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# Directional Bandgap Analysis in Phononic Crystal with Rectangular Super Cell Structure

**Abhishek Singh**
School of Engineering,
Indian Institute of Technology, Mandi, Himachal Pradesh, India.
*Corresponding author*: kulhadiyaabhishek@gmail.com

**Arpan Gupta**
School of Engineering,
Indian Institute of Technology, Mandi, Himachal Pradesh, India.
E-mail: agupta@iitmandi.ac.in

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**Abstract**
Phononic crystals have promising applications in the field of elastic waves and to attenuate the vibrations with their extraordinary feature of stop bands also known as bandgaps. Due to periodicity in structure, the wave in certain band of frequencies is not allowed to propagate. In this paper, we have proposed a phononic crystal whose unit cell is composed of super cell structure consisting of air voids and scatterer distributed in nice rectangular units. The new design is found to have a wider directional band gap in the low-frequency zone. The dispersion relationship of the model is obtained by the computational method using finite element analysis. Numerical results show that the improved phononic crystal with super cell structure enhances the directional bandgap by around 180% when compared with the conventional phononic crystal having one scatterer with same filling factor. The proposed model with super cell structure opens the directional bandgap of 84 Hz with the lower bound frequency of 125Hz and upper bound frequency of 209 Hz in the horizontal ($\Gamma \rightarrow X$) direction.

**Keywords**-Phononic Crystal (Pnc), Scatterer, Eigenfrequencies, Bandgap.

## 1. Introduction

Periodic structures are made by repetition of unit cells; the distance between the unit cell is called as lattice constant. Phononic crystals are macroscopic periodic structures used for elastic waves that possess the unique property of bandgap. These are the structures composed of materials having periodically arranged elastic constants and mass densities which are collectively responsible for bandgaps. These bandgaps are the regions where ideally, propagation of the elastic wave is completely forbidden. For a finite sized structure, the wave can propagate with very low amplitude or very high transmission loss.

Based on the arrangement of the material and geometry, there are two types of phononic crystals, Bragg's scattering structure and locally resonating structure. The phenomenon of wave attenuation in both structures has a different principle. In Bragg's scattering the frequency attenuation is the result of interference of scattered and incident waves while in locally resonating crystals the attenuation of the wave is due to the local resonance in the individual scatterer. Bragg's diffraction has one limitation that it is possible only when the incident wavelength and subwavelength of the structure are comparable. The local resonance metamaterial overcomes this drawback. In this paper, we have modeled Bragg's scattering crystal having super cell structure and obtained the dispersion relationship by computational methods using COMSOL

Multiphysics. Super cell structure implies having some periodic unit cells within a given unit cell. This will lead to lowering of the low frequency band gap limit and also to widen the frequency band gap.

### (a) Bragg's Scattering based Phononic Crystal

Bragg's Scattering based Phononic Crystal are periodic structures composed of two different materials having a difference in elasticity. Based on the elasticity, generally, these materials have been divided into two components one with high elasticity is the scatterer, and the other is the matrix. Scatterers are embedded periodically in the matrix material and form Bragg's phononic crystal. Bragg's scattering crystals are further divided into two categories based on their unit cell structure and bandgap they produce. The conventional symmetric Bragg's phononic crystal is shown in Figure 1.

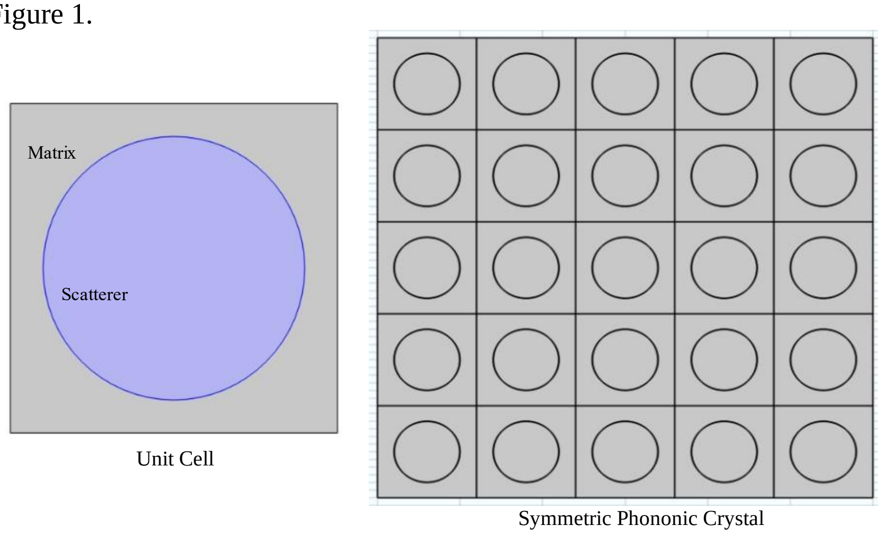


**Figure 1.** Symmetric Bragg's scattering panel with unit cell.

Kushwaha et al. (1993) introduced the concept of phononic crystal for the very first time in 1993. He used the term periodic elastic composite for the phononic crystals, and his experimental study shows the evidence of the first full bandgap (Bilal & Hussein, 2011; Li et al., 2015) structure which attracted the attention of the researchers in this domain. Sigalas et al. (1992) studied the elastic wave propagation in the inhomogeneous solid and stated the equation of wave diffusion equation in solids. Yu et al. (2013) introduced a 2D phononic crystal made up of rectangular rods as scatterer impinging in a homogeneous material through the neck-like structure which shows that low-frequency bandgap generation is due to local resonance of the rods and lamb modes. Cheng & Shi (2014) studied Bragg's scattering panels and found that elastic waves can be effectively attenuated if three units of periodic structures are used parallelly and his study shows promising results. Computation analysis also shows that vibration could be attenuated if excitation frequency falls under the bandgap region. Hsu & Wu (2006) studied the effect of thickness of the crystal and concluded that the tunning of 2D crystal thickness have an impact on the width of frequency bandgap. Cheng & Shi (2014) have also done the parametric study of the phononic crystal and stated that the asymmetric Bragg's scattering panels shown in Figure 2 result in directional bandgaps which are very useful for engineering applications where frequency attenuation is desired only in one direction.

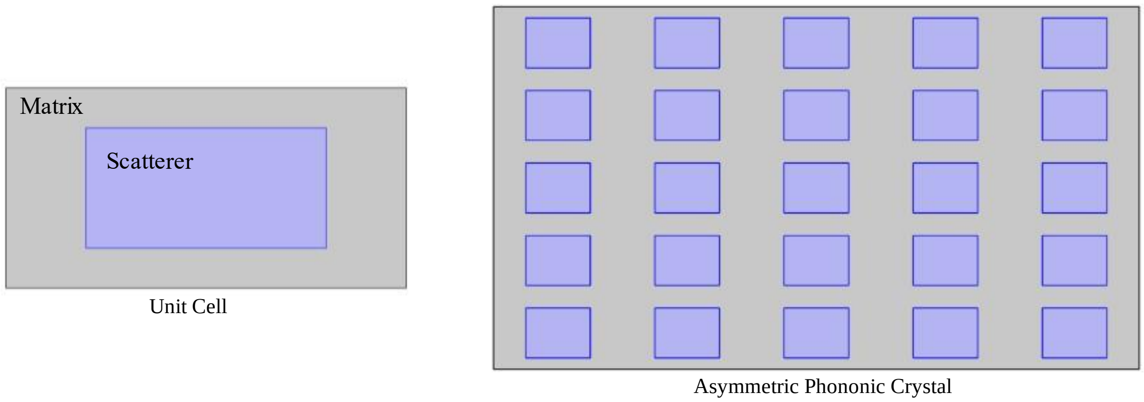


**Figure 2.** Asymmetric Bragg's scattering Phononic crystal with unit cell. The lattice constant is different in horizontal and the vertical directions making the structure asymmetric.

## (b) Locally Resonant Phononic Crystal

Local resonating phononic crystals are different from Bragg's scattering based phononic crystal as they can open up the band of frequencies below the Bragg's limit. Unlike Bragg's scattering panels, local resonating crystals have a spectral gap due to the local resonance of the coated core. In this type of phononic crystals, frequency attenuation is the result of local resonance in which the disturbance is absorbed by the core material. Local resonating panels can also attenuate the elastic waves having wavelengths more than the lattice constant. Cheng et al. (2013) studied the local resonating panel and shows that the lower bound frequency of the spectral gap of his model coincides with the second characteristic frequency of the inner scatterer. Generally, locally resonating panels have dual periodicity whose physical significance is that a second mass (core) is attached with the scatterer which also shows the periodic substructure. Figure 3 shows the schematic of a 2D locally resonating phononic crystal. These are the phononic crystals of periodic structure which consist of effective negative attributes such as negative effective mass density tensor and bulk modulus for certain frequency ranges. This type of periodic crystal realizes that most of the external force applied by the excitation source is absorbed by the effective negative attribute of the core material. The drawback of a locally resonating periodic structure is that they have narrow bandwidth when compared with the conventional Bragg's panel but with further research. Krushynska et al. (2017) designed the single-phase solid metamaterial having quasi-resonant bandgaps and theoretically proved that the model contains a wider bandgap than Bragg's panel.

Hu et al. (2014) studied the two-dimensional binary locally resonating crystal structures which are crystals having cylindrical periodic rubber scatterers embedded in an epoxy matrix. A simple quasi 1D mechanical analog model is used to easily understand the physical interpretation of locally resonating structures. Their numerical study on this binary structure proved that the sub-frequency gap also occurred in this model because of the high contrast in rubber density and bulk modulus. They discovered for the first time that in any locally resonating structure resonant mode will result in subfrequency gaps.

Assouar et al. (2016) studied a locally resonating scatterer made up of rubber stubs stuck on the plate of aluminum. They have numerically studied the model using finite element analysis within the frequency range 0.4 kHz to 6 kHz and obtain the extraordinary results of global bandgap in the range of 1.9 - 2.6 kHz. The central frequency of this resonating panel is far away from Bragg's frequency. This study is also verified by the experiments. Ma et al. (2015) gives a membrane-type phononic crystal's model which overcomes the problem of rigidity of the phononic crystal. Liu et al. (2000) designed a locally resonating structure having a periodic constant of two orders smaller than the spectral wavelength and obtained the two bandgaps which were not practically possible with Bragg's periodic panel due to subwavelength constraint. The structure is a lead ball as a scatterer covered with the silicone rubber as the core material and then this arrangement is fixed in the honeycomb lattice in epoxy.

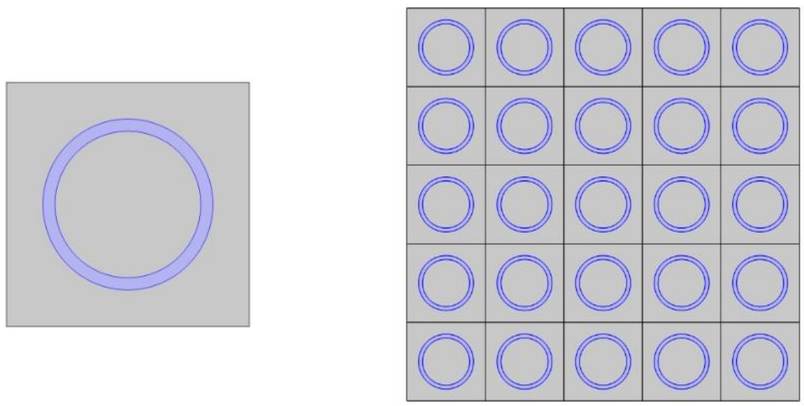

**Figure 3.** Locally resonating Phononic crystal with unit cell.

## 2. Model

The proposed model of phononic crystal is a periodic structure made of the rectangular unit cell. The unit cell is a super-cell structure. One unit cell consists of 25 conventional rectangular unit cells having 9 unit cells with rectangular metallic scatterers in the center and 16 unit cells with rectangular air voids on the periphery. These air voids help in elastic wave attenuation by the mode of change of medium in wave path and metallic scatterers attenuate the waves by rigid body resonance of the scatterers in the matrix and form the bandgaps. The schematic of our proposed model is shown in Figure 4.

The parameters of our proposed model are given in Table 1. The material properties of the model are well-chosen from the Ashby chart by keeping in mind the feasibility of the model design and the cost of the material. The material property of the model is given in Table 2.

**Table 1.** Model parameters as used in the model represented in Figure 4

| **Parameter** | **Lattice constant(A×B)** | **Rectangle scatterer size(a×b)** |
|---|---|---|
| **Size(mm)** | 80×45 | 8×6 |

**Table 2.** Material properties

| **Name** | **Young's modulus (Pa)** | **Poisson's ratio (υ)** | **Density(ρ/Kg.m$^{-3}$)** |
|---|---|---|---|
| **Steel (Scatterer)** | $21.06\times10^{10}$ | 0.26 | 7780 |
| **Vulcanized rubber (Matrix)** | $1\times10^{6}$ | 0.47 | 1300 |

To study the dispersion relationship of our proposed model and to analyze the bandgap of the proposed super cell structure, finite element analysis has been used through the software COMSOL Multiphysics.

## 3. Finite Element Analysis

Phononic crystals are studied in the study domain of solid mechanics (elastic wave) with required boundary conditions. The governing equation for the elastic wave propagation in phononic crystal is given as

$$\rho(r)\,\frac{\partial^2 u}{\partial t^2} = \nabla\,\{[\lambda(r) + 2\mu(r)](\nabla \cdot u)\} - \nabla \times [\mu(r)\nabla \times u] \tag{1}$$

where $u$ is the displacement vector in coordinate vector $r$, $\rho(r)$ is the density, $\lambda(r)$, $\mu(r)$ are Lame constants. The geometry over which the equation is solved is shown in Figure 4 below.

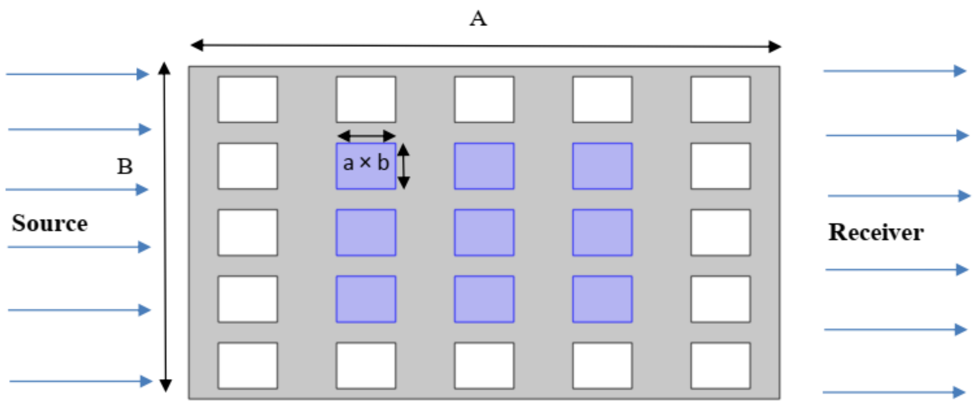


**Figure 4.** Structure of the unit cell under computational study with source and receiver surfaces. The empty white region indicates air voids, while the shaded purple rectangle indicates steel scatterers.

Floquet Periodic Boundary conditions are used as described in the next section. Eigen value analysis is performed to calculate the band gap structure of the super cell structure. Free triangular element (6 noded triangular element) was used with maximum element size of 0.00296 m.

## 4. Eigen Frequency Study for Phononic Crystal

Eigen frequency study is done to establish the dispersion relationship. The dispersion relationship is the relation between the eigenfrequency and the wave vector. In this study we have uses the Bloch-Floquet periodic condition in the unit cells for establishing the relationship between displacement (*U*) and wave vector (*k*) which is expressed by Eq.4.

$$U(r + a) = \mathrm{e}^{i(ka)}\; U(r) \tag{2}$$

where $U$ is the displacement at nodes, $k$ is the wave vector, $r$ is the position vector in the cell and $a$ is the lattice constant after which cells start repeating. Band gap represents a region of frequency where no wave vector is allowed to exist or propagate. To obtain the band gap, wave vector is varied in the first Brillouin zone to obtain the allowable frequency that can propagate through the structure. The Brillouin zone is a primitive cell in the reciprocal lattice used to locate the scattering plane. The first Brillouin zone is the smallest possible primitive cell in reciprocal space. It is also known as the irreducible Brillouin zone because it's enclosed the minimum possible volume which is enclosed by the Bragg's scattering planes. If the calculations are done beyond this zone, then due to symmetry the results start repeating. The schematic of the Brillouin zone for the 2D periodic unit cell is shown in Figure 5.

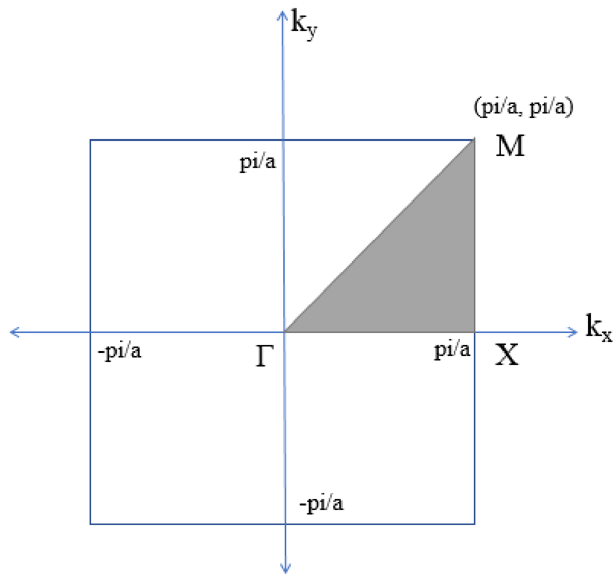


**Figure 5.** Schematic of the Brillouin zone for 2D geometry.

For the 2D geometry Brillouin zone is divided into three directions as shown in the above figure. The compaction of the Brillouin zone is done because of the symmetry and is known as the irreducible Brillouin zone. The division is as follows **Γ-X, X-M,** and **Γ-M** region in which the wave vector is varied to get the eigenmodes. If eigenfrequencies are not available in any direction, then it will be termed as global bandgap and if there is unavailability in some particular direction only then it is termed as directional bandgaps which are presented in this paper. Bloch-Floquet theorem is valid only if the structure is periodic. The waves propagate in the phononic crystals are called Bloch waves and represented as:

$$p(x) = \varphi(x)e^{ikx} \tag{3}$$

where, $\varphi(x)$ is a Bloch-Floquet periodic function. If the lattice constant of the phononic crystal is $a$ then $\varphi(x)$ is defined as:

$$\varphi(x) = \varphi(x + a) \tag{4}$$

Combining Eq. (5) and (6) we get the Bloch- Floquet equation.

$$p(x + a) = \varphi(x)e^{ik(x+a)} \tag{5}$$

By applying this boundary condition to the surfaces of the unit cell the analysis was restricted to one unit cell with the benefit of getting a result for infinite unit cell structure.

## 5. Results and Discussion

The motivation behind this study is to design a phononic crystal that can open the wider directional bandgaps in lower frequency bands. Directional bandgaps had received less attention and can have applications where attenuation of waves is required in one particular direction. To overcome this issue, we have designed a new kind of asymmetric phononic crystal that can eliminate this limitation and will open the paths for the application where a directional bandgap is required in a wider frequency band. The computational study is done, and bandgaps are generated from the procedure explained in the previous sections. To analyze the effectiveness of our model over the conventional asymmetric phononic crystals we have compared our results with the conventional phononic crystal's unit cell and found a subsequent improvement in the bandgaps.

Firstly, to validate our computational method, initial validation simulation results were performed with (Sun et al., 2019). Our results are found in good agreement as shown in figure 6. (Sun et al., 2019) also uses super cell structure, but with symmetric square geometry. In the current work, this super cell structure is made asymmetric with rectangular scatterers.

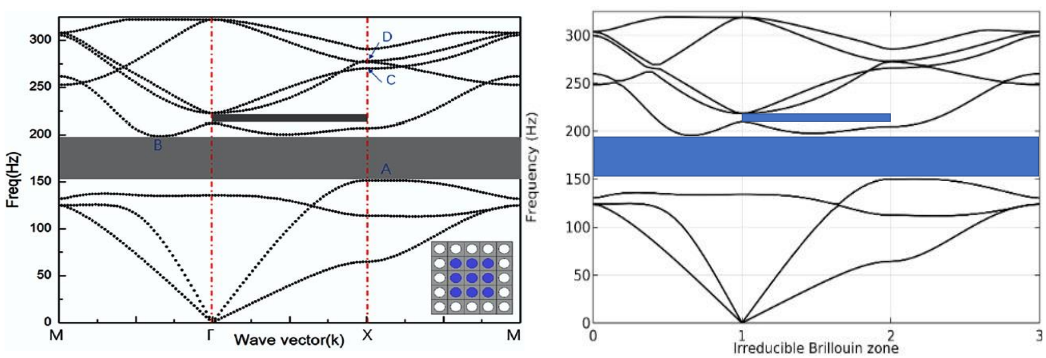


**Figure 6.** Result from Sun et al.(L) and regenerated result (R).

The frequency band gap of the proposed asymmetric super cell phononic crystal (Figure 4) is shown in figure 7. The results are promising as the it opens the directional bandgap in the lower frequency band with lower band frequency of 125Hz and upper band frequency of 209 Hz. A symmetric super cell structure (Sun et al., 2019) produces complete band gap, however, the asymmetric structure produces directional band gap.

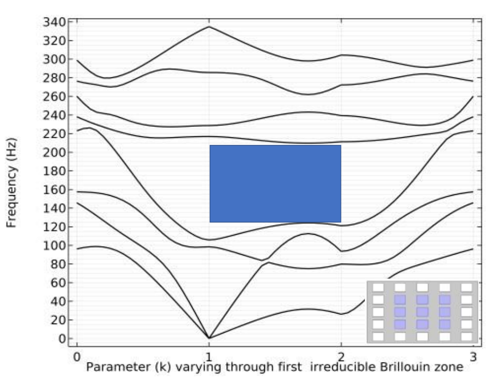


**Figure 7.** Dispersion relationship of the proposed model.

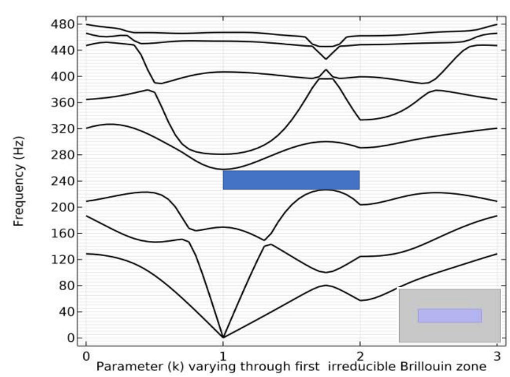


**Figure 8.** Dispersion relationship of the conventional unit cell.

The model is compared with a unit cell having same filling fraction but with only one scatterer. Figure 8 shows the frequency band gap of the conventional phononic crystal having a rectangular unit cell with the same filling factor. The inset shows the geometry used for the simulation. This conventional phononic crystal possesses a bandgap of 30Hz with lower band frequency 225Hz and upper band frequency 255Hz. Our study shows that distributing the scatterer to nine small scatterers, and having a super cell structure, opens the wider bandgaps with the same material requirement. The band gap is widened by around 2.8 times and the lower frequency is shifted from 225 Hz to 125 Hz.

## 6. Conclusion

In this work, a novel asymmetric super cell phononic crystal is proposed for wider band gap. Phononic crystals attenuate the elastic wave in solid due to periodic presence of scatters. The band width and lower frequency of band gap can be improved by including a super cell structure which has periodic units in a unit cell. A novel rectangular super cell structure is analysed using the finite element method. The computed band gaps show increase in frequency band width by around 180%. The lower band gap frequency reduced to 125 Hz from 22 Hz. The proposed model has promising application for elastic wave attenuation in structure by designing structure of finite size and still be able to attenuate low frequency waves.

### Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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