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Reliability Analysis of Probabilistic Competing Failure Systems with Both Selective and Global Failure Propagation Effects

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Abstract
This paper investigates the reliability modeling of systems subject to probabilistic competing failure behaviors with complicated failure propagation effects. To be specific, besides local failure (LF) that only affects the component itself, a component can also experience propagated failure with global effect (PFGE) leading to the system-wide failure, and propagated failure with selective effect (PFSE) that affects the set of other components. There exists a probabilistic functional dependence dynamic between system components, where some components (referred to as the probabilistic-dependent components) functionally depend on other components (designated as trigger components) and can be isolated by the trigger component failure in a probabilistic manner. Trigger component LF and PF of probabilistic-dependent components compete in the time domain. Different occurrence orderings of these component failures can lead to dramatically different system states. However, the existing reliability assessment methods are not applicable to addressing such probabilistic competing failure behaviors with considering both PFGEs and PFSEs in the system reliability analysis. A novel combinatorial reliability methodology is presented to tackle this issue with its applicability and effectiveness being demonstrated through step-by-step reliability analysis on a smart home sensor system. The proposed method is verified and the methodology complexity is discussed by comparing with the Markov method.

Keywords- Probabilistic competing failure, Propagated failure with selective effect, Propagated failure with global effect, Combinatorial method.

1. Introduction

In many real-world complex systems, component failures fall into two categories: local failures (LFs) that only cause malfunction of the component itself, and propagated failures (PFs) which also affect other system components (Xing & Levitin, 2010). PFs are further classified into two types based on the extent of impact: propagated failure with selective effect (PFSE) and propagated failure with global effect (PFGE).

PFSE influences a specific subset of system components, while PFGE causes failure of the whole system. Meanwhile, probabilistic functional dependence behavior can also exist between system components, where some components (referred to as probabilistic-dependent components) functionally depend on other components (referred to as trigger components) in a probabilistic manner, that is, the trigger component failure can lead to the inability or restricted access of probabilistic-dependent (PDEP) components within the same PDEP group with a certain probability p (referred to as isolation factor) (Wang et al., 2015). Systems with probabilistic functional dependence involve a time-domain competition between trigger component LF and PDEP component PFs. Different failure sequences result in distinct system states, illustrating the complexity of failure dynamics in such systems. (Xing et al., 2012a, 2012b, 2018). Specifically, when the trigger component experiences LF first, the probabilistic isolation effect is induced, and each PDEP component is isolated with a specific isolation factor p. The PFs originating from the isolated PDEP components are prevented from causing further impact to other system components. Once isolated, the PDEP component is inaccessible and regarded as functionally failed, and the system state is evaluated according to system structure and the remaining components. On the contrary, if a PF from PDEP components precedes the trigger component LF, the failure propagation effect will take place. In such case, a PFGE of the relevant PDEP component will result in system-wide failure and a PFSE from the relevant PDEP component will affect certain system components. Particularly, if the isolation factor $p = 1$, this dynamic behavior is simplified to deterministic competing failure behavior.

Many real-world systems exhibit probabilistic competing failure behaviors with both PFGEs and PFSEs (Levitin & Xing, 2010). For instance, in a smart home sensor system (SHSS) with n sensors connected in parallel, due to signal attenuation or battery-saving plans, the sensors normally achieve long-distance signal transmission to intelligent terminals through relay nodes. Sensors and relay nodes can experience LF (from malfunction), PFGE (via jamming attacks), and PFSE (from targeted signal interference). When a relay node fails, connected sensors may boost transmission power to facilitate direct terminal interconnection. The likelihood of this response is dynamically contingent on their residual power. When the residual power fails to sustain direct interconnection, the SHSS implements isolation of the sensor. As a result, the sensors probabilistically depend on the relay nodes and form one or more PDEP groups with the relay nodes. Relay node LF and sensor PFs within the same PDEP group engage in time-domain competition. In the case where relay node LF occurs priorly, each corresponding sensor and its PF is isolated from the SHSS with a probability modeled by a specific isolation factor p. While if any PF from sensors occur before the relay node LF, the PF would propagate to other sensors even crash the entire SHSS (Luo et al., 2013). Such complex failure competition behaviors pose challenges for the reliability assessment of systems, necessitating comprehensive consideration of probabilistic competing effects in reliability modeling.

As far as we know, there is currently no effective method to comprehensively address the reliability modeling of probabilistic competing failures with system components subject to both PFGEs and PFSEs. This paper makes a significant contribution by introducing a combinatorial method for analyzing the reliability of systems subject to probabilistic competing failure, explicitly addressing both PFGEs and PFSEs. The proposed method does not restrict failure rate distributions for system components, which improves its versatility. A SHSS case study is carried out to illustrate the effectiveness and applicability of the proposed method.

The organization of this paper is as follows: a literature review is presented in Section 2. Section 3 shows the proposed combinatorial method. Section 4 presents a case study of a SHSS and employs the proposed combinatorial method to perform reliability analysis on the example SHSS. The proposed method is verified through comparison with the Markov method. Section 5 further generalizes the proposed method. Section 6 discusses efficiency of the method through a comparison with the Markov method. The paper is concluded in Section 7, which also delineates future research directions.

2. Literature Review

Existing reliability studies have covered different types of competing failures. For example, developing a competition model for degradation mechanisms and random external shock events (Wang et al., 2020; Lyu et al., 2025a); investigating competing processes within the framework of accelerated life testing (Moustafa et al., 2021), system maintenance strategies (Yousefi et al., 2020), and system size optimization (Song et al., 2014); studying the competition between unexposed and covered failure modes of components in systems with incomplete failure coverage (Xing, 2007; Xiang et al., 2014). Unlike prior studies, this paper focuses on the competition between failure propagation and isolation effects, which is induced by functional dependences and different failure modes between different system components.

Reliability modeling of systems with competing failure behaviors caused by functional dependences has been investigated in many works, and several methods have been developed including simulation methods (Yeh, 2022; Oszczypała et al., 2024), Markov analysis methods (Zhou et al., 2021; Mittal et al., 2024; Lyu et al., 2025b), and combinatorial methods (Xing et al., 2019). Combinatorial methods exceed the limitations of simulation methods that only provide approximate results, as well as the limitations of Markov analysis that may suffer from state space explosion. Existing research has introduced combinatorial method for system reliability modeling with probabilistic competing failures, validating their effectiveness via case studies across various system categories. These methods integrate the strengths of different methods to enable comprehensive system reliability analysis. A novel combinatorial method is developed for reliability analysis of probabilistic competing failure systems with a single PDEP group (Wang et al., 2015). Then a combinatorial procedure is generated for modeling the impacts of correlated and probabilistic competing failures in reliability assessment for nonrepairable binary-state systems (Wang et al., 2018). To explore the *s*-independent and *s*-dependent dependencies between a component's LF and PF, a combinatorial method is developed to evaluate reliability of non-repairable systems with probabilistic failure isolation effects and failure propagation (Wang et al., 2017b). Competitions of probabilistic isolation and failure propagation effect is also modeled in relay-assisted wireless sensor networks reliability analysis with multi-level performance (Wang et al., 2018a). By incorporating multi-valued decision diagrams, reliability of systems subject to phase-dependent probabilistic competing failures is further addressed (Wang et al., 2018b). Then a hierarchical combinatorial method is developed to assess reliability of cascading probabilistic competing failure systems with random failure propagation time (Zhao & Xing, 2020). The aforementioned studies focused mainly on the PFGEs. With focusing on deterministic competing failures, a new combinatorial method for handling deterministic competing effects in reliability assessment of systems exhibiting both PFGEs and PFSEs is developed (Wang et al., 2012).

Previous studies indicate that existing system reliability analysis methods for probabilistic competing failure systems have certain limitations. Specifically, methods applicable to systems with PFGE cannot be directly applied to systems where both PFGE and PFSE coexist. Considering the complex scenarios in practical applications where probabilistic competing failures are combined with PFGE and PFSE, this work introduces a new combinatorial method to analyze such systems' reliability, thereby making an original contribution.

3. Proposed Combinatorial Methodology

A five-step general procedure is presented in this section to evaluate the reliability of probabilistic competing failure systems experiencing both PFGEs and PFSEs. Non-repairable systems and independent PDEP groups are assumed when presenting the methodology.

3.1 PDEP Behavior Modeling

When developing a dynamic fault tree (DFT), the probabilistic functional dependence (PFD) gate provides an effective approach to model the PDEP behavior, as depicted in **Figure 1** (Wang et al., 2017b). The PFD gate includes a trigger event and at least one PDEP events. Once the trigger event is activated, the corresponding PDEP events are initiated with certain (normally different) probabilities, and the switch symbols within the gate are used to model this probabilistic behavior. In terms of the time-domain competition between failure propagation and isolation effects considered in this paper, the trigger component LF is the trigger event, and the isolation of PDEP component is PDEP event. when the trigger component fails, each corresponding PDEP component is isolated, i.e., losing its accessibility or availability, with a certain probability.

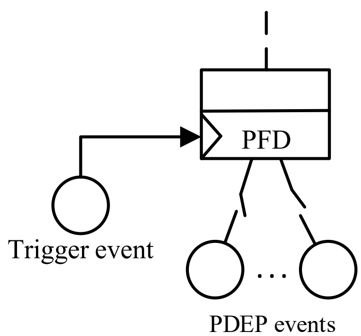

Figure 1. PFD gate.

3.2 Method Description

The proposed methodology assumes that propagated failures only originate from PDEP components, which will be relaxed in Section 5. The proposed five-step reliability analysis method can be stated as follows.

Step 1. Define two disjoint trigger events according to whether the trigger component fails.

Designated as a trigger component failure event ($TCFE$), each event captures the dual possibilities of trigger component failure, encompassing both its failure and non-failure conditions.
$TCFE_1$: Trigger component does not fail.
$TCFE_2$: Trigger component fails.

By invoking the law of total probability, system unreliability can be defined as

$$\Pr(\text{sytem fails}) = \Pr(\text{system fails}|\mathrm{TCFE_1}) \times \Pr(TCFE_1) + \Pr(\text{system fails}|TCFE_2) \times \Pr(TCFE_2) \qquad (1)$$

Step 2: Separate the impact of PFGEs for $TCFE_1$.

When the trigger component operates without malfunction, both failure competition and failure isolation are absent. In this state, any PFGE stemming from a PDEP component has the potential to trigger an entire system failure. $\Pr(\text{system fails}|TCFE_1)$ is calculated by decomposing PFGEs.

$TCFE_1$ can be decomposed into two complementary events, each event, referred to as a PFGE occurrence event (PGE), represents the occurrence and non-occurrence of PFGEs in PDEP components.
$PGE_{1,1}$: No PFGE occurs.

$PGE_{1,2}$: At least one PFGE is generated by the PDEP component.

Therefore,

$$\Pr(\text{system fails}|TCFE_1) \times \Pr(TCFE_1) = \sum_{i=1}^{2}\left[\Pr(\text{system fails}|PGE_{1,i}) \times \Pr(PGE_{1,i})\right] \quad (2)$$

According to the definition of PFGE, the system will inevitably fail when $PGE_{1,2}$ occurs, that is, $\Pr(\text{system fails}|PGE_{1,2}) = 1$. Therefore,

$$\Pr(\text{system fails}|TCFE_1) \times \Pr(TCFE_1) = \Pr(\text{system fails}|PGE_{1,1}) \times \Pr(PGE_{1,1}) + \Pr(PGE_{1,2}) \quad (3)$$

Due to the consideration of both PFGEs and PFSEs in this paper, the calculation of $\Pr(\text{system fails}|PGE_{1,1})$ will be handled via an effective method later in Step 4.

Step 3: Separate the impact of PFGEs for $TCFE_2$.

Time domain failure competition as well as the probabilistic failure isolation effects should be addressed during system reliability modeling in the following steps.

Step 3.1: Establish the probabilistic dependence event space.

In order to target the PDEP component of which PF has a time domain competition with the trigger component LF, different probabilistic dependence events are first identified, and each event is a different combination of whether the PDEP component remain functional given that the corresponding trigger component LF occurs. Assume there are *n* distinct PDEP components c_x $(x = 1, 2, \ldots, n)$. When the trigger component experiences LF, the PF from component c_x is isolated with a probability of p_x (isolation factor), and thus cannot be isolated with a probability of $(1 - p_x)$. If the component can be isolated, it is in state S_x and is referred to as an isolable component. If the component cannot be isolated, it is in state $\overline{S_x}$ and is referred to as a non-isolable component. The following dependence events are defined and each event is represented by I_k $(k = 1, 2, \ldots, 2^n)$.

$$I_1 = S_1 \cap S_2 \cap \ldots \cap S_n,$$

$$I_2 = \overline{S_1} \cap S_2 \cap \ldots \cap S_n,$$

$$\vdots$$

$$I_{2^n} = \overline{S_1} \cap \overline{S_2} \cap \ldots \cap \overline{S_n} \quad (4)$$

where, I_1 represents all PDEP components are isolable components and can be isolated by the trigger component LF; I_2 represents that component c_1 is a non-isolable component and cannot be isolated by the trigger component LF; I_{2^n} represents that all PDEP components are non-isolable and could not be isolated by their respective trigger component LF. The probability of occurrence for each dependence event is expressed as

$$\Pr(I_1) = p_1 \times p_2 \times \ldots \times p_n,$$

$$\Pr(I_2) = (1 - p_1) \times p_2 \times \ldots \times p_n,$$

$$\vdots$$

$$\Pr(I_{2^n}) = (1 - p_1) \times (1 - p_2) \times \ldots \times (1 - p_n) \quad (5)$$

Table 1 shows the isolable component set (D_{I_k}) and non-isolable component set (ND_{I_k}) of each dependence event.

Table 1. The $\boldsymbol{D_{I_k}}$ and $\boldsymbol{ND_{I_k}}$ of each dependence event.

Event	D_{I_k}	ND_{I_k}
I_1	$\{c_1, c_2, \cdots, c_n\}$	$\emptyset$
I_2	$\{c_2, \cdots, c_n\}$	$\{c_1\}$
$\vdots$	$\vdots$	$\vdots$
I_{2^n}	$\emptyset$	$\{c_1, c_2, \cdots, c_n\}$

The components in D_{I_k} do not possess sufficient capabilities to operate independently of their corresponding trigger component, so the trigger component LF. can render the isolable components unavailable. In addressing the time domain competition between trigger component LF and isolable components PFs, it becomes crucial to recognize that the sequence of these two occurrences exerts a substantial effect on the system's overall state. Conversely, the components in ND_{I_k} have sufficient capabilities to operate without relying on the trigger component. The LF of trigger component has no impact on the non-isolable components, and the PFs from the non-isolable components will affect other system components. Therefore, separating the impact of PFGEs involves considering the occurrence sequences of the PFs of isolable components and the LF of trigger component, as well as whether the PFs of non-isolable components occur.

Step 3.2: Separate the impact of PFGEs.
Similar to Step 2, we define two events $PGE_{2,1}$ and $PGE_{2,2}$.
$PGE_{2,1}$: No PFGE occurs or all PFGEs of the isolable components (referred to the components in D_{I_k}) occur after the LF of trigger component. In this event, assessing the system unreliability requires further consideration of the PFSEs originating from non-isolable components, and the time domain competition between PFSEs of the isolable components and LF of the trigger component. This analysis will take into account the varying dependence conditions and be thoroughly elaborated in Step 4.

$PGE_{2,2}$: Either no fewer than one PFGE derived from the isolable components transpires prior to the trigger component LF, or no fewer than one PFGE derived from non-isolable components takes place. The occurrence of $PGE_{2,2}$ alone is sufficient to induce total system failure, overriding any influence from PFSEs or associated competition effects. Consequently, the conditional probability $\Pr(\text{system fails}|PGE_{2,2})$ is definitively equal to 1.

According to the established dependence events I_k $(k = 1, 2, \ldots, 2^n)$ in Step 3.1, each PGE can be further defined as:
$I_k \cap PGE_{2,1}$: Components in ND_{I_k} do not experience PFGEs, and components in D_{I_k} either experience PFGEs after the trigger component LF or do not experience PFGEs. Note that for event $I_{2^n} \cap PGE_{2,1}$, D_{I_n} is empty, i.e., no component can be isolated due to the trigger component LF appeared. When $I_{2^n} \cap PGE_{2,1}$ occurs, the probability of system failure is equal to the probability of system failure under event $PGE_{1,1}$, that is, $\Pr(\text{system fails}|I_{2^n} \cap PGE_{2,1}) = \Pr(\text{system fails}|PGE_{1,1})$.

$I_k \cap PGE_{2,2}$: Either no fewer than one PFGE from a component within D_{I_k} occurs prior to the LF of the trigger component, or at least one PFGE from a component in ND_{I_k} takes place.

We have

$$\Pr(\text{system fails}|TCFE_2) \times \Pr(TCFE_2)$$
$$= \sum_{k=1}^{2^n} \sum_{i=1}^{2} \left[\Pr(\text{system fails}|I_k \cap PGE_{2,i}) \times \Pr(I_k \cap PGE_{2,i})\right] \tag{6}$$

where, $\Pr(\text{system fails}|I_k \cap PGE_{2,2}) = 1$, as the occurrence of event $I_k \cap PGE_{2,2}$ leads to the dominance of the global failure propagation effect, which will lead to the overall failure of the system. Therefore,

$$\Pr(\text{system fails}|TCFE_2) \times \Pr(TCFE_2) = \sum_{k=1}^{2^n}\left[\Pr\left(\text{system fails}\middle|I_k \cap PGE_{2,1}\right) \times \Pr\left(I_k \cap PGE_{2,1}\right) + \Pr(I_k \cap PGE_{2,2})\right] \tag{7}$$

When evaluating $\Pr(\text{system fails}|I_k \cap PGE_{2,1}) \times \Pr(I_k \cap PGE_{2,1})$ $(k = 1, 2, 3, \dots, 2^n - 1)$, the impact of PFSEs cannot be overlooked, and a detailed discussion of this aspect is provided in Step 4. In this step, we only calculate $\Pr\left(I_k \cap PGE_{2,2}\right)$ $(k = 1, 2, 3, \dots, 2^n)$ and $\Pr\left(I_{2^n} \cap PGE_{2,1}\right)$.

For random variables X_1, $X_2, \dots X_n$ representing components' time to failure, the sequential failure probability for *n* components is calculated through multiple integration as Equation (8) (Xing et al., 2013). Define $X_1 \to X_2$ as the occurrence of event X_1 prior to event X_2.

$$\Pr(X_1 \to X_2 \to \cdots \to X_n) = \int_0^t \int_{\tau_1}^t \dots \int_{\tau_{n-1}}^t \prod_{i=1}^n f_{X_i}(\tau_i) d\tau_n \dots d\tau_1 \tag{8}$$

Let c_{xpg} $(x = 1, 2, \dots, n)$ denote the event of the PFGE of occurrence for component c_x, and let event Tl represent the trigger component LF. The probability of occurrence of events $I_1 \cap PGE_{2,2}$, $I_k \cap PGE_{2,2}(k = 2, \dots, 2^n - 1)$ and $I_{2^n} \cap PGE_{2,2}$ can be calculated using Equation (8) in a straightforward manner as

$$\Pr\left(I_1 \cap PGE_{2,2}\right) = \Pr(I_1) \times \Pr\left[\left(c_{1pg} \cup c_{2pg} \cdots \cup c_{npg}\right) \to Tl\right]$$
$$= \Pr(I_1) \times \int_0^t \int_{\tau_1}^t f_{(c_{1pg} \cup c_{2pg} \cdots \cup c_{npg})}(\tau_1) \times f_{Tl}(\tau_2) d\tau_2 d\tau_1 \tag{9}$$

$$\Pr\left(I_k \cap PGE_{2,2}\right) = \Pr(I_k) \times \Pr\begin{bmatrix}\left(\left(c_{1pg} \cdots \cup c_{(k-1)pg}\right) \cap Tl\right) \\ \cup \left(\left(c_{kpg} \cdots \cup c_{npg}\right) \to Tl\right)\end{bmatrix}$$
$$= \Pr(I_k) \times \left[\Pr\left(\left(c_{1pg} \cdots \cup c_{(k-1)pg}\right) \cap Tl\right) + \Pr\left(\left(c_{kpg} \cdots \cup c_{npg}\right) \to Tl\right) - \Pr\left(c_{1pg} \cdots \cup c_{(k-1)pg}\right) \times \Pr\left(\left(c_{kpg} \cdots \cup c_{npg}\right) \to Tl\right)\right] \tag{10}$$

$$\Pr\left(I_{2^n} \cap PGE_{2,2}\right) = \Pr(I_{2^n}) \times \Pr\left[\left(c_{1pg} \cup c_{2pg} \cdots \cup c_{npg}\right) \cap Tl\right] \tag{11}$$

According to the definition of event $I_{2^n} \cap PGE_{2,1}$, $\Pr\left(I_{2^n} \cap PGE_{2,1}\right)$ is calculated by Equation (12) as

$$\Pr\left(I_{2^n} \cap PGE_{2,1}\right) = \Pr(I_{2^n}) \times \Pr\left[\left(\overline{c_{1pg}} \cap \overline{c_{2pg}} \cdots \cap \overline{c_{npg}}\right) \cap Tl\right] \tag{12}$$

Step 4: Define the events to address PFSEs and evaluate the conditional system failure probabilities.

For calculating $\Pr(\text{system fails}|PGE_{1,1})$ and $\Pr(\text{system fails}|I_{2^n} \cap PGE_{2,1})$, since no PFGE occurs under these two events, it is only necessary to consider how the PFSEs of PDEP components affect the system state. An event space is constructed to address PFSEs, with each element, called a combinational PFSE occurrence event (PSE), encoding the presence or absence of the PFSE event for each PDEP component. PSE_i $(i = 1, 2, \dots, 2^m)$ consists of 2^m events, corresponding to the maximum of *m* independent PFSE events could occur during trigger component operation. Let $c_{xps}(x = 1, 2, \dots, n)$ represent event that the PFSE of component c_x occurring.

$PSE_1 = \overline{c_{1ps}} \cap \overline{c_{2ps}} \cap \dots \cap \overline{c_{nps}}$,

$PSE_2 = c_{1ps} \cap \overline{c_{2ps}} \cap \ldots \cap \overline{c_{nps}},$
$\vdots$
$$PSE_{2^n} = c_{1ps} \cap c_{2ps} \cap \ldots \cap c_{nps} \tag{13}$$

where, PSE_1 represents no PFSE occurring; PSE_2 represents that the PFSE of component c_1 occurs; PSE_n represents all PFSEs occurring.

$\Pr(\text{system fails}|PGE_{1,1})$ and $\Pr(\text{system fails}|I_{2^n} \cap PGE_{2,1})$ can be computed as
$\Pr(\text{system fails}|PGE_{1,1}) = \Pr(\text{system fails}|I_{2^n} \cap PGE_{2,1})$
$$= \sum_{i=1}^{2^m} [\Pr(\text{system fails}|PSE_i) \times \Pr(PSE_i)] \tag{14}$$

For events of $I_k \cap PGE_{2,1}$, $(k = 1, 2, \ldots, 2^n - 1)$, although the PFGEs of isolable components are isolated, PFSEs could occur either before or after the trigger component LF. Therefore, the PFSEs should be addressed in the calculation of $\Pr(\text{system fails}|I_k \cap PGE_{2,1})$ $(k = 1, 2, \ldots, 2^n - 1)$. All propagation effects are isolated if no PFSE occurs or PFSEs of components in D_{I_k} occur after trigger component LF. However, when no fewer than one PFSE deriving from a component in D_{I_k} occurs before the trigger component LF or any PFSE deriving from the component in ND_{I_k} happens, the PFSEs dominate. In order to consider the combination of PFSE events and the sequential occurrence between the trigger component LF and PFSEs of the components in D_{I_k}, an event space $PSE_{k,i}$ $(i = 1, 2, \ldots, 2^{a_k+b_k})$ consisting of $2^{a_k+b_k}$ events should be set up. This event space encompasses the combinatorial occurrence and non-occurrence of a_k competing PFSE events for components in D_{I_k} (representing that PFSEs of components in D_{I_k} happen before the trigger component LF), and b_k PFSE events for components in ND_{I_k}. Therefore,
$\Pr(\text{system fails}|I_k \cap PGE_{2,1}) \times \Pr(I_k \cap PGE_{2,1})$
$$= \Pr(I_k) \times \sum_{i=1}^{2^{a_k+b_k}} [\Pr(\text{system fails}|PSE_{k,i}) \times PSE_{k,i}] \tag{15}$$

To determine the conditional failure probability of system, within the fault tree (FT) model of system, events denoting the trigger component LF and the corresponding isolation of isolable component are substituted with constant "1" (TRUE). Simultaneously, considering the impact of PFSEs on the system, component failure events induced by PFSEs are also replaced with constant "1" to derive a reduced system FT. Following the reduced FT, a BDD is built to model the simplified system, from which the conditional failure probabilities under *PSE*s occurrences are obtained through BDD evaluation.

Step 5: Integrate to obtain the system failure probability.

Step 2 to Step 4, the unreliability of system is determined by Equation (16).

$$\begin{aligned} Pr(system\ fails) &= Pr(system\ fails|TCFE_1) \times Pr(TCFE_1) + Pr(system\ fails|TCFE_2) \times Pr(TCFE_2) \\ &= \sum_{i=1}^{2^m} [\Pr(\text{system fails}|PSE_i) \times \Pr(PSE_i)] \times \Pr(PGE_{1,1}) + \Pr(PGE_{1,2}) + \\ &\sum_{k=1}^{2^n-1} \left\{\Pr(I_k \cap PGE_{2,2}) + \Pr(I_k) \times \sum_{i=1}^{2^{a_k+b_k}} [\Pr(\text{system fails}|PSE_{k,i}) \times \Pr(PSE_{k,i})]\right\} + \\ &\Pr(I_{2^n} \cap PGE_{2,1}) \times \sum_{i=1}^{2^m} [\Pr(\text{system fails}|PSE_i) \times \Pr(PSE_i)] + \Pr(I_{2^n} \cap PGE_{2,2}) \end{aligned} \tag{16}$$

This method achieves effective decomposition and processing of the problem by breaking down the original reliability problem into multiple simplified sub-problems. The flowchart is shown in **Figure 2**.

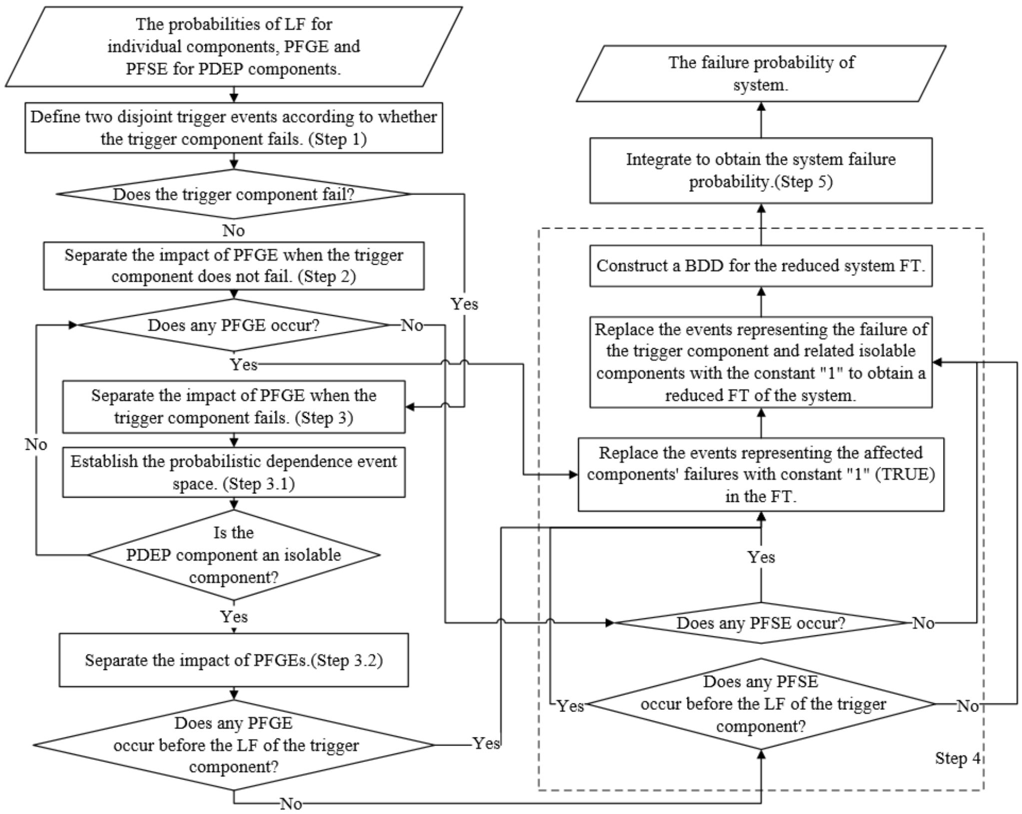

Figure 2. The flowchart of the proposed combinatorial methodology.

4. Case Study

To demonstrate the detailed application and effectiveness of the proposed combinatorial method, a case study on reliability analysis of SHSS is presented in this section since SHSS has been gradually embedded in all aspects of modern life with extremely wide applications. **Figure 3** shows the considered example SHSS for security application, which can prevent intrusion by strangers and protect personal safety (Kodali et al., 2016; Hoque & Davidson, 2019; Zhao & Xing, 2023). A human infrared sensor (*A*) and a door magnetic sensor (*B*) are set up outside the door. *A* can monitor the activities of people passing through the doorway in real-time through sensing the heat of the human body using infrared technology. *B* is used to detect the opening and closing statuses of the door and sends a signal immediately when the status of the door is changed. In this way, even if one of the sensors fails, the system is still able to monitor the situation outside the door. To achieve long-distance signal transmission, both *A* and *B* need to transmit the signals to the central control system via a relay node (*R*). A PFD behavior exists between *R* and (*A*, *B*), and the LF of *R* can cause isolation effect to the two sensors in a probabilistic manner. Specifically, when *R* is locally failed, if *A* or *B* has enough residual power to transmit the signals to the central control system, no isolation effect will occur; conversely, *A* or *B* has insufficient residual power to enable direct signal transmission to the central control system, isolation effect will occur. A human body sensing sensor (*C*) is set indoors to monitor the real-time changes in human movements. When the sensor detects abnormal movement patterns, such as a sudden fall or prolonged immobility, it will immediately trigger an alarm and send the information to the central control system. The system is also equipped with a smart bracelet (D) that integrates multiple sensors capable of monitoring the wearer's physiological status and activity in real-time. Additionally, the wristband featuring fall detection and an emergency call button further enhances the safety and protection capability of the SHSS. In SHSS, all components experience LF (malfunction). Only sensors A and B can experience both PFGEs (jamming attacks through signal interference) and PFSEs (targeted attacks). Define events Xl, Xpg and Xps as the event of LF, PFGE, and PFSE for component X, where X could be components R, A, B, C, or D. **Table 2** enumerates component failure events and the components affected by each event categorized by global or selective failure propagation effects. This paper assumes PFSEs only cause LFs in affected components. For instance, PFSE originating from A only causes the LF of B, while the PF (either PFGE or PFSE) originating from B can still happen due to the independence of LF and PF for each component. As long as either A or B is operational, the outdoor security module is considered reliable. When both the outdoor and indoor sensors are unavailable, the outdoor and indoor security modules are deemed to have failed. The entire SHSS for security is considered to have failed only when the smart bracelet, outdoor, and indoor sensors all malfunction. The system FT model is built as in **Figure 4.**

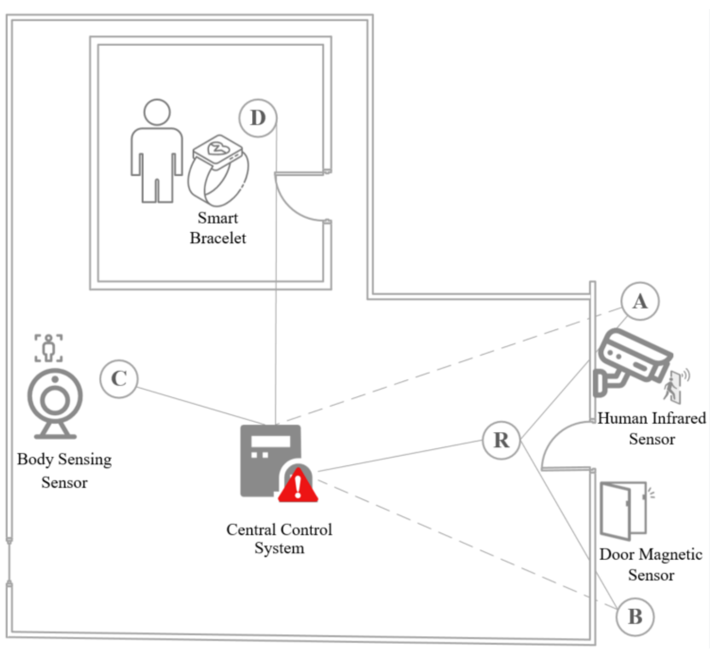

Figure 3. An example SHSS.

Table 2. SHSS component failure events and affected components.

Component	Event	Affected components by failure propagation
R	Rl	None
A	Al	None
	Apg	All
	Aps	$\{B, C\}$
B	Bl	None
	Bpg	All
	Bps	$\{A, D\}$
C	Cl	None
D	Dl	None

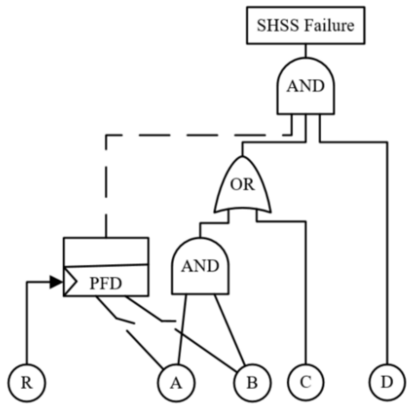

Figure 4. The FT model.

All component failure times are assumed to follow exponential distribution in this paper. **Table 3** lists the LF and PF rates of all components (Wang et al., 2022; Chen et al., 2024). The probability density function (pdf) of the failure rate is presented in Equation (17) and cumulative distribution function (cdf) is presented in Equation (18). The analysis is conducted at a mission time of $t = 1000$ hours.

Table 3. Failure rates of components(h^{-1}).

Component	PFGE rate	PFSE rate	LF rate
A, B	0.00005	0.00005	0.0001
R, C, D	0	0	0.0001

$$f(t) = \lambda e^{-\lambda t} \tag{17}$$

$$F(t) = 1 - e^{-\lambda t} \tag{18}$$

Using the combinatorial method introduced in Section 3, the SHSS reliability analysis steps are shown below:

Step 1: Define two disjoint trigger events according to whether the trigger component fails.

$TCFE_1$: The relay node R does not experience LF during working time.

$TCFE_2$: The relay node R experiences LF during working time.

Thus, the unreliability of SHSS can be expressed as

$$\Pr(\text{SHSS fails}) = \Pr(\text{SHSS fails}|TCFE_1) \times \Pr(TCFE_1) + \Pr(\text{SHSS fails}|TCFE_2) \times \Pr(TCFE_2) \tag{19}$$

Step 2: Separate the impact of PFGEs for $TCFE_1$.

Since $TCFE_1$ means that R does not fail and the failure competition does not occur, two events $PGE_{1,1}$ and $PGE_{1,2}$ are defined for whether PFGEs occur within the SHSS.

$PGE_{1,1}$: No PFGE occurs within the SHSS.

$PGE_{1,2}$: At least one PFGE occurs within the SHSS.

Therefore,

$$\Pr(\text{SHSS fails}|TCFE_1) \times \Pr(TCFE_1) = \Pr\left(\text{SHSS fails}|PGE_{1,1}\right) \times \Pr\left(PGE_{1,1}\right) + \Pr\left(\text{SHSS fails}|PGE_{1,2}\right) \times \Pr(PGE_{1,2}) \tag{20}$$

In Equation (20),

$$\Pr(PGE_{1,1}) = \Pr(\overline{Apg} \cap \overline{Bpg} \cap \overline{Rl}) = (1 - q_{Apg}) \times (1 - q_{Bpg}) \times (1 - q_{Rl}) \quad (21)$$

$$\Pr(PGE_{1,2}) = \Pr(TCFE_1) - \Pr(PGE_{1,1}) = (1 - q_{Rl}) - \Pr(PGE_{1,1}) \quad (22)$$

Since the failure probabilities of the components are as follows,

$q_{Apg} = q_{Bpg} = q_{Aps} = q_{Bps} = 1 - e^{-0.00005\times1000} = 0.04877$,

$q_{Rl} = q_{Al} = q_{Bl} = q_{Cl} = q_{Dl} = 1 - e^{-0.0001\times1000} = 0.09516$,

$\Pr(PGE_{1,1}) = 0.81873$ and $\Pr(PGE_{1,2}) = 0.08611$. From the definition of $PGE_{1,2}$, in the case that R does not fail, as long as PFGE occurs in the system, the whole SHSS fails, so $\Pr(\text{SHSS fails}|PGE_{1,2}) = 1$. The only value unknown in Equation (20) is $\Pr(\text{SHSS fails}|PGE_{1,1})$, in the case that R is functional and no PFGE occurs in the system, evaluating the SHSS reliability should not only consider the component LF, but also address the impact of PFSEs on the system. The evaluation of $\Pr(\text{SHSS fails}|PGE_{1,1})$ will be given in Step 4.

Step 3: Separate the impact of PFGEs for $TCFE_2$.

Step 3.1: Establish the probabilistic dependence event space.

Four events are decomposed as shown in **Table 4**, where $p_A = 0.1$ and $p_B = 0.9$ are the isolation factors modeling the probabilistic isolation effects from LF of R to components A and B.

Table 5 shows the D_{I_k} and ND_{I_k} of each dependence event. When the LF of R occurs, the sensors within D_{I_k} have insufficient remaining power to enable direct transmission to the central control system and are isolated from the SHSS. There is a time domain competition between the LF of R and the PFs of the isolable components. Conversely, the sensors within ND_{I_k} have sufficient remaining power to enable direct transmission to the central control system. The LF of R has no impact on the non-isolable components. Once the PFs from the non-isolable components occur, they will affect the SHSS.

Table 4. Probabilistic dependence events for the example SHSS.

Event	Definition	Occurrence probability
I_1	$A \cap B$	$p_A \times p_B$
I_2	$\bar{A} \cap B$	$(1 - p_A) \times p_B$
I_3	$A \cap \bar{B}$	$p_A \times (1 - p_B)$
I_4	$\bar{A} \cap \bar{B}$	$(1 - p_A) \times (1 - p_B)$

Table 5. The $\boldsymbol{D_{I_k}}$ and $\boldsymbol{ND_{I_k}}$ of each dependence event.

Event	$\boldsymbol{D_{I_k}}$	$\boldsymbol{ND_{I_k}}$
I_1	$\{A, B\}$	$\emptyset$
I_2	$\{B\}$	$\{A\}$
I_3	$\{A\}$	$\{B\}$
I_4	$\emptyset$	$\{A, B\}$

Step 3.2: Separate the impact of PFGEs.

$PGE_{2,1}$: No PFGE occurs within the SHSS or all PFGEs originating from the isolable components (i.e., components in D_{I_k}) occur after the LF of R.

$PGE_{2,2}$: No fewer than one PFGE of the isolable component happens before the LF of R or at least one PFGE of the non-isolable component (i.e., components in ND_{I_k}) occurs.

According to the established dependence events I_k $(k = 1,2,\dots,4)$ in Step 3.1, each PGE can be further distinguished as two disjoint events $I_k \cap PGE_{2,1}$ and $I_k \cap PGE_{2,2}$.

We have

$$Pr(\text{SHSS fails}|TCFE_1) \times \Pr(TCFE_1) = \sum_{k=1}^{4} \begin{bmatrix} \Pr(\text{SHSS fails}|I_k \cap PGE_{2,1}) \times \Pr(I_k \cap PGE_{2,1}) \\ +\Pr(\text{SHSS fails}|I_k \cap PGE_{2,2}) \times \Pr(I_k \cap PGE_{2,2}) \end{bmatrix} \tag{23}$$

where, $\Pr(\text{SHSS fails}|I_k \cap PGE_{2,2}) = 1$.

Therefore,

$$\Pr(\text{SHSS fails}|TCFE_1) \times \Pr(TCFE_1) = \sum_{k=1}^{4}[\Pr(\text{SHSS fails}|I_k \cap PGE_{2,1}) \times \Pr(I_k \cap PGE_{2,1}) + \Pr(I_k \cap PGE_{2,2})] \tag{24}$$

where, $\Pr(\text{SHSS fails}|I_k \cap PGE_{2,1}) \times \Pr(I_k \cap PGE_{2,1})$ $(k = 1,2,3)$ and $\Pr(\text{SHSS fails}|I_4 \cap PGE_{2,1})$ will be solved in the Step 4 due to the consideration of PFSEs. In this step, $\Pr(I_k \cap PGE_{2,2})$ $(k = 1,2,3,4)$ and $\Pr(I_4 \cap PGE_{2,1})$ will be solved as the following:

$I_1 \cap PGE_{2,2}$: No fewer than one PFGE originating from *A* or *B* happens before the LF of *R*. $\Pr(I_1 \cap PGE_{2,2})$ is calculated through the following formula.

$$\Pr(I_1 \cap PGE_{2,2}) = \Pr[(Apg \cup Bpg) \to Rl] \times \Pr(I_1) \tag{25}$$

Due to

$$\Pr(Apg \cup Bpg) = 1 - \Pr(\overline{Apg} \cap \overline{Bpg}) = 1 - e^{-(\lambda_{Apg}+\lambda_{Bpg})t} \tag{26}$$

$$f_{Apg\cup Bpg} = (\lambda_{Apg} + \lambda_{Bpg})e^{-(\lambda_{Apg}+\lambda_{Bpg})t} \tag{27}$$

the solution of $\Pr[(Apg \cup Bpg) \to Rl]$ in Equation (25) is

$$\Pr[(Apg \cup Bpg) \to Rl] = \int_0^t \int_{\tau_1}^t f_{Apg\cup Bpg}(\tau_1) f_{Rl}(\tau_2) d\tau_2 d\tau_1 = \int_0^t \int_{\tau_1}^t (\lambda_{Apg} + \lambda_{Bpg})e^{-(\lambda_{Apg}+\lambda_{Bpg})\tau_1}\lambda_{Rl}e^{-\lambda_{Rl}\tau_2} d\tau_2 d\tau_1 = 0.00453 \tag{28}$$

therefore, $\Pr(I_1 \cap PGE_{2,2}) = 0.00041$.

$I_2 \cap PGE_{2,2}$: Either PFGE from *A* occurs, or PFGE from *B* occurs prior to the LF of *R*. $\Pr(I_2 \cap PGE_{2,2})$ is calculated through the following formula.

$$\Pr(I_2 \cap PGE_{2,2}) = \Pr[(Apg \cap Rl) \cup (Bpg \to Rl)] = \{\Pr(Apg \cap Rl) + \Pr(Bpg \to Rl) - \Pr[Apg \cap (Bpg \to Rl)]\} \times \Pr(I_2) \tag{29}$$

Similar to Equation (28), $\Pr(Bpg \to Rl) = 0.00230$. Therefore, $\Pr(I_2 \cap PGE_{2,2}) = 0.00553$.

$I_3 \cap PGE_{2,2}$: Either PFGE from *B* occurs, or PFGE from *A* occurs prior to the LF of *R*. $\Pr(I_3 \cap PGE_{2,2})$ is calculated through the following formula.

$$\Pr(I_3 \cap PGE_{2,2}) = \Pr[(Apg \to Rl) \cup (Bpg \cap Rl)] \times \Pr(I_3) = \{\Pr(Bpg \to Rl) + \Pr(Apg \cap Rl) - \Pr[(Apg \to Rl) \cap Bpg]\} \times \Pr(I_3) \quad (30)$$

where, $\Pr(Apg \to Rl) = 0.00230$. Therefore, $\Pr(I_3 \cap PGE_{2,2}) = 0.00007$.

$I_4 \cap PGE_{2,2}$: Any PFGE originating from *A* or *B* occurs. $\Pr(I_4 \cap PGE_{2,2})$ is calculated through the following formula.

$$\Pr(I_4 \cap PGE_{2,2}) = \Pr[(Apg \cup Bpg) \cap Rl] \times \Pr(I_4) = 0.00082 \quad (31)$$

$I_4 \cap PGE_{2,1}$: No PFGE occurs to *A* or *B*. $\Pr(I_4 \cap PGE_{2,1})$ is calculated through the following formula.

$$\Pr(I_4 \cap PGE_{2,1}) = \Pr[(\overline{Apg} \cap \overline{Bpg}) \cap Rl] \times \Pr(I_4) = 0.00775 \quad (32)$$

Step 4: Define the events to address PFSEs and evaluate the conditional system failure probabilities.

$\Pr(\text{SHSS fails}|PGE_{1,1})$: *R* always remains operational and no PFGE occurs during working hours, it is only necessary to consider how PFSEs of *A* and *B* affect the system state.

Table 6. Event space for considering PFSEs under the condition $PGE_{1,1}$.

Event	Event definition	Unaffected components	Occurrence probabilistic
PSE_1	$\overline{Aps} \cap \overline{Bps}$	$\{A, B, C, D\}$	$(1 - q_{Aps}) \times (1 - q_{Bps})$
PSE_2	$Aps \cap \overline{Bps}$	$\{D\}$	$q_{Aps} \times (1 - q_{Bps})$
PSE_3	$\overline{Aps} \cap Bps$	$\emptyset$	$(1 - q_{Aps}) \times q_{Bps}$
PSE_4	$Aps \cap Bps$	$\emptyset$	$q_{Aps} \times q_{Bps}$

Table 6 outlines the construction of an event space, with the second column defining events via PFSE occurrence combinations, the third column identifying unaffected components for each *PSE*, and the fourth column quantifying *PSE* occurrence probabilities. By invoking the law of total probability, it follows that

$$\Pr(\text{SHSS fails}|PGE_{1,1}) = \sum_{i=1}^{4}[\Pr(\text{SHSS fails}|PSE_i) \times \Pr(PSE_i)] \quad (33)$$

We assess the conditional failure probability of SHSS when each event occurred.

$\Pr(\text{SHSS fails}|PSE_1)$: In this case, *R* does not fail, and neither *A* nor *B* experiences PFSE during mission hours. **Figure 5** shows the reduced FT model, and **Figure 6** is the BDD model of the reduced FT. Based on the BDD model, $\Pr(\text{SHSS fails}|PSE_1) = 0.00984$.

$\Pr(\text{SHSS fails}|PSE_2)$: The LF of component *D* determines the failure state of the SHSS. **Figure 7** shows the reduced FT, and thus $\Pr(\text{SHSS fails}|PSE_2) = 0.09516$.

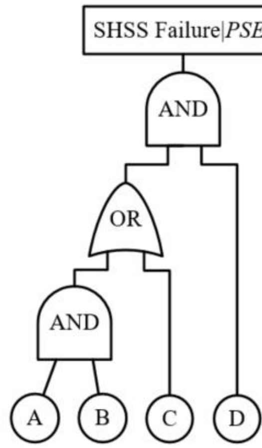

Figure 5. Reduced FT for evaluating Pr(SHSS fails|PSE_1).

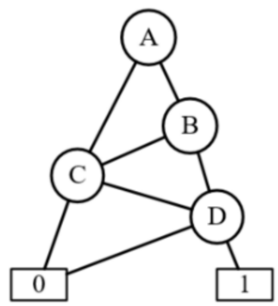

Figure 6. BDD model for evaluating Pr(SHSS fails|PSE_1).

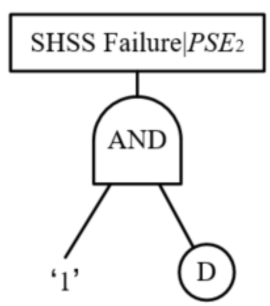

Figure 7. Reduced FT for evaluating Pr(SHSS fails|PSE_2).

Pr(SHSS fails|PSE_3) or Pr(SHSS fails|PSE_4): All components within the SHSS fail due to the occurrence of PSE events. Therefore, Pr(SHSS fails|PSE_3) $=$ Pr(SHSS fails|PSE_4) $= 1$.

Integrating the results obtained above, the result of Equation (33) is
$\Pr\left(\text{SHSS fails}|PGE_{1,1}\right) = \sum_{i=1}^{4}[\Pr(\text{SHSS fails}|PSE_i) \times \Pr(PSE_i)] = 0.06209$.

$\Pr(\text{SHSS fails}|I_1 \cap PGE_{2,1}) \times \Pr(I_1 \cap PGE_{2,1})$: A and B will become unavailable due to the failure of R. In this situation, it is necessary to consider the time domain competition between the PFSEs of A and B and the LF of R.

Build an event space as shown in **Table 7**. It provides the definition of each event and the components that remain unaffected within the system when the event occurs. Therefore,

$$\Pr\left(\text{SHSS fails}\middle|I_1 \cap PGE_{2,1}\right) \times \Pr\left(I_1 \cap PGE_{2,1}\right)$$

$$= \Pr(I_1) \times \sum_{i=1}^{4}\left[\Pr\left(\text{SHSS fails}|PSE_{1,i}\right) \times \Pr\left(PSE_{1,i}\right)\right] \quad (34)$$

Table 7. Event space for considering PFSEs under the condition $I_1 \cap PGE_{2,1}$.

Event	Event definition	Unaffected components
$PSE_{1,1}$	No PF occurs before the occurrence of Rl	$\{D\}$
$PSE_{1,2}$	Only Aps occurs before Rl occurs	$\{D\}$
$PSE_{1,3}$	Only Bps occurs before Rl occurs	$\emptyset$
$PSE_{1,4}$	Aps and Bps both occur before Rl occurs	$\emptyset$

The occurrence probability of each event and the SHSS failure probability under each event are evaluated.

$PSE_{1,1}$: No PF occurred before the occurrence of Rl.

$$\Pr\left(PSE_{1,1}\right) = \Pr(Rl) - \Pr(\text{at least one PF happens before } Rl \text{ happens}) = \Pr(Rl) - \Pr[(Aps \cup Apg \cup Bps \cup Bpg) \to Rl] \quad (35)$$

In Equation (35),

$$\Pr[(Aps \cup Apg \cup Bps \cup Bpg) \to Rl] = \int_0^t \int_{\tau_1}^t f_{Aps\cup Apg\cup Bps\cup Bpg}(\tau_1) \times f_{Rl}(\tau_2)d\tau_2 d\tau_1 = \int_0^t \int_{\tau_1}^t \left[\left(\lambda_{Aps} + \lambda_{Apg} + \lambda_{Bps} + \lambda_{Bpg}\right) \times e^{-(\lambda_{Aps}+\lambda_{Apg}+\lambda_{Bps}+\lambda_{Bpg})\tau_1} \times \lambda_{Rl} \times e^{-\lambda_{Rl}\tau_2}\right] d\tau_2 d\tau_1 = 0.00877,$$

therefore, $\Pr\left(PSE_{1,1}\right) = 0.08639$. Since the failure status of the SHSS in this case only depends on whether D fails, $\Pr\left(\text{SHSS fails}\middle|PSE_{1,1}\right) = q_{Dl} = 0.09516$.

$PSE_{1,2}$: Only Aps occurred before Rl.

$$\Pr\left(PSE_{1,2}\right) = \Pr(Aps \to Rl) - \Pr\{[Aps \cap (Apg \cup Bps \cup Bpg)] \to Rl\} \quad (36)$$

In Equation (36),

$$\Pr\{[Aps \cap (Apg \cup Bps \cup Bpg)] \to Rl\} = \int_0^t \int_{\tau_1}^t f_{Aps\cap(Apg\cup Bps\cup Bpg)}(\tau_1) \times f_{Rl}(\tau_2)d\tau_2 d\tau_1 = \int_0^t \int_{\tau_1}^t \left[\lambda_{Aps} \times e^{-\lambda_{Aps}\tau_1} + \begin{pmatrix}\lambda_{Apg} + \\ \lambda_{Bps} + \lambda_{Bpg}\end{pmatrix} \times e^{-\begin{pmatrix}\lambda_{Apg} \\ +\lambda_{Bps}+\lambda_{Bpg}\end{pmatrix}\tau_1 - \begin{pmatrix}\lambda_{Aps}+\lambda_{Apg} \\ +\lambda_{Bps}+\lambda_{Bpg}\end{pmatrix}\times e^{-\begin{pmatrix}\lambda_{Aps}+\lambda_{Apg} \\ +\lambda_{Bps}+\lambda_{Bpg}\end{pmatrix}\tau_1}}\right] \times \lambda_{Rl} \times e^{-\lambda_{Rl}\tau_2} d\tau_2 d\tau_1 = 0.00022,$$

Therefore, $\Pr\left(PSE_{1,2}\right) = 0.00209$. Since in this case the failure status of the SHSS only depends on whether D fails, $\Pr\left(\text{SHSS fails}|PSE_{1,2}\right) = q_{Dl} = 0.09516$.

$PSE_{1,3}$: Only Bps occurred before Rl.

$$\Pr(PSE_{1,3}) = \Pr(Bps \to Rl) - \Pr\{[Bps \cap (Apg \cup Aps \cup Bpg)] \to Rl\} \quad (37)$$

Similarly, $\Pr(PSE_{1,3}) = 0.00209$. Since all system components are affected by the occurrence of $PSE_{1,3}$ and failed, $\Pr(\text{SHSS fails}|PSE_{1,3}) = 1$.

$PSE_{1,4}$: Aps and Bps both occur before Rl.

$$\Pr(PSE_{1,4}) = \Pr[(Aps \cap Bps) \to Rl] - \Pr\{[Aps \cap Bps \cap (Apg \cup Bpg)] \to Rl\} \quad (38)$$

It is easily known that $\Pr(PSE_{1,4}) = 0.00007$ and $\Pr(\text{SHSS fails}|PSE_{1,4}) = 1$.

Integrating the results obtained above, the result of Equation (34) is
$\Pr(\text{SHSS fails}|I_1 \cap PGE_{2,1}) \times \Pr(I_1 \cap PGE_{2,1}) = 0.00095$.

$\Pr(\text{SHSS fails}|I_2 \cap PGE_{2,1}) \times \Pr(I_2 \cap PGE_{2,1})$: Only *B* will become unavailable due to the LF of *R*, while the PFSE of *A*, if happens, can still affect other components. In this case, the time domain competition between the PFSE of *B* and the LF of *R* should be addressed.

Establish an event space considering the PFSEs as shown in **Table 8**. We have

$$\Pr(\text{SHSS fails}|I_2 \cap PGE_{2,1}) \times \Pr(I_2 \cap PGE_{2,1}) = \Pr(I_2) \times \sum_{i=1}^{4}[\Pr(\text{SHSS fails}|PSE_{2,i}) \times \Pr(PSE_{2,i})] \quad (39)$$

Table 8. Event space for considering PFSEs under the condition $I_2 \cap PGE_{2,1}$.

Event	Event definition	Unaffected components
$PSE_{2,1}$	Aps does not occur and Bps occurs after Rl occurs	$\{A, C, D\}$
$PSE_{2,2}$	Aps occurs and Bps occurs after Rl occurs	$\{D\}$
$PSE_{2,3}$	Aps does not occur and Bps occurs before Rl occurs	$\emptyset$
$PSE_{2,4}$	Aps occurs and Bps occurs before Rl occurs	$\emptyset$

Similarly, we evaluate the probability of each event occurring and the corresponding conditional failure probability of the SHSS given each event.

$PSE_{2,1}$: Aps does not occur and Bps occurs after Rl occurs. $\Pr(PSE_{2,1})$ is calculated through the following formula.

$$\Pr(PSE_{2,1}) = \{\Pr(Rl) - \Pr[(Bps \cup Bpg) \to Rl]\} \times \Pr(\overline{Aps} \cap \overline{Apg}) = 0.08201 \quad (40)$$

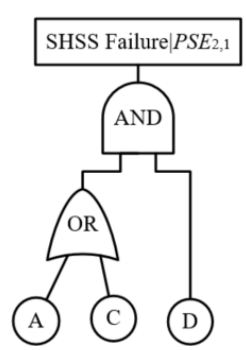

Figure 8. Reduced FT for evaluating $\Pr(\text{SHSS fails}|PSE_{2,1})$.

$\Pr(\text{SHSS fails}|PSE_{2,1})$: In this case, the components *A*, *C*, and *D* in the SHSS are not affected. **Figure 8** shows the reduced FT, and **Figure 9** is the BDD model of the reduced FT.

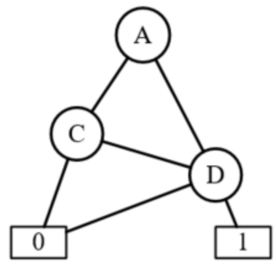

Figure 9. BDD model for evaluating $\Pr(\text{SHSS fails}|PSE_{2,1})$.

According to the BDD model, $\Pr(\text{SHSS fails}|PSE_{2,1}) = 0.01725$.

$PSE_{2,2}$: Aps occurs and Bps occurs after Rl occurs. $\Pr(PSE_{2,2})$ is calculated through the following formula.

$$\Pr(PSE_{2,2}) = \{\Pr(Rl) - \Pr[(Bps \cup Bpg) \rightarrow Rl]\} \times \Pr(Aps \cap \overline{Apg}) = 0.00420 \tag{41}$$

$\Pr(\text{SHSS fails}|PSE_{2,2})$: If Aps occurs, *B* and *C* will be affected and fail; Bps occurs after the occurrence of Rl and is isolated. Only *D* is functioning normally, $\Pr(\text{SHSS fails}|PSE_{2,2}) = 0.09516$.

$PSE_{2,3}$: Aps does not occur and Bps occurs before Rl occurs. $\Pr(PSE_{2,3})$ is calculated through the following formula.

$$\Pr(PSE_{2,3}) = \{\Pr(Bps \rightarrow Rl) - \Pr[(Bps \cap Bpg) \rightarrow Rl]\} \times \Pr(\overline{Aps} \cap \overline{Apg}) = 0.00201 \tag{42}$$

$\Pr(\text{SHSS fails}|PSE_{2,3})$: Aps does not occur, *B* and *C* will not be affected; however, Bps occurs before the occurrence of Rl, causing *A* and *D* to fail due to failure propagation effect. The whole system fails and $\Pr(\text{SHSS fails}|PSE_{2,3}) = 1$.

$PSE_{2,4}$: Aps occurs and Bps occurs before Rl occurs. $\Pr(PSE_{2,4})$ is calculated through the following formula.

$$\Pr(PSE_{2,4}) = \{\Pr(Bps \rightarrow Rl) - \Pr[(Bps \cap Bpg) \rightarrow Rl]\} \times \Pr(Aps \cap \overline{Apg}) = 0.00010 \tag{43}$$

$\Pr(\text{SHSS fails}|PSE_{2,4})$: If Aps occurs, *B* and *C* will be affected and fail; if Bps occurs before Rl occurs, *A* and *D* will fail as a result of the propagation effect of Bps. The whole system fails, $\Pr(\text{SHSS fails}|PSE_{2,4}) = 1$.

Integrating the results obtained above, the result of Equation (39) is
$\Pr(\text{SHSS fails}|I_2 \cap PGE_{2,1}) \times \Pr(I_2 \cap PGE_{2,1}) = 0.00319$.

$\Pr(\text{SHSS fails}|I_3 \cap PGE_{2,1}) \times \Pr(I_3 \cap PGE_{2,1})$: Only *A* will become unavailable due to the LF of *R*, and the PF of *B* is not affected by the LF of *R*. Under this scenario, the time domain competition of the PFSE of *A* with the LF of *R* and whether the PFSE of *B* occurs need to be considered.

An event space considering the PFSEs is established as shown in **Table 9**. It is derived that

$$\Pr\left(\text{SHSS fails}\middle|I_3 \cap PGE_{2,1}\right) \times \Pr\left(I_3 \cap PGE_{2,1}\right) = \Pr(I_3) \times \sum_{i=1}^{4}\left[\Pr\left(\text{SHSS fails}|PSE_{3,i}\right) \times \Pr\left(PSE_{3,i}\right)\right] \qquad (44)$$

Table 9. Event space for considering PFSEs under the condition $I_3 \cap PGE_{2,1}$.

Event	**Event definition**	**Unaffected components**
$PSE_{3,1}$	Aps occurs after Rl occurs and Bps does not occur	$\{B, C, D\}$
$PSE_{3,2}$	Aps occurs after Rl occurs and Bps occurs	$\emptyset$
$PSE_{3,3}$	Aps occurs before Rl occurs and Bps does not occur	$\{D\}$
$PSE_{3,4}$	Aps occurs before Rl occurs and Bps occurs	$\emptyset$

Assess the occurrence probability of each event and the SHSS failure probability under each event.

$PSE_{3,1}$: Aps occurs after Rl occurs and Bps does not occur. $\Pr\left(PSE_{3,1}\right)$ is calculated through the following formula.

$$\Pr\left(PSE_{3,1}\right) = \{\Pr(Rl) - \Pr[(Aps \cup Apg) \rightarrow Rl]\} \times \Pr(\overline{Bps} \cap \overline{Bpg}) = 0.08201 \qquad (45)$$

$\Pr\left(\text{SHSS fails}|PSE_{3,1}\right)$: Due to the failure of *R*, *A* which depends on *R* will become unavailable, and the PFSE of *B* does not occur and the PFSE of *A* does not occur before the failure of *R*. In this case, the components not affected in SHSS are *B*, *C*, and *D*. **Figure 10** shows the reduced FT, and **Figure 11** shows the BDD model of the reduced FT. From the BDD model, $\Pr\left(\text{SHSS fails}|PSE_{2,1}\right) = 0.01725$.

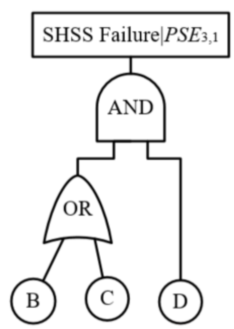

Figure 10. Reduced FT for evaluating $\Pr\left(\text{SHSS fails}|PSE_{3,1}\right)$.

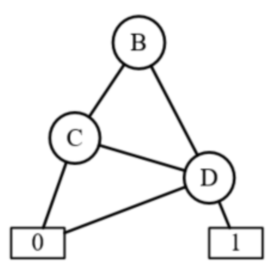

Figure 11. BDD model for evaluating $\Pr\left(\text{SHSS fails}|PSE_{3,1}\right)$.

$PSE_{3,2}$: Aps occurs after Rl occurs and Bps occurs. $\Pr\left(PSE_{3,2}\right)$ is calculated through the following formula.

$$\Pr\left(PSE_{3,2}\right) = \{\Pr(Rl) - \Pr[(Aps \cup Apg) \rightarrow Rl]\} \times \Pr(Bps \cap \overline{Bpg}) = 0.00420 \quad (46)$$

$\Pr\left(\text{SHSS fails}|PSE_{3,2}\right)$: If Aps does not occur before Rl occurs, *B* and *C* will not be affected; if Bps occurs, *A* and *D* will fail as a result of the propagation effect of Bps. The whole system fails, $\Pr\left(\text{SHSS fails}|PSE_{3,2}\right) = 1$.

$PSE_{3,3}$: Aps occurs before Rl occurs and Bps does not occur. $\Pr\left(PSE_{3,3}\right)$ is calculated through the following formula.

$$\Pr\left(PSE_{3,3}\right) = \{\Pr(Aps \rightarrow Rl) - \Pr[(Aps \cap Apg) \rightarrow Rl]\} \times \Pr(\overline{Bps} \cap \overline{Bpg}) = 0.00201 \quad (47)$$

$\Pr\left(\text{SHSS fails}|PSE_{3,3}\right)$: If Aps occurs before Rl occurs, *B* and *C* will be affected and fail; if Bps does not occur, *A* and *D* will not be affected since the propagation effect of Bps is avoided. Only *D* is still running normally, $\Pr\left(\text{SHSS fails}|PSE_{3,3}\right) = 0.09516$.

$PSE_{3,4}$: Aps occurs before Rl occurs and Bps occurs. $\Pr\left(PSE_{3,4}\right)$ is calculated through the following formula.

$$\Pr\left(PSE_{3,4}\right) = \{\Pr(Aps \rightarrow Rl) - \Pr[(Aps \cap Apg) \rightarrow Rl]\} \times \Pr(Bps \cap \overline{Bpg}) = 0.00010 \quad (48)$$

$\Pr\left(\text{SHSS fails}|PSE_{3,4}\right)$: If Aps occurs before Rl occurs, *B* and *C* will be affected and fail; if Bps occurs, *A* and *D* will fail as a result of the propagation effect of Bps. Therefore, the whole system fails and $\Pr\left(\text{SHSS fails}|PSE_{3,4}\right) = 1$.

Integrating the results obtained above, the result of Equation (44) is
$\Pr\left(\text{SHSS fails}\middle|I_3 \cap PGE_{2,1}\right) \times \Pr\left(I_3 \cap PGE_{2,1}\right) = 0.00006$.

$\Pr\left(\text{SHSS fails}\middle|I_4 \cap PGE_{2,1}\right)$: Neither *A* nor *B* will become unavailable by the LF of *R*. Similar to the event that *R* remains operational during working hours, the time domain competition of the PFSE of *A* and *B* with the LF of *R* is not considered. That is, the event space to consider the PFSEs is PSE_i $(i = 1, 2, 3, 4)$. $\Pr\left(\text{SHSS fails}\middle|I_4 \cap PGE_{2,1}\right)$ can be obtained as

$$\Pr\left(\text{SHSS fails}\middle|I_4 \cap PGE_{2,1}\right) = \textstyle\sum_{i=1}^{4}[\Pr(\text{SHSS fails}|PSE_i) \times \Pr(PSE_i)] = 0.06209 \quad (49)$$

Step 5: Integrate to obtain the system failure probability.

According to Step 2 to Step 4, the unreliability of the SHSS is obtained by

$$\begin{aligned} Pr(SHSS\,fails) &= Pr(SHSS\,fails|TCFE_1) \times Pr(TCFE_1) + Pr(SHSS\,fails|TCFE_2) \times Pr(TCFE_2) \\ &= \sum_{i=1}^{4}[Pr(SHSS\,fails|PSE_i) \times Pr(PSE_i)] \times Pr\left(PGE_{1,1}\right) + Pr\left(PGE_{1,2}\right) \\ &+ \sum_{k=1}^{3}\left\{Pr(I_k) \times \sum_{i=1}^{4}\left[Pr\left(SHSS\,fails\middle|PSE_{k,i}\right) \times Pr\left(PSE_{k,i}\right)\right] + Pr\left(I_k \cap PGE_{2,2}\right)\right\} \\ &+ Pr\left(I_4 \cap PGE_{2,1}\right) \times \sum_{i=1}^{4}[Pr(SHSS\,fails|PSE_i) \times Pr(PSE_i)] + Pr\left(I_4 \cap PGE_{2,2}\right) \\ &= 0.14844. \end{aligned}$$

By solving the Markov model generated for the SHSS with the same component failure rates, we derived the probability of the system in failed state 21, which is in full agreement with the results derived by the combinatorial method proposed in Section 3.2 (Wang et al., 2017a).

In the analysis of SHSS, the event space of $TCFE$ is 2. For $TCFE_1$, since there are two PDEP groups, four event spaces are decomposed, generating four reduced problems to be dealt with for $TCFE_1$. For $TCFE_2$, there is a more complex time domain competition, which can be decomposed into 4 sub-events according to the results of competition between PFGE and isolation effect. Among them, 3 events need to be further decomposed into $2^2 = 4$ event spaces when considering 2 PFSE events, while the other event needs to be considered with the same issues as $TCFE_1$ when considering PFSEs. In other words, for $TCFE_2$, there are 12 reduced problems to be dealt with. When analyzing the SHSS reliability with the combinatorial method proposed in this paper, only 16 reduced problems need to be considered. Compared to evaluating the 22-state Markov model, which corresponds to 22 differential equations, the proposed approach in this paper is computationally more efficient.

The proposed method also does not restrict the failure time distribution of components. For SHSS, we assume that the failure times follow the Weibull distribution in **Table 10**. The pdf and cdf of the Weibull distribution with scale parameter λ_W and shape parameter α_W are shown in Equation (50) and Equation (51). The analysis considers a mission time of *t* = 1000 *h*.

Table 10. The Weibull time-to-failure parameters of components.

Component	PFGE		PFSE		LF	
	α_W	λ_W	α_W	λ_W	α_W	λ_W
A, B	2	0.00005	2	0.00005	2	0.0001
R, C, D	-	-	-	-	2	0.0001

$$f_W(t) = \alpha_W \lambda_W{}^{\alpha_W} t^{\alpha_W - 1} e^{-(\lambda_W t)^{\alpha_W}} \tag{50}$$

$$F_W(t) = 1 - e^{-(\lambda_W t)^{\alpha_W}} \tag{51}$$

Applying the Section 3.2 method steps to numerical analysis yields an SHSS failure probability of 0.00758 for Weibull-distributed component failure times.

5. Generalization of the Proposed Combinatorial Methodology

In Section 3.2, we only consider the PFs from PDEP components. In fact, including the trigger component, those non-PDEP components can also experience PFs. This section generalizes the combinatorial method to analyze the case where non-PDEP components of the system also have both PFGEs and PFSEs. The steps comprising the generalized methodology are detailed below:

Step 1: Separate the PFGEs from the non-PDEP components.

Define event E_1, referred to as no fewer than one PFGE from original non-PDEP components occurs.
Define event E_2 that is disjoint with E_1, referred to as no PFGE from original non-PDEP components occurs.

The system unreliability is defined as follows
$\Pr(\text{system fails}) = \Pr(\text{system fails}|E_1) \times \Pr(E_1) + \Pr(\text{system fails}|E_2) \times \Pr(E_2)$,

where, $\Pr(\text{system fails}|E_1) = 1$.

Step 2: Handle the PFSEs from the original non-PDEP components.

Similar to the Step 4 in Section 3.2, assume that there are u independent PFSEs from original non-PDEP components, and establish an event space that comprising 2^u combined events. Each event, referred to as a combinational PFSE occurrence event ($NPSE$), embodies a unique combination of PFSE event presences and absences. The system unreliability is evaluated by

$\Pr(\text{system fails}) = \Pr(E_1) + \sum_{i=1}^{2^u}[\Pr(\text{system failure}|NPSE_i) \times \Pr(NPSE_i)] \times \Pr(E_2)$

Step 3: Evaluate the conditional system failure probability for combined events via the proposed methodology in Section 3.2.

Step 4: Compute system unreliability via the total probability law.

In particular, if only the PFGEs from original non-PDEP components are considered, only Step 1 needs to be added before the proposed method in Section 3.2. If only the PFSEs from original non-PDEP components are considered, only Step 2 needs to be added before the proposed method in Section 3.2.

6. Complexity Analysis

The proposed method is verified by conducting SHSS reliability analysis with a Markov-model. Compared with evaluating a 22-state Markov model equivalent to 22 differential equations, the proposed combinatorial method requires only 16 simplifications for analyzing the example SHSS. In comparison with Markov methods, the proposed method offers higher analytical efficiency when dealing with the SHSS subject to probabilistic competing failures with complex propagated effects. To be more general, the space and time complexity are discussed as follows:

Consider a system consisting of x independent trigger components, m PDEP components, and n components that do not belong to any PDEP groups. Assume that the total number of system components is N, then $N = x + m + n$. There are a PFSE events within the system.

For the proposed combinatorial method, when establishing the $TCFE$ event space according to whether the trigger components fail or not, 2^x $TCFE_i$ events are generated. Among them, for the event $TCFE_1$ where all trigger components function normally, the discussion on probabilistic dependent behavior is reduced. For $TCFE_1$, the PGE space is constant 2. Under the event $PGE_{1,1}$, a PFSE event space with size of 2^a is established and the PFSE events are evaluated separately. In the worst-case scenario, a BDD model is established for each PFSE event. The reduced FT has fewer nodes than $n + \frac{m}{2}$, so the size of the BDD model for each combinatorial PFSE event must be less than $2^{n+\frac{m}{2}}$ (Shrestha et al., 2009), and the space required for analyzing conditional system unreliability given the occurrence of $TCFE_1$ is less than $2^{n+\frac{m}{2}+a}$. For other events $TCFE_i$ ($i > 1$) with m PDEP components, the dependence event space (i.e., size of I_k) is 2^m. For the combined event $I_k \cap PGE_{1,i}$, the PFSEs of isolable and non-isolable components are considered. PFSE event space with a size of 2^a is established. In the worst-case scenario, the BDD model size for each combinatorial PFSE event must be less than $2^{n+\frac{m}{2}}$. At this time, the space required for analyzing $TCFE_i$ ($i > 1$) is less than $2^{a+n+\frac{m}{2}} + (2^x - 1) * 2^{m+a+n+\frac{m}{2}} = 2^{\left(x+m+a+n+\frac{m}{2}\right)} = 2^{\left(N+\frac{m}{2}+a\right)}$. When the event $PGE_{2,i}$ occurs, causing the system to fail, and further decomposing analysis under this event is reduced.

Thus, the proposed combinatorial method exhibits a space complexity lower than $O\left(2^{\left(N+\frac{m}{2}+a\right)}\right)$. For the Markov method, the worst-case number of states for k variables are 2^k. Therefore, for $x+m+m+n+a$ input variables, that is, $N+a+m$ input variables, there are $2^{(N+m+a)}$ states in the Markov model (Reibman et al., 1989). So, the Markov method space complexity is $O\left(2^{(N+m+a)}\right)$.

The proposed method uses divide-and-conquer to split the reliability problem into independent simplified ones. For the simplified problem, the "depth-first traversal" algorithm on the BDD is applied to derive root-to-terminal node paths. This traversal exhibits a computational complexity of $O(k)$, with k denoting the BDD model's node count ($k < 2^n/n$). This implies that the time complexity of the proposed combinatorial method incorporating the traditional BDD is less than $O(2^n/n)$. While for the Markov method, the computational complexity is $O(w * v)$, with w being the number of solution steps ($w = O(m^2)$ for equilibrium in acyclic chains), $v = O(m^3)$ as the per-step complexity, and $m = O\left(2^{(N+m+a)}\right)$ defining the system state count (Amari & Misra, 1997).

Table 11 presents the complexity comparison between the proposed combinatorial approach and the Markov method. Based on the afore-mentioned discussion and **Table 11**, the combinatorial method proposed in this paper is superior to the Markov method in terms of both space complexity and time complexity.

Table 11. The complexity comparison.

Method	Space complexity	Time complexity
The proposed combinatorial method	Less than $O\left(2^{\left(N+\frac{m}{2}+a\right)}\right)$	Less than $O(2^n/n)$
The Markov method	$O\left(2^{(N+m+a)}\right)$	$O\left(2^{(N+m+a)^2} * 2^{(N+m+a)^3}\right)$

7. Conclusions and Future Work

Since the existing methods are limited when performing reliability analysis on probabilistic competing failure systems with both PFGEs and PFSEs simultaneously, this paper introduces a novel combinatorial method to address such issue. The proposed method allows for the application of any type of distribution to model system component failure times and probability failure isolation factors. This flexibility greatly enhances the method's adaptability to various real-world scenarios, where component failure characteristics may vary significantly. The effectiveness of the proposed method is demonstrated via a case study of SHSS reliability analysis. In comparison to the SHSS reliability analysis procedure using the Markov method, the correctness and efficiency of the proposed method are verified. When being adopted to the reliability analysis of probabilistic competing failure systems with both PFGEs and PFSEs, the proposed combinatorial method is superior to the Markov method in terms of space complexity as well as time complexity. Furthermore, the proposed method can be applied to probabilistic competing failure systems with single PFD group or multiple independent, non-overlapping PFD groups. The proposed method can also be generalized to the systems with non-PDEP components experiencing LF, PFGE, and PFSE.

In the future, more complex scenarios will be explored by extending the method to analyze systems with multiple dependent PFD groups with shared trigger or PDEP components (Wang et al., 2013). Another future research direction is to investigate methods for multi-state systems (Wang et al., 2018a), phased-mission systems (Tang et al., 2023; Wang et al., 2025), and cascading functional dependence systems (Zhao & Xing, 2019) with components affected simultaneously by PFGEs and PFSEs. In addition to extending toward more complex scenarios, expansions can also be directed toward new application domains. With the continuous evolution of technology and the constant expansion of application fields, novel system characteristics and requirements have emerged successively. Subsequent research can attempt to adapt and apply existing methods to more emerging application fields, such as the Industrial Internet of Things (Li et al., 2025), vehicle networks (Du et al., 2025), and smart medical systems (Lin et al., 2025). These fields are characterized by complex system architectures, sophisticated business logic, and more stringent reliability requirements.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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The author(s) declare that no assistance is taken from generative AI to write this article.

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