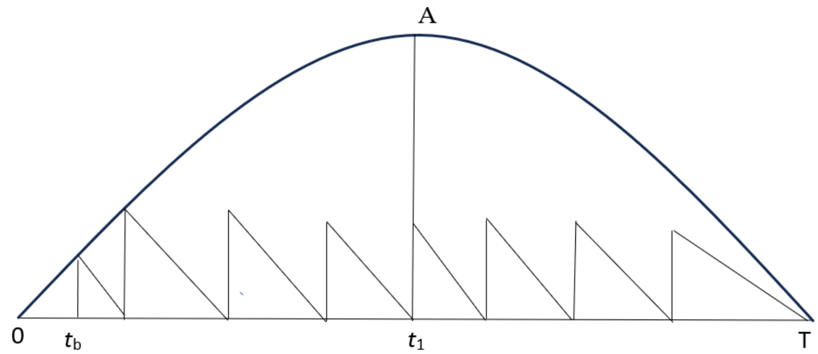

Figure 2. Stock within a closed, integrated vendor-buyer system.

The differential equations for the vendor and buyer are as follows.

$$\frac{dI_m(t)}{dt} = P - D; 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_m(t)}{dt} = -D; \quad t_1 \le t \le T \tag{2}$$

$$\frac{dI_b(t)}{dt} = -\alpha D; \quad 0 \le t \le t_b \tag{3}$$

$$I_m(0) = 0,\ I_m(T) = 0,\ I_b(t_b) = 0$$

By solving Equations (1), (2), and (3) with the provided boundary conditions, we obtain the following solutions.

$$I_m(t) = (P - D)t \ ; 0 \le t \le t_1 \tag{4}$$

$$I_m(t) = D(T - t) \ ; t_1 \le t \le T \tag{5}$$

$$I_b(t) = D(t_b - t)\ ; 0 \le t \le t_b \tag{6}$$

By using the condition $I_b(0) = q$ and with the help of Equation (6) we get

$$q = \alpha D t_b \tag{7}$$

From Equations (4) and (5) we get the

$$t_1 = \frac{DT}{P} \tag{8}$$

From **Figure 2**, we can say that,

$$T = Nt_b \tag{9}$$

Upon substituting the value of T into Equation (8), the result obtained is

$$t_1 = \frac{DNt_b}{P} \tag{10}$$

4.1 Vendor's Profit Function

The vendor's total profit includes costs for production, procurement, inspection, and storage, as well as income from both primary and secondary markets. Below is the detailed breakdown of these costs.

4.1.1 Production and Procurement Cost

The vendor acquires raw materials for a specific production system at a unit cost denoted as c_p per unit known as procurement cost. The product is manufactured at a rate of P in the duration 0 to t_1, which is shown in **Figure 2**. The unit cost of manufacturing is c_m. Consequently, the sum of production and procurement cost is as follow:

$$Pt_1(c_p + c_m) \tag{11}$$

4.1.2 Holding Cost (H.C.)

The vendor stores manufactured items, which incur essential costs. This cost is known as holding cost. Carbon emission cost during holding refers to the environmental cost added to inventory storage due to energy use, such as lighting, refrigeration, or machinery. These activities release carbon emissions. The total holding cost includes both the regular holding cost h_m and the carbon emission cost, h_{cm} associated with storing the items. This cost is represented as $(h_m + h_{cm})$. The vendor's inventory is held between [0, T], so the holding cost can be calculated as

$$H.C. = (h_m + h_{cm})[\int_0^{t_1} I_m(t)dt + \int_{t_1}^{T} I_m(t)dt]$$

$$H.C. = (h_m + h_{cm})[\int_0^{t_1}(P - D)t\,dt + \int_{t_1}^{T} D(T - t)dt]$$

$$H.C. = (h_m + h_{cm})[(P - D)\frac{{t_1}^2}{2} + \frac{D(t_1 - T)^2}{2}] \tag{12}$$

4.1.3 Inspection Cost

Once produced, all items undergo inspection, which incurs a cost. The inspection cost for newly produced items is

$$\text{Inspection cost} = C_i P t_1 \tag{13}$$

4.1.4 Carbon Emission Cost (CEC)

Carbon emissions during production occur due to the use of energy intensive processes, machinery, and raw materials, often powered by fossil fuels. Similarly, during transportation, emissions result from the fuel consumption of vehicles used to move goods between facilities, warehouses, and customers. Carbon emissions occur during both the production process and the transportation of goods. Therefore, the total associated cost will be

$$\text{CEC} = c_{ev} p t_1 + N c_{ef} \tag{14}$$

4.1.5 Total Revenue

After the inspection, perfect items are sold to the buyer in *N* shipments at a cost of C_1 \$/unit, while imperfect items are sold in the secondary market at reduced price of C_2 \$/unit. Consequently, the vendor accumulates total revenue of $C_1 \alpha DT$ from the primary market and $C_2(1 - \alpha)DT$ from the secondary market. Now the vendor's total profit function will be the difference between total revenue and all associated costs.

The vendor's profit function

$$TP_s = \frac{1}{T}[\alpha C_1 DT + C_2(1-\alpha)DT - \left(C_p + C_m + C_i\right)Pt_1 - (h_m + h_{cm})[(P-D)\frac{{t_1}^2}{2} + D\frac{(t_1-T)^2}{2} - \alpha c_{ev}pt_1 + Nc_{ef}] \quad (15)$$

Substituting the value of T and t_1 from Equations (9) and (10) into the Equation (15) the total profit function for vendor will be

$$TP_s = [DC_1\alpha + DC_2(1-\alpha) - D\left(C_p + C_m + C_i\right) - (h_m + h_{cm})\left(\frac{(P-D)NDt_b}{2P}\right) - Dc_{ev} + \frac{c_{ef}}{t_b}] \quad (16)$$

4.2 Buyer’s Profit Function

The buyer’s profit is determined by subtracting all relevant costs from the sales revenue. The different associated costs are calculated as follows.

4.2.1 Ordering Cost

The ordering cost for procuring perfect products from the vendor is represented by O.

4.2.2 Purchasing Cost

Purchasing q units of perfect items from the vendor over N cycles at c_1 per unit leads to the buyer’s total purchasing cost, which is calculated as c_1Nq.

4.2.3 Holding Cost

The vendor delivers the perfect items to the buyer in N separate shipments. The cost incurred for storing these items is known as the holding cost. Since storing perfect items also results in carbon emissions, the corresponding carbon emission cost, denoted as h_{cb} is added to the buyer’s holding cost h_b. Therefore, the total holding cost for the buyer over N cycles is

$$H.C. = (h_b + h_{cb})N\int_0^{t_b} \mathrm{I}_b(t)dt,$$

$$H.C. = (h_b + h_{cb})N\int_0^{t_b} \alpha D(t_b - t)dt,$$

$$H.C. = (h_b + h_{cb})\alpha ND\frac{{t_b}^2}{2} \quad (17)$$

4.2.4 Total Revenue

The buyer orders perfect units from the vendor and sells this quantity of αDT units in the primary market at a price of c_3 per unit, generating a total revenue of $c_3\alpha DT$.

The buyer’s profit function is a difference between total revenue earned by buyer and all associated costs.

$$TP_b = \frac{1}{T}[c_3\alpha DT - O - c_1\alpha DT - (h_b + h_{cb})\alpha ND\frac{{t_b}^2}{2}],$$

$$TP_b = [c_3\alpha D - \frac{O}{Nt_b} - c_1\alpha D - (h_b + h_{cb})\alpha D\frac{t_b}{2}] \quad (18)$$

4.3 Total Profit Function of Supply Chain

The overall profit function of the supply chain is derived from adding the buyer's profit function and the vendor's profit function.

$$TP = TP_b + TP_s$$

$$TP = DC_1\alpha + DC_2(1-\alpha) - D\left(C_p + C_m + C_i\right) - (h_m + h_{cm})\frac{(P-D)NDt_b}{2P} + c_3\alpha D - \frac{O}{Nt_b} - DC_1\alpha - (h_b + h_{cb})\alpha D\frac{t_b}{2} - Dc_{ev} - \frac{c_{ef}}{t_b},$$

$$TP = DC_2(1-\alpha) - D\left(C_p + C_m + C_i\right) - (h_m + h_{cm})\frac{(P-D)NDt_b}{2P} + c_3\alpha D - \frac{O}{Nt_b} - (h_b + h_{cb})\alpha D\frac{t_b}{2} - Dc_{ev} - \frac{c_{ef}}{t_b} \quad (19)$$

5. Solution Procedure

The mathematical framework used for optimal delivery scheduling is based on an extended Economic Production Quantity (EPQ) model. It incorporates multiple cost components including production, holding, transportation, and carbon emission costs into a unified total cost function.

This study aims to determine the optimal values for N and t_b that maximize the overall profit, focusing on determining the ideal number of deliveries per cycle and the corresponding cycle time $t_b{}^*$ and N^*.

In this study, the Hessian matrix method is utilized to determine the optimal values of N^* (the ideal number of deliveries per cycle) and $t_b{}^*$ (the corresponding cycle time) that maximize overall profit. This method is well-suited for the problem due to the following reasons:

- The function is continuous and differentiable with respect to both decision variables.
- The Hessian matrix enables the accurate identification of maxima by analyzing the curvature of the function using second-order derivatives.
- It offers a structured and reliable approach to classify critical points as local maxima, minima, or saddle points.

The optimization process consists of the following steps:
- Formulating the profit function in terms of the decision variables N and t_b.
- Computing the first-order partial derivatives with respect to N and t_b to locate critical points.
- Deriving the second-order partial derivatives to construct the Hessian matrix.
- Examining the Hessian matrix to determine the nature of each critical point, specifically verifying whether it represents a maximum by assessing the matrix's definiteness.

Now the objective is to
$Max\ TP$

Subject to constraint
$t_b > 0$ and $N > 0$.

For the optimal value of N and t_b, the total profit function is first differentiated with respect to N and t_b and the resulting equations are then set equal to zero. To find the initial point we consider N as the continuous variable.

To optimize, the total profit function is differentiated with respect to N and t_b.
$\frac{\partial TP}{\partial t_b} = 0, \frac{\partial TP}{\partial N} = 0.$

$$\frac{\partial TP}{\partial t_b} = \frac{O}{N{t_b}^2} - \frac{(h_b+h_{cb})\alpha D}{2} + \frac{c_{ef}}{{t_b}^2} - (h_m + h_{cm})\frac{(P-D)ND}{2P} \tag{20}$$

$$\frac{\partial TP}{\partial N} = \frac{O}{N^2 t_b} - (h_m + h_{cm})\frac{(P-D)Dt_b}{2P} \tag{21}$$

Equating Equations (20) and (21) to zero and solving them, we obtain the optimal solution as $(N^*, {t_b}^*)$

$${t_b}^* = \sqrt{\frac{2P(O + Nc_{ef})}{N[\alpha PD(h_b + h_{cb}) + ND(P - D)(h_m + h_{cm})]}},$$

$$N^* = \frac{1}{t_b}\sqrt{\frac{2OP}{D(P-D)(h_m + h_{cm})}},$$

$$\frac{\partial^2 TP}{\partial N^2} = -\frac{2O}{N^3 t_b},$$

$$\frac{\partial^2 TP}{\partial {t_b}^2} = -\frac{2O}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3},$$

$$\frac{\partial^2 TP}{\partial N \partial t_b} = -\frac{O}{N^2{t_b}^2} - (h_m + h_{cm})\frac{(P-D)D}{2P},$$

$$\frac{\partial^2 TP}{\partial t_b \partial N} = -\frac{O}{N^2{t_b}^2} - (h_m + h_{cm})\frac{(P-D)D}{2P},$$

$$H = \begin{pmatrix} \frac{\partial^2 TP}{\partial {t_b}^2} & \frac{\partial^2 TP}{\partial t_b \partial N} \\ \frac{\partial^2 TP}{\partial N \partial t_b} & \frac{\partial^2 TP}{\partial N^2} \end{pmatrix},$$

$$= \begin{pmatrix} -\frac{2O}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3} & -\frac{O}{N^2{t_b}^2} - (h_m + h_{cm})\frac{(P-D)D}{2P} \\ -\frac{O}{N^2{t_b}^2} - (h_m + h_{cm})\frac{(P-D)D}{2P} & -\frac{2O}{N^3 t_b} \end{pmatrix}.$$

We know that, $-\frac{2O}{N{t_b}^3} - \frac{2c_{ef}}{{t_b}^3} < 0$, $\quad -\frac{2O}{N^3 t_b} < 0$,

$$|H| = \left(\frac{2O}{N{t_b}^3} + \frac{2c_{ef}}{{t_b}^3}\right)\left(\frac{2O}{N^3 t_b}\right) - \left(\frac{O}{N^2{t_b}^2} + (h_m + h_{cm})\frac{(P-D)D}{2P}\right)^2 > 0.$$

As a result, the Hessian matrix of H shows negative definite. Therefore, we can conclude that the total profit function TP determines concavity with respect to variables N and t_b.

6. Numerical Analysis

To demonstrate the practical relevance of the proposed model, a case study is developed, based on realistic supply chain settings. Consider a vendor–buyer system in the textile manufacturing industry, where large production volumes, imperfect items, and significant carbon emissions coexist. In such a system, defective products (e.g., garments with minor stitching or dyeing defects) are inevitable. These imperfect items are inspected with 100% reliability and sold at discounted prices in a secondary market, while perfect items are sold in the primary market. At the same time, production and storage activities generate carbon emissions, which are regulated through carbon tax policies.

The parameter values adopted in this case reflect industry consistent ranges (e.g., production costs, holding costs, inspection expenses, and carbon tax values) obtained from secondary data in prior studies (Sana, 2010; Sarkar et al., 2016; Mishra et al., 2020) and market reports. The model integrates both economic and environmental dimensions, ensuring that cost optimization and sustainability are considered together.

These following are the numerical values used for different parameters.
α =0.8, C_1 =$ 50 per unit, C_2 =$ 30 per unit, C_p =$ 8 per unit, C_m =$ 15 per unit, C_i =$ 0.5 per unit, C_3 =$70 per unit, h_b =$ 1.5 per unit per unit time, h_{cb} =$ 0.5 per unit per unit time, h_m =$ 1 per unit per unit time, h_{cm} =$ 1 per unit per unit time, O = $ 1000 per setup, P =500 units per unit item, D =1000 units per unit item, C_{ef} =$ 50 per unit, C_{ev} =$ 0.2 per unit.

We derive the optimal solution and results in the following **Table 2**.

Table 2. Solution search procedure for optimal value of N.

N	t_b	TP	TP_b	TP_v
1	0.962533	36118.3	14191	21927.2
2	0.612372	36503.7	14693.6	21810.1
3	0.461479	36638.7	14908.5	21730.2
4	0.375	36700	15033.3	21666.7
5	0.318357	36729.4	15117.1	21612.3
6	0.278174	36742.2	15178.3	21563.9
7	**0.248093**	**36745.3**	**15225.7**	**21519.6**
8	0.224679	36742.2	15263.9	21478.3
9	0.205907	36735.1	15295.7	21439.5
10	0.1905	36725.2	15322.7	21402.5

Using the given parameters, the system was optimized under different shipment policies. Results show that the optimal solution occurs at seven deliveries per cycle, with a cycle time of 0.2481. This yields a maximum integrated profit of $36,745.3, with $15,225.7 accruing to the buyer and $21,519.6 to the vendor. Compared to a single-shipment policy, the coordinated multi-shipment strategy increases system profit by about 1.74%, while also balancing carbon-related expenses.

This case study illustrates how the model can guide firms in real-world decision-making. In practice, managers face trade-offs between shipment frequency, product quality, and carbon taxes. The findings demonstrate that adopting coordinated replenishment policies not only improves profits but also encourages environmentally sustainable practices. Furthermore, the inclusion of imperfect items reflects the true nature of manufacturing systems, making the model more relevant to industries such as textiles, electronics, and food processing, where both defective items and emission regulations are common.

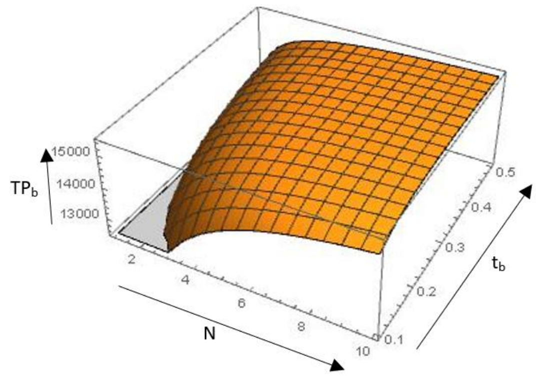

Figure 3. Concavity for the buyer's profit function TP_b.

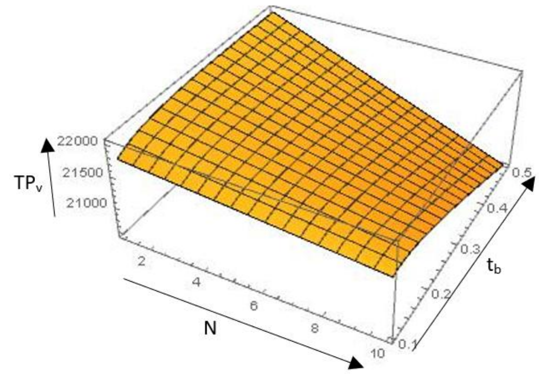

Figure 4. Concavity for the vendor's profit function TP_s.

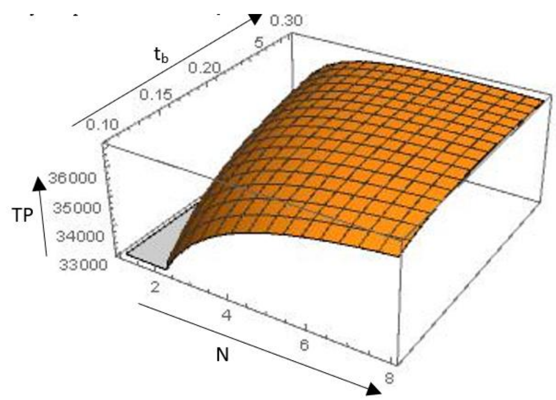

Figure 5. Concavity for the total profit function TP.

- In **Figure 3**, the downward curving surface indicates that the profit function attains its maximum at a particular optimal point. As the decision variables shift away from this point, the buyer's profit diminishes. This implies the existence of a unique optimal decision that maximizes the buyer's profit.
- In **Figure 4**, the surface is again concave, signifying the presence of a maximum profit point. This maximum lies within the interior of the decision space. It suggests that the vendor has an optimal strategy, determined by specific parameters, that maximizes their profit.
- The **Figure 5** surface is likewise concave, indicating that the total system profit reaches a global maximum. Effective coordination between the buyer and vendor results in a Pareto-optimal solution that maximizes the overall profit of the system.

7. Sensitivity Analysis

This section examines how changes in key parameters influence the performance of the supply chain model. We assess the effect of different key parameters on delivery cycle time, profits of the buyer and vendor, and the overall system profit. The analysis highlights which parameters have the most significant impact and provides insights into maintaining an efficient and profitable supply chain. The main findings from this analysis are presented below.

Table 3. Variation in total profit functions with respect to ordering cost.

O	t_b	TP_b	TP_v	TP
800	0.228979	15317.7	21547.4	36865.1
850	0.233904	15293.7	21540.5	36834.2
900	0.238728	15270.4	21533.5	36804
950	0.243455	15247.8	21526.6	36774.3
1000	0.248093	15225.7	21519.6	36745.3
1050	0.252646	15204.2	21512.6	36716.8
1100	0.257118	15183.1	21505.6	36688.7
1150	0.261513	15162.6	21498.6	36661.2
1200	0.265836	15142.5	21491.6	36634.1

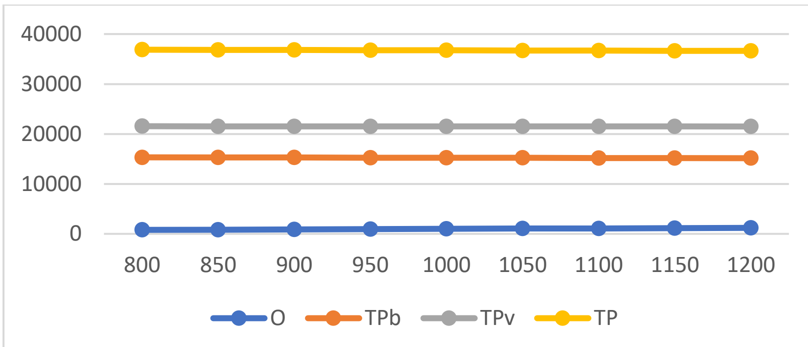

Figure 6. Variation with respect to ordering cost.

Table 4. Variation in total profit functions with respect to demand parameter.

D	t_b	TP_b	TP_v	TP
800	0.243475	12057.4	16998.4	29055.8
850	0.243288	12847.4	18122.2	30969.9
900	0.243975	13638.8	19250.2	32889
950	0.245559	14431.6	20382.6	34814.2
1000	0.248093	15225.7	21519.6	36745.3
1050	0.251666	16021	22661.4	38682.4
1100	0.256411	16817.2	23808.5	40625.7
1150	0.262523	17614.3	24961.4	42575.7
1200	0.270281	18412	26120.9	44532.9

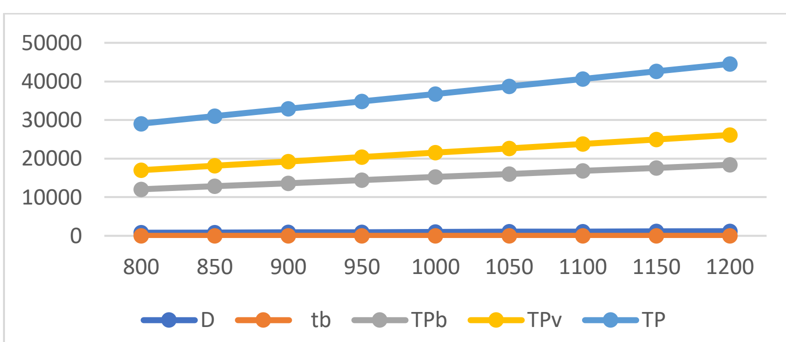

Figure 7. Variation with respect to demand parameter.

Table 5. Variation in total profit functions with respect to holding cost parameter h_m.

h_m	t_b	TP_b	TP_v	TP
0.8	0.257881	15239.7	21564.6	36804.3
0.85	0.255326	15236.2	21553.1	36789.3
0.9	0.252845	15232.7	21541.8	36774.5
0.95	0.250435	15229.2	21530.6	36759.8
1	0.248093	15225.7	21519.6	36745.3
1.05	0.245815	15222.2	21508.7	36730.9
1.1	0.243599	15218.7	21497.9	36716.6
1.15	0.241442	15215.2	21487.3	36702.5
1.2	0.239341	15211.7	21476.8	36688.5

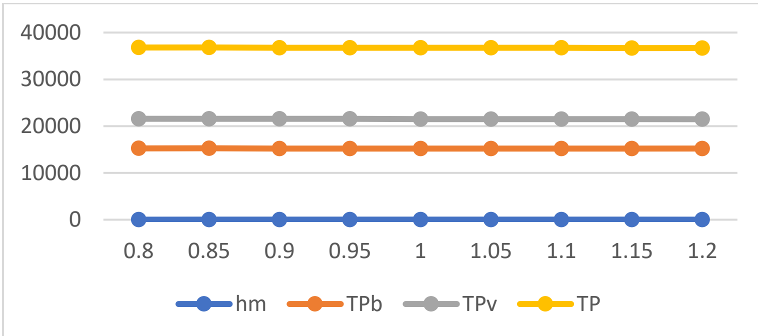

Figure 8. Variation with respect to holding cost parameter.

Table 6. Variation in total profit functions with respect to production rate.

P	t_b	TP_b	TP_v	TP
1200	0.31315	15293.3	21775	37068.3
1275	0.288955	15274.4	21690.7	36965.1
1350	0.27518	15266.7	21623	36879.7
1425	0.258428	15240	21567.4	36807.5
1500	0.248093	15225.7	21519.6	36745.3
1575	0.239737	15212.3	21478.8	36691.1
1650	0.232831	15200.2	21443.2	36643.4
1725	0.22702	15189.1	21411.9	36601
1800	0.222059	15179	21384	36563

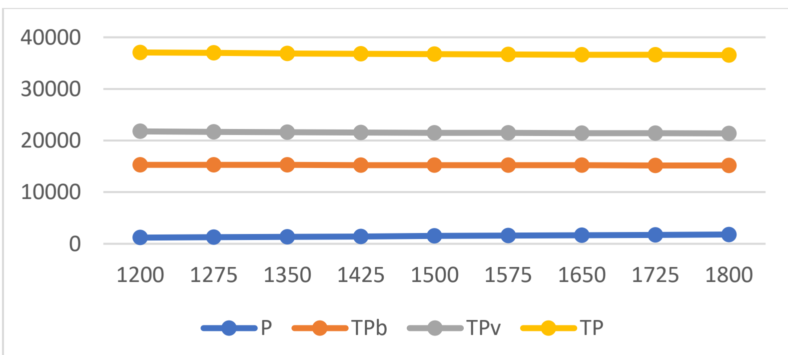

Figure 9. Variation with respect to production rate.

Table 7. Variation in total profit functions with respect to carbon emission cost.

C_{ef}	t_b	TP_b	TP_v	TP
40	0.241575	15215.4	21570.7	36786.1
42.5	0.243221	15218.1	21557.7	36775.8
45	0.244856	15220.7	21544.9	36765.6
47.5	0.24648	15223.2	21532.2	36755.4
50	0.248093	15225.7	21519.6	36745.3
52.5	0.249696	15228.1	21507.1	36735.2
55	0.251288	15230.5	21494.8	36725.3
57.5	0.252871	15232.8	21482.6	36715.3
60	0.254444	15235	21470.5	36705.5

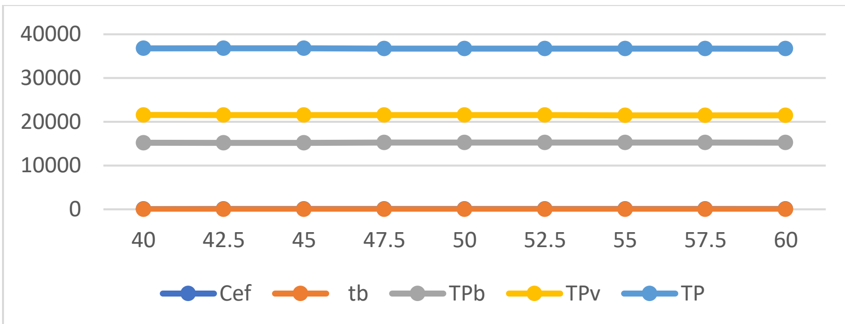

Figure 10. Variation with respect to carbon emission cost due to shipping of the items.

Table 8. Variation in total profit functions with respect to selling price for buyer.

C_3	t_b	TP_b	TP_v	TP
56	0.248093	4425.7	21519.6	25945.3
59.5	0.248093	6825.7	21519.6	28345.3
63	0.248093	9625.7	21519.6	31145.3
66.5	0.248093	12425.7	21519.6	33945.3
70	0.248093	15225.7	21519.6	36745.3
73.5	0.248093	18025.7	21519.6	39545.3
77	0.248093	20825.7	21519.6	42345.3
80.5	0.248093	23625.7	21519.6	45145.3
84	0.248093	26425.7	21519.6	47945.3

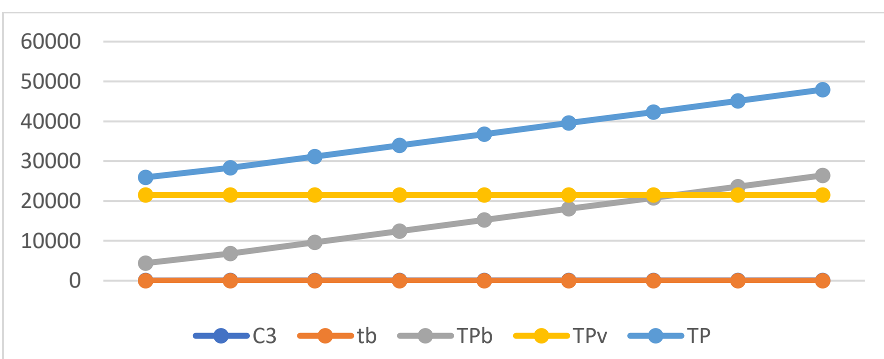

Figure 11. Variation with respect to selling price for the buyer.

Table 9. Variation in total profit functions with respect to carbon emission cost for storage at the buyer's place.

h_{cb}	t_b	TP_b	TP_v	TP
0.4	0.249692	15238.1	21517.1	36755.2
0.425	0.249289	15235	21517.8	36752.7
0.45	0.248889	15231.9	21518.4	36750.3
0.475	0.24849	15228.8	21519	36747.8
0.5	0.248093	15225.7	21519.6	36745.3
0.525	0.247698	15222.6	21520.2	36742.8
0.55	0.247305	15219.6	21520.8	36740.3
0.575	0.246914	15216.5	21521.4	36737.9
0.6	0.246524	15213.4	21522	36735.4

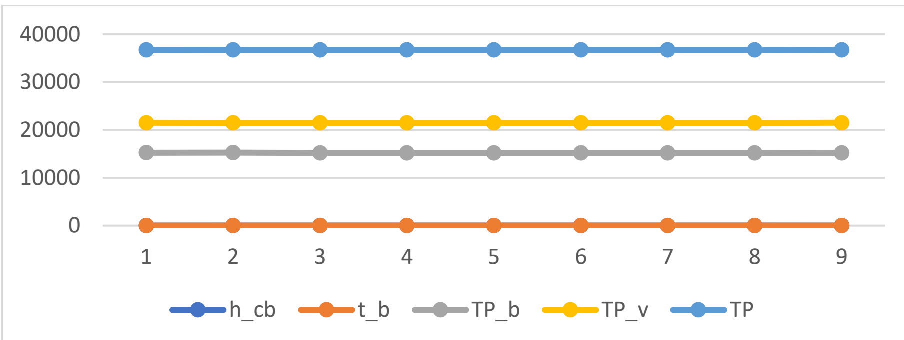

Figure 12. Variation with respect to carbon emission cost for keeping good items in storage at the buyer's place.

Table 10. Variation in total profit functions with respect to percentage of perfect items.

α	t_b	TP_b	TP_v	TP
0.64	0.254681	12076.1	18309.4	30385.5
0.68	0.252985	12863.3	19112.1	31975.3
0.72	0.251322	13650.6	19914.6	33565.3
0.76	0.249692	14438.1	20717.1	35155.2
0.8	0.248093	15225.7	21519.6	36745.3
0.84	0.246524	16013.4	22322	38335.4
0.88	0.244985	16801.3	23124.3	39925.6
0.92	0.243475	17589.3	23926.5	41515.8
0.96	0.241991	18377.3	24728.7	43106.1

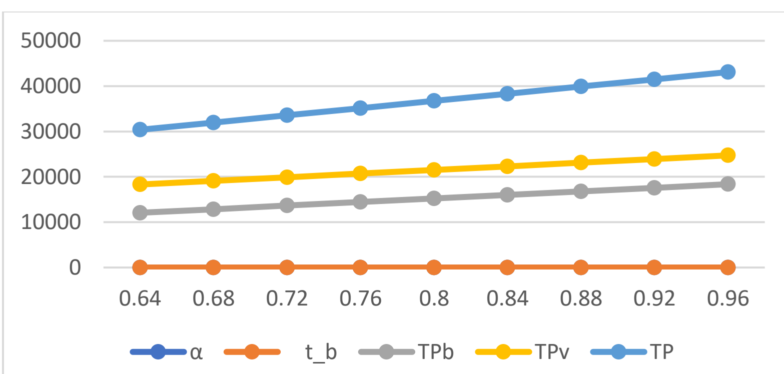

Figure 13. Variation with respect to percentage of perfect items.

8. Observations and Managerial Insights

8.1 Observations

The results of the sensitivity analysis lead to the following conclusions.

i) As the buyer's ordering cost rises, the delivery time increases, as shown in **Table 3**. This indicates that the buyer places orders less frequently to manage higher expenses. Consequently, the profits of both the buyer and the vendor, along with the overall system profit, decline. These results suggest that higher ordering costs disturb the balance between order frequency and inventory holding, reducing overall system efficiency, as illustrated in **Figure 6**.

ii) **Table 4** shows that as the demand rate increases, the buyer's cycle time and the profits of both the buyer and vendor rise, ultimately leading to an increase in the total profit of the system. **Figure 7** demonstrates that the demand rate and profitability are directly correlated.

iii) **Table 5** illustrates that an increase in the vendor's holding costs leads to a slight decline in the buyer's profit. Furthermore, this increase adversely affects the vendor's profit, the overall system profit and the delivery cycle time, all of which show a downward trend. These findings indicate that higher holding costs at the vendor level create inefficiencies that impact the entire supply chain, ultimately reducing its overall effectiveness. The relationships presented in this table are further supported by the graphical representation in **Figure 8**.

iv) **Table 6** shows that an increase in the production rate leads to a shorter delivery cycle time, indicating improved responsiveness. However, this acceleration in operational speed comes at the cost of profitability, as it is associated with a decrease in profits for the buyer, the vendor and the entire supply chain. This inverse relationship highlights the trade-offs linked to increased production capacity, as illustrated in **Figure 9**.

v) Additionally, insights from **Tables 5** and **6** reveal that the buyer's profit, denoted as TP_b, demonstrates slight negative sensitivity to changes in both the vendor's holding cost h_m and the production rate P. This suggests that even small modifications in these parameters can adversely affect the buyer's profitability. Therefore, it is crucial to maintain a balanced and cost-effective operational strategy to sustain economic performance throughout the supply chain.

vi) **Table 7** shows how changes in the carbon emission cost (C_{ef}) affect the supply chain. As carbon costs rise, the delivery cycle time also increases, likely because production and logistics are adjusted to reduce carbon use. At the same time, the vendor's profit drops due to the extra costs, while the buyer gains slightly. Overall, the total system profit decreases, showing a negative impact on supply chain performance. These trends are shown in **Figure 10**, which illustrates the rising cycle time and the different profit changes for the buyer and vendor.

vii) In **Table 8**, as the value of selling price for buyer (C_3) increases from 56 to 84, TP_b shows a clear linear rise from 4,425.7 to 26,425.7. Given that TP_v remains constant throughout, the total value TP also increases linearly, mirroring the growth in TP_b. This steady rise in TP is primarily driven by the increase in TP_b while TP_v stays unchanged. The parameter t_b is small, has a fixed value and has minimal impact on the overall trend of TP. This pattern suggests a scenario were increasing the variable C_3 enhances productivity or output TP_b, which in turn raises the total output TP, while other baseline factors remain stable. These relationships and their dynamics are clearly illustrated in **Figure 11**.

viii) In **Table 9**, the carbon emission cost for storage items by the buyer (h_{cb}) increases from 0.4 to 0.6, the cycle time t_b shows a slight decrease from 0.249692 to 0.246524. This suggests that the buyer may shorten the cycle time in response to higher carbon emission cost, likely as a strategy to minimize the overall impact of carbon-related expenses. The buyer's profit TP_b also shows a slight decline, dropping from 15,238.1 to 15,213.4, indicating a modest reduction in profitability due to increased carbon holding costs. On the other hand, the vendor's profit TP_v remains very stable, with only a minimal increase from 21,517.1 to 21,522, suggesting it is largely unaffected by changes in h_{cb}. The total profit TP shows only minor fluctuations, ranging between 36,752.3 and 36,755.2. This stability indicates that the overall system is robust against small variations in the buyer's carbon-related holding cost, as shown in **Figure 12**.

ix) In **Table 10** and **Figure 13**, As the percentage of perfect items α increases from 0.64 to 0.96, all profit functions (TP_b, TP_v and TP) show a fixed increasing trend. This clearly indicates that improving product quality directly enhances overall profitability. The cycle time (t_b) decreases slightly from 0.2547 to 0.2420 with the rise in α, suggesting that higher quality reduces the need for longer cycle times and leads to more efficient inventory management. The buyer's profit (TP_b) increases from 12,076.1 to 18,377.3, showing that buyers benefit from fewer defective items since lower returns and rejections translate into higher gains. Similarly, the vendor's profit (TP_v) rises from 18,309.4 to 24,728.7 as α improves. Vendors thus gain from producing higher-quality items, as production costs are utilized more effectively and waste is minimized. It is also observed that the vendor's profit (TP_v) remains consistently higher than the buyer's profit (TP_b); however, both profits grow steadily with increasing α. This highlights the mutual benefit for both partners, with the vendor enjoying a slightly greater share of profit.

8.2 Managerial Insight

The analysis reveals how variations in different parameters influence profits and delivery times, offering practical guidance for enhancing supply chain efficiency, profitability, and cooperation between buyers and vendors.

i) An increase in demand positively influences cycle time and profit margins for both buyer and vendor. This highlights the importance of demand stimulation strategies, such as marketing or pricing incentives, to enhance overall profitability.

ii) While higher production rates shorten cycle times, they can also reduce profitability. Managers should determine the optimal production rate that balances responsiveness with cost-effectiveness to maintain strong financial performance.

iii) Buyer profit is moderately sensitive to both vendor holding costs and production rate changes. Therefore, collaborative planning and transparent cost-sharing mechanisms are critical to preserving buyer profitability and supply chain harmony.

iv) Increasing carbon costs lead to longer delivery cycles and lower vendor profits, despite slight gains for buyers. This suggests the need for environmentally conscious production strategies that do not compromise overall supply chain profitability. Investments in low-emission technologies and carbon-efficient logistics can help mitigate this trade-off.

v) Improving product quality directly enhances profitability for both buyers and vendors while reducing cycle time, leading to more efficient inventory management. Although vendors consistently gain slightly higher profits, both partners benefit substantially, highlighting the importance of collaborative quality improvement as a strategic driver of mutual growth and competitiveness.

The findings suggest that both policymakers and firms must strike a balance between profitability and sustainability. For policymakers, fair carbon taxation and incentives for green technologies can encourage firms to reduce emissions without severely affecting profits. For companies, results highlight that faster production does not always improve profitability; instead, optimal production planning, cost-sharing between vendor and buyer, and adoption of eco-friendly methods are key. Overall, the study shows that sustainable practices, if supported by sound policies and smart business strategies, can help achieve both economic and environmental goals.

9. Conclusion

This study examines an inventory model that accounts for imperfect production, where defective items are sold at a discounted rate in a secondary market to minimize waste. A key environmental concern in this economic production framework is carbon emission, and prior research has often suggested the use of carbon taxes to mitigate their impact. The proposed model focuses on a single-vendor, single-buyer supply chain and incorporates costs associated with carbon emissions during production, inventory storage, and shipment. It also considers inspection costs to distinguish between perfect and imperfect items.

The model provides guidance to both the vendor and buyer on maximizing their combined profit by determining the optimal delivery frequency and timing for each shipment. Certain assumptions are made, including constant demand and production rates, no lead time for deliveries, and fixed profit parameters. Sensitivity analysis indicates that total profit increases with higher demand but decreases with rising ordering costs, production rates, and vendor holding costs. Implementing carbon taxes can effectively reduce emissions, while optimizing delivery cycle time and the number of shipments enhances overall profitability for both parties.

The study recognizes some limitations, such as the assumptions of constant demand and production rates with zero lead time, which open avenues for future research. Subsequent studies could incorporate variable or stochastic demand, multiple vendors, dynamic pricing, and price-dependent demand to better capture real-world market behavior. Expanding the model to multi-echelon supply chains including suppliers, manufacturers, distributors, and retailers would enable coordinated decision making across all levels, improving efficiency. Additionally, integrating environmental factors such as carbon emissions, energy use, and waste reduction would further align the supply chain with sustainability objectives.

Conflict of Interest
The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments
The authors would like to express their sincere thanks to the reviewers for their valuable comments and constructive suggestions, which greatly helped in improving the quality of the manuscript. We also extend our gratitude to the editor for their guidance and support throughout the review process.

AI-Disclosure
During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

References

AlDurgam, M., Adegbola, K., & Glock, C.H. (2017). A single-vendor single-manufacturer integrated inventory model with stochastic demand and variable production rate. *International Journal of Production Economics*, *191*, 335-350. https://doi.org/10.1016/j.ijpe.2017.05.017.

Alfares, H.K. (2014). Production-inventory system with a finite production rate, stock-dependent demand, and variable holding cost. *RAIRO-Operations Research*, *48*(1), 135-150. https://doi.org/10.1051/ro/2013058.

Alfares, H.K., & Ghaithan, A.M. (2022). A generalized production-inventory model with variable production, demand, and cost rates. *Arabian Journal for Science and Engineering*, *47*(3), 3963-3978. https://doi.org/10.1007/s13369-021-06516-4.

Bazan, E., Jaber, M.Y., & El Saadany, A.M.A. (2015). Carbon emissions and energy effects on manufacturing–remanufacturing inventory models. *Computers & Industrial Engineering*, *88*, 307-316. https://doi.org/10.1016/j.cie.2015.07.002.

Ben-Daya, M., Asad, R., & Nabi, K.A. (2019). A single-vendor multi-buyer production remanufacturing inventory system under a centralized consignment arrangement. *Computers & Industrial Engineering*, *135*, 10-27. https://doi.org/10.1016/j.cie.2019.05.032.

Ceylan, M., & Aydemir, E. (2025). A new sustainable economic production quantity model with imperfect production and shortage. *Arabian Journal for Science and Engineering*, 1-12. https://doi.org/10.1007/s13369-025-10109-w.

Datta, T.K., Nath, P., & Choudhury, K.D. (2020). A hybrid carbon policy inventory model with emission source-based green investments. *Opsearch*, *57*(1), 202-220. https://doi.org/10.1007/s12597-019-00430-y.

Fahimnia, B., Sarkis, J., Choudhary, A., & Eshragh, A. (2015). Tactical supply chain planning under a carbon tax policy scheme: a case study. *International Journal of Production Economics*, *164*, 206-215. https://doi.org/10.1016/j.ijpe.2014.12.015.

Ghadimi, P., Wang, C., & Lim, M.K. (2019). Sustainable supply chain modeling and analysis: past debate, present problems and future challenges. *Resources, Conservation & Recycling, 140*, 72-84. https://doi.org/10.1016/j.resconrec.2018.09.005.

Giri, B.C., & Masanta, M. (2019). Optimal production policy for a closed-loop supply chain with stochastic lead time and learning in production. *Scientia Iranica*, *26*(5), 2936-2951. https://doi.org/10.24200/sci.2019.21537.

Hovelaque, V., & Bironneau, L. (2015). The carbon-constrained EOQ model with emission dependent demand. *International Journal of Production Economics, 164*, 285-291. https://doi.org/10.1016/j.ijpe.2014.11.022.

Hsu, J.T., & Hsu, L.F. (2012). An integrated single-vendor single-buyer production-inventory model for items with imperfect quality and inspection errors. *International Journal of Industrial Engineering Computations, 3*(5), 703-720. https://doi.org/10.5267/j.ijiec.2012.04.004.

Hsu, L.F., & Hsu, J.T. (2016). Economic production quantity (EPQ) models under an imperfect production process with shortages backordered. *International Journal of Systems Science*, *47*(4), 852-867. https://doi.org/10.1080/00207721.2014.906768.

Jauhari, W.A., Pujawan, I.N., & Suef, M. (2021). A closed-loop supply chain inventory model with stochastic demand, hybrid production, carbon emissions, and take-back incentives. *Journal of Cleaner Production*, *320*, 128835. https://doi.org/10.1016/j.jclepro.2021.128835.

Lin, H.J. (2021). An economic production quantity model with backlogging and imperfect rework process for uncertain demand. *International Journal of Production Research*, *59*(2), 467-482. https://doi.org/10.1080/00207543.2019.1696491.

Medina-Santana, A.A., & Cárdenas-Barrón, L.E. (2020). A sustainable inventory model considering a discontinuous transportation cost function and different sources of pollution. *Optimization and Control.* https://doi.org/10.48550/arXiv.2005.03781.

Mishra, M., Hota, S.K., Ghosh, S.K., & Sarkar, B. (2020). Controlling waste and carbon emission for a sustainable closed-loop supply chain management under a cap-and-trade strategy. *Mathematics*, *8*(4), 466. https://doi.org/10.3390/math8040466.

Mishra, S. (2022). Sustainability of inventory models along with carbon emission for deteriorating goods. *Journal of Physics: Conference Series, 2267*(1), 012131. https://doi.org/10.1088/1742-6596/2267/1/012131.

Mishra, U., Wu, J.Z., & Sarkar, B. (2021). Optimum sustainable inventory management with backorder and deterioration under control carbon emissions. *Journal of Cleaner Production*, *279*, 123699. https://doi.org/10.1016/j.jclepro.2020.123699.

Mou, Q., Cheng, Y., & Liao, H. (2017). A note on "lead time reduction strategies in a single-vendor-single-buyer integrated inventory model with lot size-dependent lead times and stochastic demand". *International Journal of Production Economics*, *193*, 827-831. https://doi.org/10.1016/j.ijpe.2017.09.012.

Rani, N., Maheshwari, S., & Sharma, M.K. (2025). A sustainable inventory model: integrating eco-packaging, advertising, and pricing strategies under carbon tax policies. *International Journal of System Assurance Engineering and Management, 16*(3), 1339-1356. https://doi.org/10.1007/s13198-024-02644-w.

Ruidas, S., Seikh, M.R., & Nayak, P.K. (2021). A production inventory model with interval-valued carbon emission parameters under price-sensitive demand. *Computers & Industrial Engineering, 154*, 107154. https://doi.org/10.1016/j.cie.2021.107154.

Sana, S.S. (2010). A production-inventory model in an imperfect production process. *European Journal of Operational Research, 200*(2), 451-464. https://doi.org/10.1016/j.ejor.2009.01.041.

Sarkar, B., Saren, S., Sarkar, M., & Seo, Y.W. (2016). A Stackelberg game approach in an integrated inventory model with carbon-emission and setup cost reduction. *Sustainability, 8*(12), 1244. https://doi.org/10.3390/su8121244.

Saxena, L.K., Jain, P.K., & Sharma, A.K. (2018). Tactical supply chain planning for tyre remanufacturing considering carbon tax policy. *The International Journal of Advanced Manufacturing Technology*, *97*(1), 1505-1528.

Saxena, N., Singh, S.R., & Sana, S.S. (2017). A green supply chain model of vendor and buyer for remanufacturing. *RAIRO-Operations Research*, *51*(4), 1133-1150. https://doi.org/10.1051/ro/2016077.

Shee, S., & Chakrabarti, T. (2024). Impact of carbon emission policies on a production inventory model with stock and demand dependent production rate and preservation technology related deterioration. *International Journal of Mathematics in Operational Research*, *28*(4), 409-430. https://doi.org/10.1504/IJMOR.2024.140350.

Suvetha, R., Rangarajan, K., Dey, B.K., Alrasheedi, A.F., Ivkovic, N., & Jana, C. (2025). A sustainable production inventory model for power-pattern demand with carbon emissions and shelf life considerations. *International Journal of Computational Intelligence Systems*, *18*(1), 129. https://doi.org/10.1007/s44196-025-00861-0.

Tshinangi, K., Adetunji, O., & Yadavalli, S. (2025). A two-echelon supply chain inventory model for perishable products with a shifting production rate, stock-dependent demand rate, and imperfect quality raw material. *AppliedMath, 5*(2), 50. https://doi.org/10.3390/appliedmath5020050.

Utama, D.M., Santoso, I., Hendrawan, Y., & Dania, W.A.P. (2025). A sustainable production-inventory model with CO2 emission, electricity and fuel consumption under quality degradation and stochastic demand: a case study in the agri-food industry. *Environment, Development and Sustainability*, *27*(4), 9569-9608.

Wang, Y., Xu, X., & Zhu, Q. (2021). Carbon emission reduction decisions of supply chain members under cap-and-trade regulations: a differential game analysis. *Computers & Industrial Engineering, 162*, 107711.

Yadav, A.S., Kumar, A., Yadav, K.K., & Rathee, S. (2025). Optimization of an inventory model for deteriorating items with both selling price and time-sensitive demand and carbon emission under green technology investment. *International Journal on Interactive Design and Manufacturing*, *19*(2), 1297-1313.

Zhang, C., Xu, Y., & Zheng, Y. (2024). Blockchain traceability adoption in low-carbon supply chains: an evolutionary game analysis. *Sustainability, 16*(5), 1817. https://doi.org/10.3390/su16051817.