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Applied Picture Fuzzy Sets for Group Decision-Support in the Evaluation of Pedagogic Systems

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Abstract

Evaluation of E-Learning resources plays a significant role in the context of pedagogic systems. Resource evaluation is important in both conventional 'talk-and-chalk' teaching and in blended learning. In on-line (e-learning) teaching [an enforced feature of pedagogic systems in tertiary education during the Covid-19 pandemic] the effective evaluation of teaching resources has obtained importance given the lack of 'face-to-face' student-teached interaction. Moreover, the enforced use of e-learning has demonstrated the effectiveness of on-line pedagogic systems, which has been argued in blended learning pedagogic systems. Additionally, in e-learning, the lack of 'face-to-face' meetings [between teaching staff and students and in staff meetings] makes feedback (positive and negative) important for all actors in the pedagogic system. In this paper we present a novel approach to enable effective evaluation of teaching resources, which provides effective group decision-support designed to evaluate e-learning resources, enhancing students' satisfaction. The proposed approach employs Picture Fuzzy Sets to quantify survey responses from actors, including: agree, disagree, neutral, and refuse to answer. In our approach, the system can manage the evaluation of e-learning resources based on both explicit and tacit knowledge using a picture fuzzy rule-based approach in which linguistic semantic terms are used to express rules and preferences. The proposed system has been tested using e-learning case studies with the goal of enhancing the learning experience and increasing students' satisfaction. Experimental results demonstrate that our proposed approach achieves a significant improvement in performance in the evaluation of e-learning resources.

Keywords- Picture fuzzy sets, Pedagogic systems, Group-decision support, e-learning resource evaluation, e-learning course evaluation.

1. Introduction

Overtime, e-learning and m-learning (the terms are frequently used interchangeably – hereafter termed e-learning) has gained traction as an effective mode of delivery in a broad range of domains and systems as an effective mode of delivery where especially pedagogic systems that can be implemented flexibly *anywhere* and *anytime*, especially pedagogic systems (Moore & Pham, 2012). This flexibility: (1) provides a basis for and communication between students and between student and tutors, (2) improves the teaching/learning experience, (3) provides a platform upon which a sense of community can be created, (3) mitigates the isolation often experience by students studying on a distance learning basis (such as when taking an Open University course of study with often at most one weeks residential study), (4) accommodates the three learning styles identified in Stead and Colley (2008) enabling students to learn at their own pace, and (5) provides for improved time management to enable learning and networking at a time and place to suit the students availability to study. Moreover, the isolation due to Covid-19 pandemic has enforced the use of e-learning in pedagogic systems, proving that e-learning can be both effective and useful. This realization has been made in the applies to educational domain and other domains (e.g., healthcare) where there has been an historical reluctance to employ such methods.

There are, however, potential problems when using the e-learning paradigm (Moore & Pham, 2012), including assessment of: (a) the course, (b) the course teaching resources, (c) student engagement, (d) student satisfaction, and (e) the reaction and acceptance of on-line pedagogic systems by tutors. The overall evaluation of points (a) to (e) presents significant challenges. In this paper, we present a method designed to provide an effective basis upon which e-learning courses and the related course resources can be assessed and evaluated. In conventional pedagogic systems, which have traditionally used the 'talk-and-chalk' approach, tutors can evaluate student engagement and satisfaction relatively easily in via 'face-to-face' interactions and student surveys. Surveys are also be used to assess tutor attitudes and satisfaction. In this paper, we propose a novel model which employs fuzzy set theory to evaluate pedagogic systems to evaluate such systems. In this study, we limit our research to evaluating the e-learning approach with the goal of assessing the online learning experience and student satisfaction. However, our proposed approach provides an effective basis upon which points (a) to (e) may be evaluated.

For students, effective evaluation of pedagogic systems (including e-learning systems) is vital to ensure successful delivery, effective implementation, positive impact, and continuous upgrading and improvement in the quality of course teaching materials. For instructors, evaluation of pedagogic systems (all types of delivery including e-learning courses) can improve the clarity and quality of course teaching resources. Furthermore, institutions (e.g., schools, universities, and educational providers generally) are frequently evaluated to test the key performance indicators (KPI). For example, universities are evaluated and assessed to grade performance (often using league tables) and authorize the awarding of degrees.

The novel approach proposed in this study extends on the traditional fuzzy set theory and Picture Fuzzy Sets (PFS) (Cuong, 2014; Cuong & Kreinovich, 2013). PFS are applied to quantify qualitative and quantitative factors with respect to e-learning courses. We defer a description of the proposed approach to Section 4. However, in summary, the evaluation of e-learning teaching resources can handle both explicit and tacit knowledge using a picture fuzzy rule-based approach in which linguistic and semantic terms are used to express picture fuzzy rules in relations and preferences. Our study contributions lie in:

- The quantifications of qualitative and quantitative factors including neutral responses for the learners when students study online to express courses as well as learning resources
- The traditional fuzzy set theory and the extensions of PFS to express preferences in terms of: (a) positive responses, (b) negative responses, (c) neutral responses, and (d) refusal to respond. The ability to evaluate the four potential responses extends the capabilities inherent in the traditional fuzzy sets and of 'intuitionistic' fuzzy sets concepts using fuzzy weights. While corresponding E-learning feedbacks of students, they may be responded as neural and refusal responses.
- A Group Decision-Support in the Evaluation of Pedagogic Systems has quantified sensibilities of learners including positive responses, negative responses, and neutral responses using PFS while studying learning resources online. An evaluating group will be the enhanced quality of learning resources, together with their learning expectations.

In this paper, we provide a practical case study in which we evaluate the e-learning pedagogic system with the focus on the course topics and resources. The proposed model has been tested with the aim of enhancing the learning experience and student satisfaction. These methods have demonstrated the effectiveness of the lecture evaluation based on multi-criteria with the attending of students.

The paper is structured as follows: related research is considered in Section 2. In Section 3 we introduce the preliminaries with the proposed model defined and discussed in Section 4. Section 5 presents the case study with open research questions and future directions for research addressed in Section6. The paper closes with Section 7 where we present concluding observations.

2. Picture Fuzzy Sets and Related Research

There is a large volume of published research addressing pedagogic systems which have presented an analysis of such systems. The research has considered pedagogic systems from many perspectives including socio-economic perspectives (Adebisi & Oyeleke, 2018). However, the principal focus lies in addressing access to education and improving the effectiveness of pedagogic systems. Considered from the access to education, e-learning is a pedagogic system that provides democratic and equitable access to education at all levels *anytime* and *anywhere* (given a suitable computer and Internet access) Moore (2011).

While the published research has addressed access to education and socio-economic limitations, the open research question is the ability to effectively assess and evaluate the courses and teaching resources. The focus of this paper is e-learning pedagogic systems; however, the research addressed resource evaluation in all types of pedagogic systems. As introduced in Section 1, in this paper we present a novel approach to resource evaluation using PFS which is a novel approach introduced in (Cuong, 2014; Garg, 2017), Cuong and Kreinovich (2013) which is a direct extension to the traditional fuzzy set theory (FST) introduced by Lofti Zadeh (Klir & Yuan, 1996; Brown, 1971) and 'intuitionistic' fuzzy sets (IFS) which introduced extensions to traditional fuzzy set theory by enabling *positive*, *negative* and *neutral* degrees of membership of a set. Fuzzy set theory plays an important role in decision making under uncertainty. PFS extend IFS by enabling *positive*, *negative*, *neutral* and *refusal* degrees of membership of a set. Moreover, PFS provides a general approach to integrating calculated numerical knowledge and human linguistic as fuzzy rule base.

IFS provides an important generalization of FST and IFS has been applied to multiple domains of interest. For example, De et al. (2001) published a paper entitled "An **application** of **intuitionistic**

fuzzy sets in medical diagnosis" and Szmidt and Kacprzyk (2001) considered IFS in "some medical applications". Pattern recognition has been a feature of IFS where Dengfeng and Chuntian (2002) published a study entitled "New similarity measures of intuitionistic fuzzy sets and application to pattern recognitions". Pattern recognition using IFS has been studied to address similarity assessment in a paper entitled "A biparametric similarity measure on intuitionistic fuzzy sets with applications to pattern recognition" Boran and Akay (2014).

It can be seen from the foregoing discussion that one of the important concepts related to the degree of neutrality is lacking in IFS theory. Cuong (2014) studied some properties of PFSs and suggested distance measures between PFSs. Phong et al. (2014) studied some compositions of picture fuzzy relations.

- Cuong and Hai (2015) investigated main fuzzy logic operators: negations, conjunctions, disjunctions, and implications on PFS and also constructed main operations for fuzzy inference processes in picture fuzzy systems. Cuong et al. presented properties of picture fuzzy sets, Viet et al. (2015) presented picture fuzzy inference system based on membership graph. Pan et al. (2021) established some new operational laws of PFS.
- Suman and Neeraj Gandotra (2021) introduced a new information measure under the PFS known as (R, S) norm picture fuzzy information measure for application in multi-criteria decision making.
- Peng and Dai (2017) proposed an algorithm for PFS and applied in decision making based on new distance measures.
- Wei (2017) presented some processes to measure similarity between PFS, Garg (2017) studied some picture fuzzy aggregation operations and their applications to multicriteria decision making.
- Qin et al. (2020) discussed some limitations of the existing operational laws and aggregation operator of PFVs and developed a set of novel operational laws of PFVs in the framework of Dempster-Shafer theory.

2.1. Group Decision-Support

A conventional Group Decision-Support Model (GDM) is commonly known as collaborative decision-making when decision-makers arrive at the correct decision(s) in the evaluation pedagogic systems, courses, topics, and resources. GDM can be used to combine users' preferences to enable effective evaluation. Such methods have demonstrated the effectiveness of the lecture evaluation based on multi-criteria with the assistance of experts. To further apply these methods, Hong et al. (2020) have investigated complex fuzzy inference systems combined with fuzzy knowledge graph in decision making, Marketa et al. (2012) designed a complex model of learning resources' evaluation with adaptive e-learning courses.

There are many example of research addressing evaluation of pedagogic systems:

- Sung et al. (2011) address a *quality certification system* adopted by the *e-Learning Quality Service Centre* (eLQSC) in Taiwan; the approach uses the e-Learning Courseware Quality Checklist (eLCQC).
- Salahli et al. (2012) applied Fuzzy logic methods to create a Knowledge Management System for Personalized E-learning.
- Mohammed Megahed and Ammar Mohammed (2020) modelled adaptive e-learning environments using facial impressions and fuzzy logic.

- Hai et al. (2012) have proposed a model using knowledge graph for the improvement of clustering user's behavior on Social Networks in E-learning domain.
- Al-Fraihat et al. (2020) identified factors to evaluate the satisfaction of students of e-Learning coursewares.
- Jia-Wei Gong applied a multi-criteria decision making approach using fuzzy logics for e-Learning websites evaluation. Hai et al. (2012) studied intelligent context-aware system intelligent context processing with reasoning, decision support, and *Kansei* evaluation.

From the examples cited it can be seen that the use of FST and IFS provides an effective basis upon which the evaluation of pedagogic systems can be realised. Moreover, the extensions to FST and IFS by PFS provided interesting potential opportunities for extending the benefits derived from FST and IFS is the evaluation of pedagogic systems and related resources.

3. Preliminaries

3.1. Picture Fuzzy Sets and Relations

- ***Definition 1.*** A fuzzy set (FS). A on a universe X is an object of the form $A = \{\langle x, \mu_A(x)\rangle | \forall x \in X, \mu_A(x) \ \in [0,1]\}$ where $\mu_A(x)$ is called the degree of membership of x in A.
- ***Definition 1.*** Cuong and Kreinovich (2013) have considered 2 set of space X,Y is not empty. The picture fuzzy relation (PFR) R is a picture fuzzy set (PFS) on $X \times Y$ $(R \in PFR(X \times Y)$ of the form:
    - $R = \{((x,y), \mu_R(x,y), \eta_R(x,y), \nu_R(x,y) \vee (x,y) \in X \times Y)\}$, where $\mu_R: X \times Y \to [0,1]$ is the level of positive membership of (x, y) in R, $\eta_R: X \times Y \to [0,1]$ is the level of neutral membership of (x, y) in R, $\nu_R: X \times Y \to [0,1]$ is the level of negative membership of (x, y) in R, $0 \leq \mu_R(x,y) + \eta_R(x,y) + \nu_R(x,y) \leq 1$ with $\forall (x,y) \in X \times Y$ and $\pi_R(x,y) = 1 - \mu_R(x,y) - \eta_R(x,y) - \nu_R(x,y)$ is the level of declined membership (x, y) in R.

We have the formula to convert the 3-membership function of the PFR set into a real value that shows the level of relationship between x and y:

$$S_R(x,y) = \mu_R(x,y) - \nu_R(x,y) * \pi_R(x,y) \quad (1)$$

$$\text{Where } \pi_R(x,y) = 1 - \mu_R(x,y) - \eta_R(x,y) - \nu_R(x,y)$$

3.2. Picture Fuzzy Cross-Entropy Measure

- ***Definition 2.*** Cuong and Kreinovich (2013) are given 2 picture fuzzy sets $\alpha = (\mu_{\alpha_j}(x_j), \eta_{\alpha_j}(x_j), \nu_{\alpha_j}(x_j))$, and $= (\mu_{\beta_j}(x_j), \eta_{\beta_j}(x_j), \nu_{\beta_j}(x_j))$ with j = 1, 2, …, n on the background space X with $x_j \in X$, with the weight vector of α and β being $w = (w_1, w_2, \dots, w_n)$ with $w_j \in [0,1]$, j = 1, 2, …, n such that $\sum_{j=1}^{n} w_j = 1$,, the cross-entropy measure between α and β.

4. The Proposed Model and Problem Description

This section presents some key concepts to illustrate the proposed method. We consider the problem being addressed and present our novel proposed model for evaluating policymaking under uncertainty.

4.1. The Speaking Problem

In this section we present the process to establish: (i) a system of evaluation, (ii) a system to advise course tutors of the course metrics, and (iii) a system to implement the ranking of highly rated courses in descending order.

Input: Assessments from students, lecturers and experts with the course are based on the set of criteria ICT and the course evaluation criteria set by HUST in the appendix.
Output: (1) advice for lecturers who prepare courses to improve the course delivery, and (2) offer highly rated courses for students.

4.1.1. General Model

The system consists of 3 main blocks:

Block 1 “Picture fuzzy relation”:

- Input: Evaluation vectors are obtained through the course evaluation interface.
- Output: Set $PFR(C \times F)$ is a set of picture fuzzy relation between the course and criteria, where C is a set of courses, F is a set of criteria.

Block 2 “Deductive motor”:

- Input: Set $PFR(C \times F)$ is calculated in block 1.
- Output: Giving advice to lecturers on lesson preparation. The advice is given by calculating $S_R(C \times F)$ from $PFR(C \times F)$ and taken $S_R(C \times F)$ as input to the deductive motor then advice is given form there..

Block 3 “Course Rank”:

- Input: Set $PFR(C \times F)$ is calculated in block 1.
- Output: Provide highly rated courses in descending order for students.
- In the next section, it shows in details in the processing blocks, blocks 1 and 2 in section 3. 2, block 3 in section 3. 3 below and the process of building blocks.

4.2 Course Consultant Algorithm

Here we consider: (a) the *speaking* problem and provide a problem description.

Speaking problem:
Develop a rating and consulting online courses system.

- **Input**: Assessments from students, lecturers and experts with the course based on the criteria set ICT Newhouse (Paul Newhouse, 2002) and the HUST's set of criteria. In practice, we have applied HUST’s set of criteria as an official course evaluation for the proposed model in the case study of HUST students.
- **Output**: The proposed model is given as an advice for preparing lecturers, mostly based on results of group decision support in evaluations to make the course resources better.

Proposes solutions to solve the problem:
Our proposed solution the effectively address the evaluation issue lies in the use of PFS to perform the evaluation and build deductive apparatus (apply progressive inference) to give consultant (advice and analysis learning courses) from the assessments of students, lecturers, and experts (Figure 1).

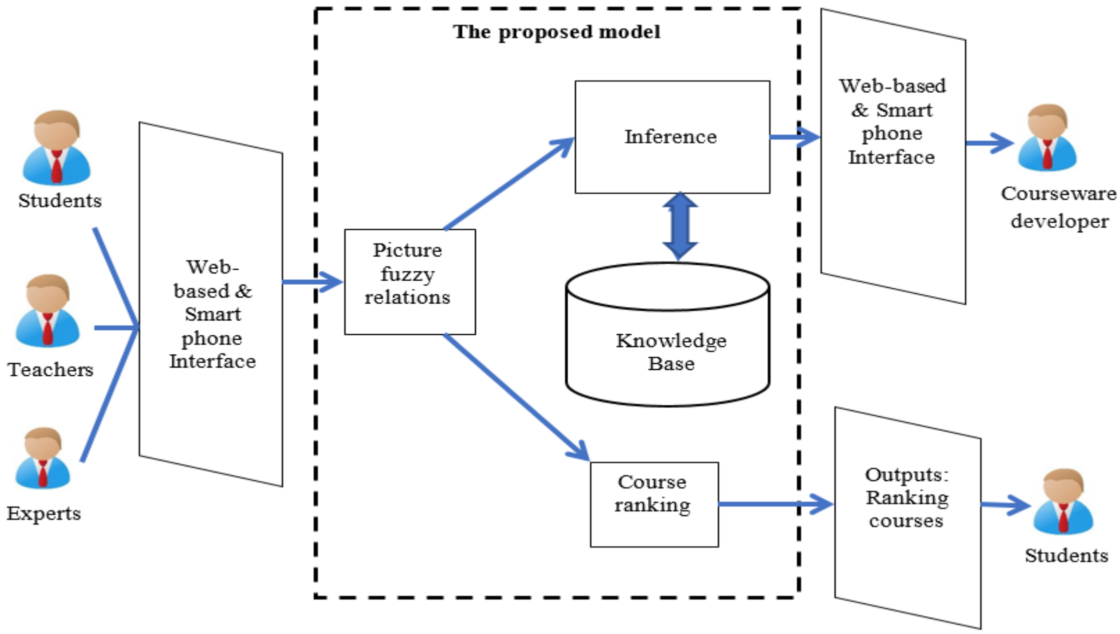

Figure 1. The proposed Pedagogic System.

Problem description:
The evaluation of students, lecturers and experts will be conducted based on predefined criteria. Each criterion in the set of criteria is a pedagogical aspect of the courses. Each of these criteria will be graded from many students, lecturers, experts with the levels are (Agree, Neutral, Disagree, Refuse to answer).

Set $E = \{e_1, e_2, \dots, e_g\}$ is the set of experts with g is the number of experts, $T = \{t_1, t_2, \dots, t_p\}$ is the set of teachers with p is the number of teachers, $S = \{s_1, s_2, \dots, s_q\}$ is the set of students with q is the number of students and $P = E \cup T \cup S$ is the set of n elements is the number of participants in the evaluation with $n = g + p + q$, $W = \{w_1, w_2, \dots, w_n\}$ is the weight of reliability level for the evaluator $w_i \in [0,1]$. Called $C = \{c_1, c_2, \dots, c_h\}$ is a space set of course consisting of h elements which are being evaluated courses, $F = \{f_1, f_2, \dots, f_m\}$ is a space set of course evaluating criteria consisting of m elements with a number of criteria to evaluate the course and set of criteria taken from section 5.

The basic algorithm has 5 steps as follows:
Step 1: Determine the evaluation vector of the students, lecturers and experts (Conducting an evaluation on the interface)
At this step, with a course $c_t \in C$ through the program interface, students, lecturers and experts will evaluate the criteria, the result is a matrix:

$$\begin{bmatrix} v_{11} & \cdots & v_{1m} \\ \vdots & \ddots & \vdots \\ v_{n1} & \cdots & v_{nm} \end{bmatrix} \quad (2)$$

where v_{ij} is the evaluation of the i-th evaluator for the j-th criterion of the course c_t and $v_{ij} \in \{Agree, Neutral, Disagree, Refuse\ to\ anwser\}$.

Vector $v_i(v_{i_1}, v_{i_2}, \dots, v_{i_m})$ is considered the evaluation of the i-th students or teachers or experts for the being considered course.

Step 2: Determine $PFR(C \times F)$ is the picture fuzzy relation in order to evaluate a course mostly based on criteria.

For each course, suppose *n* assessors corresponding to *n* vectors of the form:

$$v_i\left(v_{i_1}, v_{i_2}, \dots, v_{i_m}\right)\ với\ i = 1,2, \dots, n \quad (3)$$

Then we will proceed to determine $PFR(C \times F)$ is R. The result is as follows:

$$R = \{((c_t, f_j), \mu_R(c_t, f_j), \eta_R(c_t, f_j), \nu_R(c_t, f_j))\} \quad (4)$$

with c_t is the being evaluated course with $t = 1,2, \dots, h$ and f_j is the j-th criterion with $j = 1,2, \dots, m$. Functional members $\mu_R(c_t, f_j), \eta_R(c_t, f_j), \nu_R(c_t, f_j)$ corresponding to the level of agree, neutral and disagreement are determined by:

$$\frac{\sum_{i=1}^{k} w_i}{\sum_{j=1}^{n} w_j} \quad (5)$$

with $\sum_{i=1}^{k} w_i$ is the total of the weights of the people chose answer,, $\sum_{j=1}^{n} w_j$ the total of the weight of all the evaluating participants

Step 3: Determine the general evaluation vector which reveals the degree of Agree, Neutral, Disagree, Refuse to answer between criterial and the course

For each course c_t we have set:

$$R = \left\{\left(\left(c_t, f_j\right), \mu_R\left(c_t, f_j\right), \eta_R\left(c_t, f_j\right), \nu_R\left(c_t, f_j\right)\right)\right\} \quad (6)$$

with $j = 1,2, \dots, m$.

To determine the general evaluation vector we use the formula:

$$S_R(c_t, f_j) = \mu_R(c_t, f_j) - \nu_R(c_t, f_j) * \pi_R(c_t, f_j) \quad (7)$$

with $\pi_R(c_t, f_j) = 1 - \mu_R(c_t, f_j) - \eta_R(c_t, f_j) - \nu_R(c_t, f_j)$ to convert data from the PFS set into real values, as input to the deductive motor. From there we obtain the general evaluation vector as follows:

$$S_R(C \times F) = \{(S_R(c_t, f_1), S_R(c_t, f_2), \dots, S_R(c_t, f_m))\} \quad (8)$$

with c_t is the being evaluated course and m is the number of criteria.

Step 4: Conduct inferences to give advice

Step 4. 1: Conduct inferences from the evaluation of course criteria.

- Inference using rule sets has the form of rule base as follows:

$$(S_R(c_t, f_1) = a_1) \wedge (S_R(c_t, f_2) = a_2) \wedge \dots \wedge (S_R(c_t, f_i) = a_i) \rightarrow (c, p) \quad (9)$$

Where $S_R(c_t, f_i) = a_i$ means the score of course dependency level kh_t depends on criteria tc_i is a_i (The equals in the law can also be replaced by other signs such as =, <, >, ≤, ≥). The conclusion of the law is an assertion p which is a course statement with coefficient c is the certain level of that assertion.

- At this step, the system compares the scores of the criteria in the average assessment with the levels to make course comments. Evaluation is uncertain, so each rating should give a certain level of that evaluation.

Step 4. 2: At this step, the system will take step 4. 1 results as input to provide other comments or appropriate advice. The given advice not only on the assessment of the course, but also on the certainty of that assessment.

- This deductive step uses the rule set form:

$$(c_1, p_1, q_1)^\wedge\ (c_2, p_2, q_2)\ ^\wedge\ (c_3, p_3, q_3)\ ^\wedge \ldots\ (c_i, p_i, q_i) \quad \rightarrow (r, c) \tag{10}$$

- r is an event (course comment or advice), c is the reliability of r.
- Here (c_i, p_i, q_i) is a comment of the course with the allowable reliability range for that assessment. **If the event c_i has certainty within that range, it satisfies the premise of this type of law.**
- In addition, r is continued to be used during the inference of step 4. 2 until no rules are satisfied.

Step 5: Giving advice of course resources and contents
If r is an advice and has reliability greater than a threshold (e.g. 7) then that advice will be given.

5. The Case Study

Assuming there are n = 6 participants including: 2 experts are $E = \{e_1, e_2\}$, 1 lecturer is $T = \{t_1\}$, 3 students are $S = \{s_1, s_2, s_3\}$ with reliability weights being

$W = \{w_{e_1}, w_{e_2}, w_{t_1}, w_{s_1}, w_{s_2}, w_{s_3}\} = \{1{,}0.8{,}0.5{,}0.2{,}0.2{,}0.2\}$, evaluation of a course is $C = \{c\}$ with 3 criteria are $F = \{f_1, f_2, f_3\}$. The answers to each of these criteria are **Agree, Neutral, Disagree, Refuse to answer.** The reliability threshold for giving advice is 0. 7.

The number of criteria is large, so for example, I would like to use 3 criteria in the ICT Newhouse as follows (Tables 1–3):

Table 1. Example criteria.

Criteria code	Name of criteria
f_1	Discover the truth and develop knowledge.
f_2	Providing scale for high level thinking skills.
f_3	Positively engage students with encouragement and challenge.

The available events in the knowledge base:

Table 2. Example events.

Event code	Event description
F_{11}	The course has no practical significance.
F_{12}	The course with much dry knowledge
F_{13}	The course does not give students the opportunity to be creative.
F_{14}	The course has not been helpful for students on the knowledge base.
F_n	…………………………………………………………………

The rules of knowledge base:

Table 3. Example rules.

Code of law	Law description	
	Premise	Conclude
R_{11}	$0.5 < S_R(c, f_1) \leq 0.75$	$(F_{11}, 0.9)$
R_{12}	$0.5 < S_R(c, f_1) \leq 0.75$	$(F_{12}, 0.9)$
R_{13}	$0.5 < S_R(c, f_1) \leq 0.75$	$(F_{13}, 1.0)$
R_{14}	$0.25 < S_R(c, f_2) \leq 0.5$	$(F_{14}, 0.7)$
R_{15}	$0.25 < S_R(c, f_2) \leq 0.5$	$(F_{15}, 0.9)$
R_n	$0.25 < S_R(c, f_n) \leq 0.5$	$(F_n, 0.9)$

The algorithm is started as follows:

Step 1: Determine the evaluation vector of the evaluator.

Assuming after evaluating on the evaluation interface, the evaluation vectors are obtained as follows:

Expert e_1: $v_{e_1} = (Agree, Agree, Neutral)$
Expert e_2: $v_{e_2} = (Agree, Refuse, Refuse)$
Lecturer t_1: $v_{t_1} = (Neutral, Neutral, Refuse)$
Student s_1: $v_{s_1} = (Disagree, Disagree, Refuse)$
Student s_2: $v_{s_2} = (Agree, Disagree, Refuse)$
Student s_3: $v_{s_3} = (Disagree, Disagree, Neutral)$

Step 2: Determine $PFR(C \times F)$ the picture fuzzy relation between the course and the criteria.

Then we will proceed to determine $PFR(C \times F)$ which is R. The result will be:

$$R = \left\{\begin{matrix} ((c, f_1), \mu_R(c, f_1), \eta_R(c, f_1), \nu_R(c, f_1)), \\ ((c, f_2), \mu_R(c, f_2), \eta_R(c, f_2), \nu_R(c, f_2)), \\ ((c, f_3), \mu_R(c, f_3), \eta_R(c, f_3), \nu_R(c, f_3)) \end{matrix}\right\} \quad (11)$$

Step 3: Determine the general evaluation vector.

We have the general evaluation vector

$$S_R(C, F) = (S_R(c, f_1), S_R(c, f_2), S_R(c, f_3)) = (0.69, 0.29, 0) \quad (12)$$

Step 4: Conduct inferences to give advice.

Step 4.1: From the general evaluation vector above we see the applicated rules: $\mathbf{R_{11}}$, $\mathbf{R_{12}}$, $\mathbf{R_{13}}$, $\mathbf{R_{14}}$, $\mathbf{R_{15}}$, $\mathbf{R_{16}}$, $\mathbf{R_{17}}$.

After applying those laws, we have conclusions with corresponding certainties.

- (F_{11}, 0.9): "The course has no practical significance." with certainty is 0.9
- (F_{12}, 0.9): "The course with much dry knowledge" with certainty is là 0.9
- (F_{13}, 1.0): "The course does not give students the opportunity to be creative." with certainty is 1
- (F_{14}, 0.7): "The course has not been helpful for students on the knowledge base." with certainty is 0.7
- (F_{15}, 0.9): "Providing scale for high level thinking skills is not good." with certainty is 0.9

- (F_{16}, 0.9): “The course has few exercises and group exercises for students to interact with each other.” with certainty is 0.9
- (F_{17}, 0.5): “The course has few exercises.” with certainty is 0.5

Step 4.2: Continue to make inferences with the premise that is the conclusion in step 4. 1. For example, when there is a conclusion F_{11} with certainty of 0.9, which is in the range (0.8, 1.0) means that it satisfies the condition (F_{11}, 0.8, 1.0). Similarly, we have the previous step conclusions (F_{12}, 0.9) satisfy (F_{12}, 0.9, 1.0) and (F_{13}, 1.0) satisfy (F_{13}, 1.0, 1.0).

In comparison with the conventional evaluation, it is easy to see that the proposed model has the following advantages:

- More professional: The model of the project performs evaluation based on a set of criteria. The evaluation based on the criteria also shortens the time for evaluation and consultancy so much compared to the fact that experts have to enter text advice for each course.
- Resolve conflicts between assessors: With the common assessment, when many people evaluate the same course, there will be cases where these assessments are contradiction, even contradictory. The project model solves this problem by using an assessment that represents all evaluation participants.

In the case study of Hanoi University of Science and Technology, a total of 43 participants responded to the criteria set (HUST) for students. The course evaluation data is summarized in Table 4 as follows:

Table 4. Course evaluation data.

No.	Criteria	Agree	Neutral	Disagree	Refused to answer
1	Is the mixed-learning class the same quality as the face-to-face class?	21	16	3	3
2	Is electronic courseware (video, e-book, slides etc.) of the course suitable for the content and training objectives of the subject and produce good results in learning?	21	19	2	1
3	Is the distribution of classroom and online activities appropriate to support the learning process?	11	16	14	2
4	Does mixed training have a positive impact on learning results: knowledge is increased in quantity and better organized, and can lessons be easily reviewed?	28	10	0	5
5	Do teachers in mixed classes often interact and answer students' questions on the LMS system?	13	16	10	4
6	Do the tests / assessments / exercises used by teachers on LMS bring positive results?	19	10	7	7
7	Is it easier to find documents that suit your needs (exercises, books, reference materials) through a mixed class than traditional classes?	22	7	11	3
8	The website loads fast and is easy to view and download learning resources?	13	15	8	7
9	Class organization on LMS is logical, visually clear, and makes switching between courses and lessons easy?	21	11	8	3
10	Are you satisfied with the mixed training and will continue to participate in the mixed training classes in the future (if any)?	20	9	7	7

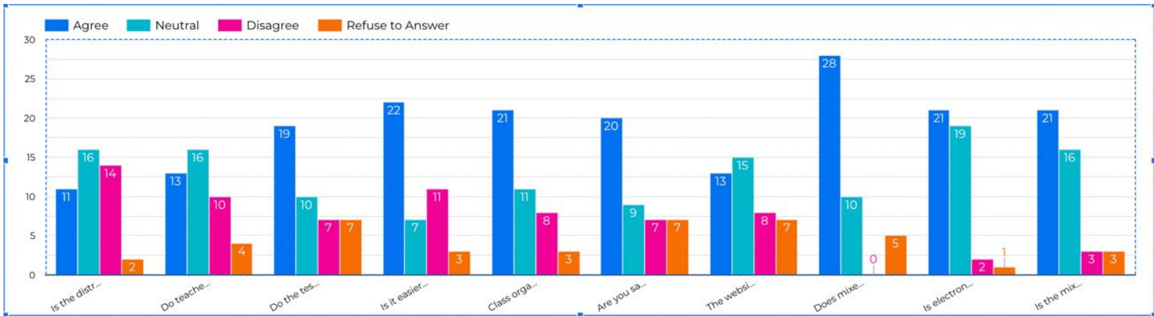

Figure 2. Experimental results for students' responses.

6. Results and Discussion

A total of 43 participants responded to the criteria set for students. The results may be summarized as follows:

(i) Each criteria received 43 answers being 'agree', 'neutral', 'disagree' or 'refuse to answer', as shown in Figure 2.

(ii) The ten criteria are as in Table 4. Overall, there was an average of 4.2 refusal to answer and 12.9 neutral responses for each category.

(iii) Except for criteria "Is the distribution of classroom and online activities appropriate to support the learning process?" (Where there was 14 disagree and 11 agree) every criterion achieved more agree responses than disagree responses.

(iv) The biggest difference being from "Does mixed training have a positive impact on learning results: knowledge is increased in quantity and better organized, and can lessons be easily reviewed?" having 28 agree and 0 disagree.

(v) There is an average of 18.9 agree and 7 disagree for a category, making for a 11.9 mean difference.

In summary, we may conclude that the method proposed in this paper provides an effective basis upon which e-learning courses may be assessed and evaluated. However, while the current study has resolved many issues, we have identified open research questions (ORQ) along with areas where improvements in our proposed system are achievable including:

- We are considering investigating extensions using large rules including conflicted rules and fired rules in the knowledge base.
- In future studies, we propose to focus on the 'real-time' learning environment with automated consultant responses to provide advanced technical support using adaptable interfaces designed for web-based, smart phone and other devices which is an essential feature in the developing mobile technology landscape.
- As discussed in Section 1 there are potential problems inherent in the e-learning paradigm. In future studies we intend to investigate autonomous assessment of student engagement, student satisfaction, and the reaction and acceptance of on-line pedagogic systems by tutors. These research objectives present significant challenges in current online pedagogic systems.

7. Conclusion

In this article we have presented a new method for improvement of knowledge-based consultant systems with an illustrative case study predicated on an e-learning pedagogic system. To address a range of typical situations we use dynamic questions and responses based on discussions with advice from experts and consultants. Experimental results demonstrate that the proposed system can provide increased accuracy in dynamic situations for e-learning based on the HUST criterial, ICT Newhouse approach when matched with expert advice. Furthermore, the results derived from the study confirm that overall, the knowledge consultant system relies on reasoning rule levels to complete the consultant process.

Evaluation of the proposed 'real-time' consultant system supports the conclusions drawn that the proposed system performs better than conventional methods. However, the reported results demonstrate the potential for the proposed system presented in this paper as useful dialogs in the case study at the Hanoi University of Science and Technology (HUST).

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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