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A Novel Feasibility Based Approach for Redundancy Allocation and Reliability Based Cost Optimization

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Abstract

In the field of reliability and cost optimization, the cost function plays a crucial role in all real-life applications. Conevtionally, cost of a component is assumed directly proportional to the component reliability only. However, this assumption ignores the crucial impact of the component feasibility which determines manufacturability, design adaptability, resource availability, and operational suitability on system performance and cost. This indicates the need of a cost function that considers the component reliability along with the component feasibility. To address this issue this work proposes a modified cost function for the component which integrates the feasibility parameter along with component reliability. The proposed work aims to strike a balance between system's reliability and cost by introducing the modified cost optimization problem while the system's reliability with multiple redundancy arrangements taken as a constraint. By leveraging the general model of Abrasive Jet Machining (AJM), this study explores the trade-off between reliability and cost across different levels of feasibility. To tackle this complex non-linear mixed-integer mathematical optimization problem and system designing, a well-established metaheuristic Particle Swarm Optimization (PSO) algorithm is developed. Additionally, a comparative study with a widely used benchmark series system, is presented for method validation. This research offers a practical framework for industries to enhance the reliability of systems while optimizing costs with a sincere consideration of component's feasibility.

Keywords- Feasibility, Redundancy, Non-linear programming, PSO, Optimization.

Notations

Following notations are used in the proposed work:

t	Time
R_S	System reliability
R_{Si}	System reliability with one redundant component in i^{th} subsystem
R_{Sij}	System reliability with one redundant component in i^{th} and j^{th} subsystem
R_{Sijk}	System reliability with one redundant component in i^{th}, j^{th}, and k^{th} subsystem
C_s	System's cost
c_i	Cost of the i^{th} component
R_g	Goal reliability for the system

$R_{i,min}$	Initial reliability for i^{th} component
$R_{i,max}$	Maximum achievable reliability for i^{th} component
m	The total number of components
F_i	Feasibility of increasing i^{th} component's reliability
n_i	Number of redundant components in i^{th} subsystem
V	The upper bound of sum of the product of weight, volume, and redundancy level of the components in the subsystems
W	The upper bound of the weight of costs of the subsystems
v_i	The volume of each component in i^{th} subsystem.
c_i	The cost of each component in i^{th} subsystem
w_i	The weight of each component in i^{th} subsystem
α_i, β_i	Physical features of i^{th}subsystem

1. Introduction

To expand the general reliability of an explicit system, the reliability allocation problem (RAP) has become a matter of great concern and interest in recent years. RAP is a sort of enhancement issue for limiting the total system costs subject to the goal reliability objective limitations (Yang et al., 1989). RAP intends to decide two choice factors, the sort of components and the number of components, which is known as the redundancy level for every subsystem. In RAP, it is consistently accepted that there are some restricted selections of parts with predefined properties, for example, reliability, weight, cost, and volume. In 1968 a detailed strategy to tackle RAP was depicted, which was intended for the choice of ideal arrangement (Fyffe et al., 1968). After this, some researchers proposed various techniques to address RAP in which the exact methods were carried out at first which were long processing methods (Bellman and Dreyfus, 1958; Tillman, 1969; Hyun, 1975; Luus, 1975) and later heuristic strategies like GA, PSO, hybrid GA-PSO, hybrid PSO, and so on taken into consideration (Nakagawa and Nakashima, 1977; Yang et al., 1999; Sheikhalishahi et al., 2013; Liang et al., 2016). These heuristic strategies were giving approximated outcomes in very little time when contrasted with exact methods.

Then again, the reliability redundancy allocation problem (RRAP) endeavors to track down the best design for a system while the attributes of components are considered as choice factors. This issue makes the RRAP a serious testing issue for system designers and a more convoluted issue for researchers. Therefore, in the field of reliability allocation, most of the work and study is concentrated on the RRAP. The RRAP is a more challenging problem that can be formulated as a non-linear mixed-integer programming problem with a certain set of non-linear constraints. The basic objective of RRAP is to find the number of redundant components and the reliability levels of each component in order to maximize the system's reliability. It has already been proven that some RRAP can be NP-hard and for solving such RRAP several authors have developed many strategies and algorithms (Chern, 1992; Hsieh et al., 1998; Chen, 2006; Yeh and Hsieh, 2011; Wu et al., 2011; Wang and Li, 2012; Afonso et al., 2013; Garg and Sharma, 2013; Khalili-Damghani et al., 2013; Kanagaraj et al., 2013; Ardakan and Hamadani, 2014a; Ardakan and Hamadani, 2014b; He et al., 2015; Ardakan et al., 2016; Zhang and Chen, 2016; Kim and Kim, 2017; Zhang et al., 2019, Lin et al., 2019). In recent years, several other strategies have been incorporated to solve RRAP in which Lin et al., (2021) adopted the fuzzy theory and credibility theory to solve the components uncertainty in the constraints of RRAP, Yeh et al. (2022) proposed a novel general active reliability redundancy allocation problems, Zhang et al. (2023) proposed k-out-of-n: G systems to optimize system reliability with mixed redundancy strategy, Bhandari et al. (2024a) implemented RRAP with Opposition Based Levy Flight Moth Flame Optimization algorithm to optimize reliability of SONAR system, Zhang et al. (2025) developed a reliability model of the wind-photovoltaic power system and proposed a maintenance optimization model with energy complementarity strategies and Choudhary et al. (2025b) used component mixing in RRAP to maximize the reliability of hybrid energy system containing wing and solar energy.

Generally, there are two normal and perceived kinds of redundancy called active and cold standby redundancy. In the late 90s, much work has been reported in active redundancy allocation. In the domain of active redundancy, a penalty-guided artificial immune algorithm was presented to solve the reliability design problem. This algorithm was capable to find the feasible region for optimal solution (Chen, 2006). A penalty-guided Artificial Bee Colony algorithm (ABC) was also implemented to solve RRAP where the objective was the reliability of components, and the constraint was the cost of the system. Authors simultaneously analyzed the number of redundant components and their respective reliability to maximize the system's reliability (Yeh and Hsieh, 2011). Many authors have also worked on the Multi-objective Reliability Redundancy Problem (MORRAP) where reliability and cost had been taken as two different objectives for a system and both objectives were optimized. A software reliability model was presented by Zhu and Pham (2025). In MORRAP, the implementation of PSO provided Pareto optimal solutions instead of a single optimal solution (Garg and Sharma, 2013; Khalili-Damghani et al., 2013; Ardakan and Hamadani, 2014a; Ardakan et al., 2016; Zhang and Chen, 2016). Hsieh et al. (1998) implemented GA, Wu et al. (2011) implemented improved PSO, Kim and Kim (2017) implemented parallel GA, and Bhandari et al. (2023a) implemented Hybrid PSO-GWO. RRAP was solved by using cold standby redundancy strategy (Coit, 2001; Ardakan and Hamadani, 2014a; and Bhandari et al., 2022). The cold standby redundancy strategy with a Modified Genetic Algorithm (GA) was applied to solve non-linear mixed-integer problems and the results were better than previous solutions (Wu et al., 2011; Wang and Li, 2012; Bhunia et al., 2017; Sahoo, 2017). In consideration of the optimal redundancy strategy, advanced RRAP was introduced for a cold-standby subsystem. This advanced RRAP with a non-linear mixed-integer problem was solved by parallel GA (Kim and Kim, 2017), Hybrid PSO-GWO (Bhandari et al., 2022). Later, a mixed strategy (Ardakan and Hamadani, 2014b; Ouyang et al., 2019) was implemented other than active and standby redundancy to solve RRAP. Undoubtedly, this strategy had shown remarkable improvement in optimal solutions (Ardakan et al., 2016). Many other metaheuristics like MayFly optimization algorithm, NSGA-II, hybrid intelligent algorithm, Gurobi optimizer, whale optimization algorithm, hybrid PSO-GWO etc (Bhandari et al., 2023b; Mellal, 2024; Chen et al., 2025; Choudhary et al., 2025a; Yazdi, 2025).

Mettas (2000) proposed a component reliability and feasibility dependent new cost function and implemented it to optimize system cost by allocating reliability to components. The method determined the minimum required reliability level for each component, taking into account feasibility, current reliability, and maximum achievable reliability. The author considered a non-linear mathematical model in which cost function was the objective function and minimum system reliability and components reliability were considered as constraints.

In all the previous studies of reliability optimization, cost of a component or the system is conventionally assumed to be only reliability dependent Equation (1). This conventional assumption does not always satisfy the real-life engineering scenarios where multiple factors like manufacturability, design adaptability, and operational constraints significantly influence both cost and reliability. Although several studies focus on reliability–redundancy allocation and cost optimization but very few of them has considered component feasibility as a quantifiable determinant of system cost. This oversight may result in impractical, and economically inefficient in real-world applications.

$$C = \sum_{i=1}^{m} \alpha_i \left(-\frac{T}{\ln r_i}\right)^{\beta_i} \quad (1)$$

Here, r_i is the component reliability, T is the operating time and α_i and β_i are the physical features of the m components in the subsystem (Chen, 2006).

To tackle this issue, this research work introduces the modified cost function for optimization problems where component feasibility plays a vital role along the component's reliability. The modified cost function, contingent on feasibility, is adopted as the objective function to equip the cost-efficient performance of the Abrasive Jet Machining (AJM) system under the consideration of desires system reliability with multiple redundancies.

In the pursuit of a reliable system, the component reliability is imposed as a constraint, alongside limitations on the weight and volume of the system. The AJM system is systematically classified into fourteen distinct designs, each representing different levels of redundancy. A non-linear mixed-integer mathematical model is employed to solve the RRAP for each design, with varying degrees of feasibility. The outcomes are meticulously presented through bar graphs, providing a visual representation of the intricate trade-off between system feasibility and cost for each AJM design.

To validate the efficacy of the proposed approach a comparative study is conducted. The outcomes of the RRAP for the benchmark series system are compared with existing results in the literature. This thorough analysis confirms the applicability and superiority of the presented methodology in addressing real-world challenges in the reliability of AJM systems.

In essence this study not only introduces a novel parameter in the form of component feasibility but also extends its application to the practical domain by addressing reliability challenges in the specific context of AJM systems. The meticulous categorization of system designs, coupled with the systematic representation of outcomes, establishes a robust framework for industry practitioners seeking to optimize reliability while managing costs in series component systems. The research provides a comparative analysis with a benchmark series system commonly used in RRAP. This comparative study validates the proposed methodology and demonstrates its effectiveness in achieving higher reliability with reduced costs.

The reliability allocation issue tended to in this paper is of great functional significance. System designers work to settle on choices concerning whether to work on the feasibility or redundancy to accomplish goal reliability and minimal cost for the system. For instance, consider a system with four components associated with reliability savvy in series. The reliability of every one of the components is 0.75, 0.8, 0.85, and 0.9, and feasibility is 0.9, 0.85, 0.8, and 0.6 separately. Under identical distribution and independence, the reliability of the system will be 0.459. Accepting that a system reliability execution of 0.9 was looked for, the current plan is insufficient. Designers can overcome this issue either by utilizing redundant components to make the system desired reliable or by further improving the components' feasibility which will bring about reliability improvement of components. However, at that point, the vital issue emerges which is, the expense of the entire system. Clearly, the two arrangements will build the expense of the system. Subsequently, our point in this work is to choose which arrangement is more effective for the designers in keeping the expense low. For the instance of redundancy, an RRAP model is addressed with various potential cases and for feasibility, the RAP model is settled with all potential cases.

The outcomes of the work will be profoundly valuable for the system designers to deliver an exceptionally dependable item with a productive expense. For optimization purposes, the well-known PSO algorithm is developed to solve non-linear mixed-integer programming problems on MATLAB. The PSO has a great search efficiency which can perform the exploitation and exploration of the whole search space with very high convergence speed. The working procedure of PSO is explained in Section 7.

The rest of the article is organized as follows: Section 2 explains the term feasibility and its impact on the cost of the system. In Section 3, the arrangement of components and working procedure of AJM is presented. Section 4 explains the reliability models and their reliability functions of AJM with and without redundancy. In Section 5, the mathematical model for cost optimization with constraints is presented. Section 6 presents a detailed explanation of PSO with working flowchart and Section 7 contains all the outcomes for different cases and a comparative study is explained in Section 8. Finally, conclusions are presented in Section 9.

2. Feasibility

The feasibility parameter is a constant that measures the difficulty of raising a component's reliability in comparison to the rest of the system's components. Certain components can be exceedingly difficult to improve in comparison to other components in the system, depending on the design complexity, technology limits, and so on. A feasibility value can be calculated using a variety of approaches. Many authors have proposed weighting variables for allocating reliability, which can be used to measure feasibility. These weights are influenced by aspects such as the component's complexity, state of the art, operational profile, criticality, and so on. Dimitri and Kececioglu (2002) summarizes some of the methods used to calculate weighting factors. Engineering judgment based on previous experience, supplier quality, supplier availability, and so on can also be utilized to calculate a feasibility value.

- Feasibility F_i is introduced as a parameter that measures how difficult it is to improve a component's reliability. The lower the feasibility value, the more challenging it is to enhance the reliability of that component.
- The idea is that some components in a system may be inherently harder or more expensive to improve in terms of reliability, due to factors like design complexity or technological limitations. Feasibility quantifies this difficulty.

An empirical relationship can be derived based on past experiences and/or data for similar components. In many cases, however, such data is not available. To overcome this problem, a general behavior for the cost function is proposed in this paper, as follows (Mettas, 2000):

$$c_i\left(R_i, F_i, R_{i,min}, R_{i,max}\right) = e^{\left[(1-F_i)\left(\frac{R_i-R_{i,min}}{R_{i,max}-R_i}\right)\right]}, 1 \leq i \leq m, i \in \mathbb{N} \tag{2}$$

Equation (2) provides a mathematical representation of how the cost of improving the reliability of a component c_i is influenced by the component's current reliability R_i, its feasibility f_i, the initial value of the reliability of the component ($R_{i,min}$), and the achievable maximum reliability ($R_{i,\max}$). It uses an exponential function with parameters to capture how the cost increases as you aim for higher reliability levels. Specifically, it models the exponential behavior of cost as you approach the maximum achievable reliability.

Equation (2) shows an exact relationship between c_i, and R_i. Clearly, the lower the feasibility value, the more difficult it is to improve the component's reliability. When the influence of feasibility on the cost function of Equation (2) is examined, it can be observed that in the mid-range of reliability values the cost function reaches to higher values for components with less feasibility. As shown in **Figure 1(b)** the cost of the component with feasibility level of 0.7 changes a litter as reliability goes from 0.9 to 0.95 whereas the cost of the component with 0.5 feasibility level jumps to 8000 from 2500 (approx) in the same interval of reliability. This shows the role of feasibility in determining cost of components, here it should be noted that the cost at minimum level of reliability as well as the maximum level of reliability are independent of the feasibility of component due to the nature of the cost function. The behavior of Equation (2) is demonstrated in **Figure 1**. **Figure 1(a)** shows the general behavior of cost function with any level of feasibility where the desired reliability kept under 0.95. **Figure 1(b)** demonstrates the general relationship between cost and reliability at different feasibility levels with maximum reliability at 0.95. However, if we see the behavior of Equation (2) for minimum to lower reliability values (0.2) the system's cost increases slowly (**Figure 1(c)**) whereas for higher reliability values (from 0.95 to 1), the systems cost increases vary rapidly and approaches to infinity regardless of any feasibility value (**Figure 1(d)**). These figures explain that the cost tends towards infinity regardless of the feasibility value as reliability tends to 1. Therefore, the feasibility parameter does not change the limiting behavior of the cost function at the boundary; instead, feasibility alters the rate at which the cost increases within the allowable interval like [0.8, 0.95].

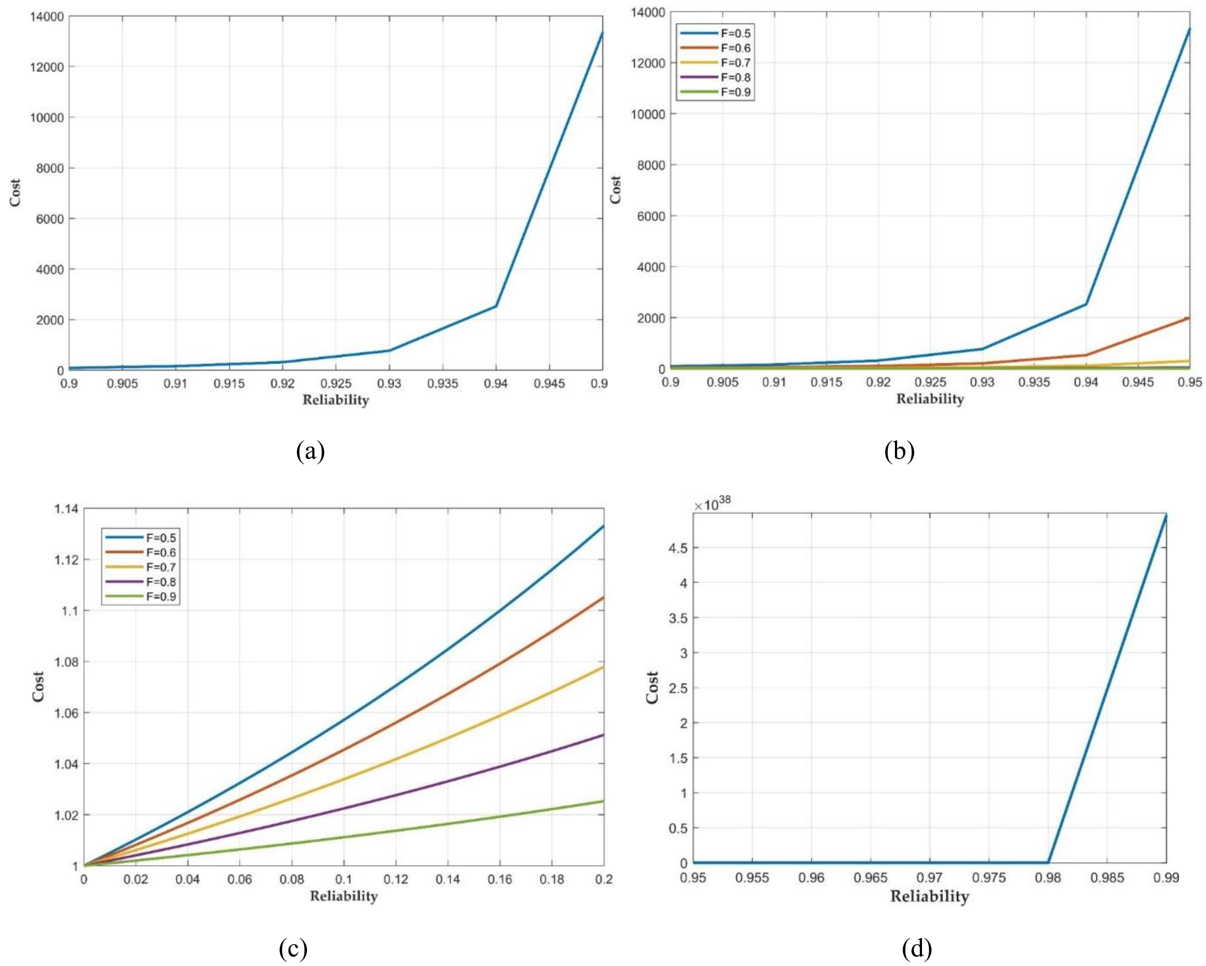


Figure 1. Component reliability vs. system cost.

The proposed cost function given by Equation (2) satisfies the following requirements (Aggarwal and Gupta, 1975):

- Cost is a monotonically increasing function of component reliability.
- The cost of a high-reliability component is very high.
- Cost of a low reliability component.is very low.
- The derivative of cost (with respect to reliability) is a monotonically increasing function of reliability.

However, if we see the behavior of Equation (2) for higher reliability values which are nearer to 1, the systems cost increases vary rapid and approaches to infinity regardless of any feasibility value.

In presented work, feasibility is taken into account as a constant between 0 to 1 as it aligns with the specific requirements and constraints of the optimization problem. Modelling feasibility as a constant between 0 and 1 enhances interpretability by clearly highlighting its significance within the optimization framework, making it easily understandable for both researchers and practitioners. This explicit representation ensures that feasible solutions are prioritized, simplifying constraint enforcement without the need for additional mechanisms that could complicate the optimization process. Additionally, treating feasibility as a constant provides flexibility, allowing its value to be adjusted based on the specific problem requirements to balance feasibility with other objectives effectively.

3. Arrangement of Components and Working of AJM

Abrasive Jet Machining (AJM) is non-conventional machining that can be advantageously used for multifarious purposes including surface cleaning like color, coating, oxide, etc., deburring, abrading, and even making holes without cutting the basic substrate. AJM consists of five components namely gas supplier, pressure regulator, power supplier, foot control valve, and spout, all in simple series.

In AJM, an engaging stream of abrasives (0.025 mm), conveyed by high-pressure air or gas is made to encroach on the work surface through a spout, and work material is eliminated by disintegration by high-speed grating particles. A high-speed (100 – 300m/s) abrasive jet sped up by dehumidified compressed gas (called transporter gas) is made to strike the work surface to bit by bit eliminate material. It is viewed as that work material is taken out basically by disintegration and at some point, helped by weak break brought about by the effect of fine grating cornmeal (size between 10 – 50µm). Sorts of rough just as its shape, size, and different properties impact machining ability. Aluminum oxide (Al2O3), silicon carbide (SiC), sodium bicarbonate ($NaHCO_3$), and glass dots are ordinarily utilized abrasives in AJM.

This is an interaction of expulsion of material by sway disintegration through the activity of focused high-speed stream of coarseness abrasives entrained in a high-speed gas stream. AJM is not the same as shot or sand impacting, as in AJM, better grating cornmeal is utilized, and boundaries can be controlled even more adequately giving better command over item quality. **Figure 2** is the representation of AJM as a series combination of components.

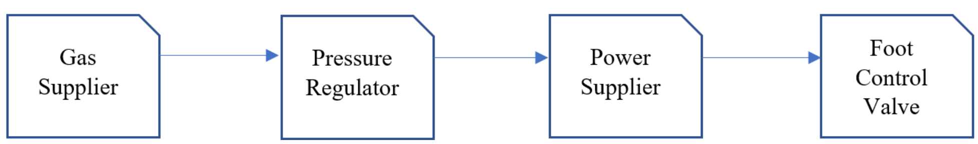


Figure 2. Block diagram of arrangement of components in AJM.

4. Reliability Models for AJM

4.1 With Redundancy

In the case of redundancy, each subsystem can choose any of the strategies which are active, cold, and mixed redundancy.

A. Active redundancy strategy

The system with the active redundancy subsystem is comparable to an equal design where parallelly arranged component works simultaneously. The reliability function of i^{th} subsystem with n_i independent active redundant components with reliability $r_i(t)$ are formulated as:

$$R_i(t) = 1 - \left(1 - r_i(t)\right)^{n_i} \quad (3)$$

B. Cold redundancy strategy

A cold standby redundancy strategy involves one subsystem as a backup for another identical primary system. The cold standby system comes in a working state only on the failure of the primary system. The lifetime of a subsystem with the cold standby strategy is the number of lifetimes of all the segments in the subsystem. Assume that the switching framework works ceaselessly, the reliability function of the i^{th}subsystem with the cold standby strategy can be given by (Coit, 2001; Ardakan and Hamadani, 2014a):

$$R_i(t) = r_i(t) + \sum_{j=1}^{n_i-1} \int_0^t \rho_i(u) r_i(t-u) f_i^{(j)}(u) du \quad (4)$$

Here in Equation (4), $R_i(t)$ and $r_i(t)$ represent the reliability of the subsystem *I* and that of the components used in the subsystem i at time t and n_i is the number of components in the subsystem i. $\rho_i(u)$ is the reliability of the continuously working switching system which is used to activate standby component on failure of working component and $f_i^{(j)}(u)$ is the pdf of j^{th} failure arrival at time u. If the switching system activates after the failure of the component, then the reliability function of the j^{th} subsystem can be given as:

$$R_i(t) = r_i(t) + \rho_i(t) \sum_{j=1}^{n_i-1} \int_0^t r_i(t-u) f_i^{(j)}(u) du \quad (5)$$

C. Mixed redundancy strategy

In a mixed redundancy strategy, a subsystem contains active and cold standby strategies together at the same time. Here the cold component activates and starts working only when all the active components fail. In the case of continuously working switching framework and heterogeneous active and cold redundant components, the reliability of j^{th} subsystem can be given as (Ardakan and Hamadani, 2014b; Ouyang et al., 2019):

$$R_i(t) = (1 - \left(1 - r_i(t)\right)^{n_{Ai}}) + \int_0^t \rho_i(u) r_i(t-u) f_i^{Max,n_{Ai}}(u) du + \sum_{j=1}^{n_i-1} \int_0^t \int_{t_1}^t \rho_i(u) r_i(t-u) f_i^{(j)}(u - t_1) f_i^{Max,n_{Ai}}(t_1) du\, dt_1 \quad (6)$$

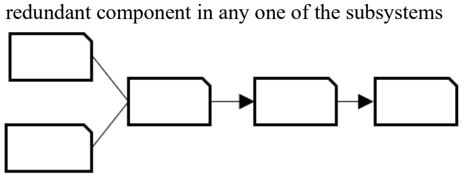
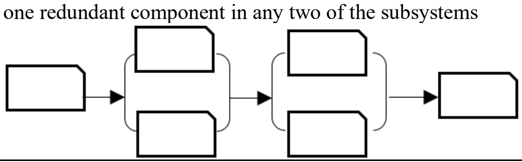
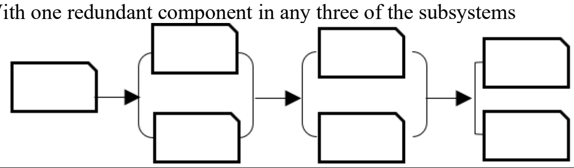
The first part of Equation (6) is the probability of at least one of the active components is working at time t. The second part determines the probability of failure of all redundant components before u time while switching first standby component and working till time t. The third part is to determine the probability of the failure of all active redundant components with first j cold standby components before time u_1 while the next cold component comes to an active state and works up to time t.

With a non-continuous switching framework and heterogeneous active and cold redundant components, the reliability of i^{th}subsystem can be given as (Gholinezhad and Hamadani, 2017):

$$R_i(t) = (1 - \left(1 - r_i(t)\right)^{n_{Ai}}) + \rho_i(t) \int_0^t r_i(t-u) f_i^{Max,n_{Ai}}(u) du + \rho_i(t) \sum_{j=1}^{n_i-1} \int_0^t \int_{t_1}^t r_i(t-u) f_i^{(j)}(u - t_1) f_i^{Max,n_{Ai}}(t_1) du\, dt_1 \quad (7)$$

In Equations (5) and (6), $f_i^{(j)}(u - t_1)$ is the pdf of j^{th} failure arrival and $f_i^{Max,n_{Ai}}(t_1)$ is the pdf for the maximum active component failure time in subsystem i. In this work, active redundancy of subsystems is taken into consideration where all the possible cases with one redundant component are studied. Recently, RRAP is solved to find the number of components to be used for maximum reliability while the cost, volume, and weight of the system are fixed as a constraint. Unlike this, our work is in search of the best design with minimum redundancy while keeping the cost of the system as low as possible without compromising the whole system's reliability. All the possible cases with different combinations of redundancies and their reliability functions are shown in following **Table 1**.

Table 1. Redundancy arrangement and reliability function.

S. no.	Redundancy arrangement	Reliability function
1.	With one redundant component in any one of the subsystems 	$R_{S1} = (2R_1 - R_1^2).R_2.R_3.R_4$ $R_{S2} = R_1.(2R_2 - R_2^2).R_3.R_4$ $R_{S3} = R_1.R_2.(2R_3 - R_3^2).R_4$ $R_{S4} = R_1.R_2.R_3.(2R_4 - R_4^2)$
2.	With one redundant component in any two of the subsystems 	$R_{S12} = (2R_1 - R_1^2).(2R_2 - R_2^2).R_3.R_4$ $R_{S13} = (2R_1 - R_1^2).R_2.(2R_3 - R_3^2).R_4$ $R_{S14} = (2R_1 - R_1^2).R_2.R_3.(2R_4 - R_4^2)$ $R_{S23} = R_1.(2R_2 - R_2^2).(2R_3 - R_3^2).R_4$ $R_{S24} = R_1.(2R_2 - R_2^2).R_3.(2R_4 - R_4^2)$ $R_{S34} = R_1.R_2.(2R_3 - R_3^2).(2R_4 - R_4^2)$
3.	With one redundant component in any three of the subsystems 	$R_{S123} = (2R_1 - R_1^2).(2R_2 - R_2^2).(2R_3 - R_3^2).R_4$ $R_{S124} = (2R_1 - R_1^2).(2R_2 - R_2^2).R_4.(2R_4 - R_4^2)$ $R_{S134} = (2R_1 - R_1^2).R_2.(2R_3 - R_3^2).(2R_4 - R_4^2)$ $R_{S234} = R_1.(2R_2 - R_2^2).(2R_3 - R_3^2).(2R_4 - R_4^2)$

4.2 Without Redundancy

Normally, redundancy can increase the reliability of any system, however, system designers' quest is for a cost-productive high reliable system. The approach to the high reliability of any system makes the cost exponentially high (**Figure 1(a)**). The idea is to work on different attributes of the components to further develop reliability without utilizing any redundant parts. In this work, feasibility is improved to accomplish objective reliability of the system recognizing the way that high feasibility will cost less for improvement and low feasibility will cost high for improvement (**Figure 1(b)**). Here we have checked for the expense of the system in every possible case.

5. A Mathematical Formulation of Reliability of the AJM

In literature ample of work on RRAP has conducted treating the system reliability as an objective function taking cost, weight, and volume as a constraint. This research formulates a new non-linear mixed-integer mathematical model that has cost function as an objective function with system reliability as a constraint. This mathematical model also incorporates the component feasibility as a parameter to calculate the objective function. This new mathematical model for RRAP enables researcher to optimize the system cost with desired reliable system with various levels of redundancy in system design.

Consider a system consisting of m components. Goal reliability is sought for this system. The objective is to allocate reliability to all or some of the components of that system, to meet that goal with a minimum cost. The problem is formulated as a nonlinear programming problem as follows:

Minimize

$$C_s = \sum_{i=1}^{m} n_i c_i(R_i, F_i, R_{i,min}, R_{i,max}) \tag{8}$$

Subject to:

$$R_g \leq R_s \tag{9}$$

$$R_{i,min} \leq R_i \leq R_{i,max}, for\ i = 1,2,\dots,m \tag{10}$$

$$g_1(r,n) = \sum_{i=1}^{m} w_i . v_i^2 . n_i^2 \leq V \tag{11}$$

$$g_2(r,n) = \sum_{i=1}^{m} w_i . n_i . e^{0.25 n_i} \leq W \tag{12}$$

$$n_i \in Z^+,\ 1 \leq i \leq m \tag{13}$$

This formulation is designed to achieve a minimum total system cost, subject to R_g, a lower limit on the system reliability and each component's reliability. The first step will be to obtain the system's analytical reliability function (in terms of its component's reliability). Several methods exist for obtaining the system's reliability equation. In this paper, the PSO Algorithm will be used, which is designed to solve for the system's analytical reliability function. The next step is to obtain a relationship for the cost of each component as a function of its reliability.

This LPP is intended to accomplish a base system cost, subject to R_g, the minimum required reliability of the system and minimum component reliability. The initial step will be to get R_s the system's scientific reliability. Since all the four components of AJM are arranged in the series method, therefore

$$R_s = R_1 . R_2 . R_3 . R_4 \tag{14}$$

6. Solution Method (Particle Swarm Optimization)

The PSO which emulates the social practices of the flock of birds or fishes tutoring was initially presented by Kennedy and Eberhard (1995). Being a heuristic method, PSO has shown remarkable outcomes in form of global optimum, therefore PSO has been broadly used to determine complex nonlinear programming and advancement issues. Several authors have applied PSO in RAP and RRAP which can be seen in Garg and Sharma (2013) and Khalili-Damghani et al. (2013). Generally, the essential techniques of PSO are summed up as follows (**Figure 3**):

Stage 1: Solution representation and initialization:

N particles are arbitrarily created in the feasible area and the position of i^{th} particle is defined as

$$X_i = \left(x_i^1, x_i^2, \dots, x_i^d, \dots x_i^n\right) \text{for } i = 1, 2, 3, \dots , N \tag{15}$$

In this paper, *N* = 100 and each X_i has four dimensions and the lower and the upper bounds of the particles are set to be 0.8 and 0.999 respectively. (0.920827600893549, 0.898000960813807, 0.844413770575820, 0.900195830071960) is an example of the component's reliability. The cost of the system is a reliability-dependent function.

Stage 2: Solution update

To find the best solution in the search space or the feasible region, particles update their positions. **Figure 4** shows the process of determining the new positions of the particles which depends on the current position of the particle, the very last iteration's best particle position, and the position of the best particles in all iterations. The updated position is the vector sum of the current velocity vector, personal influence vector, and social influence vector. In the j^{th} iteration, the n^{th} particle moves towards two positions, i.e., the

${P_{best}}_n^{j-1}$ and G_{best}^{j-1}. ${P_{best}}_n^{j-1}$ is the local best position that the i^{th} particle passed before, and G_{best}^{j-1} is the global best position among all the local best positions, i.e. $\{{P_{best}}_1^{j-1}, {P_{best}}_2^{j-1}, \ldots, {P_{best}}_N^{j-1}\}$.

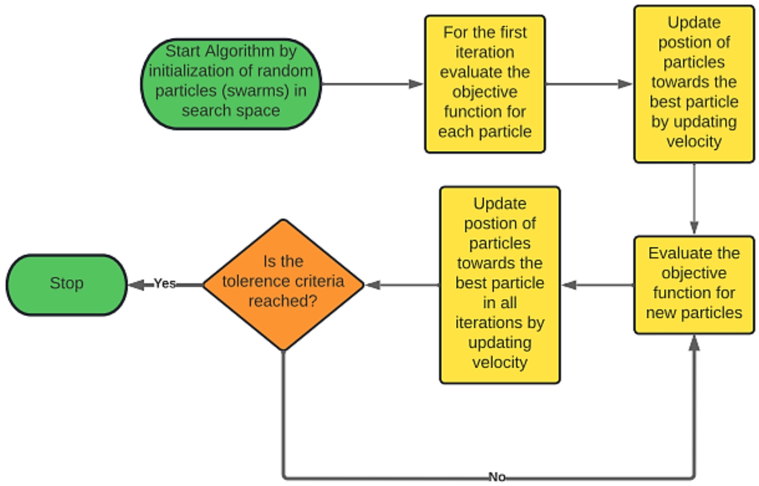


Figure 3. Working flowchart of PSO.

The moving velocity V_n^j of the n^{th} particle is determined by its present position together with V_{n-1}^j, ${P_{best}}_n^{j-1}$, and G_{best}^{j-1}. All the particles update their position after each iteration according to the following two formulae:

$$V_n^j = \omega V_n^{j-1} + c_1 r_1 \left({P_{best}}_n^{j-1} - X_i^{j-1}\right) + c_2 r_2 (G_{best}^{j-1} - X_n^{j-1}) \tag{16}$$

$$X_n^j = V_n^j + X_n^{j-1} \tag{17}$$

Here, ω is the inertia weight and it is a positive constant. This parameter is important for balancing the global search, also known as exploration (for higher values of ω) and exploitation (for lower values of ω). In Equation (16), the term "ωV_n^{j-1}" explains the diversification of the PSO where the particles search for new solutions in the regions with a high potential of the best solution. r_1 and r_2 are two random quantities complying with the standard uniform distribution. c_1 and c_2 are two acceleration coefficients that influence the searching behaviors. If c_1 is greater than c_2, particles have better exploration abilities. On the contrary, if c_1 is less than c_2, the exploitation abilities of particles will be improved. According to the literature, typical values of ω range between 0.4 and 0.9 (Shi and Eberhart, 1998). Following Khalili-Damghani et al. (2013) approach, in this work a linearly decreasing strategy from 0.9 to 0.4 over the iterations is used to gradually shift the algorithm from global search to local refinement.

In Equation (16), the term "$c_1 r_1 \left({P_{best}}_n^{j-1} - X_i^{j-1}\right)$" explains the intensification of the personal influence of the PSO where the particles explore the previous solutions and improve the individual by returning to the previous positions which are better than the current. The last term in Equation (16) i.e., "$c_2 r_2 (G_{best}^{j-1} - X_n^{j-1})$" explains the intensification of the social influence of the PSO which makes the particles follow the best neighbor's direction. Equation (17) determines the updated positions of particles by using the updated velocity of particles and the current positions of particles.

Stage 3: Termination

If the search process reaches a pre-specified total iteration number J, $Gbest^j$ in the last iteration can be viewed as the optimal solution to the optimization problem.

7. Outcomes

In this work, significant and encouraging results have been achieved in form of cost and reliability for several possible arrangement in AJM. Implementation of PSO helped to find accurate reliability of components and optimized cost of the system with several different constraints. In **Tables 2** to **7**, active redundancies in subsystems are taken into the consideration with different values of feasibility and minimum reliability for each component. Also, the goal reliability for the system and the maximum achievable reliability for each component are the same throughout this work. To validate the proposed parameter of cost different cases with varying level of feasibility are considered. Following are the details of all the six considered cases to explain the trade-off between feasibility and optimized cost for the series system:

	Component's Feasibility level f_i	Component's Minimum Reliability $R_{i,min}$
Case I	$F_i = 0.5$ for $i = 1,2,3,4$.	$R_{i,min} = 0.8$ for $i = 1,2,3,4$.
Case II	$F_1 = 0.9, F_2 = 0.8, F_3 = 0.7, F_4 = 0.6$	$R_{i,min} = 0.8\ for\ i = 1, 2, 3, 4$
Case III	$F_i = 0.9, i = 1, 2, 3, 4$	$R_{1,min} = 0.75, R_{2,min} = 0.8,$ $R_{3,min} = 0.85, R_{4,min} = 0.90$
Case IV	$F_1 = 0.9, F_2 = 0.8, F_3 = 0.7, F_4 = 0.6$	$R_{1,min} = 0.75, R_{2,min} = 0.8,$ $R_{3,min} = 0.85, R_{4,min} = 0.90$
Case V	$F_1 = 0.6, F_2 = 0.7, F_3 = 0.8, F_4 = 0.9$	$R_{i,min} = 0.8\ for\ i = 1, 2, 3, 4$
Case VI	$F_i = 0.9\ for\ i = 1, 2, 3, 4$	$R_{i,min} = 0.8\ for\ i = 1, 2, 3, 4$

The decision variable R_i provides the best possible reliability for each component in all above-mentioned cases where all the cases have different possible allocation of feasibility and minimum component reliability. In **Table 2** and **Figure 5**, feasibility for the each component is kept at 0.5 and the minimum required reliability for each component at 0.8.

Table 2. Component's reliability and cost of AJM for case I.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.9351	0.9683	0.9683	0.9683	44.3451
	R_{S2}	0.9651	0.9425	0.9652	0.9693	43.6181
	R_{S3}	0.9639	0.9684	0.9350	0.9682	44.2926
	R_{S4}	0.9651	0.9694	0.9651	0.9426	43.9099
With any two redundant components	R_{S12}	0.9123	0.9202	0.9519	0.9589	16.3198
	R_{S13}	0.9076	0.9568	0.9074	0.9569	16.4693
	R_{S14}	0.9123	0.9589	0.9519	0.9202	16.3204
	R_{S23}	0.9518	0.9205	0.9125	0.9589	16.3157
	R_{S24}	0.9542	0.9239	0.9542	0.9242	15.7486
	R_{S34}	0.9519	0.9589	0.9122	0.9203	16.3238
With any three redundant components	R_{S123}	0.8736	0.8860	0.8735	0.9417	7.5950
	R_{S124}	0.8812	0.8925	0.9343	0.8928	7.3260
	R_{S134}	0.8734	0.9418	0.8735	0.8858	7.5950
	R_{S234}	0.9343	0.8927	0.8808	0.8930	7.3280

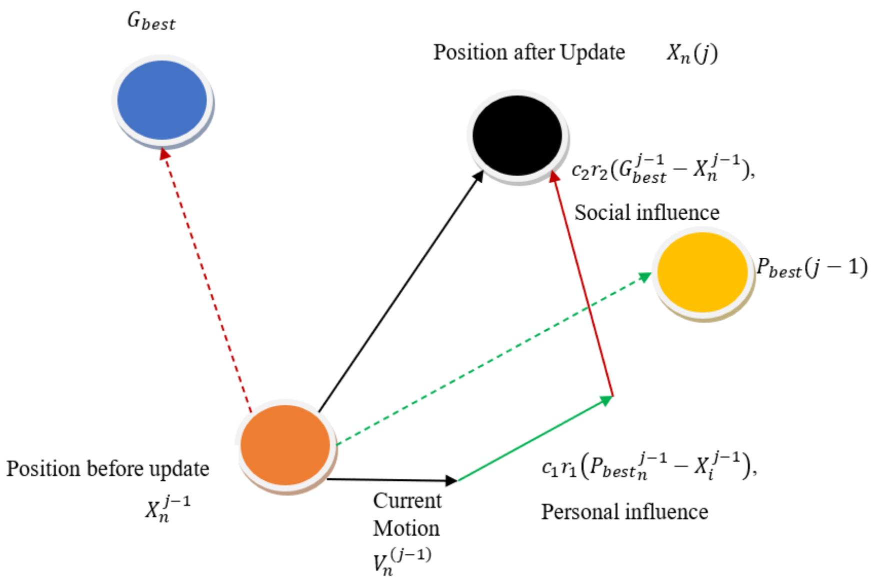


Figure 4. Process of position update in PSO.

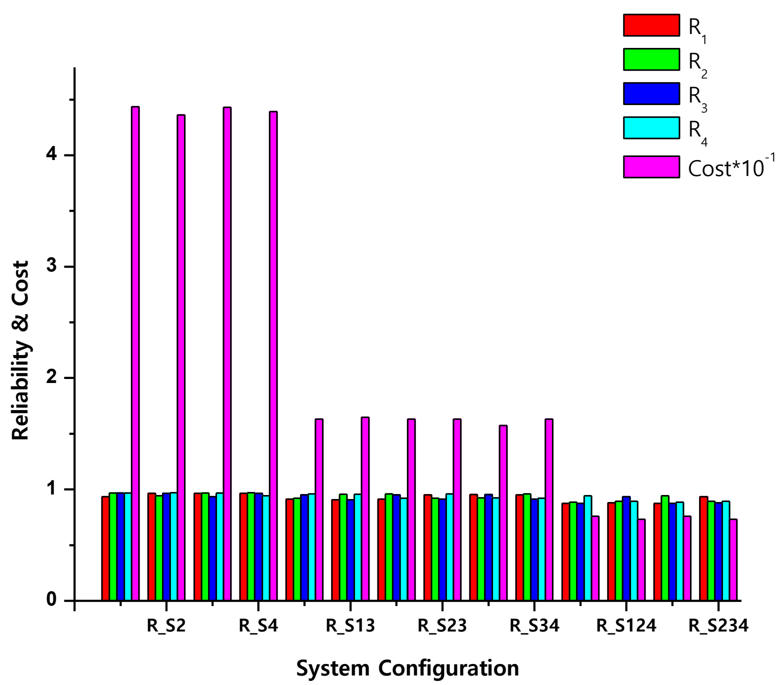


Figure 5. Component's reliability and cost of AJM for case I.

In **Table 3** and **Figure 6**, feasibility for the first component is 0.9, for the second is 0.8, for the third is 0.7, and the fourth is 0.6. With this scenario the cost of every systems configuration varies between 4.8725 – 21.7425 with a minimum required reliability for a component is 0.8484 to achieve the goal reliability.

Table 3. Component's reliability and cost of AJM for case II.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.9485	0.9754	0.9620	0.9616	21.7425
	R_{S2}	0.9797	0.9378	0.9604	0.9601	15.1316
	R_{S3}	0.9769	0.9715	0.9095	0.9562	11.4461
	R_{S4}	0.9770	0.9716	0.9558	0.9098	10.3349
With any two redundant components	R_{S12}	0.9358	0.9262	0.9528	0.9537	9.2335
	R_{S13}	0.9267	0.9656	0.8922	0.9481	8.3866
	R_{S14}	0.9281	0.9661	0.9470	0.8943	7.7113
	R_{S23}	0.9701	0.9138	0.8887	0.9463	7.4232
	R_{S24}	0.97053	0.9153	0.94518	0.8914	6.7882
	R_{S34}	0.9659	0.9602	0.8769	0.8785	6.4435
With any three redundant components	R_{S123}	0.9069	0.8952	0.8652	0.9349	5.8033
	R_{S124}	0.9092	0.8981	0.9326	0.8708	5.4109
	R_{S134}	0.8963	0.9504	0.8513	0.8548	5.2289
	R_{S234}	0.9550	0.8820	0.8484	0.8521	4.8275

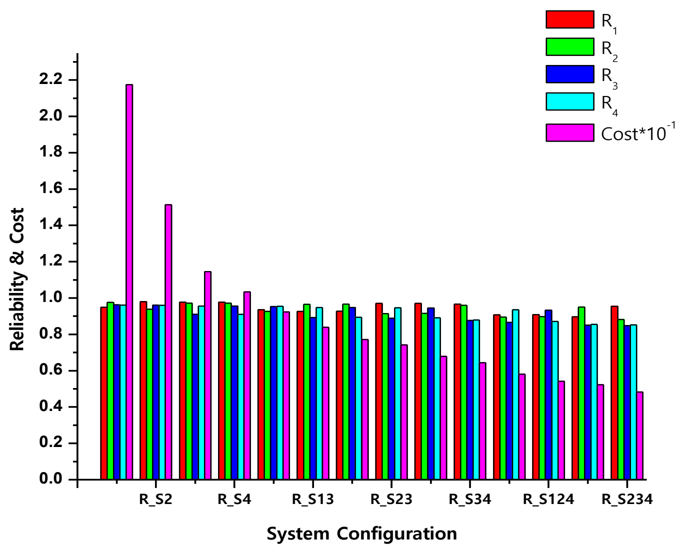


Figure 6. Component's reliability and cost of AJM for case II.

Table 4. Component's reliability and cost of AJM for case III.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.9088	0.9710	0.9626	0.9710	5.6844
	R_{S2}	0.9648	0.9234	0.9648	0.9725	5.7662
	R_{S3}	0.9625	0.9710	0.9092	0.9710	5.9884
	R_{S4}	0.9650	0.9725	0.9646	0.9239	6.2283
With any two redundant components	R_{S12}	0.8913	0.9038	0.9532	0.9643	4.7127
	R_{S13}	0.8844	0.9616	0.8843	0.9615	4.8728
	R_{S14}	0.8911	0.9643	0.9532	0.9046	5.0032
	R_{S23}	0.9534	0.9042	0.8916	0.9641	4.9188
	R_{S24}	0.9571	0.9103	0.9576	0.9000	5.0676
	R_{S34}	0.9536	0.9646	0.8916	0.9000	5.2200
With any three redundant components	R_{S123}	0.8606	0.8759	0.8592	0.9510	4.2657
	R_{S124}	0.8660	0.8812	0.9390	0.9000	4.3222
	R_{S134}	0.8567	0.9494	0.8500	0.9000	4.4305
	R_{S234}	0.9389	0.8815	0.8659	0.9000	4.4533

In **Table 4** and **Figure 7**, the feasibility for each component is 0.9 but the minimum reliability for the first component is 0.75, for the second is 0.80, for the third is 0.85, and the fourth is 0.90. With this scenario the cost of every systems configuration varies between 4.2657–6.2283 with a minimum required reliability for a component is 0.85 to achieve the goal reliability.

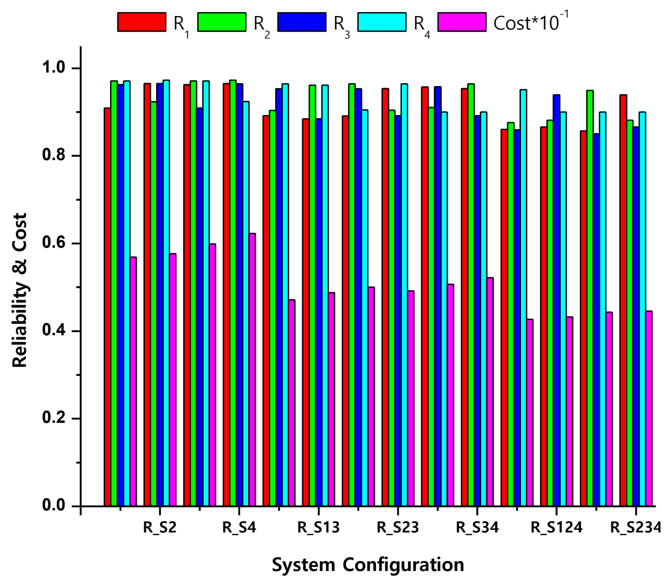


Figure 7. Component's reliability and cost of AJM for case III.

In **Table 5** and **Figure 8**, feasibility for the first component is 0.9, for the second is 0.8, for the third is 0.7, and the fourth is 0.6 and the minimum reliability for the first component is 0.75, for the second is 0.80, for the third is 0.85, and the fourth is 0.90. With this scenario the cost of every systems configuration varies between 4.5649–10.324 with a minimum required reliability for a component is 0.85 to achieve the goal reliability.

Table 5. Component's reliability and cost of the AJM for case IV.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.9493	0.9754	0.9619	0.9617	10.324
	R_{S2}	0.9798	0.9373	0.9602	0.9604	9.0847
	R_{S3}	0.9769	0.9716	0.9096	0.9560	9.1985
	R_{S4}	0.9770	0.9716	0.9558	0.9098	9.4467
With any two redundant components	R_{S12}	0.9357	0.9262	0.9528	0.9537	6.3116
	R_{S13}	0.9269	0.9657	0.8921	0.9480	6.5637
	R_{S14}	0.9271	0.9657	0.9464	0.9000	6.7196
	R_{S23}	0.9700	0.9139	0.8887	0.9464	5.9793
	R_{S24}	0.9700	0.9137	0.9442	0.9000	6.1171
	R_{S34}	0.9642	0.9585	0.8721	0.9000	6.0930
With any three redundant components	R_{S123}	0.90707	0.8952	0.8652	0.9349	4.6657
	R_{S124}	0.9043	0.8924	0.9282	0.9000	4.7596
	R_{S134}	0.8827	0.9430	0.8500	0.9000	4.7871
	R_{S234}	0.9470	0.8663	0.8500	0.9000	4.5649

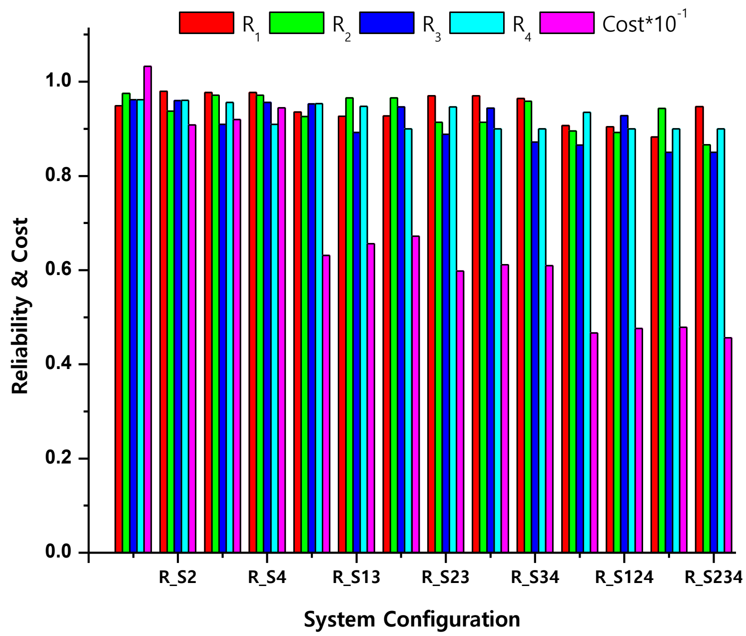


Figure 8. Component's reliability and cost of AJM for case IV.

In **Table 6** and **Figure 9**, feasibility for the first component is 0.6, for the second is 0.7, for the third is 0.8, and the fourth is 0.9. Although the minimum component reliability is the same as 0.8. With this scenario the cost of every systems configuration varies between 4.9568–16.6507 with a minimum required reliability for a component is 0.8287 to achieve the goal reliability.

Table 6. Component's reliability and cost of AJM for case V.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.8974	0.9623	0.9637	0.98080	9.0206
	R_{S2}	0.9511	0.9245	0.9682	0.9829	10.8528
	R_{S3}	0.9519	0.9663	0.9292	0.9832	13.1363
	R_{S4}	0.9568	0.9696	0.9720	0.9568	16.6507
With any two redundant components	R_{S12}	0.8675	0.8922	0.9515	0.9741	6.2697
	R_{S13}	0.8674	0.9509	0.8955	0.9740	7.0924
	R_{S14}	0.8823	0.9567	0.9578	0.9361	7.5447
	R_{S23}	0.9373	0.9044	0.9076	0.9774	8.1685
	R_{S24}	0.9445	0.9147	0.9630	0.9431	8.8392
	R_{S34}	0.9450	0.9618	0.9186	0.9440	10.4042
With any three redundant components	R_{S123}	0.8287	0.8605	0.8621	0.9639	4.9568
	R_{S124}	0.8483	0.8761	0.9424	0.9150	5.0348
	R_{S134}	0.8473	0.9420	0.8783	0.9161	5.6014
	R_{S234}	0.9269	0.8899	0.8926	0.9250	6.2150

Table 7 and **Figure 10** are the same case as **Table 2**, but the feasibility of each component is increased from 0.5 to 0.9. With this scenario the cost of every systems configuration varies between 4.4803–6.3742 with a minimum required reliability for a component is 0.8595 to achieve the goal reliability.

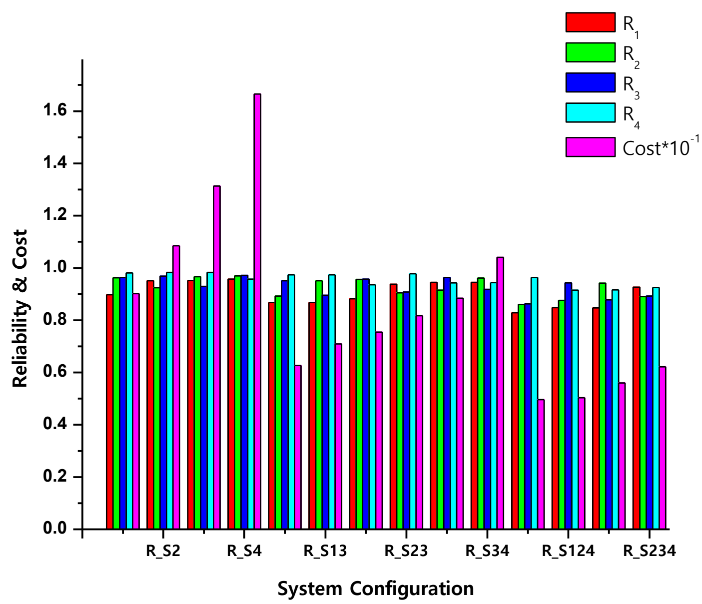


Figure 9. Component's reliability and cost of AJM for case V.

Table 7. Component's reliability and cost of AJM for case VI.

System configuration	Case	R_1	R_2	R_3	R_4	Cost
With only one redundant component	R_{S1}	0.9088	0.9710	0.9625	0.9710	6.3742
	R_{S2}	0.9647	0.9240	0.9651	0.9723	6.3295
	R_{S3}	0.9626	0.9709	0.9088	0.9710	6.3723
	R_{S4}	0.9647	0.9726	0.9649	0.9234	6.3369
With any two redundant components	R_{S12}	0.8910	0.9043	0.9534	0.9642	5.2066
	R_{S13}	0.8844	0.9614	0.8841	0.9617	5.2319
	R_{S14}	0.8912	0.9642	0.9533	0.9044	5.2068
	R_{S23}	0.9533	0.9039	0.8911	0.9643	5.2080
	R_{S24}	0.9565	0.9098	0.9566	0.9090	5.1512
	R_{S34}	0.9535	0.9641	0.8913	0.9037	5.2059
With any three redundant components	R_{S123}	0.8601	0.8764	0.8598	0.9509	4.5210
	R_{S124}	0.8692	0.8840	0.9407	0.8846	4.4801
	R_{S134}	0.8595	0.9511	0.8597	0.8764	4.5224
	R_{S234}	0.9406	0.8846	0.8688	0.8846	4.4803

In **Table 8**, no redundancy i.e., a purely series AJM is taken where all possible cases are considered for optimal reliability. In the first case, all the components of AJM have minimum reliability as 0.8 and feasibility as 0.5. In the second case, minimum reliability for all the components of AJM is fixed at 0.8 but the feasibility for the first component is 0.9, for the second is 0.8, for the third is 0.7, and the fourth is 0.6. In the third case the minimum reliability for the first component is 0.75, for the second is 0.80, for the third is 0.85, and the fourth is 0.90, and feasibility for each component is 0.9. In the fourth case the minimum reliability for the first component is 0.75, for the second is 0.80, for the third is 0.85, and the fourth is 0.90, and feasibility for the first component is 0.9, for the second is 0.8, for the third is 0.7, and the fourth is 0.6, and in the fifth case, the minimum reliability for the first component is 0.75, for second is 0.80, for third is 0.85, and the fourth is 0.90 and feasibility for the first component is 0.6, for second is 0.7, for third is 0.8, and the fourth is 0.9. Among all the above-mentioned cases case third has the minimum cost of 8.325 and 183.4579 is the maximum cost for the same system with 90% reliability.

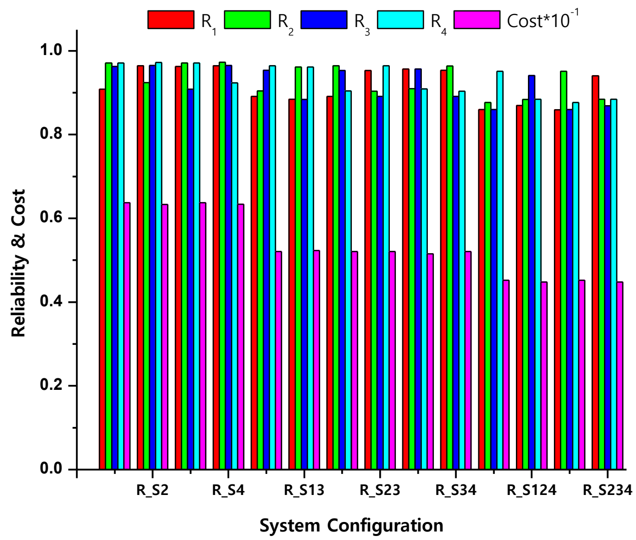


Figure 10. Component's reliability and cost of AJM for case VI.

Table 8. Component's reliability and cost of AJM with different $R_{i,min}$ and F_i.

Case	Component	$R_{i,min}$	Feasibility	$R(i)$	Cost
1	1	0.8	0.5	0.9659	183.4579
	2	0.8	0.5	0.9804	
	3	0.8	0.5	0.9744	
	4	0.8	0.5	0.9759	
2	1	0.8	0.9	0.9852	24.8826
	2	0.8	0.8	0.9751	
	3	0.8	0.7	0.9647	
	4	0.8	0.6	0.9730	
3	1	0.75	0.9	0.9721	8.325
	2	0.8	0.9	0.9677	
	3	0.85	0.9	0.9810	
	4	0.9	0.9	0.9771	
4	1	0.75	0.9	0.9826	25.0199
	2	0.8	0.8	0.9729	
	3	0.85	0.7	0.9788	
	4	0.9	0.6	0.9628	
5	1	0.75	0.6	0.9641	29.1571
	2	0.8	0.7	0.9773	
	3	0.85	0.8	0.9842	
	4	0.9	0.9	0.9722	

To ensure comparability across all scenarios, a unified summary of Cases I–VI (and the pure series case) is now presented in **Table 9**. This enables direct observation of how feasibility patterns and minimum reliability constraints jointly influence cost. For example, when feasibility is uniformly high (Case IV), the optimized cost range is substantially lower than heterogeneous-feasibility scenarios (Cases II and III), demonstrating that uniform feasibility encourages more cost-efficient reliability allocation. Conversely, reversing feasibility (Case VI) increases the maximum cost relative to Case III, confirming that components with lower feasibility (i.e., more difficult to improve) introduce higher penalty even under identical system topology. This comparative presentation enables a consistent interpretation of all scenarios, including Case I, where feasibility is now varied systematically to illustrate its isolated effect.

Table 9. Comparison of all cases.

Case	Feasibility level	Minimum reliability pattern	Cost range (Optimized)	R_{min}
Case I	0.9, 0.8, 0.7, 0.6	0.8 (same for all components)	4.8725 – 21.7425	0.8484
Case II	0.9, 0.9, 0.9, 0.9	0.75, 0.80, 0.85, 0.90	4.2657 – 6.2283	0.85
Case III	0.9, 0.8, 0.7, 0.6	0.75, 0.80, 0.85, 0.90	4.5649 – 10.324	0.85
Case IV	0.6, 0.7, 0.8, 0.9	0.8 (same for all components)	4.9568 – 16.6507	0.8287
Case V	Feasibility increased from 0.5 to 0.9 for all four components	0.8 (same for all components)	4.4803 – 6.3742	0.8595
Pure Series System (no redundancy)	Five subcases as tested (from 0.5–0.9 feasibility distributions)	As defined per subcase	Cost varies from 8.325 (best) to 183.4579 (worst)	0.90 (system level)

From the comparison above, it is evident that uniform high feasibility (Case II) yields the most cost–efficient configuration with the narrowest cost range. In contrast, heterogeneous feasibility distributions (Cases I, III and IV) significantly expand the cost spread because component-specific improvement difficulty is uneven. Increasing feasibility values across all components (Case V) again reduces cost closer to the uniform-high-feasibility case, which confirms that feasibility directly drives cost escalation. Finally, the pure series system demonstrates the highest overall cost risk, proving that redundancy yields substantial economic benefit when meeting a fixed reliability goal.

8. Comparative Study

For the generalization of the idea of taking feasibility as a parameter of cost and reliability, this work provides a comparative analysis with a benchmark series system commonly used in RRAP. This comparative study validates the proposed methodology and demonstrates its effectiveness in achieving higher reliability with reduced costs. The mathematical model for the benchmark system is considered same as in the literature (Afonso et al., 2013; Ardakan and Hamadani, 2014a; He et al., 2015; Huang, 2015; Mellal and Zio, 2016; Kim and Kim, 2017; Dobani et al., 2019; Hsieh, 2022).

Maximize

$$f(r,n) = \prod_{i=1}^{m} R_i(n_i) \tag{18}$$

Subject to

$$g_2(r,n) = \sum_{i=1}^{m} \alpha_i . \left(-\frac{1000}{\ln r_i}\right)^{\beta_i} . [n_i + e^{0.25 n_i}] \leq C \tag{19}$$

With Equation (11) and (12), where, $n_i \in \mathbb{Z}^+$, $0 \leq r_i \leq 1, r_i \in \mathbb{R}$.

Equation (18) represents the objective function for the benchmark series system that maximizes the overall system reliability. The constraint in Equation (19) is the total cost of the system and volume and weight constraints are same as previous model.

Following **Table 10** is used for input parameters for the benchmark series system with five components (Ardakan and Hamadani, 2014a). The cost function is presented as a feasibility dependent function, where the value of feasibility is set to be as 0.9.

Table 10. Data used in benchmark series system.

Stage	$10^5 . \alpha_i$	β_i	$w_i . v_i^2$	w_i	R_g	W	V
1	2.330	1.5	1	7	0.98	200	110
2	1.450	1.5	2	8			
3	0.541	1.5	3	8			
4	8.050	1.5	4	6			
5	1.950	1.5	2	9			

The results obtained from the proposed idea are compared to pre-existing results in literature in **Table 10**. The comparison clearly shows that by using the proposed methodology, the maximum reliability for the benchmark series system is higher than previously achieved reliability. Also, the required cost to achieve the optimized reliability is very much less than the previous outcomes.

Table 11. Comparison of system reliability and cost with literature.

Parameter	**Afonso et al. (2013)**	**Huang (2015)**	**He et al. (2015)**	**Mellal and Zio (2016)**	**Dobani et al. (2019)**	**Ardakan and Hamadani (2014a)**	**Kim and Kim (2017)**	**Hsieh (2022)**	**Bhandari et al. (2023a)**	**Choudhary et al. (2025c)**	**This study**
n	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]	[3 2 2 3 3]
R_s	0.9316793 9	0.931,6 823000	0.931682 2685	0.9316 82387 9	0.94601 73497	0.96957 758	0.9740 1432	0.9773843 0	0.9316823 8268047	**0.9784440 100874**	**0.9999975 77284312**
r_1	0.779 874	0.7794 66450	0.779388 414	0.7793 98883	[0.8403 027533, 0.83985 1,6282, 0.67929 71860]	0.76459 335	0.7641 5009	[0.800284 62087006 09, 0.4595827 65663791 64, 0.8452110 72694642]	0.7796388 06429359	0.7672182 25992554	0.9955014 19635678
r_2	0.872 057	0.8717 32780	0.871720 982	0.871, 83701 3	[0.917,5 839254, 0.838,9 254091]	0.88752 892	0.8887 2646	[0.941100 68282849 85, 0.8472493 83290202 5]	0.8717188 32922107	0.8874287 573451	0.9994415 63555609
r_3	0.903 426	0.902,8 49510	0.903,03 3392	0.902, 88535 7	[0.935,5 937943, 0.88019 04810]	0.91539 527	0.9159 9469	[0.945552 92513038 86, 0.8897383 56016844]	0.9029665 67728454	0.9115439 460854	0.9987249 73194559
r_4	0.710 960	0.7114 87800	0.711418 362	0.7114 02517	[0.7906 675800, 0.79537 50054, 0.56337 34998]	0.693,5 05,44	0.6932 7193	[0.747288 80818123 2, 0.5706130 36017206 3, 0.7845490 82108649 4]	0.7112787 46833696	0.7360009 167252	0.9987249 73194559
r_5	0.786 902	0.7878 16440	0.787789 288	0.7877 99485	[0.8466 926244, 0.68579 91922, 0.84965 83327]	0.77603 145	0.7717 0347	[0.752037 06412811 3, 0.6933622 0260537, 0.8687224 9551762]	0.7878795 56305224	0.7813259 05033	0.9943857 71674505
Cost	174.9 9990 1	174.99 995092	174.9999 9999328	174.95 9	174.999 999865	174.999 97522	174.99 988392	174.73476 555	174.99982 7038216	174.98579 47634798	**43.895828 8745057**
Slack g_1	27	27	27	27	27	27	27	27	27	27	27
Slack g_2	0.000 099	0.0000 4908	6.72E-09	4.1E-1	1.35E-07	0.00002 478	0.0001 1608	0.2652344 5	1.7296178 39032471e -04	.01420523 652023	**131.10417 112549**
Slack g_3	7.518 91	7.5189 182	7.51891	7.5189 182	7.51891 8	7.51891 82	7.5189 182	7.5189 1824	7.5189	7.5189182 41159	7.5189

9. Conclusion

This work considers the need of feasibility in determining the cost of a component. The feasibility of a component has a direct economic impact and must therefore be explicitly embedded in the cost formulation. To tackle this challenge, the concept of component feasibility is proposed as a parameter for system reliability and cost optimization in the context of the Abrasive Jet Machining (AJM) system, where four components are arranged in series. The paper formulates a non-linear mixed integer mathematical model that optimizes system costs while considering feasibility as a parameter, system reliability as a constraint, and various levels of redundancy in system design. To tackle this complex optimization problem, the well-established Particle Swarm Optimization (PSO) algorithm is employed.

Furthermore, the study provides a comparative analysis with a benchmark series system commonly used for Reliability Redundancy Allocation Problems (RRAP), validating the proposed methodology. The results of case I and VI for one redundant component show that system cost can be dropped down from 43.6181 to 6.3295 with improved feasibility of components. Similarly, all results validate the fact that the level of feasibility plays an important role as decision factor in cost computation. Implementation of proposed method in the conventional RRAP for the series system also improves the system reliability to 0.999997577284312 which is best among the reported results in the literature.

This research offers a significant and practical managerial value with capability of deciding the rational budget to the system deployment at design stage. This work contributes to the broader field of reliability engineering by addressing an important real-world problem and providing a systematic methodology for its resolution. Computational efficiency of the proposed work can be implemented in reliability critical systems such as nuclear plants, aerospace systems etc. to enhance the reliability of systems while taking into account the cost implications. By introducing feasibility as a key parameter and utilizing PSO for optimization, it provides a valuable framework for industries to make informed decisions regarding system design and redundancy allocation.

Future research may use the methodology to calibrate the generalized cost function using the empirical cost data. Also, this work can be further explored with multi criteria optimization with redundancy, repair cost, multi state systems, maintainability etc.

Conflicts of Interest

The authors declare no conflict of interest.

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Not Applicable.

AI Disclosure

During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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