International Journal of Mathematical, Engineering and Management Sciences
Vol. 11, No. 2, 788-803, 2026
https://doi.org/10.33889/IJMEMS.2026.11.2.033

Numerical Solution of Newell-Whitehead-Segel Equation via Quintic B-spline Collocation Method

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(Received on June 23, 2025; Revised on November 13, 2025 & January 8, 2026; Accepted on January 13, 2026)

Abstract
The Newell-Whitehead-Segel (NWS) equation plays a significant role in nonlinear systems, including mathematical biology, plasma physics, solid-state physics, optics, quantum mechanics, cosmology, fluid dynamics, and many others. In this work, we proposed the quintic B-spline collocation method to find the numerical solution of the nonlinear NWS-type equation. Crank-Nicolson finite difference method (FDM) is used to discretize the equation in time space, and quasi-linearization is employed to linearize the nonlinear term. The stability analysis has been discussed using the Von Neumann Method, and stability conditions have been obtained. The numerical results are compared with existing techniques, which demonstrate the effectiveness and applicability of the proposed technique. The proposed method has been applied to four numerical test problems at various time levels and mesh sizes to demonstrate the effectiveness, which involves quadratic, cubic, and quartic order nonlinear terms. The comparison shows good agreement with the exact solution, as demonstrated by absolute error tables and graphs. Moreover, the proposed method is easy to implement and produces good results.

Keywords- Nonlinear partial differential equation, Newell-Whitehead-Segel equation, Crank-Nicolson finite difference method, Quasi-linearization, Quintic B-spline collocation method.

1. Introduction

It is well known that non-linear partial differential equations (PDEs) and their numerical solutions play a significant and increasingly important role in modeling a wide range of phenomena encountered in engineering and science, such as fluid flow, chemical reactions, biological pattern formation, optical systems, and many others (Cross and Hohenberg, 1993; Debnath, 2005; Hoyle, 2006; Kumar and Arora, 2022). Among these nonlinear PDEs, the Newell–Whitehead–Segel (NWS) equation stands out as a canonical model for studying the emergence and evolution of spatially periodic structures near the onset of a finite-wavelength instability.

Newell and Whitehead (1969) were the first to introduce the non-linear Newell-Whitehead (NW) partial differential equation. Segel (1969) modified the NW equation by altering the source term, resulting in what became known as the NWS equations. The NWS equation has been widely studied and applied across multiple scientific disciplines. In fluid dynamics, the NWS equation describes the amplitude of convection in binary fluid mixtures near the onset of a finite-wavelength instability point in Rayleigh–Bénard convection. This phenomenon occurs when a fluid layer is heated from its lower boundary; it becomes unstable and forms organized Bénard cells, where warm fluid rises and cool fluid sinks due to density differences induced by thermal expansion. Rayleigh–Bénard convection typically produces two main flow patterns, such as rolls or stripes and hexagonal (Cross and Newell,1984). The NWS equation also arises in reaction-diffusion systems where a uniform state becomes unstable and spatial patterns such as stripes, spots form, with the slowly varying amplitude of the pattern. Also, it is used as a generic amplitude equation for stripe patterns that occur in biology and ecology, such as zebra stripes, human fingerprints, visual cortex structures, shell patterns, etc. (Getling, 1998; Cross and Greenside, 2009). Because the NWS equation captures the slow modulation of the amplitude of a pattern in space/time, it is widely used in the pattern-formation literature.

In the present paper, we consider a general NWS-type equation that is given in Hariharan (2014) of the form:

$$\upsilon_t(x,t) = \alpha\upsilon_{xx}(x,t) + a\upsilon + b\upsilon^{\gamma}, \qquad x \in [L_1, L_2], \quad t \in [0,T] \tag{1}$$

with boundary conditions:

$$\upsilon(L_1,t) = \pi_1(x), \qquad \upsilon(L_2,t) = \pi_2(x)\,,$$
$$\upsilon_x(L_1,t) = \pi_1{}'(x), \quad \upsilon_x(L_2,t) = \pi_2{}'(x), \quad t \geq 0 \tag{2}$$

and initial condition:

$$\upsilon(x,0) = \xi(x) \tag{3}$$

where, $\alpha > 0$, *a, b* are real constants, and γ is a positive integer. The term $\alpha\upsilon_{xx}(x,t)$ represents diffusion or spatial spreading of the amplitude, the linear term $a\upsilon$ gives growth (or decay) of the amplitude near threshold, and the nonlinear term $b\upsilon^{\gamma}$ saturates the growth and creates a non-trivial steady amplitude/pattern. For $\gamma = 3$, Equation (1) becomes the Newell-Whitehead (NW) equation. For $\gamma = 3$ and $\alpha = a = -b = 1$, Equation (1) represents the Allen-Cahn equation. This is utilized in the analysis of phase separation in isothermal, isotropic binary mixtures, such as molten alloys, and is also applied to oil pollution in the ocean environments (Hariharan and Kannan, 2010; Ahmad et al., 2021). Due to its broad applicability across various physical and chemical systems, the NWS equation serves as a powerful framework for analyzing the dynamics of systems near critical instability thresholds.

Multiple approaches have been developed in recent years to obtain both numerical and analytical solutions of the NWS-type equation. Pue-on (2013) applied the Laplace Adomian decomposition method, which is a semi-analytical method for the solution of the NWS equation. Jassim (2015) employed a combination of the Laplace transform method and the homotopy perturbation method to obtain an analytical solution. He's variational iteration method was used by Prakash and Kumar (2016) to find an approximate solution of the NWS equation. So, analytical solutions to the NWS equation have been obtained by numerous researchers (Inan et al., 2020; Nadeem et al., 2020; Areshi et al., 2022). To determine the numerical solution of the NWS equation, Ezzati and Shakibi (2011) employed two numerical methods: Adomian's decomposition and quasi-interpolation technique. Hariharan (2014) applied the Legendre Wavelet-Based method to obtain the numerical solution by converting the problem into an algebraic system of equations. Zahra (2017) implemented a three-time-level implicit finite difference method combined with the trigonometric cubic B-spline collocation method. The scheme is second-order convergent, and the authors employed a θ-weighted approach for both spatial and temporal discretization. Devipriya and Priya (2019) applied the Galerkin finite element method, with the equation discretized using the Crank–Nicolson scheme, to obtain a numerical solution of the NWS equation. Hilal et al. (2020) used two different explicit and fully explicit exponential finite difference methods. Since implicit methods yield nonlinear systems, the authors solved these systems using Newton's method. Iqbal et al. (2023) numerically solved the stochastic NWS equation and investigated its measurable properties. Recently, Gebril et al. (2024) discretized the spatial derivatives using Chebyshev collocation and time-fractional derivatives discretized using finite difference methods to solve the fractional NWS equation. Tarei et al. (2025) obtained numerical solution of generalized NWS equation using Jaiswal-Laguerre collocation method. In such a way, researchers have employed various techniques to obtain numerical solutions of the NWS equation (Angadi and Deshi, 2023; Ray and Chand, 2023; Pathak et al, 2024; Wang et al., 2024).

The concept of the B-spline was originally introduced by Schoenberg (1946). The B-spline collocation technique is often used to solve both linear and nonlinear ordinary and partial differential equations, as B-splines offer the advantage of providing solutions at any point within the interval while ensuring a higher order of convergence. To implement the B-spline collocation technique, special treatment may be needed near boundaries to maintain high-order accuracy, and the nonlinear terms must be linearized to simplify the computation. Numerous researchers have employed the quintic B-spline (QBS) approach to obtain numerical solutions for a wide range of differential equations (Zaki, 2000; Sepehrian and Lashani, 2008; Mittal and Arora, 2010; Mohammadi, 2015; Korkmaz and Dag, 2016; Özer, 2022). The third-order singularly perturbed delay equation, an ordinary differential equation, was solved using the QBS collocation method by Vaid and Arora (2019). Lin (2021) employed QBS technique to find a numerical solution of the third-order singular Emden–Fowler boundary value problem. Eldanaf et al. (2022) applied QBS for space discretization, while the forward difference formula was used for time discretization to solve the Sharma-Tasso-Oliver equation numerically. Recently, Haq et al. (2024) presented the QBS technique to solve the seventh-order KdV equation by converting it into two equations that reduce the order of the equation to fourth order. The authors have used a θ-weighted approach for discretization. The three-dimensional chaotic system has also been solved using QBS combined with the differential quadrature technique by Gupta and Rohila (2024).

With this inspiration, in the present paper, we have proposed a quintic B-spline collocation approach for obtaining a numerical solution of the Newell-Whitehead-Segel type equation, aiming to obtain a more accurate solution with minimum error. The proposed method has been applied to four numerical problems. The absolute errors for the first numerical problem are compared with the results of Hariharan (2014), who obtained numerical solutions of the NWS equation using an efficient Legendre Wavelet-Based method, while the absolute errors for the second numerical problem are compared with the results of Tarei et al. (2025), who obtained the numerical solution of the NWS equation by Jaiswal–Laguerre collocation method. In the third example, maximum absolute errors are compared with the results of Gebril et al. (2024), and in the fourth example, the exact and numerical solutions are compared. The physical importance of the obtained numerical results has also been illustrated through the use of graphical representations. These studies can be useful in many practical science domains, such as fluid dynamics, mechanics, chemistry, and bioengineering.

A brief outline of the remaining sections of the paper is as follows: Section 2 describes the formulation of the quintic B-spline collocation approach, followed by the implementation details are provided in Section 3. Section 4 focuses on the numerical stability analysis, whereas Section 5 presents the numerical experiments that demonstrate the accuracy and efficiency of the proposed technique. The concluding remarks of the work are given in the final Section 6.

2. Formulation of Quintic B-spline

In this section, we have discussed the formation and properties of the quintic B-spline. A piecewise polynomial function can be used to generate the B-spline basis functions by referring to the De Boor (1978) recursion formula. A spline function of order d contains the piecewise polynomial of order $(d-1)$. Quintic B-splines are piecewise-defined quintic polynomial functions of degree five that ensure continuity up to the fourth derivative at the knots. The domain $[L_1, L_2]$ is partitioned uniformly by the knots $x_i,\ i = 0{:}M$ such that $x_i - x_{i-1} = \rho$ is the length of each partition. Uniform knots provide a simple and efficient implementation of the method and reduce computational cost. Now define the constant (zeroth-degree) B-splines:

$$\mathfrak{B}_{i,0}(x) = \begin{cases} 1, & x \in [x_i, x_{i+1}) \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

From Degree-0 B-splines, B-splines of higher degree (d) are defined by the recurrence relation,

$$\mathfrak{B}_{i,d}(x) = \left(\frac{x-x_i}{x_{i+d}-x_i}\right)\mathfrak{B}_{i,d-1}(x) + \left(\frac{x_{i+d+1}-x}{x_{i+d+1}-x_{i+1}}\right)\mathfrak{B}_{i+1,d-1}(x) \tag{5}$$

Now, by using the above recursive relations (4) and (5), obtain the basis function for the quintic B-spline $\mathfrak{B}_{i,5}(x)$ on a uniform grid, refer to Haq et al. (2024).

$$\mathfrak{B}_{i,5}(x) = \frac{1}{120\rho^5}\begin{cases} (x-x_{i-3})^5, & if\ x\epsilon[x_{i-3},\ x_{i-2}) \\ -5(x-x_{i-2})^5 + 5\rho(x-x_{i-2})^4 + 10\rho^2(x-x_{i-2})^3 & \\ \quad +10\rho^3(x-x_{i-2})^2 + 5\rho^4(x-x_{i-2}) + \rho^5, & if\ x\epsilon[x_{i-2},\ x_{i-1}) \\ 10(x-x_{i-1})^5 - 20\rho(x-x_{i-1})^4 - 20\rho^2(x-x_{i-1})^3 & \\ \quad +20\rho^3(x-x_{i-1})^2 + 50\rho^4(x-x_{i-1}) + 26\rho^5, & if\ x\epsilon[x_{i-1},\ x_i) \\ 10(x_{i+1}-x)^5 - 20\rho(x_{i+1}-x)^4 - 20\rho^2(x_{i+1}-x)^3 & \\ \quad +20\rho^3(x_{i+1}-x)^2 + 50\rho^4(x_{i+1}-x) + 26\rho^5, & if\ x\epsilon[x_i,\ x_{i+1}) \\ -5(x_{i+2}-x)^5 + 5\rho(x_{i+2}-x)^4 + 10\rho^2(x_{i+2}-x)^3 & \\ \quad +10\rho^3(x_{i+2}-x)^2 + 5\rho^4(x_{i+2}-x) + \rho^5, & if\ x\epsilon[x_{i+1},\ x_{i+2}) \\ (x_{i+3}-x)^5, & if\ x\epsilon[x_{i+2}, x_{i+3}) \\ 0, & otherwise \end{cases} \tag{6}$$

The set $\{\mathfrak{B}_{-2,5}(x), \mathfrak{B}_{-1,5}(x), \mathfrak{B}_{0,5}(x), \mathfrak{B}_{1,5}(x), \dots, \mathfrak{B}_{M,5}(x), \mathfrak{B}_{M+1,5}(x), \mathfrak{B}_{M+2,5}(x)\}$ construct a basis for the linear space of dimension $M+5$ of all quintic spline functions over the interval $[L_1, L_2]$. The values of $\mathfrak{B}_{i,5}(x)$ and its first three derivatives at the knots are shown in **Table 1** below.

Table 1. Values of $\mathfrak{B}_{i,5}(x)$ and its derivatives.

QBS	x_{i-3}	x_{i-2}	x_{i-1}	x_i	x_{i+1}	x_{i+2}	x_{i+3}
$\mathfrak{B}_{i,5}(x)$	0	$\frac{1}{120}$	$\frac{26}{120}$	$\frac{66}{120}$	$\frac{26}{120}$	$\frac{1}{120}$	0
$\mathfrak{B}'_{i,5}(x)$	0	$\frac{1}{24\rho}$	$\frac{5}{12\rho}$	0	$\frac{-5}{12\rho}$	$\frac{-1}{24\rho}$	0
$\mathfrak{B}''_{i,5}(x)$	0	$\frac{1}{6\rho^2}$	$\frac{1}{3\rho^2}$	$\frac{-1}{\rho^2}$	$\frac{1}{3\rho^2}$	$\frac{1}{6\rho^2}$	0
$\mathfrak{B}'''_{i,5}(x)$	0	$\frac{1}{2\rho^3}$	$\frac{-1}{\rho^3}$	0	$\frac{1}{\rho^3}$	$\frac{-1}{2\rho^3}$	0

3. Implementation of the Proposed Method

This section presents the implementation of the proposed method for obtaining the numerical solution of the NWS-type equation. Now, the approximate solution of $v(x,t)$ can be defined as a linear combination of quintic B-spline basis given in Prenter (2008) as:

$$v(x,t) \approx U(x,t) = \sum_{i=-2}^{M+2} w_i \, \mathfrak{B}_{i,5}(x) \tag{7}$$

where, w_i denotes the time-dependent unknowns, which are determined through the collocation procedure, using boundary and initial conditions specified in Equations (2) - (3).

Now, using Equation (7) and **Table 1**, values of $v(x,t)$ and its first and second derivatives at knots x_i in terms of coefficients are as follows:

$$v = \frac{1}{120}[w_{i-2} + 26w_{i-1} + 66w_i + 26w_{i+1} + w_{i+2}] \tag{8}$$

$$v_x = \frac{1}{24\rho} w_{i-2} + \frac{5}{12\rho} w_{i-1} - \frac{5}{12\rho} w_{i+1} - \frac{1}{24\rho} w_{i+2} \tag{9}$$

$$v_{xx} = \frac{1}{6\rho^2} w_{i-2} + \frac{1}{3\rho^2} w_{i-1} - \frac{1}{\rho^2} w_i + \frac{1}{3\rho^2} w_{i+1} + \frac{1}{6\rho^2} w_{i+2} \tag{10}$$

3.1 Numerical Algorithm

To obtain the numerical solution, the following algorithm is applied.

Step 1: Discretized the spatial domain $[L_1, L_2]$ into equal intervals to create a uniform mesh.

Step 2: Apply the Crank-Nicolson scheme to discretize the problem in the time domain.

Step 3: Use the quasi-linearization technique to linearize the nonlinear terms.

Step 4: Approximate v and its derivatives in the equation obtained in Step 3 using quintic B-splines, thereby converting the problem into a system of linear equations.

Step 5: Use the initial condition, discretize it through quintic B-spline formulation, resulting them into a system of linear equations, which is then solve to find the initial time level numerical solution of the problem.

Step 6: Finally, solve the system in step 4 to compute the numerical solution at subsequent time levels.

To solve the system of linear equations of the form $AW = \beta$, use MATLAB R2024b version by utilizing the $A\backslash\beta$ command.

3.2 Discretization

The time derivative is discretized using FDM, while the space discretization is done using the quintic B-spline function. Here n and $(n+1)$ represent successive time levels. The Crank-Nicolson finite difference scheme is used to discretize Equation (1) as follows:

$$\frac{v^{(n+1)} - v^{(n)}}{\Delta t} = \alpha \frac{v_{xx}^{(n+1)} + v_{xx}^{(n)}}{2} + a \frac{v^{(n+1)} + v^{(n)}}{2} + b \frac{(v^{\gamma})^{(n+1)} + (v^{\gamma})^{(n)}}{2} \tag{11}$$

3.3 Linearization

To linearizing the nonlinear term, the quasi-linearization method proposed by Rubin and Graves (1975) is used, as follows:

$$(v^{\gamma})^{(n+1)} = (v^{\gamma})^n + (v^{n+1} - v^n)\, \gamma \, (v^{\gamma-1})^n \tag{12}$$

3.4 Implementation of Quintic B-spline

Now, simplifying Equation (11) and using Equation (12), we get,

$$\left[1-\frac{\alpha\Delta t}{2}-\frac{\gamma b\Delta t}{2}\left(v^{(n)}\right)^{\gamma-1}\right]v^{(n+1)}-\frac{\alpha\Delta t}{2}v_{xx}^{(n+1)} = \left[1+\frac{\alpha\Delta t}{2}-\frac{\gamma b\Delta t}{2}\left(v^{(n)}\right)^{\gamma-1}\right]v^{(n)}+b\Delta t\left(v^{(n)}\right)^{\gamma}+\frac{\alpha\Delta t}{2}v_{xx}^{(n)} \quad (13)$$

Using Equations (8) - (10) in Equation (13), the solution at $(n+1)^{th}$ level is,

$$\left[1-\frac{\alpha\Delta t}{2}-\frac{\gamma b\Delta t}{2}\left(v_i{}^{(n)}\right)^{\gamma-1}\right]\left(\frac{1}{120}\left[w_{i-2}{}^{n+1}+26w_{i-1}{}^{n+1}+66w_i{}^{n+1}+26w_{i+1}{}^{n+1}+w_{i+2}{}^{n+1}\right]\right) -\frac{\alpha\Delta t}{2}\left(\frac{1}{6\rho^2}w_{i-2}{}^{n+1}+\frac{1}{3\rho^2}w_{i-1}{}^{n+1}-\frac{1}{\rho^2}w_i{}^{n+1}+\frac{1}{3\rho^2}w_{i+1}{}^{n+1}+\frac{1}{6\rho^2}w_{i+2}{}^{n+1}\right)=\beta_i \quad (14)$$

for $i=0{:}M$.

Equation (14) can be written as:

$$\delta_1\, w_{i-2}{}^{(n+1)}+\delta_2\, w_{i-1}{}^{(n+1)}+\delta_3\, w_i{}^{(n+1)}+\delta_2\, w_{i+1}{}^{(n+1)}+\delta_1\, w_{i+2}{}^{(n+1)}=\beta_i \quad (15)$$

for $i=0{:}M$.

where, $\delta_1,\delta_2,\delta_3$ can be calculated from Equation (14).

From Equation (15) we obtain the following system of linear equations,

$$AW^{n+1}=\beta,$$

where,

$$A=\begin{bmatrix}\delta_1 & \delta_2 & \delta_3 & \delta_2 & \delta_1 & \cdots & \cdots & \cdots\\ \cdots & \delta_1 & \delta_2 & \delta_3 & \delta_2 & \delta_1 & \cdots & \cdots\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ \cdots & \cdots & \delta_1 & \delta_2 & \delta_3 & \delta_2 & \delta_1 & \cdots\\ \cdots & \cdots & \cdots & \delta_1 & \delta_2 & \delta_3 & \delta_2 & \delta_1\end{bmatrix}_{(M+1)\times(M+5)},\quad W^{(n+1)}=\begin{bmatrix}w_{-2}\\ w_{-1}\\ w_0\\ w_1\\ \vdots\\ w_{M-1}\\ w_M\\ w_{M+1}\\ w_{M+2}\end{bmatrix}_{(M+5)\times 1},\quad \beta=\begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \vdots\\ \beta_{M-2}\\ \beta_{M-1}\\ \beta_M\end{bmatrix}_{(M+1)\times 1}.$$

In this system, we get $(M+1)$ equations and $(M+5)$ unknowns. Using boundary conditions given in Equation (2), can add four more equations, then the system becomes $(M+5)$ equations with $(M+5)$ unknowns to get a unique solution. The primary objective is to compute a solution $\{w_0{}^{n+1},w_1{}^{n+1},\dots,w_{M-1}{}^{n+1},w_M{}^{n+1}\}$ at the $(n+1)^{th}$ time level. To achieve this, an initial solution W^0 at level 0 is required. Using the initial condition by approximating the B-Spline, we obtain the initial vector W^0, and then we can solve the above system of linear equations by any standard analytical technique at any required time level.

3.5 Initial Vector

Using the initial condition given in Equation (3), the initial vector W^0 is obtained as follows:

$$v(x,0)=\xi(x),$$
$$v_x\left(x_{0,0}\right)=\xi'(x_0),\;\; v_x(x_M,0)=\xi'(x_M),$$
$$v_{xx}\left(x_{0,0}\right)=\xi''(x_0),\;\; v_{xx}(x_M,0)=\xi''(x_M).$$

Discretizing the above equations with B-splines yields the following system:

$$A^0 W^0 = \xi^0,$$

where,

$$A^0 = \begin{bmatrix} 1/24\rho & 5/12\rho & 0 & -5/12\rho & 1/24\rho & & \cdots & \cdots & \cdots \\ 1/6\rho^2 & 1/3\rho^2 & -1/\rho^2 & 1/3\rho^2 & 1/6\rho^2 & & \cdots & \cdots & \cdots \\ 1/120 & 26/120 & 66/120 & 26/120 & 1/120 & \cdots & \cdots & \cdots & \\ 0 & 1/120 & 26/120 & 66/120 & 26/120 & 1/120 & & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ \cdots & \cdots & 1/120 & 26/120 & 66/120 & 26/120 & 1/120 & & \cdots \\ \cdots & \cdots & \cdots & 1/120 & 26/120 & 66/120 & 26/120 & 1/120 & \\ \cdots & \cdots & \cdots & 1/6\rho^2 & 1/3\rho^2 & -1/\rho^2 & 1/3\rho^2 & 1/6\rho^2 & \\ \cdots & \cdots & \cdots & 1/24\rho & -5/12\rho & 0 & 5/12\rho & 1/24\rho & \end{bmatrix}_{(M+3)\times(M+3)}$$

$$\boldsymbol{W^0} = \begin{bmatrix} w_{-2} \\ w_{-1} \\ w_0 \\ w_1 \\ \vdots \\ w_{M-1} \\ w_M \\ w_{M+1} \\ w_{M+2} \end{bmatrix}_{(M+3)\times 1} \qquad \xi^0 = \begin{bmatrix} \xi'(x_0) \\ \xi''(x_0) \\ \xi(x_0) \\ \xi(x_1) \\ \vdots \\ \xi(x_{M-1}) \\ \xi(x_M) \\ \xi''(x_0) \\ \xi'(x_M) \end{bmatrix}_{(M+3)\times(M+3)}.$$

Now, solving the above system by using any analytical technique, we get the initial vector W^0.

4. Stability Analysis

We have used the Von Neumann method to analyse the stability of the proposed technique.

Consider Equation (13) with rewriting, we get,

$$\left[1 - \frac{\alpha\Delta t}{2} - \frac{\gamma b\Delta t}{2}\left(v^{(n)}\right)^{\gamma-1}\right] v^{(n+1)} - \frac{\alpha\Delta t}{2} v_{xx}^{(n+1)} = \left[1 + \frac{\alpha\Delta t}{2} - \frac{(\gamma-2)\, b\Delta t}{2}\left(v^{(n)}\right)^{\gamma-1}\right] v^{(n)} + \frac{\alpha\Delta t}{2} v_{xx}^{(n)} \tag{16}$$

Let, $\left(v^{(n)}\right)^{\gamma-1} = z$ be assumed as locally constant for the solution at the previous time level, then Equation (16) become,

$$\left[1 - \frac{\alpha\Delta t}{2} - \frac{\gamma b\, \Delta t\, z}{2}\right] v^{(n+1)} - \frac{\alpha\Delta t}{2} v_{xx}^{(n+1)} = \left[1 + \frac{\alpha\Delta t}{2} - \frac{(\gamma-2)\, b\, \Delta t\, z}{2}\right] v^{(n)} + \frac{\alpha\Delta t}{2} v_{xx}^{(n)} \tag{17}$$

Assume,

$$\left[1 - \frac{\alpha\Delta t}{2} - \frac{\gamma b\Delta t\, z}{2}\right] = d_1,$$
$$\left[1 + \frac{\alpha\Delta t}{2} - \frac{(\gamma-2)\, b\, \Delta t\, z}{2}\right] = d_2.$$

We get Equation (17) as,

$$d_1 v^{(n+1)} - \frac{\alpha\Delta t}{2} v_{xx}^{(n+1)} = d_2 v^{(n)} + \frac{\alpha\Delta t}{2} v_{xx}^{(n)} \tag{18}$$

Now, using B-Spline approximation in Equation (18), we get,

$$d_1 \left\{\frac{1}{120}\left[w_{i-2}{}^{(n+1)} + 26w_{i-1}{}^{(n+1)} + 66w_i{}^{(n+1)} + 26w_{i+1}{}^{(n+1)} + w_{i+2}{}^{(n+1)}\right]\right\} - \frac{\alpha\Delta t}{2}\left\{\frac{1}{6\rho^2} w_{i-2}{}^{(n+1)} + \frac{1}{3\rho^2} w_{i-1}{}^{(n+1)} - \frac{1}{\rho^2} w_i{}^{(n+1)} + \frac{1}{3\rho^2} w_{i+1}{}^{(n+1)} + \frac{1}{6\rho^2} w_{i+2}{}^{(n+1)}\right\}$$
$$= d_2 \left\{\frac{1}{120}\left[w_{i-2}{}^{(n)} + 26w_{i-1}{}^{(n)} + 66w_i{}^{(n)} + 26w_{i+1}{}^{(n)} + w_{i+2}{}^{(n)}\right]\right\} + \frac{\alpha\Delta t}{2}\left\{\frac{1}{6\rho^2} w_{i-2}{}^{(n)} + \frac{1}{3\rho^2} w_{i-1}{}^{(n)} - \frac{1}{\rho^2} w_i{}^{(n)} + \frac{1}{3\rho^2} w_{i+1}{}^{(n)} + \frac{1}{6\rho^2} w_{i+2}{}^{(n)}\right\} \tag{19}$$

Assume, $y = \frac{\alpha\Delta t}{2\rho^2}$, Equation (19) becomes,

$$\left(\frac{d_1}{120} - \frac{y}{6}\right) w_{i-2}{}^{(n+1)} + \left(\frac{26d_1}{120} - \frac{y}{3}\right) w_{i-1}{}^{(n+1)} + \left(\frac{66d_1}{120} + y\right) w_i{}^{(n+1)} + \left(\frac{26d_1}{120} - \frac{y}{3}\right) w_{i+1}{}^{(n+1)} + \left(\frac{d_1}{120} - \frac{y}{6}\right) w_{i+2}{}^{(n+1)} = \left(\frac{d_2}{120} + \frac{y}{6}\right) w_{i-2}{}^{(n)} + \left(\frac{26d_2}{120} + \frac{y}{3}\right) w_{i-1}{}^{(n)} + \left(\frac{66d_2}{120} - y\right) w_i{}^{(n)} + \left(\frac{26d_2}{120} + \frac{y}{3}\right) w_{i+1}{}^{(n)} + \left(\frac{d_2}{120} + \frac{y}{6}\right) w_{i+2}{}^{(n)} \tag{20}$$

Now, substituting $w_j^n = A\lambda^n \, exp(ij\eta\rho)$, in Equation (20), A is the amplitude, $i = \sqrt{-1}$, ρ is the length of each step and η represents the mode number, then we get,

$$\lambda = \frac{\left(\frac{d_2}{120}+\frac{y}{6}\right)e^{-2\eta\rho i}+\left(\frac{26d_2}{120}+\frac{y}{3}\right)e^{-\eta\rho i}+\left(\frac{66d_2}{120}-y\right)+\left(\frac{26d_2}{120}+\frac{y}{3}\right)e^{\eta\rho i}+\left(\frac{d_2}{120}+\frac{y}{6}\right)e^{2\eta\rho i}}{\left(\frac{d_1}{120}-\frac{y}{6}\right)e^{-2\eta\rho i}+\left(\frac{26d_1}{120}-\frac{y}{3}\right)e^{-\eta\rho i}+\left(\frac{66d_1}{120}+y\right)+\left(\frac{26d_1}{120}-\frac{y}{3}\right)e^{\eta\rho i}+\left(\frac{d_1}{120}-\frac{y}{6}\right)e^{2\eta\rho i}} \tag{21}$$

After simplification of Equation (21), we obtain,

$$\lambda = \frac{2\left(\frac{d_2}{120}+\frac{y}{6}\right)cos(2\eta\rho)+2\left(\frac{26d_2}{120}+\frac{y}{3}\right)cos(\eta\rho)+\left(\frac{66d_2}{120}-y\right)}{2\left(\frac{d_1}{120}-\frac{y}{6}\right)cos(2\eta\rho)+\left(\frac{26d_1}{120}-\frac{y}{3}\right)cos(\eta\rho)+\left(\frac{66d_1}{120}+y\right)} \tag{22}$$

Let,

$$\lambda = \frac{P}{Q}.$$

The proposed method will be stable if $|\lambda| < 1$, i.e. $\left|\frac{P}{Q}\right| < 1$

Or

$$-1 < \frac{P}{Q} < 1$$

i.e., if $P + Q > 0 \; and \; Q - P > 0$.

Now,

$$P + Q = \frac{d_1+d_2}{60}\left[cos(2\eta\rho) + 26\,cos(\eta\rho) + 33\right] \tag{23}$$

$$Q - P = \left(\frac{d_1-d_2}{60} - \frac{y}{3}\right) cos(2\eta\rho) + \left(26\frac{d_1-d_2}{60} - \frac{y}{3}\right) cos(\eta\rho) + 33\frac{d_1-d_2}{60} + 2y \tag{24}$$

As, $d_1 + d_2 = 2 - (\gamma - 1)b\Delta t\, z$ and $d_1 - d_2 = -(a\Delta t + b\Delta t\, z)$,

Equation (23) and Equation (24) can be written as:

$$P + Q = \frac{2-(\gamma-1)b\,z\,\Delta t}{60}[cos(2\eta\rho) + 26\,cos(\eta\rho) + 33] \tag{25}$$

$$Q - P = \frac{2\alpha\Delta t}{\rho^2}sin^2(\eta\rho) - \frac{(a+bz)\Delta t}{30}[17 + 13\,cos(\eta\rho) - sin^2(\eta\rho)] + (5 - cos(\eta\rho))\frac{\alpha\Delta t}{\rho^2} \tag{26}$$

Now, for $\gamma > 1$,

If $bz \leq 0\ and\ (a + bz) \leq 0$, then $P + Q > 0$ and $Q - P > 0$ is always true.
If $bz > 0$ and $(a + bz) > 0$,
For $P + Q > 0$, required $2 - (\gamma - 1)b\,z\,\Delta t > 0$, we get,

$$\text{i.e}\ \ \Delta t < \frac{2}{(\gamma-1)b\,z},$$

For $Q - P > 0$, required $\frac{2\alpha\Delta t}{\rho^2}sin^2(\eta\rho) - \frac{(a+bz)\Delta t}{30}[17 + 13\,cos(\eta\rho) - sin^2(\eta\rho)] > 0$

$$\text{i.e.}\quad \rho^2 < \frac{60\,\alpha\,sin^2(\eta\rho)}{[17 + 13\,cos(\eta\rho) - sin^2(\eta\rho)]\,(a + bz)}$$

Finally, the proposed method is unconditionally stable if $bz \leq 0$ and $(a + bz) \leq 0$, otherwise conditionally stable under the conditions

$$\Delta t < \frac{2}{(\gamma-1)b\,z} \quad and\ \rho < \left(\frac{60\,\alpha\,sin^2(\eta\rho)}{[17+13\,cos(\eta\rho)-sin^2(\eta\rho)]\,(a+bz)}\right)^{\frac{1}{2}}.$$

5. Numerical Examples

In this section, to illustrate the accuracy and applicability of the proposed method, some particular cases are presented. The error is measured as the absolute difference between the numerical solution produced by the proposed method and the exact solution. The Absolute Error (AE) is expressed as:

$$AE(x_i, t) = |\upsilon(x_i, t) - U(x_i, t)|,$$

and the discrete maximum absolute norm (L_∞) is expressed as:

$$L_\infty = |\upsilon(x, t) - U(x, t)|_\infty = \max_{\forall x_i}|\upsilon(x_i, t) - U(x_i, t)|$$

where, $\upsilon(x_i, t)$ is the exact solution and $U(x_i, t)$ is an approximate solution. For computational purposes, we have used MATLAB R2024b version as the software tool on an HP Pavilion system equipped with an Intel Core i5 7th-generation processor and 8 GB of RAM.

Example 1: Consider the Newell-Whitehead-Segel Equation (1) with $\alpha = 1, a = 1, b = -1$ and $\gamma = 3$, the reduced equation is also known as the Allen-Cahn equation (Hariharan, 2014).

$$\upsilon_t(x, t) = \upsilon_{xx}(x, t) + \upsilon - \upsilon^3,$$

with initial condition:

$$\upsilon(x, 0) = -0.5 + 0.5\,tanh(0.3536x).$$

The exact solution is,

$$v(x,t) = -0.5 + 0.5\,tanh(0.3536x - 0.75t).$$

For Example 1, the obtained results are compared with the existing ones at various values of t and x, as presented in **Table 2**. The absolute errors from the proposed method are compared with those obtained using the Legendre Wavelet method (Hariharan, 2014). The results reveal that the proposed approach provides a closer agreement with the exact solution and achieves better accuracy than the existing technique. **Figure 1** illustrates the visual comparison between the exact and numerical solutions of Example 1 for different time levels of t $(t = 0.1, 0.5, 1, 2)$, with $\rho = 1,\ \Delta t = 0.1$ over the spatial domain $x \in [-15, 15]$.

Table 2. Comparison of the absolute errors by the present method for Example 1 ($M = 400, \rho = 0.01, \Delta t = 10^{-3}$).

x	AE, Hariharan (2014)	AE, Present Method	AE, Hariharan (2014)	AE, Present Method	AE, Hariharan (2014)	AE Present Method
	$t = 0.1$		$t = 0.3$		$t = 0.5$	
-25	1.1894×10^{-11}	3.2995×10^{-13}	7.2784×10^{-12}	7.5317×10^{-13}	6.474×10^{-12}	9.3969×10^{-13}
-15	2.3663×10^{-09}	2.7995×10^{-10}	9.3646×10^{-10}	6.1775×10^{-10}	1.3565×10^{-10}	7.3511×10^{-10}
25	9.8474×10^{-10}	3.2196×10^{-13}	8.4646×10^{-09}	1.2944×10^{-12}	7.4992×10^{-10}	2.8064×10^{-12}
30	3.5757×10^{-11}	9.3988×10^{-15}	7.4848×10^{-10}	3.7730×10^{-14}	2.4444×10^{-10}	8.1764×10^{-14}
Computational Time /Sec.	-	0.184595	-	0.575709	-	0.872507

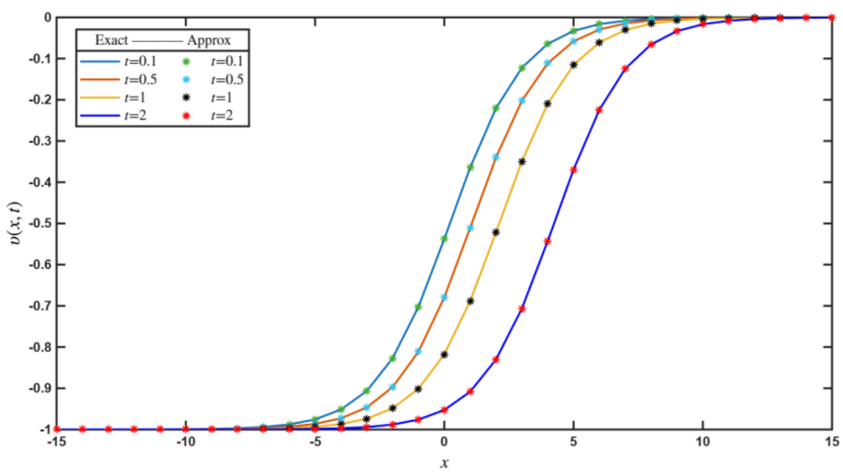

Figure 1. Exact and numerical solution by the present method ($M = 40,\ h = 1,\ \Delta t = 0.1$) for Example 1.

Example 2: Consider the NWS Equation (1) with $\alpha = 1, a = 3,\ b = -4$ and $\gamma = 3$ (Hariharan, 2014; Tarei et al., 2025).

$$v_t(x,t) = v_{xx}(x,t) + 3v - 4v^3,$$

with initial condition:

$$v(x,0) = \sqrt{\frac{3}{4}}\,\frac{e^{\sqrt{6}\,x}}{e^{\sqrt{6}\,x} + e^{\frac{\sqrt{6}}{2}x}},$$

The exact solution in closed form is,

$$v(x,t) = \sqrt{\frac{3}{4}}\left(\frac{e^{\sqrt{6}\,x}}{e^{\sqrt{6}\,x} + e^{\left(\frac{\sqrt{6}}{2}x - \frac{9}{2}t\right)}}\right).$$

For Example 2, the obtained results are compared with the existing ones at various values of t and over the spatial domain $x \in [0,1]$ with $\rho = 0.1$, as presented in **Table 3**. The absolute errors from the proposed method are compared with those obtained using the Jaiswal–Laguerre collocation method (Tarei et al., 2025). The results reveal that the proposed approach provides a closer agreement with the exact solution and achieves better accuracy than the existing technique. **Figure 2** illustrates the visual comparison between the exact and numerical solutions of Example 2 for different time levels of t, $(t = 1,\ 2,\ 5)$, with $\rho = 1, \Delta t = 0.01$ over the spatial domain $x \in [-30, 5]$.

Table 3. Comparison of the absolute errors by the present method for Example 2 ($M = 10,\ \rho = 0.1,\ x \in [0,1]$).

x	AE, Tarei et al. (2025)	AE, Present Method	AE, Tarei et al. (2025)	AE, Present Method
	$t = 0.2,\ \Delta t = 10^{-4}$		$t = 1,\ \Delta t = 10^{-2}$	
0.2	1.1873×10^{-06}	7.9484×10^{-07}	1.6373×10^{-04}	3.0869×10^{-07}
0.6	8.4248×10^{-05}	1.316110^{-06}	8.3394×10^{-04}	3.8920×10^{-07}
0.8	6.3112×10^{-05}	7.6280×10^{-07}	6.4800×10^{-04}	2.2984×10^{-07}
Computational Time /Sec.	-	0.051859	-	0.028366

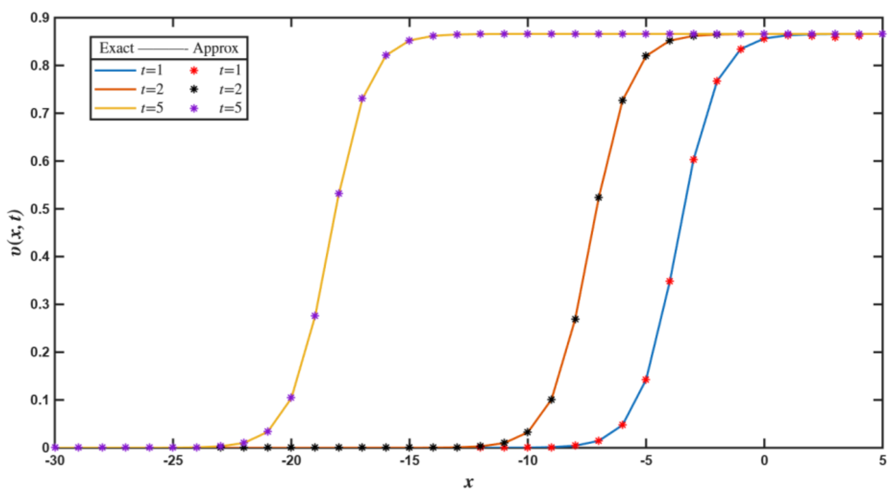

Figure 2. Exact and numerical solution by the present method ($M = 35, h = 1, \Delta t = 0.01$) for Example 2.

Example 3: Consider the NWS Equation (1) with $\alpha = 1, a = 2, b = -3$ and $\gamma = 2$ (Gebril et al., 2024).

$$v_t(x,t) = v_{xx}(x,t) + 2v - 3v^2,$$

with initial condition:

$$v(x,0) = \frac{2}{3}\left(1 + e^{\frac{x}{\sqrt{3}}}\right)^{-2}.$$

The exact solution is,

$$v(x,t) = \frac{2}{3}\left(1 + e^{\left(\frac{1}{\sqrt{3}}x - \frac{5}{3}t\right)}\right)^{-2}.$$

The L_∞ errors computed using the proposed method are compared with those obtained using the Chebyshev collocation method (Gebril et al., 2024) and presented in **Table 4** for Example 3. The results indicate that the proposed method aligns well with the existing technique while achieving better accuracy than the existing technique. Furthermore, **Figure 3** illustrates the comparison between the exact and numerical solutions of Example 3 for different time levels t $(t = 0.5,\ 1,\ 2)$, with $\rho = 1, \Delta t = 0.01$ over the spatial domain $x \in [-10, 10]$.

Table 4. Comparison of the L_∞ errors by the present method for Example 3 $(M = 10, \rho = 0.1, \Delta t = 0.05, x \in [0, 1])$.

t	L_∞, Gebril et al. (2024)	L_∞, Present Method	Computational time /sec.
0.2	6.451×10^{-04}	2.1624×10^{-05}	0.010110
0.4	5.484×10^{-04}	2.8308×10^{-05}	0.013851
0.8	1.619×10^{-04}	3.1557×10^{-05}	0.021114

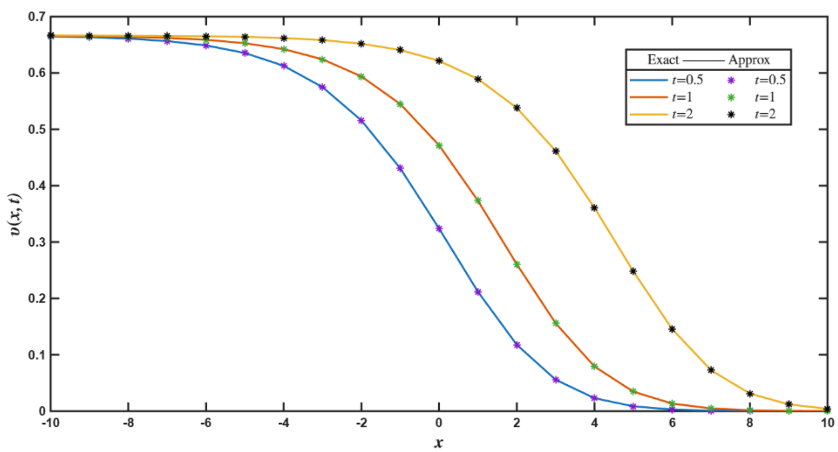

Figure 3. Exact and numerical solution by the present method $(M = 20,\ h = 1,\ \Delta t = 0.01)$ for Example 3.

Example 4: Consider the NWS Equation (1) with $\alpha = 1, a = 1, b = -1$ and $\gamma = 4$ (Hariharan, 2014).

$$v_t(x,t) = v_{xx}(x,t) + v - v^4,$$

with initial condition:

$$v(x,0) = \left(1 + e^{\frac{3x}{\sqrt{10}}}\right)^{-\frac{2}{3}}.$$

Exact solution is,

$$v(x,t) = \left(0.5 + 0.5\tanh\left(-\frac{3}{2\sqrt{10}}\left(x - \frac{7}{\sqrt{10}}t\right)\right)\right)^{\frac{2}{3}}.$$

Table 5 shows the comparison of the exact and numerical solutions for Example 4 over domain $x \in [-1, 1]$ at $t = 1$ with $\Delta t = 0.001, M = 20$ also, absolute errors are evaluated. It has been observed that the present method has great agreement with the exact solution. **Figure 4** provides a visual surface area of $\upsilon(x, t)$ of Example 4 by the present method at different values of $t \in [0, 1]$ over the space domain $x \in [-20, 20]$ with $\Delta t = 0.01, M = 40$.

Table 5. Absolute errors by the present method and comparison of numerical with exact solutions for Example 4 ($M = 20, \rho = 0.1, t = 1, \Delta t = 0.001, x \in [-1, 1]$).

x	Exact solution	Solution present method	AE, present method
-0.8	0.963518087973119	0.963518096568342	8.5952×10^{-09}
-0.6	0.956309356451123	0.956309373453088	1.7002×10^{-08}
-0.4	0.947772165608420	0.947772189786587	2.4170×10^{-08}
-0.2	0.937701719806424	0.937701749616564	2.9810×10^{-08}
0	0.925877802808888	0.925877836289203	3.3480×10^{-08}
0.2	0.912070273354114	0.912070307959848	3.4606×10^{-08}
0.4	0.896047515357764	0.896047547807118	3.2449×10^{-08}
0.6	0.877588255292332	0.877588281460749	2.6168×10^{-08}
0.8	0.856496841365324	0.856496856368054	1.5003×10^{-08}

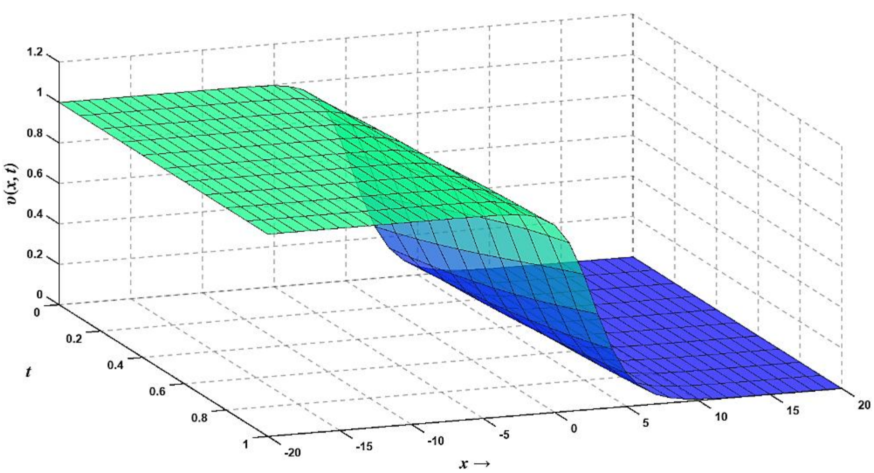

Figure 4. Surface area (Numerical solution) by the present method ($M = 40, h = 1, \Delta t = 0.01$) for Example 4.

6. Conclusion

The present study is devoted to obtaining numerical solutions to the nonlinear Newell-Whitehead-Segel type equation to examine the effectiveness and stability of the quintic B-spline collocation method combined with the Crank-Nicolson scheme. For the proposed method, stability analysis is carried out using the Von Neumann approach. The method is found to be unconditionally stable for specific arbitrary constants a and b; otherwise, it is conditionally stable, and the corresponding stability conditions are determined. To assess the applicability of the proposed method, the scheme is applied to four illustrative numerical examples. The efficiency and accuracy of the method are further verified through a comparative analysis of the obtained results with those available in the existing literature, revealing a close agreement and better performance. Also, the proposed method shows excellent agreement with the exact solution. The quintic B-spline collocation method provides accurate solutions with minimal effort. The accuracy is visually confirmed in **Figures 1-4** at various time levels. The comparisons and corresponding errors are presented in **Tables 2-5** at various time levels. The results demonstrate that the proposed numerical scheme is more accurate, well-founded and can be applied to linear and non-linear PDEs occurring in the engineering and science domains. Future research could be focus on extending this methodology to nonlinear coupled systems of PDEs, providing accurate and efficient solutions for more complex and interconnected problems. Additionally, the proposed method may be implemented using various mesh structures (e.g., Shishkin mesh) to achieve higher accuracy at specific points or within localized regions of the computational domain.

Conflicts of Interest

The authors have confirmed that there are no conflicts of interest to report with this research.

Acknowledgments

The authors sincerely thank the Editor-in-Chief, the handling editor, and the anonymous referees for their valuable comments, insightful suggestions, and constructive feedback which have significantly improved the quality and clarity of this manuscript. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

AI Disclosure

The author(s) declare that no assistance is taken from generative AI to write this article.

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