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Combinatorial Reliability Evaluation of Multi-State System with Epistemic Uncertainty

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Abstract
Multi-state systems (MSSs) are common in real-word applications, in which a system and/or its components exhibit multiple stochastically-dependent states or performance levels. Such characteristic poses challenges to the reliability evaluation of MSSs. Multi-valued decision diagrams (MDDs) have been developed to address the reliability analysis of MSSs under the assumption that the failure-time parameters of system components are deterministic. However, due to epistemic uncertainty, it is often difficult or impossible to obtain the determinate values of the component parameters. Therefore, this paper aims to address the MDD-based reliability evaluation of MSSs with epistemic uncertainty by incorporating the interval theory and fuzzy set theory. The proposed methods are verified through a detailed case study of a high-speed train bogie system. The results show that the proposed methods can obtain practical reliability evaluation results reflecting the condition of epistemic uncertainty.

Keywords- Multi-state system, Multi-valued decision diagram, Epistemic uncertainty, Interval theory, Fuzzy set theory.

1. Introduction

When a system or its components exhibit multiple performance levels (ranging from perfect operation to complete failure), the system is referred to as a multi-state system (MSS) (Xing and Amari, 2015). Existing works addressing reliability analysis of MSSs include, for example, extensions of binary-state reliability model (Ramirez et al., 2004; Shrestha and Xing, 2008), Markov-based methods (Cafaro et al., 1986), universal generation function-based methods (Levitin, 2004; Nahas and Nourelfath, 2021), Bayesian networks (Zhou et al., 2006), simulations (Pourhassan et al., 2021; Zio and Podofillini, 2003), survival signature-based methods (Bai et al., 2022), and multi-valued decision diagrams (MDDs) (Xing and Dai, 2009). As an extension of binary decision diagrams (BDDs) for the multi-valued case, the MDD-based methods can offer lower computational complexity for analyzing large-scale MSSs through using a much lower number of multi-valued variables to reduce the model size (Xing and Amari, 2015).

The MDD-based methods have been significantly studied in recent works. For example, Tian and Liu (2018) developed an MDD-based method to evaluate the reliability of an on-load tap-changer. Li et al. (2017) proposed a combinational method based on modified BDDs and MDDs for reliability analysis of MSSs considering failure mechanism correlations. Li et al. (2018) developed an MDD model for reliability analysis of a phased-mission system with non-repairable multi-state components. Wang et al. (2020) suggested MDD-based approaches for analyzing the reliability of non-repairable multi-phased systems subject to common-cause failures caused by random external shocks. Zhang et al. (2020) adapted the MDD-based method to evaluate the trust in online social networks considering distrust propagation and conflicts of opinions. Mo and Xing (2021) put forward an MDD-based analytical approach for efficient resource availability analysis of cloud computing systems with heterogeneous and multi-state computing nodes. Wang et al. (2021) suggested an MDD-based method for reliability analysis of a dynamic k-out-of-n phase-AND mission system, encompassing a fast MDD model generation algorithm that considers behaviors of all the mission phases simultaneously based on node labeling. Mo et al. (2021) proposed an MDD-based method for analyzing the mission success probability of a repairable computing system subject to scheduled checkpointing. Liu et al. (2021) proposed an analytical method integrating Markov models and MDDs for assessing survivability and vulnerability of a cloud RAID (Redundant Array of Independent Disks) storage system subject to disk faults and cyber-attacks. Xing et al. (2021) proposed a new behavior-driven reliability modeling methodology using MDDs for accurate and efficient reliability analysis of complex sensor network-based smart systems. However, to the best of our knowledge, no works on MDDs have considered uncertainty existing in the real-world systems.

Generally, uncertainty can be divided into two types: aleatory uncertainty and epistemic uncertainty (Hu et al., 2021; Sarazin et al., 2021). Aleatory uncertainty, also known as objective uncertainty, originates from the inherent contingency or variability of the system and is inevitable. The description and propagation of aleatory uncertainties are usually handled by the probability theory. Epistemic uncertainty, also known as subjective uncertainty, is typically caused by factors like incomplete knowledge, lack of data, epistemic bias, and has been addressed by techniques such as the Dempster–Shafer evidence theory (Curcurù et al., 2012), interval theory (Sankararaman et al., 2011), fuzzy set theory (Gaonkar et al., 2021; Mula et al., 2007), etc. Particularly, the fuzzy set theory proposed by Zadeh (1965) has been widely adopted to deal with the uncertainty in reliability assessment of complex systems. The interval analysis only requires the upper and lower bounds of an uncertain variable and thus mathematically simple, which makes it applicable widely in real-world applications.

This paper makes contributions by first integrating MDDs and the interval and fuzzy set theories for modeling and analyzing the reliability of MSSs with epistemic uncertainty. The interval-MDD and fuzzy-MDD methods are suggested to obtain the reliability estimates of MSSs. The feasibility and validity of the suggested methods are demonstrated through the reliability analysis of a high-speed train bogie system.

The rest of the paper is structured as follows: Section 2 presents the basics of the preliminary model, MDD including the MDD generation and evaluation. Section 3 presents the proposed interval-MDD and fuzzy-MDD methods to consider epistemic uncertainty in the reliability analysis of MSSs. Section 4 illustrates the proposed methods through a detailed case study of a high-speed train bogie system. Section 5 presents conclusions and directions for future research.

2. Preliminary Model of MDDs

An MDD is a directed acyclic graph with two leaf/sink nodes representing the system being (labeled by '1') or not being (labeled by '0') in a specific state (Xing et al., 2015; Xing et al., 2019). Each non-leaf node models a system component (e.g., having n states) and is tied to an n-valued variable, denoted by x (Figure 1). The MDD rooted at node x encodes the following n-valued expression f in the *case* format. $f_{x=i}$ (i=1,2, …, n) in (1) is f evaluated with x being i.

$$f = (x = 1)f_{x=1} + (x = 2)f_{x=2} + \cdots + (x = n)f_{x=n} = case(x, f_1, f_2, \dots, f_n) \tag{1}$$

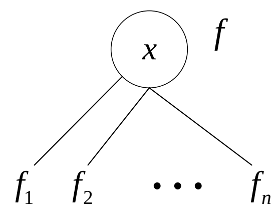

Figure 1. Illustrative general MDD structure.

To build an MDD model, we start with the basic event MDD, representing a system component being at a particular state (Figure 2). Manipulation rules of (2) are then applied to combine sub-MDDs corresponding to f and h based on the required logical relationship (AND or OR denoted by ◊ in (2)) (Xing et al., 2009). To apply these rules, we compare indexes of component variables, determined using heuristic methods (Xing et al., 2015), for the two root nodes (x for f, y for h). In the case of x and y having the same index, the logical operation is applied to their child nodes; otherwise, the logical operation is applied between child nodes of the root having a smaller index and the other root having a larger index.

$$f \diamond h = case(x, f_1, \cdots, f_n) \diamond case(y, h_1, \cdots, h_n) \\ = \begin{cases} case(x, f_1 \diamond h_1, \cdots, f_n \diamond h_n) & index(x) = index(y) \\ case(x, f_1 \diamond h, \cdots, f_n \diamond h) & index(x) < index(y) \\ case(y, f \diamond h_1, \cdots, f \diamond h_n) & index(x) > index(y) \end{cases} \tag{2}$$

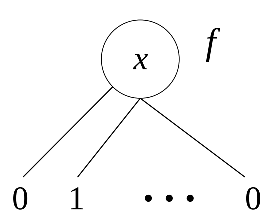

Figure 2. Illustrative basic event MDD.

With the MDD established for system state S_k, Pr(S_k) can be obtained by adding probabilities of all possible paths from the root to leaf node '1'. The recursive evaluation method with regard to the MDD in Figure 1 is given in (3).

$$P_k(f) = p_{x,1}(t)P_k(f_1) + \cdots + p_{x,n}(t)P_k(f_n) \tag{3}$$

where $P_k(f)$ is Pr(S_k) with respect to the current sub-MDD *f*, and $p_{x,i}(t)$ (*i*=1,2, …, *n*) is the probability of component *x* being in state *i*. $P_k(f)$ gives Pr(S_k) when the root of the sub-MDD is the root of the entire system MDD.

3. MDD-Based Uncertainty Reliability Analysis

This section presents the interval-MDD and fuzzy-MDD methods to consider epistemic uncertainty in the reliability analysis of MSSs.

3.1 Interval-MDD Method

Let $[a] = [\underline{a}, \overline{a}], [b] = [\underline{b}, \overline{b}]$ be real compact intervals. Let $\circ$ represent one of the basic operations of 'addition', 'subtraction', 'multiplication' and 'division' for real numbers, that is, $\circ \in \{+, -, \cdot, /\}$. In (4), we define the corresponding operations for interval $[a]$ and $[b]$, where $0 \notin [b]$ is assumed in the case of division.

$$[a] \circ [b] = \{a \circ b | a \in [a], b \in [b]\} \tag{4}$$

It is proved that the set $I(\mathcal{R})$ of real compact intervals is closed with respect to these operations (Alefeld and Mayer, 2000). $[a] \circ [b]$ can be further represented by using only the bounds of $[a]$ and $[b]$ through the following rules:

$$[\mathrm{a}] + [\mathrm{b}] = [\underline{a} + \underline{b}, \overline{a} + \overline{b}] \tag{5}$$

$$[\mathrm{a}] - [\mathrm{b}] = [\underline{a} - \overline{b}, \overline{a} - \underline{b}] \tag{6}$$

$$[\mathrm{a}] \cdot [\mathrm{b}] = [\min\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\}, \max\{\underline{a}\underline{b}, \underline{a}\overline{b}, \overline{a}\underline{b}, \overline{a}\overline{b}\}] \tag{7}$$

$$[\mathrm{a}]/[\mathrm{b}] = [\min\{\underline{a}/\underline{b}, \underline{a}/\overline{b}, \overline{a}/\underline{b}, \overline{a}/\overline{b}\}, \max\{\underline{a}/\underline{b}, \underline{a}/\overline{b}, \overline{a}/\underline{b}, \overline{a}/\overline{b}\}] \tag{8}$$

If $\underline{a} = \overline{a} = a$, then a is identified as a real number with the degenerate interval $[a, a]$ keeping the real notation. The operation for two intervals also results in an interval.

According to discussions in Section 2, when the MDD of S_k is obtained, Pr(S_k) can be calculated by (3). In the presence of epistemic uncertainty, the specific probability of a component being in a specific state cannot be obtained, but the upper and lower bounds of it. To introduce the interval theory into MDD evaluation, the probability of component *x* being in state *i* (*i*=1,2, …, *n*) at time *t* is denoted as an interval in the form of $[p_{x,i}](t)$. Thus, the probability of S_k can be evaluated as

$$[P_k](f) = [p_{x,1}](t)P_k(f_1) + \cdots + [p_{x,n}](t)P_k(f_n) \tag{9}$$

The operation rules defined in (5)-(8) can then be applied to complete the MDD evaluation.

3.2 Fuzzy-MDD Method

A fuzzy number, also called a fuzzy probability, describes 'vagueness' on equal terms with 'crispness', such as 'close to 5', 'high reliability', and 'low failure rate' (Weber, 1994). A fuzzy number in probability space represents a fuzzy number between 0 and 1, which can be assigned to the probability of an event (Misra and Soman, 1995). Different forms of fuzzy numbers, such as triangular fuzzy number (Kumar and Dhiman, 2021a) and trapezoidal fuzzy number (Kumar and Dhiman, 2021b), have been used in reliability analysis. The membership function determines the form of fuzzy numbers, which also characterizes the probability of a specific value being selected in the interval of fuzzy numbers.

Let $x, a, b, c \in \mathcal{R}$ and $f: R \to [0,1]$ represent the membership function. A triangular membership fuzzy number *A* can be defined as in (10). With $a \leq b \leq c$, this triangular fuzzy number can be denoted as $A(a, b, c)$. The parameter 'b' gives the maximal grade of $A(x)$, 'a' and 'c' are the lower and upper bounds of the value range.

$$A(x) = \begin{cases} (x-a)/(b-a), a \leq x \leq b \\ (c-x)/(c-b), b \leq x \leq c \\ 0 \qquad\qquad\qquad , otherwise \end{cases} \tag{10}$$

The operation rules on fuzzy numbers are derived based on the expansion principle proposed by Zadeh (1965), and the expansion principle is further extended by Klir and Yuan (1995). Specifically, the fuzzy set of universe *U* can be deduced from the fuzzy set of universe *V* via functions when the elements of *U* and *V* follow mapping relations. For example, when a mapping relation denoted as (11) exists between the universes *U* and *V*, and the membership function of the fuzzy set on the universe *U* is known, the membership function of universe *V* can be obtained by (12).

$$\begin{aligned} f: U \to V \\ u \mapsto v = f(u) \end{aligned} \tag{11}$$

$$f(A)(v) = \begin{cases} \vee_{v=f(u)} A(u), & f^{-1}(v) \neq \Phi \\ 0, & f^{-1}(v) = \Phi \end{cases} \tag{12}$$

Assume there are two triangular fuzzy numbers $A = [a_1, b_1, c_1,]$, and $B = [a_2, b_2, c_2]$. According to the expansion principle, the operation rules can be defined as (13) to (16) (Chen, 1994):

$$A + B = [a_1 + a_2, b_1 + b_2, c_1 + c_2] \tag{13}$$

$$A - B = [a_1 - c_2, b_1 - b_2, c_1 - a_2] \tag{14}$$

$$A \cdot B = [a_1 \cdot a_2, b_1 \cdot b_2, c_1 \cdot c_2] \tag{15}$$

$$A \,/\, B = [a_1/c_2, b_1/b_2, c_1/a_2] \tag{16}$$

When the parameters in the MDD evaluation function are fuzzy numbers, (13)-(16) can be applied to measure both the component and system state probabilities.

4. Case Studies

This section illustrates the proposed interval-MDD method and fuzzy-MDD method through detailed reliability analysis of a high-speed train bogie system. According to the function mechanism of high-speed trains, both the bogie system and its components can experience multiple degradation levels. Table 1 shows the state definition of major components in the bogie system. Table 2 defines the five disjoint states of the bogie system.

Table 1. Major components in the bogie system of a high-speed train.

Components	Symbol	Component States
Axle	A	1. Operational
		2. Failed
Vertical shock absorber	B	1. Operational
		2. Failed
Wheel	C	1. Operational
		2. Abrasion
		3. The wheel tread is stripped but at least one standard is not achieved
		4. The wheel tread is stripped and achieves all the three standards
Air spring	D	1. Operational
		2. Failed
Gear box	E	1. Operational
		2. Failed
Resist sinusoidal vibration absorber/ Anti-hunting damper	F	1. Operational
		2. Failed
		3. Connecting bolts are missing or loose
Axle box	G	1. Operational
		2. Failed
Traction motor	H	1. Operational
		2. Failed

Table 2. Definition of different states of the bogie system.

<table>
<tr><th>Components</th><th>State 1</th><th>State 2</th><th>State 3</th><th>State 4</th><th>State 5</th></tr>
<tr><td>Axle</td><td rowspan="8">All components are in state 1.</td><td rowspan="8">The vertical shock absorber is in state 2 and the other components are in state 1.</td><td rowspan="8">The wheel is in state 2 or 3, or the air spring is in state 2, or the Axle box is in state 2, or the Resist sinusoidal vibration absorber is in state 2 when the other components are in state 1.</td><td rowspan="8">The wheel is in state 4, or the axle box is in state 2, or the Traction motor is in state 2, or any two conditions in State 3 are both satisfied when the other components are in state 1.</td><td rowspan="8">Other conditions except state 1,2,3,4.</td></tr>
<tr><td>Vertical shock absorber</td></tr>
<tr><td>Wheel</td></tr>
<tr><td>Air spring</td></tr>
<tr><td>Gear box</td></tr>
<tr><td>Resist sinusoidal vibration absorber</td></tr>
<tr><td>Axle box</td></tr>
<tr><td>Traction motor</td></tr>
</table>

4.1 System State MDDs

The MDDs for the bogie system being in states 1 to 4 are illustrated in Figures 3 to 6. The MDD for state 5 is not provided as this state probability can be evaluated as Pr(5)=1-Pr(1)-Pr(2)-Pr(3)-Pr(4).

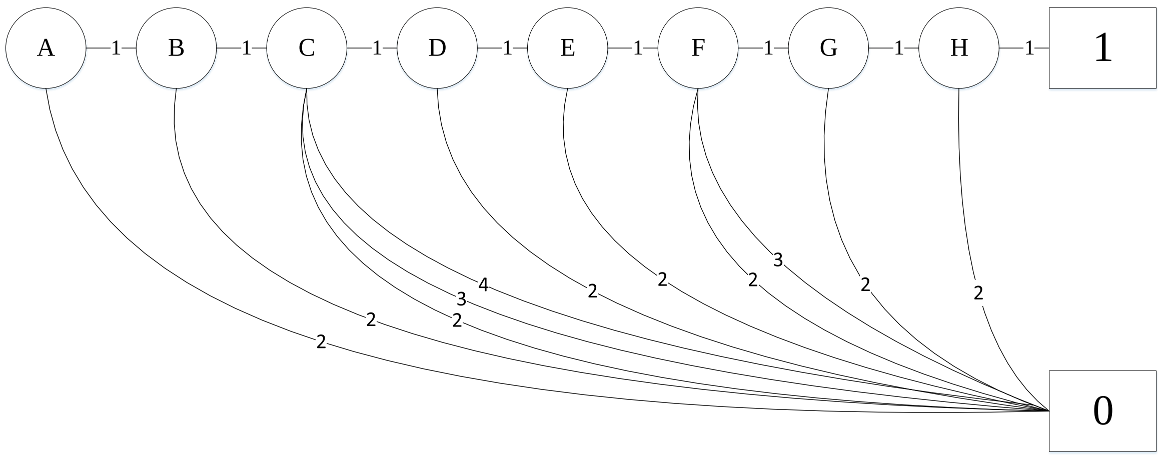

Figure 3. The MDD of the bogie system in state 1.

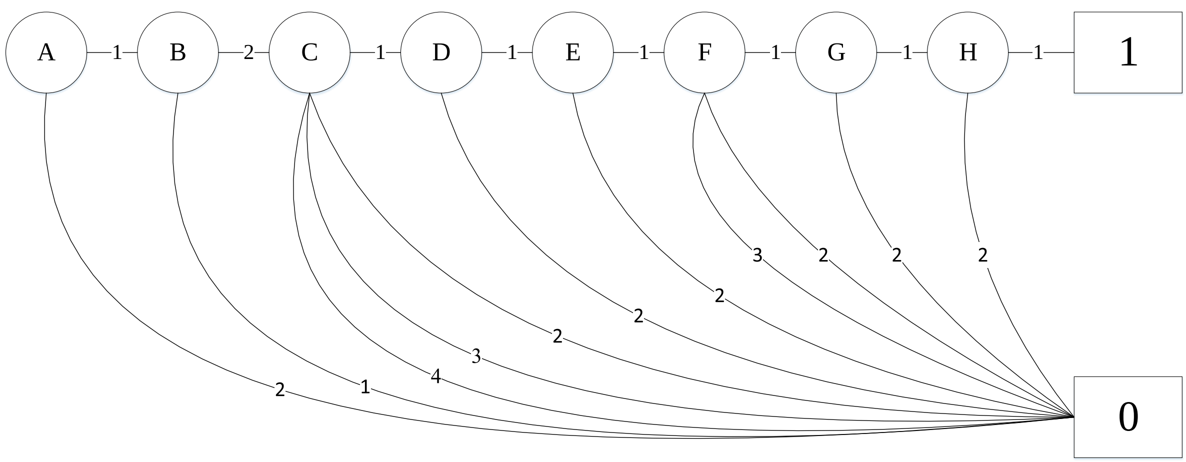

Figure 4. The MDD of the bogie system in state 2.

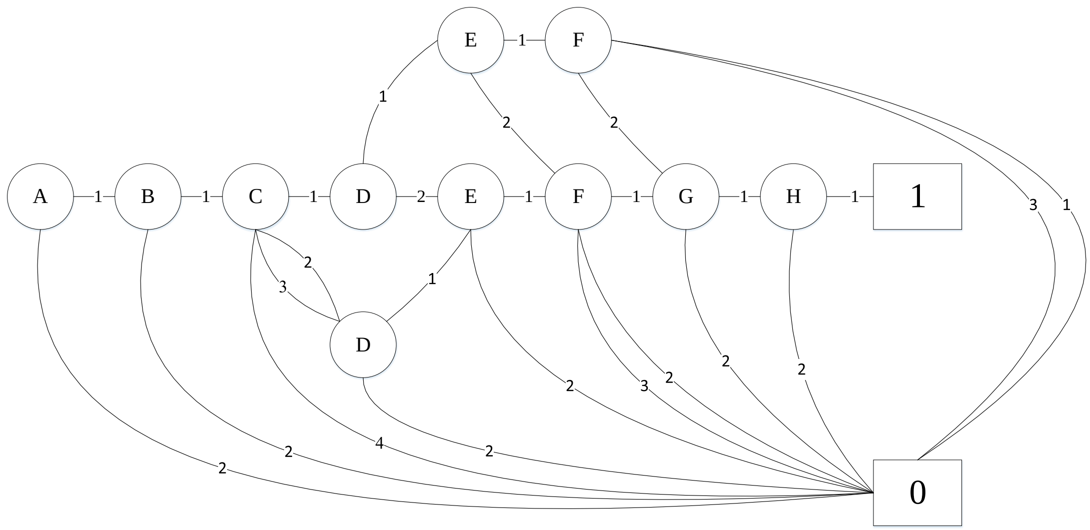

Figure 5. The MDD of the bogie system in state 3.

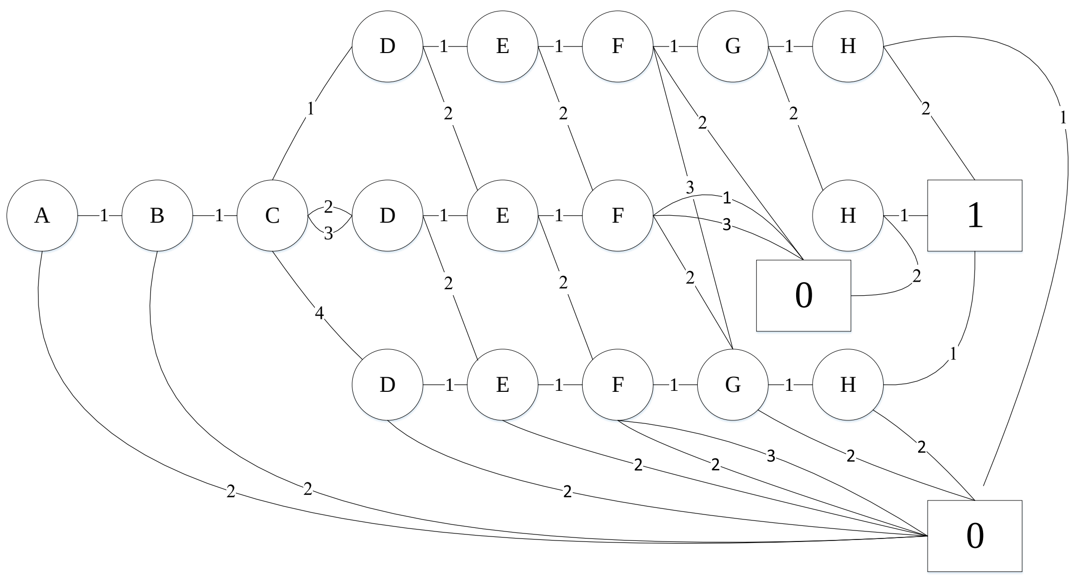

Figure 6. The MDD of the bogie system in state 4.

4.2 Reliability Evaluation Using Interval-MDD

Based on the MDDs generated in Figures 3-6, the probability function of the bogie system being in the five states can be expressed in (17)-(21).

$$\Pr(1) = \Pr(A_1)\cdot\Pr(B_1)\cdot\Pr(C_1)\cdot\Pr(D_1)\cdot\Pr(E_1)\cdot\Pr(F_1)\cdot\Pr(G_1)\cdot\Pr(H_1) \tag{17}$$

$$\Pr(2) = \Pr(A_1)\cdot\Pr(B_2)\cdot\Pr(C_1)\cdot\Pr(D_1)\cdot\Pr(E_1)\cdot\Pr(F_1)\cdot\Pr(G_1)\cdot\Pr(H_1) \tag{18}$$

$$\Pr(3)=\Pr(A_1)\cdot\Pr(B_1)\cdot(\Pr(C_1)\cdot\Pr(D_1)\cdot\left(\Pr(E_1)\cdot\Pr(F_2)+\Pr(E_2)\cdot\Pr(F_1)\right)+ \Pr(D_2)\cdot\Pr(E_1)\cdot\Pr(F_1))+(\Pr(C_2)+ \Pr(C_3))\cdot\Pr(D_1)\cdot\Pr(E_1)\cdot\Pr(F_1))\cdot\Pr(G_1)\cdot\Pr(H_1) \tag{19}$$

$$\begin{aligned}\Pr(4)=&\Pr(A_1)\cdot\Pr(B_1)\cdot(\Pr(C_1)\cdot(\Pr(D_1)\cdot(\Pr(E_1)\cdot(\Pr(F_1)\cdot(\Pr(G_1)\cdot\Pr(H_2)\\&+\Pr(G_2)\cdot\Pr(H_1))+\Pr(F_3)\cdot\Pr(G_1)\cdot\Pr(H_1))+\Pr(E_2)\cdot\Pr(F_2)\cdot\Pr(G_1)\cdot\Pr(H_1))\\&+\Pr(D_2)\cdot(\Pr(E_1)\cdot\Pr(F_2)+\Pr(E_2)\cdot\Pr(F_1))\cdot\Pr(G_1)\cdot\Pr(H_1))\\&+(\Pr(C_2)+\Pr(C_3))\cdot\Pr(D_1)\cdot(\Pr(E_1)\cdot\Pr(F_2)+\Pr(E_2)\cdot\Pr(F_1))\\&+\Pr(D_2)\cdot\Pr(E_1)\cdot\Pr(F_1))+\Pr(C_4)\cdot\Pr(D_1)\cdot\Pr(E_1)\cdot\Pr(F_1)\cdot\Pr(G_1)\cdot\Pr(H_1))\end{aligned} \tag{20}$$

$$\Pr(5)=1-(\Pr(1)+\Pr(2)+\Pr(3)+\Pr(4)) \tag{21}$$

From the data set in Lin et al. (2015) and Yin et al. (2016), we can get the lifetime distribution functions of the eight major components in the bogie system as shown in Table 3.

Table 3. Lifetime distribution functions of the bogie system components.

Components and Symbols		Lifetime Distribution Function
A. Axle		$F(k)=1-\exp\left[-\left(\frac{k+0.53}{77.81}\right)^{3.57}\right]$
B. Vertical shock absorber		0.0092
C. Wheel	State 2	$F(k)=1-\exp\left[-\left(\frac{k}{179.33}\right)^{3.21}\right]$
	State 3	$F(k)=1-\exp\left[-\left(\frac{k}{84.83}\right)^{3.21}\right]$
	State 4	$F(k)=1-\exp\left[-\left(\frac{k}{206.51}\right)^{3.21}\right]$
D. Air spring		$F(k)=1-\exp\left[-\left(\frac{k+25733.9}{25920.8}\right)^{521.68}\right]$
E. Gear box		$F(k)=1-\exp\left[-\left(\frac{k}{132.4}\right)^{2.2}\right]$
F. Resist sinusoidal vibration absorber	State 2	$F(k)=1-\exp\left[-\left(\frac{k+574.10}{748.61}\right)^{13.93}\right]$
	State 3	$F(k)=1-\exp\left[-\left(\frac{k+574.10}{1497.22}\right)^{13.93}\right]$
G. Axle box		$F(k)=1-\exp\left[-\left(\frac{k}{128.18}\right)^{2.52}\right]$
H. Traction motor		$F(k)=1-\exp\left[-\left(\frac{k-23.45}{119.88}\right)^{0.88}\right]$

We change the mileage (denoted by k) in the lifetime distribution functions to different values of 100, 300 and 800 thousand kilometers, and collect the system state probabilities calculated using (17)-(21). The results are shown in Table 4.

Table 4. System state probabilities with changing *k.*

Mileage (kilometers)	100K	300K	800K
Pr(1)	0.9269	0.7265	0.0246
Pr(2)	0.0086	0.0067	0.0002284
Pr(3)	0.0603	0.1259	0.0659
Pr(4)	0.0027	0.0871	0.0656
Pr(5)	0.0015	0.0537	0.8437

When the component failure parameters are intervals with the original value being the median value, and the interval width being 0.02, the system state probabilities calculated using (17)-(21) are shown in Table 5. Each result in Table 4 falls into its corresponding interval in Table 5. Based on the operation rules, the result of interval operations can cover all the numerical computation results within the interval. In other words, if $x_1^L < x_1 < x_1^U$, $x_2^L < x_2 < x_2^U$,…, $x_n^L < x_n < x_,^U$ then $F(x_1^L, x_2^L, \dots, x_n^L) < F(x_1, x_2, \dots, x_n) < F(x_1^U, x_2^U, \dots, x_n^U)$. Thus, there is no information reduction for the interval parameters during the calculation process.

Table 5. Interval system state probabilities with changing *k.*

Mileage (kilometers)	100K	300K	800K
Pr(1)	[0.9266,0.9273]	[0.7232,0.7297]	[0.0243,0.0249]
Pr(2)	[0.0085,0.0087]	[0.0066,0.0068]	[2.2316e-4,2.3381e-4]
Pr(3)	[0.0601,0.0605]	[0.1246,0.1273]	[0.0651,0.0666]
Pr(4)	[0.0026,0.0027]	[0.0848,0.0895]	[0.0647,0.0665]
Pr(5)	[7.617e-4,0.0022]	[0.0466,0.0608]	[08418,0.8456]

4.3 Reliability Evaluation Using Fuzzy-MDD

When the component failure parameters are triangle fuzzy numbers with the original parameter value (ignoring uncertainty) being the median value, and the weight being 0.02, the system state probabilities are calculated based on (17) to (21) and shown in Table 6.

Table 6. Fuzzy system state probabilities with changing *k.*

Mileage (kilometers)	100K	300K	800K
Pr(1)	[0.9266,0.9269,0.9273]	[0.7232,0.7265,0.7297]	[0.0243,0.0246,0.0249]
Pr(2)	[0.0085,0.0086,0.0087]	[0.0066,0.0067,0.0068]	[2.2316e-4,2.2844e-4,2.3381e-4]
Pr(3)	[0.0601,0.0603,0.0605]	[0.1246,0.1259,0.1273]	[0.0651,0.0659,0.0666]
Pr(4)	[0.0026,0.0027,0.0027]	[0.0848,0.0871,0.0895]	[0.0647,0.0656,0.0665]
Pr(5)	[7.617e-4,0.0015,0.0022]	[0.0466,0.0537,0.0608]	[0.8418,0.84370.8456]

The triangular fuzzy numbers and the interval numbers have similarities in operation rules. Since the maximum possible value is added in the triangular fuzzy numbers, the accuracy is further increased after the operations. Thus, the triangular fuzzy numbers can offer more information to decision-making and improve the accuracy of overall information compared to the interval numbers.

5. Conclusion and Future Work

This paper investigates the MDD-based methods for the reliability analysis of MSSs considering epistemic uncertainty. The contributions made include the interval-MDD and fuzzy-MDD methods, which are put forward to address the reliability evaluation when the precise failure parameters of system components are not available. The proposed MDD-based methods are illustrated through the reliability analysis of a high-speed train bogie system in detail. While the Weibull distribution and fixed probabilities are used in the case study, both methods are flexible in handling diverse types of component lifetime distributions. The numerical results demonstrate that when the upper bound, the lower bound, and the maximum possible value of each component parameter are available, the fuzzy-MDD method can achieve more accurate reliability evaluation than the interval-MDD method.

In this paper, the interval and fuzzy parameters for the illustrative bogie system are pre-determined. In the future, the proposed methods will be applied and extended to more complicated systems with practical data sets for estimating parameters required by the methods.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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