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Predator-prey Activity-Tuned Ecological System: An Exhilarating Rollercoaster of Survival

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Abstract
In natural ecosystems, the timing of daily activity in predator and prey plays a crucial role in determining their encounters and evolutionary outcomes. This study explores how coevolution in circadian activity patterns emerges from the dynamic interplay between predator pursuit and prey avoidance. Using a biologically inspired system of simplified differential equations, we formulate the system that incorporates the activity levels of both prey and predator, integrating factors such as energy demands, predation pressure, and the dynamic synchronization of predator behavior in response to prey activity patterns. In the qualitative analysis, positivity and boundedness, uniform persistence, local and global stability, stochastic p-stability are advocated. This work bridges theoretical ecology and dynamical systems, offering a mechanistic insight into how predator–prey co-adaptation shapes temporal niches and changes their routine activities. The effectiveness of predators as killing machines lies not merely in their physical traits, but in the timing and adaptability of their activities, strategically aligned to exploit prey vulnerability. On the other side, prey craft their activity rhythms to reduce detectability and outmaneuver predators, optimizing escape opportunities, which is indeed the masterpiece of predator-prey evolution and the perilous journey of survival. It provides a quantitative framework for future studies on behavioral evolution in complex environments, emphasizing that the true essence of predator–prey systems lies in understanding and capturing their adaptive activity patterns.

Keywords- Uniform permanence, Boundedness, Local stability, Global stability, Mean square stability.

1. Introduction

Predator–prey ecology has garnered significant research attention due to its complex and intellectually stimulating challenges, drawing interest from a broad spectrum of ecological researchers/scientists (Kaushik & Banerjee, 2023; Hamester et al., 2025; Mondal & Khajanchi, 2025). Recent work has shown that predator–prey models continue to produce rich nonlinear behavior, including Neimark–Sacker bifurcations and chaos, fear effect and prey refuse (Roy et al., 2025; Rocha et al., 2026; Uddin et al., 2026). These predator-prey models show how the comprehensive mathematical setup behind these models can explain the complex real-world environmental issues quite easily (Strogatz, 2024; Paul et al., 2025). In natural habitats, the threat of predation often changes throughout the day in a fairly consistent manner, influenced by environmental factors like light levels and temperature, as well as by the behavioral patterns of predators. As a result, prey species can modify their daily activity schedules to minimize exposure to the predators (Sirot et al., 2025). Moreover, predators often adapt their behavior to track the movements of their prey, leading to a synchronization in their activity patterns that can enhance the efficiency of predation (Sirot et al., 2025).

Extensive research has highlighted that the behavioral routines and daily movement cycles of predators play a crucial role in determining how effectively they can capture prey (Ables, 1969; Asa & Wallace, 1990; Geffen & MacDonald, 1993; Boal & Giovanni, 2007). In active animals, the behavior reflects a balancing between several competing needs. There exist two prime variables governing this. One is performing activity for enhancement of feeding and reproduction, while another one is about concurrently minimizing the risk of encountering a predator or detrimental consequences (Alkon & Mitrani, 1988; Razo et al., 2011). These predation patterns, influenced by environmental cues, prey availability, and evolutionary adaptations, frequently specify how and where predators can successfully hunt at the right time. Predators align the timing of their peak activities with those of their prey or when environmental conditions are at their best in order to have a successful kill. This synchronization enhances hunting success and, at the same time, also reflects a complex balance between energy conservation, risk avoidance, and ecological interactions.

Usually, prey animals exhibit clear day rhythmic activities influenced by internal biological clocks (Mondal, 2025). The entrainment of the circadian rhythm helps an animal maintain a regular pattern of activity-rest cycles. The light conditions of the day used the internal clock to keep the prey species mentally and bodily alert, increasing their foraging, predator detection, and responsiveness to the surroundings. At night, the same internal timing mechanism induces a natural sleep or reclusive phase in them, helping preserve energy and naturally avoid the danger lurking in darkness. Such rhythm in behavior is not only essential for survival but also temporally balances vigilance with recovery.

Climate change directly affects the predation and prey dynamics as a consequence of its impact on important ecological parameters like the patterns of behavior and even the temporal patterns of activity. As temperatures rise and habitats shift, both predators and prey may be forced to adapt their hunting or foraging schedules, migratory routes, and even interspecies interactions, leading to potential mismatches in their activity windows and survival strategies. How climate change influences species characteristics and predator-prey relationships largely depend on the surrounding environmental conditions and the degree to which organisms have adapted to their local habitats. These factors shape how individual species respond to shifting climates, potentially modifying their interactions and survival strategies within the ecosystem (Laws, 2017).

When prey exhibit heightened activity levels, they increase their visibility and, consequently, their vulnerability to predators. Observations have shown that prey species often display subdued or still behavior in the presence of nearby predators or when they detect a predator's scent (see **Figure 1(a)**). Such a composed and controlled reaction may make them challenging prey. Predators have also developed clever techniques. They can easily detect the scent of their prey from a far distance or act as if they did not even see them. By stealth and cautiousness, they move closer before striking prey. However, sometimes the preys may be able to recognize the predator at a far distance and run away, which results in a chase (see **Figure 1(b)**). Such actions bring a constant development in the behavioral traits of predators as well as prey.

Sirot et al. (2025) pointed out that there are two ways in which the predator may respond. Firstly, it might align its own activity with the peak times of prey movement, mimicking the prey's behavioral patterns, or secondly, it may choose to hunt when prey activity is minimal, but environmental factors make hunting more effective. Herrera et al. propose an approach to measure how significantly predators influence or alter the activity patterns of their prey (Herrera et al., 2024). Chad et al. (2025) highlighted that gaining insights into how important prey species utilize their habitats and the timing of their daily activities is essential for the effective conservation of apex predators.

The intricate relationship between predator and prey activity patterns gives rise to ecologically significant and often intricate dynamics within natural systems. Although a wide array of predator-prey models has been proposed over the years, most have remained confined to examining fluctuations in population size or biomass. However, the nuanced behavioral interactions, especially those linked to daily and situational activity rhythms, have been largely limited to experimental findings, with very rare or no known attention in mathematical modeling. Motivated by these observational insights, the present study introduces a novel and streamlined mathematical framework aimed at capturing activity-driven interactions within predator-prey relationships. A key innovation of this work is the integration of temporal activity rhythms into the ecological modeling landscape, addressing questions such as how prey adjust their responses to predator presence, at what moments predators initiate pursuit, how synchronized or mismatched behaviors can lead to systemic collapse, how environmental variables alter activity cycles, and how predators dynamically adapt their strategies in response to prey behavior. All of these facets are methodically explored within this model.

The structure of this paper is as follows: We begin with a concise overview of relevant literature, covering the theoretical foundations and recent developments in this field. Following this, in Section 2, we provide necessary assumptions and thereby introduce a stochastic prey–predator model that incorporates the predator and prey activity pattern. In Section 3, the research centers its attention on the outcome of the equivalent deterministic model, and later in Section 4, it investigates the stochastic stability of the developed stochastic system. Simulations are used to validate the theoretical derivative outcome in the case of deterministic and stochastic models in Section 5. Finally, this research will conclude in Section 6, where the significance of the obtained outcome in relation to the biological aspect will be explained. In fact, an important aspect that emerges from the obtained outcome is the significance of species activity in ensuring stability in the system.

2. Ecological Assumptions and Model Formulation

Let x be the prey activities and y be the predator activities, then the model equations can be given as follows

$$\frac{dx}{dt} = \underbrace{\lambda(x_0 - x(t))}_{\text{Required Energy acquisition activities}} - \underbrace{Ex^2(t)y(t)}_{\text{Nonlinear Capture likelihood}} + \underbrace{\sigma_1(x(t) - x^*)d\xi_1}_{\text{Environmental noise effect}} \tag{1}$$

$$\frac{dy}{dt} = \underbrace{\mu y(t)(x(t) - y(t))}_{\text{Adaptive Predator activities Response}} + \underbrace{\sigma_2(y(t) - y^*)d\xi_2}_{\text{Environmental noise effect}} \tag{2}$$

where, σ_1 and σ_2 are the real constants, ξ_1 and ξ_2 are independent weiner process. The parameters λ, μ are positive; however, parameter E is unrestricted in sign. The following assumptions have been made during formation of this model

- Prey increase their activities to meet a required energy intake level x_0. In other words, prey try to maintain a natural activity level x_0 for their energy needs.
- We assume here the prey capturing likelihood score (PCLS) E of successfully capturing prey is typically higher and positive when the prey is closer, as it allows for a more direct and efficient attack. This is because closer proximity often means less time is needed to close the distance and initiate the attack, and it also allows for more accurate targeting of the prey. However, the prey capturing likelihood score (PCLS) of a predator successfully capturing prey typically decreases and becomes negative as

the distance between them increases. This is because predators often rely on factors like surprise, speed, and close-range attack strategies, which become less effective with greater distances, especially when prey is highly active and capable of outrunning the predators.

- Prey reduce activity if the predator is active to avoid detection and increase survival, as it increases the capture risk of the prey.
- Capture likelihood per unit time is modeled as because the more active the prey is, the more exposed it is (conspicuousness, noise, movement); hence it is justified to assume that prey activity contributes nonlinearly here. Hence x^2 term has been added here, and overall nonlinear capture likelihood term is given as Ex^2y.
- However, the flip side of the coin is that prey become more active when chased by the predator, consequently, they run faster to escape. In that case, the constant E will be negative to reverse the sign of Ex^2y so that the activities of the prey x will be increased rapidly if it is chased by the predators.
- Predators increase/decrease their activity to match the activity level of the prey. However, if a predator feels full, it shows no activity $y = 0 \Rightarrow y' = 0$.
- Prey that is silent or motionless often avoids detection, which is why many prey species freeze when sensing a predator. Predators, in turn, evolve heightened senses (e.g., acute hearing or night vision) and patience to detect and silently approach such prey. Conversely, active prey generates auditory, visual, or olfactory cues, prompting predators to use those to locate and chase them and thereby increasing their activities according to the prey.
- We also assume that the activities of animals are highly influenced by environmental conditions. Hence, stochastic perturbations of the predator and prey activities around their values are of white noise type, which are proportional to the distances of x, y from equilibrium activity (steady state activities) values x^*, y^* respectively.

2.1 Biological Meaning and Feasible Range of the Parameters

The several parameters, which are used in the model (1-2), are biologically justified and have reasonable ecological meaning. Environment also decides a fixed range of every parameter. MET (Metabolic Equivalent) is a unit used to measure the energy cost of physical activities of mammals. Hence, all units and descriptions are given in **Table 1**. In general, the biological meaning of every parameter and their feasible range can be given as follows:

- $\boldsymbol{x_0}$ is the essential level of prey activity when there are no predators. It depicts both the natural behaviour of prey and the diversity of the surrounding environment. In addition to reflecting the diversity and resource availability of the surrounding environment, it records the inherent behavioural inclination of the prey, which is influenced by elements like locomotion, social interactions, and foraging. Therefore, x_0 symbolizes the combined impact of natural prey behavior and environmental richness in the absence of predators. Since $\boldsymbol{x_0}$ is the essential level of prey activity, therefore its values depend on the prey species. Some prey species are highly active (like rabbit), hence $\boldsymbol{x_0}$ value will be more for them, conversely, some prey are lazy due to their long sleep hours or energy-conserving lifestyles (like Koalas), consequently, $\boldsymbol{x_0}$ value will be smaller for them. By and large, $0 < x_0 < 5$ can be taken as moderate feasible range for this parameter.
- $\boldsymbol{\lambda}$ is the rate of increment of prey activity toward its natural activity level. It actually measures how rapidly prey activity increases toward its intrinsic or required activity level x_0 in the absence of predation pressure. In terms of biology, this phrase describes the prey's natural desire to hunt, investigate, or make use of resources, as well as recuperation of activity following a break or disruption or the behavioural drive to sustain a baseline level of energy or survival. Undoubtedly, λ should be positive; however, a large λ value means $x(t)$ converges to x_0 instantaneously, which is not looking realistic; hence feasible range of this parameter would be $0 < \lambda \leq 1$.

- **E** is the prey capturing likelihood score (PCLS). This biologically measures the impact of prey activity on predator-prey interactions. It's also ecologically vindicated that as the distance between a predator and its prey rises, the prey capture likelihood score (PCLS) of the predator successfully capturing prey usually falls and turns negative. P, being probability of capturing the prey, remains between 0 and 1, and therefore, PCLS E will be positive for $P > 0.5$, that is, higher probability and negative for $P < 0.5$, that is, lower probability of capturing the prey.
- $\boldsymbol{\sigma_1}$ **and** $\boldsymbol{\sigma_2}$ are the intensity of environmental noises, which is influenced by sporadic disruptions such as weather variations, habitat variability, and human impacts. Recall that environmental noise represents climate variability, habitat disturbance, and random encounters; hence, these effects are typically moderate perturbations, not dominant forces. $\boldsymbol{\sigma_i > 1}$ implies that random fluctuations would overwhelm biological regulation, which is an unrealistic situation. Hence, the ecologically feasible range must be $\boldsymbol{0 < \sigma_i \leq 1}$.
- $\boldsymbol{\mu}$ is the rate of predator's adjusting capacity according to the activities of the prey. It gauges how well predators modify their activity levels in reaction to shifts in their prey's activity. When prey activity increases, this includes behavioural adjustments like heightened pursuit, alertness, or hunting effort. μ, being the predator's adjusting capacity, should remain within the moderate range $(\mathbf{0}, \mathbf{1}]$ as higher values of μ mean that predator reacts to the prey's activities too rapidly, which seems a little bit fanciful.

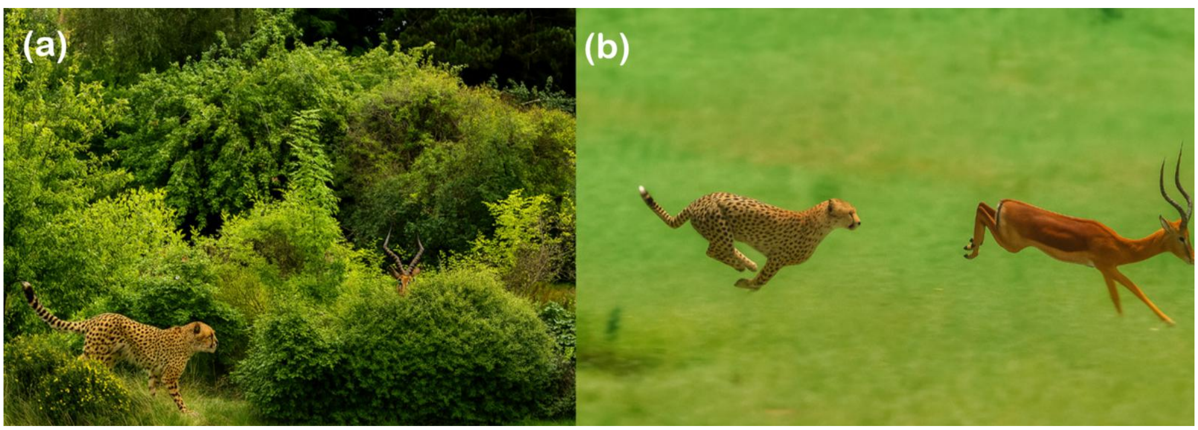

Figure 1. In **(a)**, prey is not showing any activity to avoid predator and the predator smartly approaches the prey to show that the prey is not identified, while in **(b)**, prey is showing tremendous activity in the form of running to escape from predators.

Table 1. Parameters and variables used in the system with their description and units.

Variable / Parameter	Biological/Ecological meaning	Unit
x	Prey activities: Behavioral responsiveness, daily routine activities to search food etc.	MET per individual
y	Predator activities especially hunting effort	MET per individual
dx/dt	Rate of change in prey activities	MET per individual per day
dy/dt	Rate of change in predator activities	MET per individual per day
λ	Rate of increment of prey activities to obtain natural activity level	Per day
x_0	Natural activity level	MET per individual
E = ln(P/(1 - P))	Prey capturing likelihood score (PCLS), P being probability of capturing the prey	Unitless
μ	Rate of predator's adjusting capacity according to the activities of prey	Per day
σ_1	Environmental fluctuations impact on prey activities	Per day
σ_2	Environmental fluctuations impact on predator activities	Per day

3. Analysis of the Deterministic Model without Stochastic

The model equations without stochastic perturbations will take the simplified form which can be written as follows:

$$\frac{dx}{dt} = \lambda(x_0 - x(t)) - Ex^2(t)y(t) \tag{3}$$

$$\frac{dy}{dt} = \mu y(t)\big(x(t) - y(t)\big) \tag{4}$$

Now, we discuss the positivity, boundedness, uniform persistence, local stability, and global stability for this model (3-4).

3.1 Positivity and Boundedness

Theorem 1. Any solution of the mathematical model (3-4) starting in R_+^2, stay in the positive quadrant of R^2 for all $t \geq 0$.

Proof. Assume $x(t) = 0$ for some $t_1 > 0$, therefore, we assume that $x(t)$ becomes 0 at some time $t_1 > 0$, despite starting from $x(0) > 0$. From Equation (3)

$$\left.\frac{dx}{dt}\right|_{x=0} = \lambda x_0 - 0 = \lambda x_0 > 0.$$

That means the slope at $x = 0$ is positive, so the trajectory can not reach or cross $x = 0$. This contradicts the assumption that $x(t_1) = 0$. Therefore $x(t) > 0$ for all $t \geq 0$. Again, from Equation (4)

$$\frac{1}{y}\frac{dy}{dt} = \mu(x - y),$$

$$\Rightarrow \ln\, y(t) = \ln y(0) + \mu \int_0^t \big(x(s) - y(s)\big)ds,$$

$$\Rightarrow y(t) = y(0)e^{w} > 0 \;\text{ where }\; w = \mu \int_0^t \big(x(s) - y(s)\big)ds.$$

Therefore, $y(t) > 0$ for all $t \geq 0$.

Theorem 2. All the solution of the system (3-4), which start in R_+^2 are ultimately bounded for all time $t \geq 0$ provided $E > 0$.

Proof. From Equation (3)

$$\frac{dx}{dt} = \lambda(x_0 - x) - Ex^2y.$$

Let's derive an upper bound. Since $x, y > 0$, we know $-Ex^2y \leq 0$, so

$$\frac{dx}{dt} \leq \lambda(x_0 - x) \tag{5}$$

Now consider the auxiliary equation

$$\frac{dX}{dt} = \lambda(x_0 - X) \Rightarrow X(t) = x_0 + (X(0) - x_0)e^{-\lambda t}.$$

This is a standard linear ODE and its solution tends to x_0 as $t \to \infty$. Since $\frac{dx}{dt} \leq \frac{dX}{dt}$ and comparison theorem tells us

$$x(t) \leq X(t) \Rightarrow x(t) \leq \max(x_0, x(0)) \;\forall\; t \geq 0.$$

So, $x(t)$ is bounded above. Also, from, positivity earlier $x(t) > 0$, so $x(t) \in (0, M]$ for some constant $M > 0$. From Equation (4) is

$$\frac{dy}{dt} = \mu y(x - y).$$

Suppose $x(t) \leq M$, as just shown, then

$$\frac{dy}{dt} \leq \mu y(M - y).$$

After applying the comparison principle, we get

$$\limsup_{t\to\infty} y \leq \frac{\mu M}{\mu} = M \ \Rightarrow \ y_{\max} = M.$$

Hence the result.

Remark 1. Note that the foundation of a mathematically sound and ecologically significant model is made up of positivity, which ensures the system's biological viability, and boundedness, which maintains ecological realism by preventing the unbounded development of the variables.

3.2 Uniform Persistence

Uniform persistence in mathematical biology refers to the long-term maintenance of each species' population above a strictly positive lower bound, independent of initial conditions.

Definition 1. If there exist positive constants m_x, M_x, m_y and M_y such that each solution of the system (3-4) satisfies

(i) $\liminf_{t\to\infty} x(t) \geq m_x > 0, \liminf_{t\to\infty} \ y(t) \geq m_y > 0$, and

(ii) $0 < \limsup_{t\to\infty} \ x(t) \leq M_x, 0 < \limsup_{t\to\infty} y(t) \leq M_y$.

then the system (3-4) is uniformly permanent (Xu et al., 2004).

Remark 2. Using Theorem 2, we can say that there exists a compact set $K = \left[0, X_{\text{upper}}\right] \times \left[0, Y_{\text{upper}}\right] = [0, M] \times [0, M]$ such that all solutions eventually enter and remain within K regardless of initial conditions. This demonstrates that the system is uniformly bounded. Thus, we can say that the system is dissipative, that is, all solutions eventually enter and remain in a bounded region in the $x - y$ plane.

Theorem 3. For positive E, the system (3-4) is uniformly persistent.

Proof. The system (3-4) has only one boundary steady state $P_1 = (x_0, 0)$ for $(x, y) \in \omega(S)$, the omega limit set of boundaries S, if $y = 0$, from the Equation (3), it follows that $x_{(t)} \to x_0$ as $t \to \infty$. Thus, the omega limit set of S consists of $P_1 = (x_0, 0)$. Now, choose the function

$$P(x, y) = x^{\alpha_1} y^{\alpha_2},$$

where, α_1, α_2 are positive constants to be determined later. Define

$$\rho(t) = \alpha_1 \left[\lambda \left(\frac{x_0}{x} - 1\right) - Exy\right] + \alpha_2[\mu(x - y)].$$

Now using Equation (1)

$$\begin{aligned}\dot{P}(x,y) &= \alpha_1 x^{\alpha_1-1}y^{\alpha_2}\dot{x} + \alpha_2 x^{\alpha_1}y^{\alpha_2-1}\dot{y}\\ &= \alpha_1[x^{\alpha_1}y^{\alpha_2}]\frac{\dot{x}}{x} + \alpha_2[x^{\alpha_1}y^{\alpha_2}]\frac{\dot{y}}{y}\\ &= P(x,y)\left[\alpha_1\frac{\dot{x}}{x} + \alpha_2\frac{\dot{y}}{y}\right]\\ &= P(x,y)\left[\alpha_1\left(\lambda\left(\frac{x_0}{x}-1\right) - Exy\right) + \alpha_2(\mu(x-y))\right]\\ &= P(x,y)\rho(t).\end{aligned}$$

At the equilibria P_1, the second term of $\rho(t)$ is $\mu x_0 > 0$. Hence, we can always find a value α_2 such that $\rho(t) > 0$, hence the result follows from the Theorem 4.1 of Freedman & Ruan (1995).

3.3 Stability Analysis

The system (3-4) has 2 equilibrium points
(1) Zero predator activity steady state $P_1 = (x_0, 0)$. (2) Positive steady state $P^* = (x^*, x^*)$, where x^* is the positive solution of the equation, which is obtained using Equation (3) (The detailed description can be found in Appendix).

$$Ex^3 + \lambda x - \lambda x_0 = 0.$$

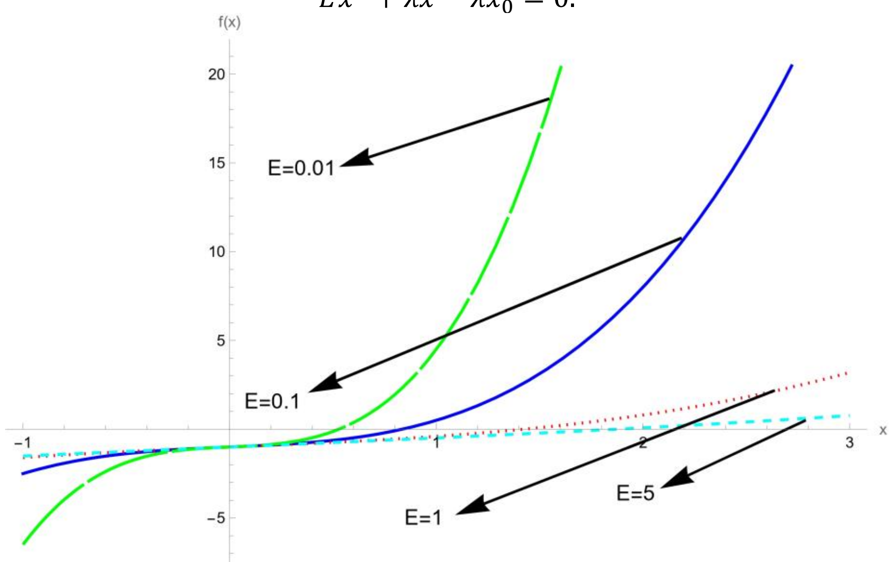

Figure 2. We plot different graphs of *f*(*x*) for multiple *E* values and keeping the rest values as λ = 0.5, x_0 = 2. We see that every graph cuts the positive x axis only once which indicates unique positive equilibrium point for E > 0.

Theorem 4. The positive steady state for the system (3-4) is unique for $E > 0$.
Proof. We define the function as follows:

$$f(x) = Ex^3 + \lambda x - \lambda x_0.$$

We analyze the behavior of $f(x)$. We have, $\lim_{x\to-\infty} f(x) = -\infty$, since $x^3 \to -\infty$ dominates.

Similarly, $\lim_{x \to +\infty} f(x) = \infty$, so the function goes from $-\infty$ to ∞, which guarantees at least one real root. Now, we can have the following derivative

$$f'(x) = 3Ex^2 + \lambda.$$

since $E, \lambda > 0$, we have $f'(x) > 0 \; \forall \; x$, which clearly states that $f(x)$ will be strictly increasing and therefore, it can be concluded that function will be strictly increasing as well as continuous which moves from $-\infty$ to ∞, hence it pass through the x axis only once. Now, $f(0) = -\lambda x_0 < 0$, and $f(x) \to \infty$ as $x \to \infty$. Hence, this root will be positive. Consequently, there will be unique positive steady state which can be graphically verified using **Figure 2**.

Theorem 5. Zero predator activity steady state is unstable.
Proof. Consider the following function

$$V(x, y) = y.$$

We now define the region (for sufficiently small $\varepsilon > 0$)

$$Q = \{(x, y) \in \mathbb{R}^2 : x > 0, y > 0, (x - x_0)^2 + y^2 < \varepsilon\}.$$

It can be easily noted that $V(x_0, 0) = 0$, $V(x, y) > 0$ for all $(x, y) \in Q$. The zero-predator activity steady state lies on the boundary of Q. We now find derivative $\dot{V}$

$$\dot{V} = \frac{d}{dt}(y) = \dot{y}.$$

We substitute the system equations and obtain

$$\dot{V} \;=\; \mu y(x - y).$$

Note that in the region Q, $x \approx x_0, y > 0$ as well as $x_0 \gg y$. Thus $x - y > 0$ and consequently, $\dot{V} > 0$ for all $(x, y) \in Q$. Therefore, all the conditions of Chetaev's instability theorem (Haddad & Chellaboina, 2024) are satisfied which proves that zero predator activity steady state is Lyapunov unstable.

Remark 3. Chetaev's instability theorem provides a sufficient condition for instability of an equilibrium point of a dynamical system (Kirillov, 2021).

Theorem 6. Positive equilibrium point $P^* = (x^*, y^*)$ is locally asymptotically stable for $E > 0$.
Proof. Jacobian matrix at the steady state $P^* = (x^*, y^*) = (x^*, x^*)$ is given as follows

$$J(x^*, y^*) = \begin{bmatrix} -\lambda - 2Ex^{*2} & -Ex^{*2} \\ \mu x^* & -\mu x^* \end{bmatrix}.$$

By Routh–Hurwitz criterion, we know that all eigenvalues have negative real parts, that is, the equilibrium is asymptotically stable if and only if $tr(J) < 0$ and $\det(J) > 0$. We find

$$tr(J) = -\lambda - 2Ex^{*2} - \mu x^* < 0,$$
$$\det(J) = \lambda\mu x^* + 3E\mu x^{*3} > 0.$$

Therefore, by the Routh–Hurwitz criterion, the equilibrium of the two-dimensional system is locally asymptotically stable with the condition $E > 0$.

For the local and global stability of the positive equilibria, we state the following theorems assuming $E > 0$. The global stability is executed using the geometric approach given in Li & Muldowney (1995). In this approach, the system is represented in vector form $\dot{X} = f(X)$, and global stability is found in a positively invariant permanence region. The variational matrix $V(X) = \partial f/\partial X$ is formed, and its second additive compound matrix $V^{[2]}(X)$ is constructed to provide the analysis of the global behavior of area elements along trajectories. A suitable diagonal matrix $M(X)$ is introduced to scale the variational system, leading to the matrix $B = M_f M^{-1} + MV^{[2]}M^{-1}$. Using an appropriate vector norm, the Lozinski measure $\Gamma(B)$ is estimated and illustrated to have a uniform upper bound. Permanence confirms boundedness of state variables, permitting this bound to be presented completely in terms of parameters. If the time-averaged Lozinski measure satisfies $\limsup_{t\to\infty} \frac{1}{t}\int_0^t \Gamma(B)ds < 0$, then all solutions contract geometrically toward the positive equilibrium, implying global asymptotic stability within the feasible region. On the basis of the geometric approach, Theorem 7 can be stated as follows:

Theorem 7. The positive equilibrium point $P^* = (x^*, y^*)$ is globally asymptotically stable.
Proof. Now, motive of the proof is to illustrate that the unique positive equilibrium (x^*, x^*) of the system (3-4) is globally asymptotically stable after applying the geometric approach. Let us rewrite the system in the vector form

$$\frac{dX}{dt} = F(X), X = \begin{bmatrix} x \\ y \end{bmatrix},$$

where, we have

$$F(X) = \begin{bmatrix} f_1(x,y) \\ f_2(x,y) \end{bmatrix} = \begin{bmatrix} \lambda(x_0 - x) - Ex^2y \\ \mu y(x-y) \end{bmatrix}.$$

For a 2×2 system, the second additive compound matrix $J^{[2]}$ is the trace of J is as follows

$$J^{[2]} = \mathrm{tr}(J) = -\lambda - 2Exy + \mu(x - 2y).$$

We define

$$M(x,y) = \mathrm{diag}(x,y), M^{-1} = \mathrm{diag}\left(\frac{1}{x}, \frac{1}{y}\right),$$
$$M' = \mathrm{diag}\left(\frac{dx}{dt}, \frac{dy}{dt}\right), M'M^{-1} = \mathrm{diag}\left(\frac{\dot{x}}{x}, \frac{\dot{y}}{y}\right).$$

Since $J^{[2]}$ is scalar and M is diagonal, the Lozinskii measure is as below

$$\Gamma(B) = \max\left\{\frac{\dot{x}}{x} + J^{[2]}, \frac{\dot{y}}{y} + J^{[2]}\right\}.$$

From the system (3-4), we have

$$\frac{\dot{x}}{x} = \frac{\lambda(x_0 - x)}{x} - Exy, \frac{\dot{y}}{y} = \mu(x - y),$$
$$J^{[2]} = -\lambda - 2Exy + \mu(x - 2y).$$

Thus, we can say

$$\Gamma(B) \le \max\left\{\frac{\lambda(x_0 - x)}{x} - Exy + J^{[2]}, \mu(x-y) + J^{[2]}\right\}.$$

At equilibrium $x = y = x^*$, we obtain

$$J^{[2]} = -\lambda - 2E(x^*)^2 + \mu(x^* - 2x^*) = -\lambda - 2E(x^*)^2 - \mu x^* < 0.$$

Since $\frac{\dot{x}}{x} \to 0, \frac{\dot{y}}{y} \to 0$ near equilibrium, and $J^{[2]} < 0$, hence, we get

$$\Gamma(B) < 0 \text{ eventually.}$$

Hence, we can say that

$$\limsup_{t\to\infty} \frac{1}{t}\int_0^t \Gamma(B(s))ds < 0.$$

By the geometric stability theorem, Lozinskii measure $\Gamma(B) < 0$ eventually, and hence, the equilibrium (x^*, x^*) is globally asymptotically stable.

Remark 4. Every admissible trajectory converges to the positive (coexistence) equilibrium since it has been demonstrated to be globally asymptotically stable in the physiologically feasible domain. In the interior of the phase space, nontrivial invariant sets like closed orbits or limit cycles cannot exist due to global stability. By definition, a Hopf bifurcation occurs when an equilibrium loses its local stability due to two complex conjugate eigenvalues that cross the imaginary axis. This causes sustained periodic solutions to develop close to the equilibrium. As a result, any observed oscillations only reflect transitory dynamics rather than permanent periodic behaviour, and the established global stability of the positive steady state eliminates the possibility of a Hopf bifurcation in the parameter range under consideration.

4. Analysis of the Stochastic Model: Stochastic Stability

The system (1-2) can be rewritten in the form as

$$dY(t) = \psi\big(t, Y(t)\big)dt + \phi\big(t, Y(t)\big)dB(t) \tag{6}$$

where, $Y(t) = Y_0, t \in [t_0, t_\psi]$. Here $Y(t)$ is a solution which is an Ito process, ψ being the drift coefficient, ϕ being the diffusion coefficient. Wiener process components which increments $\Delta B_j(t) = B_j(t + t) - B_j(t),\ j = 1, 2, 3$ are independent Gaussian random variables $N(0, \Delta t)$. Different components of Equation (6) are written as follows: We write the system in vector form:

$$Y(t) = \begin{bmatrix} x \\ y \end{bmatrix}, B(t) = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \psi(t, Y(t)) = \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix},$$

where, $\psi_1 = \lambda(x_0 - x) - Ex^2y, \psi_2 = \mu y(x - y)$

$$\phi(t) = \begin{bmatrix} \sigma_1(x - x^*) & 0 \\ 0 & \sigma_2(y - y^*) \end{bmatrix}.$$

Stochastic integral equation corresponding to Equation (6)

$$Y(t) = Y_0 + \int_{t_0}^{t} \psi\big(s, Y(s)\big)ds + \int_{t_0}^{t} \phi\big(s, Y(s)\big)dB(s) \tag{7}$$

where, the first integral is a deterministic Riemann-Stieltjes integral, while the second integral is a stochastic integral. The stochastic system (1-2) can be centered at P^* after changing the following variable $u_1 = x - x^*, u_2 = y - y^*$. After linearisation, stochastic differential equations around equilibrium point E take the form

$$du(t) = \psi\big(u(t)\big)dt + \phi\big(u(t)\big)dB(t)dt \tag{8}$$

where, $u(t) = \text{col}(u_1(t), u_2(t), u_3(t)$ and

$$\psi(u(t)) = \begin{bmatrix} (-\lambda - 2E(x^*)^2)u_1 - E(x^*)^2u_2 \\ \mu x^* u_1 - \mu x^* u_2 \end{bmatrix}$$

and

$$\phi\big(u(t)\big) = \begin{bmatrix} \sigma_1 u_1 & 0 \\ 0 & \sigma_2 u_2 \end{bmatrix}.$$

Theorem 8. Suppose there exists a function $V(t,U) \in C_2^0(\eta)$ satisfying the inequalities

$$K_1|U|^p \leq V(t,U) \leq K_2|U|^p,$$

$$LV(t,U) \leq -K_3|U|^p \tag{9}$$

where, $K_i(i = 1{,}2) > 0, p > 0$ are some suitable constants. Then the trivial solution of the system (1-2) is exponentially p stable for $t \geq 0$. If $p = 2$, then the trivial solution of system (1-2) is exponentially mean square stable. With reference to system (1-2) the expression for $LV(t,U)$ is defined by

$$LV(t,U) = \frac{\delta V(t,U)}{\delta t} + \psi^t\big(U(t)\big)\frac{\delta V(t,U)}{\delta U} + \frac{1}{2}\operatorname{Tr}\left[\phi^T\left(U(t)\frac{\delta^2 V(t,U)}{\delta^2 U} g(U(t)\right]\right. \tag{10}$$

where, $\frac{\delta V}{\delta u} = \operatorname{col}\left[\frac{\delta V}{\delta u_1}, \frac{\delta V}{\delta u_2}, \frac{\delta V}{\delta u_3}\right]$.

Now, we can state the following theorem:

Theorem 9. The zero solution of the system (1-2) is asymptotically mean square stable if

$$\sigma_1^2 < 2(\lambda + 2E(x^*)^2), \sigma_2^2 < 2\mu x^*,$$

provided $E > 0$.

Proof. We consider the Lyapunov function

$$V = \frac{1}{2}[\omega_1^2 u_1^2 + \omega_2^2 u_2^2] \tag{11}$$

where, $\omega_i, (i = 1{,}2{,}3)$ are real positive constants to be determined later. For Lyapunov function Equation (11), we have

$$\frac{\partial V(t,U)}{\partial U} = \begin{pmatrix} \omega_1 u_1 \\ \omega_2 u_1 \end{pmatrix}, \frac{\partial^2 V(t,U)}{\partial U^2} = \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix} \tag{12}$$

Hence, we get

$$\phi^T(U)\frac{\partial^2 V}{\partial U^2}\phi(U) = \begin{pmatrix} \omega_1 \sigma_1^2 u_1^2 & 0 \\ 0 & \omega_2 \sigma_2^2 u_2^2 \end{pmatrix} \tag{13}$$

with

$$\frac{1}{2}\operatorname{Tr}\left[\phi^T(U)\frac{\partial^2 V}{\partial U^2}\phi(U)\right] = \frac{1}{2}[\omega_1 \sigma_1^2 u_1^2 + \omega_2 \sigma_2^2 u_2^2] \tag{14}$$

Now, using Equations (12), (14), and (8), we get

$$\begin{aligned} LV(t,U) = {} & u_1\omega_1\big((-\lambda - 2Ex^{*2})u_1 - Ex^{*2}u_2\big) + u_2\omega_2(\mu x^* u_1 - \mu x^* u_2) \\ & + \frac{1}{2}(\omega_1 \sigma_1^2 u_1^2 + \omega_2 \sigma_2^2 u_2^2) \end{aligned} \tag{15}$$

In Equation (15), choose $\omega_1 = 1, \omega_2 = \frac{E(x^*)^2}{\mu x^*} = \frac{Ex^*}{\mu}$, then equation reduces to

$$LV(U,t) = -\left(\lambda + 2E(x^*)^2 - \frac{\sigma_1^2}{2}\right)u_1^2 - \left(\left(E(x^*)^2 - \left(\frac{Ex^*}{\mu}\right)\frac{\sigma_2^2}{2}\right)u_2^2,\right.$$
$$= -U^T P U.$$

where, matrix P is given as follows

$$P = \begin{pmatrix} \lambda + 2E(x^*)^2 - \frac{\sigma_1^2}{2} & 0 \\ 0 & Ex^{*2} - \frac{Ex^*}{2\mu}\sigma_2^2 \end{pmatrix}.$$

This square matrix is real symmetric positive matrix. The eigenvalues of this matrix will be positive when

$$\lambda_1 = \lambda + 2E(x^*)^2 - \frac{\sigma_1^2}{2} > 0,$$
$$\lambda_2 = E(x^*)^2 - \frac{Ex^*}{2\mu}\sigma_2^2 > 0,$$

then from Equation (15)

$$LV(U,t) \le -\lambda_m |U(t)|^2.$$

Hence according to the Theorem 8, the proof is completed.

Remark 5. The concept of a dynamical system's stability in the face of random disturbances, or noise, is discussed in the form of p-square stability in Theorem 9. Stochastic stability examines how solutions behave in probability or on average under unpredictability, in contrast to deterministic stability, where paths are set by initial circumstances.

5. Numerical Simulation

Numerical values used in the simulations are presented in **Table 2**. We have shown the numerical simulations for the deterministic model and for the stochastic model to capture the numerical aspects.

5.1 Numerical Dynamics of Deterministic Model

First, we study the numerical simulation for the deterministic model (3-4), taking the numerical values of the parameters given in **Table 2**. **Figure 3(A)** and **Figure 3(B)** represent the time plot and parametric plotting respectively for Prey Capturing Likelihood Score (PCLS) E value as 0.9. We see that predator-prey activities rise initially and then stabilize. **Figure 3(C)** and **Figure 3(D)** represent the scenario for smaller Capturing Likelihood Score $E = 0.01$. System again approaches a stable equilibrium but this time steady state values are comparatively higher. The decreasing capturing likelihood score shows that prey survive from the predators with comparatively high activity level. **Figure 3(E)** and **Figure 3(F)** represent the case where PCLS is $E = -0.1$. We see that activities of prey and predator skyrocket this time, indicating that the prey have been detected by the predators, they outrun the predators, and the predators try to follow them creating a hotchpotch in the habitat. We see the bifurcation diagram between prey activity equilibrium and parameter E is given in **Figure 4**. At the $E = 0$, both equilibriums have a collision point, however, stability doesn't switch. For $E < -1/27$, there is no positive equilibrium point; consequently, the system will be unbounded and unstable. Hence the system has a bifurcation point at $E = -1/27$.

5.2 Numerical Dynamics of Stochastic Model

In **Figures 5(A)-(C)**, we have shown the stochastic time plots for the values in **Table 2**. **Figure 5(A)** shows the stochastic stability for the dynamics for $E = 0.9$, while **Figure 5(B)** and **Figure 5(C)** show the dynamics for $E = 0.01$ and $E = -0.1$. Since the deterministic system is strongly stable, then even with noise, the trajectory stays near the deterministic path and stochastic fluctuations are not accumulate enough to cause visible deviation. **Figure 6(A)** and **Figure 6(B)** show the histograms corresponding to $E = 0.9$, which reflect the frequency of values taken by the variables over time. Since the plot shows both variables stabilizing near 1, it's natural the histograms peak near 1 with some early values causing lower bars. Similarly, for $E = 0.01$, time plots stabilize near 2, and same thing is reflected in the corresponding histogram. All the graphical outputs reported in this study were produced using MAHEMATICA.

6. Discussion

The deterministic and stochastic aspects of predator-prey activity tuned dynamics, including positivity and boundedness, uniform persistence, stability analysis, stochastic stability (p-stability), and numerical simulations are examined in this paper. Here, in this section, we first discuss the model validation and then provide a comprehensive ecological interpretation to illustrate how mathematical findings of the paper relate to the real ecosystem.

6.1 Model Validation

The proposed stochastic activity-based predator–prey model is validated through theoretical consistency, ecological plausibility, and dynamical coherence. Theorem 1 and 2 ensures positivity and boundedness of the model which is the foremost requirement for any biological model. It can be easily noted that environmental noise effect terms $\sigma_1(x(t) - x^*)d\xi_1$ and $\sigma_2(x(t) - x^*)d\xi_2$ will disappear at the positive equilibrium, which means that model respects biological feasibility under environmental fluctuations as well. It is obvious that in absence of predator-activities, that is, $y = 0$, using Equation (3), we get

$$\frac{dx}{dt} = \lambda(x_0 - x).$$

This suggests that prey activity is approaching its natural baseline which is consistent with ecological intuition. Now for low prey activity, that is, $x \ll y$ using Equation (4), we obtain

$$\frac{dy}{dt} < 0.$$

In keeping with adaptive predator behavior, it shows a decrease in predator activity as a result of inadequate energetic return. Hence, the proposed system reproduces adequate ecological behaviors in limiting cases. A balanced activity state is equivalent to the positive steady state, where prey activity is in line with energy needs and predator activity corresponds with the availability of prey. Using Chetaev's instability criterion, the trivial equilibrium's instability demonstrates that ecologically, zero-activity states are impractical.

Predator engagement is invariably triggered by minor prey activity. Therefore, ecological inevitability and mathematical instability are compatible in the model. Instead of acting as a driving force, ambient noise serves as a regulating mechanism. Increased noise mimics genuine ecosystems under environmental stress by broadening oscillations without destroying persistence. Hence, the proposed model is validated as it fully maintains biological viability, recovers limiting cases with clear ecological significance, preserves consistency between ecological predictions and stability results, and exhibits a realistic response to environmental noise.

6.2 Ecological Interpretation

In the model, positivity ensures that activity for any species cannot be negative at any time. The boundedness of the activity level in this case takes place only when prey capturing likelihood score (PCLS) is positive. However, when this rate is negative, prey activity increases by leaps and bounds. Indeed, the prey capturing likelihood score (PCLS) plays a double role in the model. It can decrease the prey activities when its positive, while increase prey activities when its negative. This has a deep biological meaning. The capture likelihood will always be high when prey is in the vicinity of predators. In that case, predators always smartly but slowly approach the prey so that prey could not see predators and could easily be hunted in the blink of an eye.

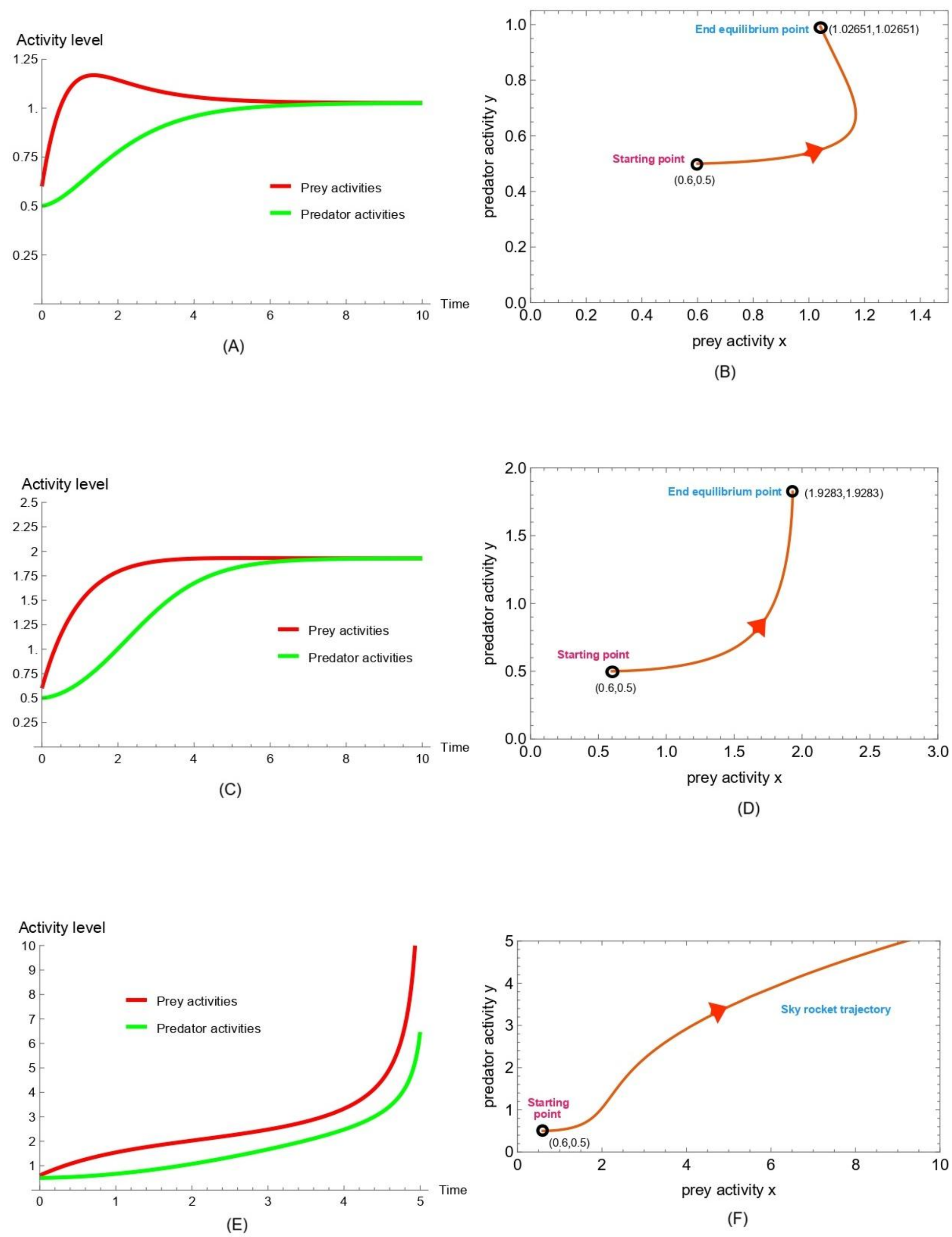

Figure 3. In A, C and D, we have shown the time plots for values in **Table 2** for the specific values of E as 0.9, 0.01 and −0.1. The corresponding parametric plots are given in B, D and F respectively. In A and C, trajectories are approaching equilibrium positions while in E, trajectory is skyrocketing. A time plot represents the evolution of ecological variables with respect to time, while the parametric plots show one ecological variable against another.

Similarly, sometimes prey also slow down their activities when they smell the predators in their neighborhood. Hence, high prey capturing likelihood score (PCLS) always reduces the activities of the prey, and hence of predators as well. However, the case will be reversed when capture likelihood is negative and prey is not easily vulnerable for predators or not within the ace of them. Consequently, the prey here identifies the predators from a while and start running, which indeed increases their activities level without any bounds.

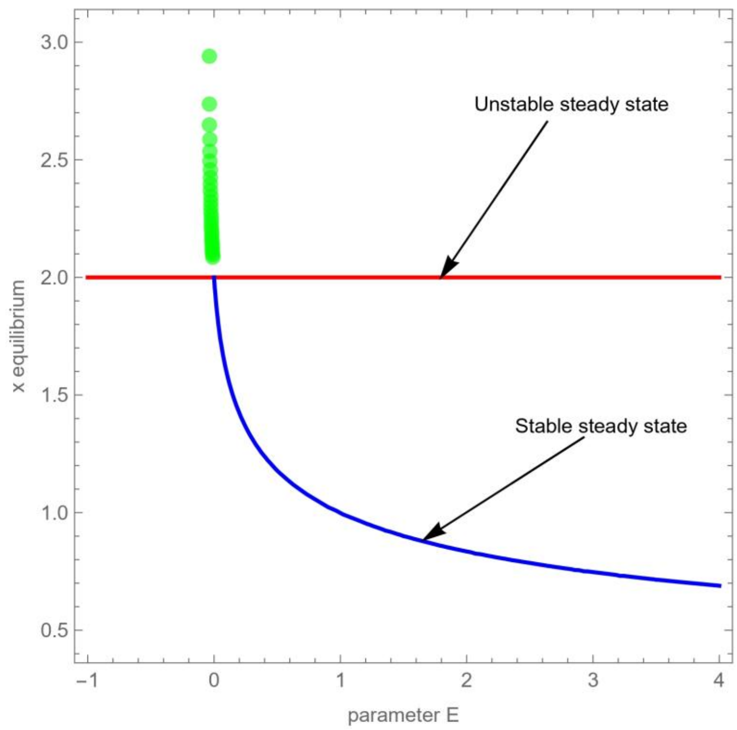

Figure 4. Bifurcation diagram for the prey equilibrium and parameter E. Blue curve represents stable steady state while red curve is for unstable steady state. The intersection point of blue and red curves gives the occurrence of bifurcation point which takes place at $E = -1/27$.

Unlike the usual predator-prey systems, where predator-free equilibrium is typically stable/unstable under certain conditions, here, in this model, the zero-predator activity steady state will always be locally unstable. This is biologically reasonable as well because inactivity of the predators can occur for some time after having the meal, but this will not remain in that stage for long consequently, this steady state has to be unstable. On other hand, the positive steady state, is not only locally stable but also it is globally asymptotically stable. Recall that predators always follow the prey in activity patterns for the survival; consequently, equilibrium values x and y values will remain the same. Stochastic stability positive steady state also follows in Section 3. Theorem 8 biologically means that if the environmental noises/fluctuations σ_1^2 and σ_2^2 are lesser than the threshold values $2(\lambda + 2\mathrm{E}(x^*)^2$ and $2\mu x^*$ respectively, then system is said to be stochastically stable. Thus, we can say that the system can tolerate small noise and still remain stochastically stable. Indeed, predator and prey behaviors are fundamentally regulated by metabolic demands and the necessity to secure food. Therefore, small stochastic fluctuations in the environment are unlikely to significantly alter their core activity patterns, however, some tremendous stochastic fluctuations, like floods, storms, unusual heat waves, can destroy the stability which is justified undoubtedly.

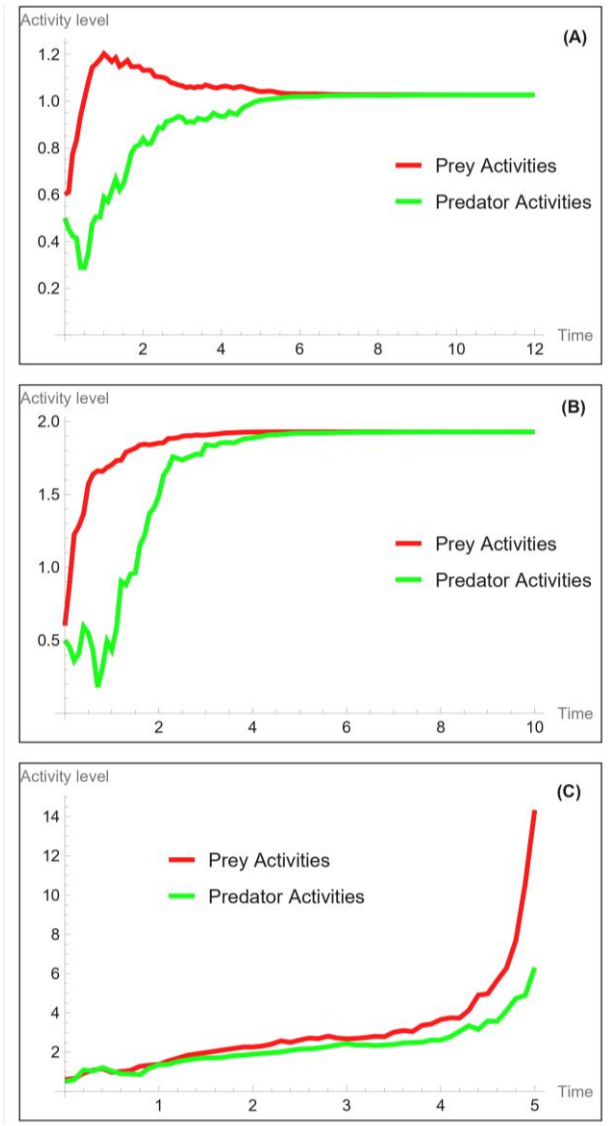

Figure 5. In (A)-(C), we have shown the stochastic time plots for values in **Table 2** for the specific values of E as 0.9,0.01 and -0.1.

We have also captured the system's fascinating dynamics through numerical simulations for deterministic as well as stochastic aspects. In each time plot, predator activity consistently remains lower than that of the prey, which is ecologically justified. This reflects a fundamental natural principle that predators depend on prey for survival and thereby adapt their behavior in response to prey activities. In **Figure 3(A)**, we obtain the stable equilibrium, whose intensity increases in **Figure 3(C)** as value of E decreases. From an ecological perspective, higher activity levels enable prey to stay ahead of predators both spatially and temporally. Thus, chances for capturing prey will be lesser this time. Prey can often detect the presence of predators from a distance, through smell, sound, or visual cues and accordingly adjust their behavior. This early detection allows them to modify their activity patterns; they can shift to another habitat. As a result, the likelihood of being captured significantly decreases.

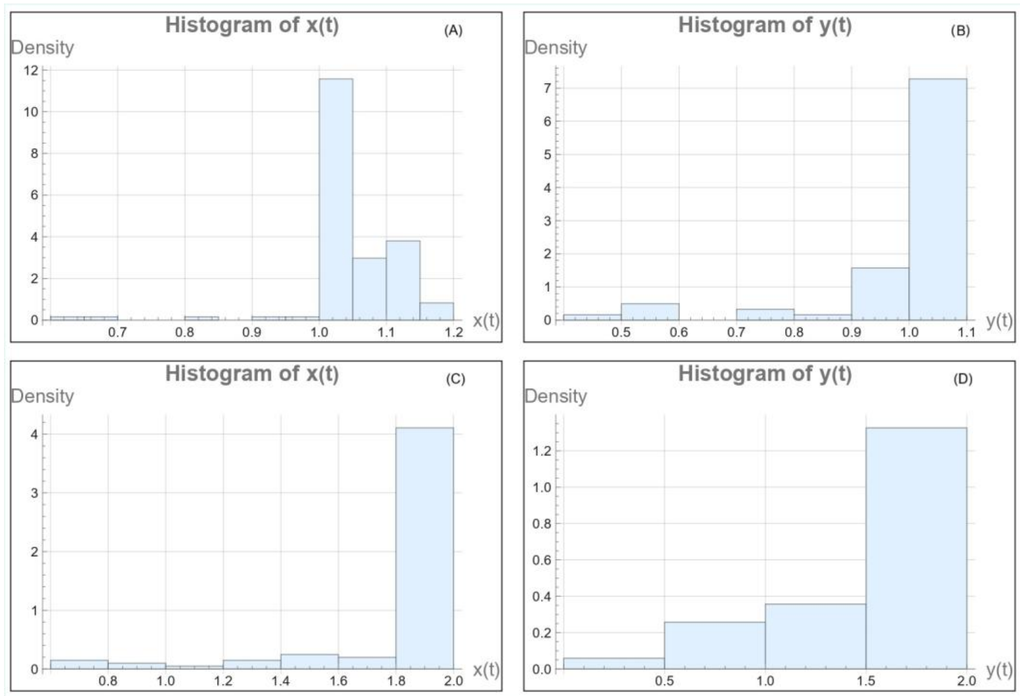

Figure 6. Histogram shown for the values in **Table 2** for the specific values of E as 0.9 and 0.01.

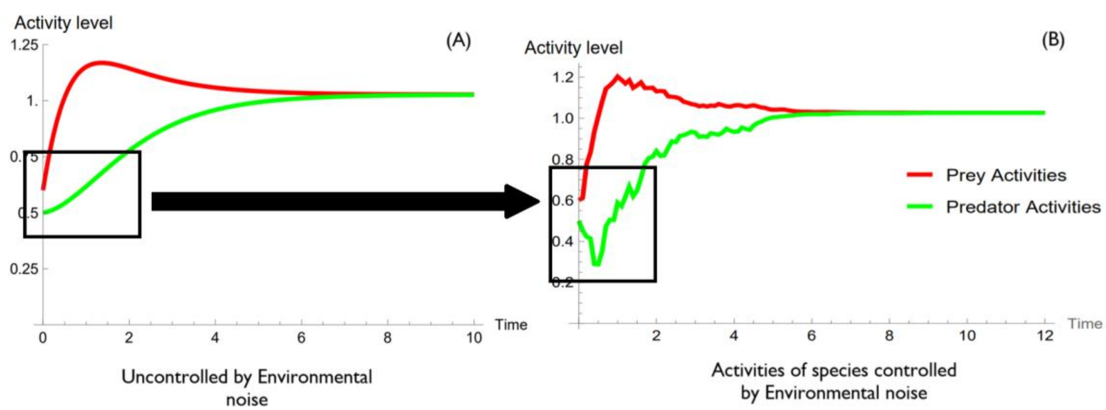

Figure 7. Comparison between the species time plot graphs, uncontrolled by environmental noises and controlled by the environmental noises showing the impact of environmental disturbance for values in **Table 2** for the specific values of E as 0.9.

A highly coordinated connection between the both species' activities is indicated by the coincidence (or near overlap) of the predator and prey curves as time increases in **Figure 3(A)** and **Figure 3(C)**. From a biological perspective, this suggests that the system has effective feedback and control mechanisms since predator activity reacts to changes in prey activity practically instantly. Such synchronization implies that prey activity and predation pressure are well-balanced, resulting in a dynamically stable interaction. Furthermore, the endurance of this overlapping behaviour over time shows that the linked predator-prey behaviours settle into a coherent and coordinated regime, demonstrating the interaction structure's resilience and bolstering the model's biological plausibility.

Table 2. Numerical values of parameters and initial conditions used in the numerical simulations.

Parameter / condition	Initial numerical value of parameters / initial condition
λ	1
x_0	2
E	0.9 / 0.01 / -0.1
μ	0.5
σ_1	0.4
σ_2	0.4
$x(0)$	0.6
$y(0)$	0.5

Now, in **Figure 3(E)**, capturing likelihood score being negative, indicates a very less chances of capturing the prey. Obviously, this is the case when the predator is straightforward identified and prey start running to escape, obviously, predator follow the prey and system goes unstable. Prey run quickly and smartly disorient predators by talented Zigzagging, running and jumping activities. In the bifurcation diagram **Figure 4**, this behavior is indicated by green bubbles. Prey outsmarted the predators totally by fast and calculated maneuvers, the latter become furious and relentless but thereby make habitat unstable.

In **Figure 5** we capture the stochastic dynamics for the values given in **Table 2**. Now, if we observe the graphs carefully, we notice that in the beginning, say, for time 0 to 2, the prey activities in **Figure 5(A)** and **Figure 5(B)** increase, but the predator activities decrease in this duration. The predator slowly and steadily approaches the prey in calculated silence, keeping sound and movements at an absolute minimum to avoid any detection. This, again, makes sure that the prey does not know from which direction the danger is actually coming and hence leads to an increased possibility of the predator hunting down the prey. It is only through evolution that the predator can go unnoticed into the environment and cover the distance without the potential targets suspecting it. Howsoever tall the grass might be, a tiger will stalk through it, or however dark the night might be, the owl will fly soundless. Surprise is the main factor in securing a meal in the wild. In **Figure 6**, histograms for E values 0.9 and 0.01 have been illustrated, which track the evolution of a distribution over time and shows how randomness drives changes, transitions, or spreading of state variables. The histograms show that although the populations are biologically persistent, their distributions are broadened by rising stochastic effects, suggesting less stability and increased ecological variability.

In the **Figure 7**, we have shown a comparison between the noise controlled and uncontrolled phenomenon in which it can be seen that in noise-uncontrolled system, that is, deterministic model, a smooth increase for both the species can be observed for small time span $0 < t < 1$, however, environmental stochasticity, such as abrupt temperature changes, habitat disruption, and resource patchiness, does not have an equal or simultaneous impact on prey and predators in the noise-controlled system. Due to the short-term environmental changes can encourage prey movement, alertness, or foraging activity as an adaptive reaction to uncertain situations, prey behaviors initially rise. On the other hand, predators are often more sensitive to environmental stress, their activities decline concurrently. For example, noise might temporarily lower hunting efficiency, raise search expenses, or promote risk-avoidance behavior.

By and large, this research combines mathematical precision and biological realism, and hence, the model is appealing both to ecologists, applied mathematicians, and behavioral scientists. Its interdisciplinary appeal is in unifying theoretical ecology, behavioral science, and applied mathematics, and as such, it is most applicable in conservation biology, temporal niche theory, and ecosystem management. The rich metaphor of this sort of rollercoaster of survival makes the intricate interplay of adaptation and competition both compelling and readable to outsiders. This work fills a major gap in existing understanding but also paves the way for a new and promising line of research. Its novelty provides fertile ground for further development in theory and interdisciplinary uses, perhaps leading to a sequence of future research intended to enrich understanding and broaden the scope of activity-based ecological modeling.

Conflicts of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

Appendix

The deterministic model $(3-4)$ is as follows

$$\frac{dx}{dt} = \lambda(x_0 - x) - Ex^2y,$$
$$\frac{dy}{dt} = \mu y(t)(x - y).$$

To obtain the equilibrium points, we substitute x' and y' equal zero, that is,

$$\frac{dx}{dt} = \frac{dy}{dt} = 0.$$

It gives

$$\lambda(x_0 - x) - Ex^2y = 0 \quad (16)$$

$$\mu y(x - y) = 0 \quad (17)$$

From Equation (17), we get

$$y = 0 \text{ or } x = y.$$

If $y = 0$, then from Equation(16)

$$\lambda(x_0 - x) - 0 = 0$$
$$\Rightarrow x = x_0$$

Hence $(x_0, 0)$ is first equilibrium point.

If $x = y$ then from Equation (16)

$$Ex^3 + \lambda x - \lambda x_0 = 0 \quad (18)$$

Hence positive equilibrium (x^*, y^*) will be given by the solution of Equation (18).

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