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Solving Fuzzy Fractional Assignment Problem using Genetic Approach

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Abstract
This manuscript highlights solving the fractional assignment problems (FAP) with parameters as triangular fuzzy numbers. The following is an explanation of the key contribution of the planned study. FAP is an AP with the ratio of objective functions. Of these objective functions, one objective function is minimization whereas the other function is maximization. Now, the fuzzy FAP is changed into a deterministic problem using the α-cut of fuzzy linear membership function on each parameter and then solved using a genetic algorithm (GA) procedure. The procedure so obtained is solved to obtain the set of efficient/non-efficient solutions and the optimal compromise solution. A numerical example is illustrated to explore the efficiency of our proposed genetic approach. A comparative study has been done between the proposed approach and the genetic algorithm tool (Matlab). This approach assists the decision-makers (DM) in selecting a preferred solution based on their economic level.

Keywords- Fractional assignment problem, Fuzzy linear membership function, Genetic approach, Efficient solutions, Optimal compromise solution.

1. Introduction
In the field of Operations research, the assignment problem is a well-studied domain. Various assignment models exist in a variety of forms in today's competitive environment and everyone intends to use the greatest resources available to optimize the activities. AP has a wide range of applications in medicine, training, mailing, education, and athletics. The AP aims to find the best way to assign a number of sources to an equal number of destinations at the lowest feasible cost or highest feasible profit. Different approaches, such as fuzzy programming, GA and neural network, have been adopted to solve AP. This article employs the genetic approach. GA are search-based algorithms that have been widely used in reality and have been effectively applied to solve AP. FAP is an extension of a linear AP in which the objective function is expressed as a ratio with linear constraints. Many practical optimization problems use quotients of two functions as objective functions to achieve the highest ratio of outcome to cost or profit to time, profit to cost, average output to workers, return to cost, actual cost to typical cost, and minimizing inventory levels to sales, cost to time, academic to instructor ratios, and so on, where the ratio represents the highest efficiency of a system. FAP is closely related to the way of life, the range and locations of activity, and the commodities and services that will be available for consumption. Our main goal is to discover the efficient solutions and optimal compromise solution in the presence of incomplete information. We have developed genetic approach that produce an effective solution to FAP with fuzzy parameters and can assist society with logistics challenges.

2. Literature Review

Fractional programming is modelled on parameters, decision variables and an objective function in rational form, subject to various forms of constraints. Fractional programming problems are being used by numerous researchers in a variety of domains. Charnes and Cooper (1962), have defined and solved different challenges as fractional problems using linear fractional functions. Joshi et al. (2011) studied a linear fractional transportation problem with varying demand and supply. Veeramani et al. (2021) solved the multiobjective fractional transportation problem through the neutrosophic goal programming approach. Pramy and Islam (2017) determined efficient solutions of multi-objective linear fractional programming problems and application. Due to unpredictable circumstances, parameter values may not be known precisely in real-world applications. So, fuzzy set theory is used in the literature to the fractional assignment problems. Liu (2016) proposed fractional transportation problem with fuzzy parameters. Garg et al. (2021) studied fractional two-stage transshipment problem under uncertainty and explained the application of the extension principle approach. GAs can tackle large-scale problem in general, owing to the simultaneous exploration of various sections of a search space, as long as "sufficient" time is available. Many researchers have applied genetic approach to solve AP. Holland (1975) proposed GA, which is based on the idea of natural selection as a biological process. Holland and his understudies have made significant contributions to the expansion of the area. Savic et al. (2008) suggested GA approach for solving the task AP. Chu (1997) proposed GA apporach for the generalised AP. Sosa (2020) applied GA in fuzzy multi-objective transportation problem. Researchers have attempted various methods to tackle those types of fractional problems. Gessesse et al. (2019) solved multi-objective linear fractional stochastic transportation problems involving normal distribution using simulation-based genetic algorithm. Gessesse et al. (2020) proposed a novel programming solution based on a genetic algorithm for solving a multi-objective linear fractional stochastic transportation problem with a four-parameter Burr distribution. Dutta et al. (2020) suggested a FPA based on ga for multi-objective fuzzy stochastic routing and hazardous waste siting. (Dutta et al., 2016) proposed a fuzzy stochastic TP with continuous random variables based on a genetic approach. Bhunia et al. (2017) presented a GA-based technique to solving the unbalanced AP in an interval domain.

The main goal of this paper is to use the genetic approach to solve a fuzzy fractional assignment problem (FFAP). Addressing the ratio of two objectives to achieve the set of efficient/non-efficient solutions and the optimal compromise solution is a common challenge. The fuzzy FAP is now turned into a deterministic problem by applying the α-cut of the fuzzy linear membership function to each parameter and then solved using a genetic algorithm (GA) procedure. The GA is a popular evolutionary algorithm in both research and commerce. There are three operators in it: selection, crossover, and mutation. The primary focus of this paper is to implement GA with the following operators to solve the problem, as an encoding scheme, as well as an evaluvation, ranking selection method, partially matched crossover (PMX) and scramble mutation. The fitness value of the resultant offspring is the set of efficient/non-efficient solutions. From the efficient/non-efficient solutions, we have to find the optimal compromise solution. The working principle of the proposed method is demonstrated with a numerical example, and its effectiveness is compared with that well-known software. It is observed that the proposed algorithm provides a better solution while requiring less processing complexity. The proposed approach assists DM in selecting a suitable solution based on their economic location.

The rest of our article is organised as follows. In section 3, preliminaries to do this work is discussed. Section 4 deals with the mathematical model of FFAP. The study of our proposed genetic approach is presented in Section 5, which includes a numerical example and discussion of the results. Lastly, Section 6 summarizes the conclusion of this study.

3. Preliminaries

The definitions of the fuzzy set, fuzzy number, triangular fuzzy number and arithmetic operations on the triangular fuzzy number which can be found in (Zimmermann, 1978).

Definition 3.1 Feasible Solution (Abd El-Wahed & Lee, 2006).

A set $X^* = \{x_{ij}^*, i = 1, 2, ..., \text{n}; j = 1, 2, ..., n\}$ is said to be feasible to the problem (P) if X^* satisfies all the constraints.

Definition 3.2 Efficient Solution (Abd El-Wahed & Lee, 2006)**.**

A feasible solution X^* is said to be an efficient solution to the problem (P) if there exists no other feasible *X* of biobjective AP (BOAP) such that $\text{Z}_1(\text{X}) \le \text{Z}_1(\text{X}^*)$ and $\text{Z}_2(\text{X}) < \text{Z}_2(\text{X}^*)$ (or) $\text{Z}_2(\text{X}) \le \text{Z}_2(\text{X}^*)$ and $\text{Z}_1(\text{X}) < \text{Z}_1(\text{X}^*)$ Otherwise, it is called non-efficient solution to the problem (P). For simplicity, a pair $(\text{Z}_1(\text{X}^*), \text{Z}_2(\text{X}^*))$ is called an efficient / a non-efficient solution to the problem (P) if $\tilde{\text{X}}^0$ is efficient / non-efficient solution to the problem (P).

Definition 3.3 An Optimal Compromise Solution (Abd El-Wahed & Lee, 2006).

$(Z_1(\text{R}), Z_2(\text{S}))$ is an efficient solution that is closest to the ideal solution $(Z_1(\text{X}), Z_2(\text{X}))$ where $Z_1(\text{X})$ is an optimal solution to the first objective problem with all constraints and $Z_2(\text{X})$ is an optimal solution of the second objective problem with all constraints.

Definition 3.4 Linear Membership Function (Behera & Nayak, 2011)

$$\text{It can be defined as, } \mu_{x_i}(\text{X}) = \begin{cases} 0 & \text{if } x_{ij} < \underline{x}_{ij} \\ \dfrac{\overline{x}_{ij} - x_{ij}}{\overline{x}_{ij} - \underline{x}_{ij}} & \text{if } \underline{x}_{ij} < x_{ij} < \overline{x}_{ij} \\ 1 & \text{if } x_{ij} > \overline{x}_{ij} \end{cases} .$$

The transformation of the fuzzy system to α -cut for the linear membership function is defined as

$$\frac{\overline{x}_{ijk} - x_{ijk}}{\overline{x}_{ijk} - \underline{x}_{ijk}} = \alpha \text{, such that } x_{ijk} = (1-\alpha)\overline{x}_{ijk} + \alpha \underline{x}_{ijk}, \text{ for all } \alpha \in [0,1].$$

4. Mathematical Formulation

Consider n resources (i = 1, 2,…,n) to be executed by n activities (j = 1, 2,…, n). The following are the notations that are involved, $\tilde{c}_{ij}$ cost of assigning i^{th} resource to j^{th} activity, $\tilde{d}_{ij}$ profit associated with assignment of i^{th} resource to j^{th} activity. The goal of the assignment problem is to allocate each activity to one and only one resource in such a way that each resource is covered by someone (specifically by one activity) and assignment efficiency is minimized for the ratio between cost and profit. As a result, a fuzzy fractional assignment problem with objective functions can be expressed as follows:

4.1 Fuzzy Fractional Assignment Problem (FFAP)

The mathematical model of FFAP is shown below.

$$(\mathrm{F})\ \text{Minimize } \tilde{z} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}\tilde{x}_{ij}}{\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{d}_{ij}\tilde{x}_{ij}} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n}\tilde{x}_{ij} = 1,\ i = 1,2,\dots,n \qquad (2)$$

$$\sum_{i=1}^{n}\tilde{x}_{ij} = 1,\ \mathrm{j} = 1,2,\dots,n, \qquad (3)$$

$$\tilde{x}_{ij} = \{0,1\} \quad \text{for all i and j} \qquad (4)$$

Applying α -cut for the fuzzy linear membership function in the problem (F), we get

$$(\mathrm{P})\ \text{Minimize z} = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{c}_{ij}(1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij})}{\sum_{i=1}^{n}\sum_{j=1}^{n}d_{ij}(1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij})}, \qquad 0 \le \alpha \le 1 \qquad (5)$$

Subject to

$$\sum_{j=1}^{n}((1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}) = 1,\ i = 1,2,\dots,n, \qquad (6)$$

$$\sum_{i=1}^{n}((1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}) = 1,\ j = 1,2,\dots,n, \qquad (7)$$

$$x_{ij} = \{0,1\} \quad \text{for all i and j}. \qquad (8)$$

Convert the problem (P) into BOAP in which one objective function is minimization (P_1) and another one is maximization (P_2).

$$(\mathrm{P}_1)\ \text{Minimize N} = \sum_{i=1}^{n}\sum_{j=1}^{n}\mathrm{c}_{ij}(1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}), \qquad (9)$$

Subject to the constraint (6) -(8).

$$(\mathrm{P}_2)\ \text{Maximize D} = \sum_{i=1}^{n}\sum_{j=1}^{n}d_{ij}(1-\alpha)\overline{x}_{ij} + \alpha\underline{x}_{ij}), \qquad (10)$$

Subject to the constraint (6) -(8).

5. Method of Optimization

This section includes a method of optimization and numerical example to demonstrate the efficacy and applicability of our FFAP study. To solve their deterministic counterparts, GA mathematical programming models were implemented manually to find the set of efficient/non efficient solutions and optimal compromise solution for the proposed FFAP models. The results are then compared to GA (Matlab solver). The working principle of the proposed genetic approach is summarized as follows.

Step 1: Check if the given FFAP is balanced. If not, add a dummy row or column. The costs of assigning dummy cells are always zero.

Step 2: Construct a deterministic fractional assignment problem (P) using the Definition 3.4 in the given problem (F).
Step 3: Convert the problem (P) into BOAP in which one objective function is minimization (P_1) and another one is maximization (P_2).

Step 4: Substitute any one of the alpha values ranging from 0 to 1 (say alpha=0) and then obtain an optimal solution to the problems P_1 and P_2 by the existing method. To create a payoff matrix, interchange the optimal solution in P_1 in P_2 and, P_2 in P_1 as a feasible solution which becomes an efficient solution to the problem BOAP.

Step 5: Encode the chromosomes for the problem P_1 and P_2 (see Section 5.1.1 for details).

Step 6: Generate chromosomes to initialize the generation, $gen=0$ (see Section 5.1.2).

Step 7: Evaluate fitness value of each chromosome (see Section 5.1.3).

Step 8: Apply the ranking selection method and assign a rank to each chromosome based on its fitness value (see Section 5.1.4).

Step 9: Select two chromosomes as parents for PMX crossover based on their rank, followed by a scramble mutation to produce offspring (see Section 5.1.5).

Step 10: Replace the old offspring ($gen=0$) with the latest one ($gen = gen + 1$) and repeat Step 4 to Step 7 for new generation to get the new offspring and proceed to the successive generations.

Step 11: Terminate the algorithm, if the subsequent generations have the same sets of offspring. Collect the offspring from the last generation and evaluate their fitness value which is an efficient / non- efficient solution to the BOAP.

Step 12: Combine all solutions (efficient / non efficient) of BOAP obtained from step 11. From this, a set of efficient solutions and a set of non-efficient solutions to the P can be obtained.

Step 13: To obtain the set of efficient/ non-efficient solutions and optimal compromise solution of BOAP for different values of alpha ranging from 0 to 1, repeat step 4 to step 12.

To have a quick reference of the work sequence in genetic approach, the flow diagram is presented in Figure 1.

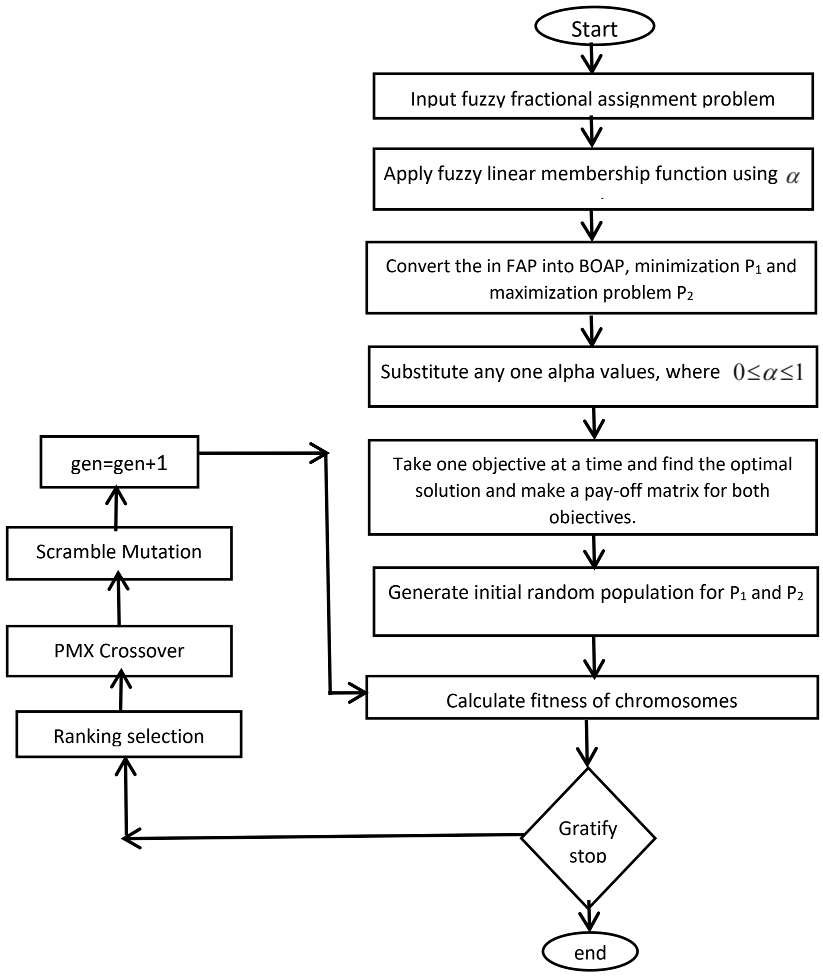

Figure 1. Flow chart of the genetic approach.

5.1 Numerical Example

Let us consider a camp conducted by the blood bank. The nurses N_1, N_2, N_3, N_4 are assigned to collect the blood corresponding to the area of the donors D_1, D_2, D_3, D_4. If the objective of this blood bank is to minimize the cost in return to the total profit of the blood bank, then assign the nurses for different areas of the donors to minimize the cost concerning profit. The data of the fuzzy cost to the fuzzy profit is shown below in Table 1.

Table1. Triangular fuzzy fractional assignment problem (F).

Donors j		D_1	D_2	D_3	D_4
Nurses i	N_1	$\frac{(1,2,4)}{(5,6,9)}$	$\frac{(0.1,0.3,0.4)}{(1,2,4)}$	$\frac{(1.2,1.5,1.6)}{(4,6,8)}$	$\frac{(1.5,2,2.5)}{(5,7,10)}$
	N_2	$\frac{(1,1.5,2.5)}{(2,4,7)}$	$\frac{(2,2.5,3.5)}{(7,9,10)}$	$\frac{(0.5,1,2)}{(3,5,6)}$	$\frac{(0.4,0.8,0.9)}{(1,3,6)}$
	N_3	$\frac{(2,3,5)}{(5,8,10)}$	$\frac{(1,1.5,2.5)}{(2,4,7)}$	$\frac{(0.5,2,3.5)}{(7,8,11)}$	$\frac{(1,2,4)}{(2,5,7)}$
	N_4	$\frac{(2,4,5)}{(5,7,9)}$	$\frac{(2.5,3.5,4)}{(7,8,10)}$	$\frac{(1,2,4)}{(3,6,8)}$	$\frac{(1.5,2.5,3)}{(2,5,7)}$

The following steps shows the solution for problem (F) using the proposed genetic approach.

Using step 1, the given problem (F) is balanced.

Using step 2, construct the problem (P) from the given problem (F) by using Definition 3.4 of α -cut for the linear membership function as shown in Table 2.

Table 2. α -cut for triangular fractional assignment problem (P).

	D_1	D_2	D_3	D_4
N_1	$\frac{(1-\alpha)4\bar{x}_{11}+1\alpha\underline{x}_{11}}{(1-\alpha)9\bar{x}_{11}+5\alpha\underline{x}_{11}}$	$\frac{(1-\alpha)0.4\bar{x}_{12}+0.1\alpha\underline{x}_{12}}{(1-\alpha)4\bar{x}_{12}+1\alpha\underline{x}_{12}}$	$\frac{(1-\alpha)1.6\bar{x}_{13}+1.2\alpha\underline{x}_{13}}{(1-\alpha)8\bar{x}_{13}+4\alpha\underline{x}_{13}}$	$\frac{(1-\alpha)2.5\bar{x}_{14}+1.5\alpha\underline{x}_{14}}{(1-\alpha)10\bar{x}_{14}+5\alpha\underline{x}_{14}}$
N_2	$\frac{(1-\alpha)2.5\bar{x}_{21}+1\alpha\underline{x}_{21}}{(1-\alpha)7\bar{x}_{21}+2\alpha\underline{x}_{21}}$	$\frac{(1-\alpha)3.5\bar{x}_{22}+2\alpha\underline{x}_{22}}{(1-\alpha)10\bar{x}_{22}+7\alpha\underline{x}_{22}}$	$\frac{(1-\alpha)2\bar{x}_{23}+0.5\alpha\underline{x}_{23}}{(1-\alpha)6\bar{x}_{23}+3\alpha\underline{x}_{23}}$	$\frac{(1-\alpha)0.9\bar{x}_{24}+0.4\alpha\underline{x}_{24}}{(1-\alpha)6\bar{x}_{24}+1\alpha\underline{x}_{24}}$
N_3	$\frac{(1-\alpha)5\bar{x}_{31}+2\alpha\underline{x}_{31}}{(1-\alpha)10\bar{x}_{31}+5\alpha\underline{x}_{31}}$	$\frac{(1-\alpha)2.5\bar{x}_{32}+1\alpha\underline{x}_{32}}{(1-\alpha)7\bar{x}_{32}+2\alpha\underline{x}_{32}}$	$\frac{(1-\alpha)3.5\bar{x}_{33}+0.5\alpha\underline{x}_{33}}{(1-\alpha)11\bar{x}_{33}+7\alpha\underline{x}_{33}}$	$\frac{(1-\alpha)4\bar{x}_{34}+1\alpha\underline{x}_{34}}{(1-\alpha)7\bar{x}_{34}+2\alpha\underline{x}_{34}}$
N_4	$\frac{(1-\alpha)5\bar{x}_{41}+2\alpha\underline{x}_{41}}{(1-\alpha)9\bar{x}_{41}+5\alpha\underline{x}_{41}}$	$\frac{(1-\alpha)4\bar{x}_{42}+2.5\alpha\underline{x}_{42}}{(1-\alpha)10\bar{x}_{42}+7\alpha\underline{x}_{42}}$	$\frac{(1-\alpha)4\bar{x}_{43}+1\alpha\underline{x}_{43}}{(1-\alpha)8\bar{x}_{43}+3\alpha\underline{x}_{43}}$	$\frac{(1-\alpha)3\bar{x}_{44}+1.5\alpha\underline{x}_{44}}{(1-\alpha)7\bar{x}_{44}+2\alpha\underline{x}_{44}}$

Now, using step 3, we obtain the minimizing function (P_1) and the other is maximizing function (P_2).

$$
\begin{aligned}
(P_1)\quad \text{Min } N = & ((1-\alpha)4\bar{x}_{11}+1\alpha\underline{x}_{11})+((1-\alpha)0.4\bar{x}_{12}+0.1\alpha\underline{x}_{12})+((1-\alpha)1.6\bar{x}_{13}+1.2\alpha\underline{x}_{13}) \\
& +((1-\alpha)2.5\bar{x}_{14}+1.5\alpha\underline{x}_{14})+((1-\alpha)2.5\bar{x}_{21}+1\alpha\underline{x}_{21})+((1-\alpha)3.5\bar{x}_{22}+2\alpha\underline{x}_{22}) \\
& +((1-\alpha)2\bar{x}_{23}+0.5\alpha\underline{x}_{23})+((1-\alpha)0.9\bar{x}_{24}+0.4\alpha\underline{x}_{24})+((1-\alpha)5\bar{x}_{31}+2\alpha\underline{x}_{31}) \\
& +((1-\alpha)2.5\bar{x}_{32}+1\alpha\underline{x}_{32})+((1-\alpha)3.5\bar{x}_{33}+0.5\alpha\underline{x}_{33})+((1-\alpha)4\bar{x}_{34}+1\alpha\underline{x}_{34}) \\
& +((1-\alpha)5\bar{x}_{41}+2\alpha\underline{x}_{41}))+((1-\alpha)4\bar{x}_{42}+2.5\alpha\underline{x}_{42})+((1-\alpha)4\bar{x}_{43}+1\alpha\underline{x}_{43}) \\
& +((1-\alpha)3\bar{x}_{44}+1.5\alpha\underline{x}_{44})) \qquad (11)
\end{aligned}
$$

Subject to (6) -(8).

(P₂) Max D = $((1-\alpha)9\overline{x}_{11}+5\alpha\underline{x}_{11})+((1-\alpha)4\overline{x}_{12}+1\alpha\underline{x}_{12})+((1-\alpha)8\overline{x}_{13}+4\alpha\underline{x}_{13})$

$$+((1-\alpha)10\overline{x}_{14}+5\alpha\underline{x}_{14})+((1-\alpha)7\overline{x}_{21}+2\alpha\underline{x}_{21})+((1-\alpha)10\overline{x}_{22}+7\alpha\underline{x}_{22})$$
$$+((1-\alpha)6\overline{x}_{23}+3\alpha\underline{x}_{23})+((1-\alpha)6\overline{x}_{24}+1\alpha\underline{x}_{24})+((1-\alpha)10\overline{x}_{31}+5\alpha\underline{x}_{31})$$
$$+((1-\alpha)7\overline{x}_{32}+2\alpha\underline{x}_{32})+((1-\alpha)11\overline{x}_{33}+7\alpha\underline{x}_{33})+((1-\alpha)7\overline{x}_{34}+2\alpha\underline{x}_{34})$$
$$+((1-\alpha)9\overline{x}_{41}+5\alpha\underline{x}_{41})+((1-\alpha)10\overline{x}_{42}+7\alpha\underline{x}_{42})+((1-\alpha)8\overline{x}_{43}+3\alpha\underline{x}_{43})$$
$$+((1-\alpha)7\overline{x}_{44}+2\alpha\underline{x}_{44}) \quad (12)$$

Subject to (6) -(8).

By step 4, solve the problem P_1 and P_2 by the existing method. The resultant optimal solutions are 9.4 and 40 and the pay-off matrix is shown in Table 3.

Table 3. Pay-off matrix of P.

	$N_{at\, x_1}$	$D_{at\, x_2}$
P_1	9.4	29
P_2	14.5	40

5.1.1 Encoding Chromosomes

The coding method utilized has a significant impact on the crossover operator's effectiveness. Modest chromosomal change should result in minor changes in the corresponding solution, and the crossover operator should preserve high-performing partial string arrangements. A permutation of n numbers from the set N = (1, 2..., n) provides a suitable encoding for the AP, with the j^{th} number representing activity on i^{th} resource. This encoding strategy has been employed in our implementation. Here, we use the permutation encoding given by Ratli. *et al*.(2013). By step 5, encode the scheme of the chromosomes as the possible solutions of both P_1 and P_2 at the value of $\alpha=0$. An example of the chromosome structure is shown in Table 4.

Table 4. Example of the chromosome structure of AP.

Nurse i	**1**	**2**	**3**	**4**
Donors j	1	4	3	2

In Table 4, the assignment problem in which 1^{st} nurse is assigned to the 1^{st} donor and 2^{nd} ,3^{rd}, 4^{th} nurses assigned to 4^{th}, 3^{rd} and 2^{nd} donors respectively. Following that, we must proceed with the remaining genetic operators at $\alpha=0$.

5.1.2 Initialization

A number of precedence feasible schedules are generated using a random initialization (Gu et al., 2019) as the initialization population. The random initialization ensures that all the chromosomes in the original population can be selected. The initial population generation scheme's randomness assures that all possible precedence viable chromosomes (schedules) have an equal chance of being chosen for the problem (P_1 and P_2) and initializes the first generation as *gen=0*. Using step 6, we construct an initial 10-chromosome population for the problem P_1 and P_2. These 10 chromosomes must be possible feasible solutions to both problems P_1 and P_2 which satisfy all the constraints. Initialize generation, *gen=0.*

5.1.3 Evaluation

By step 7, evaluate fitness value (Sahu & Tapadar, 2006) of each chromosome by calculating the objective values of problem P_1 and P_2. Represent the objective value of (P_1, P_2) of each chromosome in *gen=0* as the fitness values. The chromosomes and its fitness values are shown in Table 5.

Table 5. Chromosomes and their fitness values.

S.No	Chromosomes	Fitness Values	S.No	Chromosomes	Fitness Values
1	1 2 3 4	(14,37)	6	4 2 3 1	(14.5,40)
2	1 4 3 2	(12.4,36)	7	2 1 3 4	(9.4,29)
3	1 2 4 3	(15.5,34)	8	3 4 1 2	(11.5,34)
4	2 3 4 1	(11.4,26)	9	4 2 1 3	(15,38)
5	3 4 2 1	(10,30)	10	2 3 1 4	(10.4,27)

5.1.4 Ranking Selection Method

Now, using step 8, select all the chromosomes from the *gen = 0* and assign a rank to each chromosome by using the ranking selection method as explained below.

The ranking procedure is used to select the best chromosomes from a chromosome population. It ranks the chromosomes with the highest fitness from a set of randomly chosen chromosomes. It's worth noting that choosing a chromosome with a better fitness will result in the offspring with a high probability of fitting. The procedure for ranking selection method is to determine the midpoints of each chromosome's fitness function and to assign a ranking. Based on the ranking, the chromosome with the highest rank is chosen first, and so on. This process is repeated until the total number of chromosomes equal the population size. Depending on the ranking of the chromosomes, mating pool is processed.

Calculations of the selection process are shown in Table 6.

Table 6. Ranking selection method.

S.No	Chromosomes	Fitness value	Mid value	Rank	S.No	Chromosomes	Fitness value	Mid value	Rank
1	1 2 3 4	(14,37)	25.5	**8**	6	4 2 3 1	(14.5,40)	27.25	**10**
2	1 4 3 2	(12.4,36)	24.2	**6**	7	2 1 3 4	(9.4,29)	19.2	**3**
3	1 2 4 3	(15.5,34)	24.75	**7**	8	3 4 1 2	(11.5,34)	22.75	**5**
4	2 3 4 1	(11.4,26)	18.7	**1**	9	4 2 1 3	(15,38)	26.5	**9**
5	3 4 2 1	(10,30)	20	**4**	10	2 3 1 4	(10.4,27)	18.7	**2**

5.1.5 Crossover and Mutation

The crossover and mutation occur throughout this process. Here we use partially mapped crossover (PMX) and scramble mutation. PMX works in a similar way to two-point crossover. Two crossover points are chosen among two parents in this procedure and the genes between the two crossover points are exchanged. Then, we check the feasibility of the chromosomes. If it is infeasible, we swap the position of remaining genes. This process is explained in (Rahman & Uddin, 2019). For permutation representations, scramble mutation is also popular. A subset of genes is chosen from the entire chromosomes, and their values are randomly scrambled or shuffled as explained in (Ghodmare, 2018). Using step 9, we choose a set of chromosomes (parents) for mating based on the rank of the chromosomes p_r, p_{r+1} (r =1,2,3…n-1), where r is the rank of the chromosomes. Apply PMX crossover to the parent p_r, p_{r+1} for r=1 and obtain two offsprings. Apply the scramble mutation operator to the crossover resultant offspring and move the mutation resulting offspring to the new population. Repeat this process for selection and mating of all the parents for r = 2,3…n-1 and obtain the offsprings (O_i) (i =1,2,3…). Calculations of crossover, mutation of chromosomes are shown in Table 7. The genes involved in crossover and mutation are highlighted.

Then, using Step 10, replace the old offspring with the latest one (gen=1), and repeat Steps (4–7) for the new generation to produce the new offspring. Then proceed to successive generations to get the best offspring.

Table 7. Calculation of Crossover and mutation and their objective values.

S.No	Parents-(pi)	Partially mapped crossover (PMX)-(C_i)	Checking the feasibility of the offspring	Scramble Mutation(O_i)
1	p_1 2 3 4 1 p_2 2 3 1 4	C_1 2 3 1 4 C_2 2 3 4 1	C_1 2 3 1 4 C_2 2 3 4 1	O_1 2 3 4 1 O_2 3 2 1 4
2	p_2 2 3 1 4 p_3 2 1 3 4	C_3 2 1 3 4 C_4 2 3 1 4	C_3 2 1 3 4 C_4 2 3 1 4	O_3 2 4 1 3 O_4 2 1 3 4
3	p_3 2 1 3 4 p_4 3 4 2 1	C_5 3 4 3 4 C_6 2 1 2 1	C_5 3 4 2 1 C_6 2 1 3 4	O_5 3 4 1 2 O_6 2 1 4 3
4	p_4 3 4 2 1 p_5 3 4 1 2	C_7 3 4 1 2 C_8 3 4 2 1	C_7 3 4 1 2 C_8 3 4 2 1	O_7 3 4 2 1 O_8 3 4 1 2
5	p_5 3 4 1 2 p_6 1 4 3 2	C_9 1 4 1 2 C_{10} 3 4 3 2	C_9 1 4 3 2 C_{10} 3 4 1 2	O_9 1 2 4 3 O_{10} 1 3 4 2
6	p_6 1 4 3 2 p_7 1 2 4 3	C_{11} 1 2 4 2 C_{12} 1 4 3 3	C_{11} 1 2 4 3 C_{12} 1 4 3 2	O_{11} 2 1 4 3 O_{12} 1 3 4 2
7	p_7 1 2 4 3 p_8 1 2 3 4	C_{13} 1 2 4 3 C_{14} 1 2 3 4	C_{13} 1 2 4 3 C_{14} 1 2 3 4	O_{13} 1 3 2 4 O_{14} 2 1 3 4
8	p_8 1 2 3 4 p_9 4 2 1 3	C_{15} 1 2 1 3 C_{16} 4 2 3 4	C_{15} 4 2 1 3 C_{16} 1 2 3 4	O_{15} 2 1 3 4 O_{16} 4 2 1 3
9	p_9 4 2 1 3 p_{10} 4 2 3 1	C_{17} 4 2 3 1 C_{18} 4 2 1 3	C_{17} 4 2 3 1 C_{18} 4 2 1 3	O_{17} 4 1 2 3 O_{18} 3 4 2 1

5.1.6 Termination

By step 11, we have the same set of offspring from (gen=3) onwards. As a result, we stop generating new generation. Collect the offspring from (gen=3) and evaluate their fitness value which is an efficient or non-efficient solutions to BOAP. Table 8 shows the last generations offsprings and its fitness value.

Table 8. Resultant offspring and its fitness values.

S.No	Offsprings	Fitness value	Distance value	S.No	Offsprings	Fitness value	Distance value
1	O_1 2 3 4 1	(11.4,26)	14.14	10	O_{10} 1 3 4 2	(14,32)	9.2
2	O_2 4 3 2 1	(12,32)	8.4	11	O_{11} 2 1 4 3	(10.9,26)	14.08
3	O_3 2 4 1 3	(10.3,28)	12.03	12	O_{12} 1 3 4 2	(14,32)	9.2
4	O_4 2 1 3 4	(9.4,29)	11	13	O_{13} 1 3 2 4	(11.5,29)	11.19
5	O_5 3 4 1 2	(14.1,34)	7.6	14	O_{14} 2 1 3 4	(9.4,29)	11
6	O_6 2 1 4 3	(10.9,26)	14.08	15	O_{15} 2 1 3 4	(9.4,29)	11
7	O_7 3 4 2 1	(10,30)	10.01	16	O_{16} 4 2 1 3	(11.5,32)	8.2
8	O_8 3 4 1 2	(11.5,34)	6.35	17	O_{17} 4 1 2 3	(11.5,32)	8.2
9	O_9 1 2 4 3	(15.5,34)	8.5	18	O_{18} 3 4 2 1	(11.5,34)	6.35

By Step 12, combine all efficient/ non-efficient solutions of BOAP obtained using Table 8. Here all the solutions are efficient solutions to the BOAP. Therefore, a set of all efficient solutions to the BOAP can be obtained from the proposed approach: {(11.4,26), (12,32), (10.3,28), (9.4,29), (14.1,34), (10.9,26), (10,30), (11.5,34), (15.5,34), (14,32), (11.5,29), (11.5,32)}. We obtain (11.5,34) as the optimal compromise solution for BOAP by (Kamal et al., 2018) and the allocations are $x_{13} = x_{24} = x_{32} = x_{41} = 1$ or $x_{13} = x_{24} = x_{31} = x_{42} = 1$. Therefore, the optimal compromise solution we get for the problem (P) is $\frac{11.5}{34}$ and optimal compromise solution to the given problem (F) is $\frac{(4.6,7.8,10)}{(12,20,30)}$.

Using step 13, a set of all efficient solutions to the BOAP at $\alpha = 1$can be obtained from proposed approach are {(5,17), (6.2,18), (4.4,20), (6.1,17), (5,17), (3.6,11), (3.1,12), (3.4,11), (4.5,12), (3.4,11)}. We obtain (4.4,20) is an optimal compromise solution for BOAP $\alpha = 1$ and the allocation are $x_{11} = x_{24} = x_{33} = x_{42} = 1$. Therefore, the optimal compromise solution we get for the problem (P) is $\frac{4.4}{20}$ and optimal compromise solution to the given problem (F) is $\frac{(4.9,8.3,12.4)}{(20,25,36)}$.

5.2 Results and Discussion

A numerical example is used to investigate the efficiency of the proposed genetic approach. In this regard, the FFAP is constructed and solved using the genetic approach to obtain the optimal compromise solution. Using this approach, we obtain the solution for $\alpha = 0$. Figure 2 displays the set of all solutions obtained for BOAP using our proposed approach. Table 9 displays the comparison between the optimal compromise solution for the problem (P) and GA Matlab tool for different alpha levels. The optimal compromise solution for (P) at alpha that equals to 0, 0.5 and 1.0 are 0.338, 0.3 and 0.22. Whereas the optimal compromise solution obtained using GA Matlab tool for alpha that equals to 0, 0.5 and 1.0 are 0.433, 0.4 and 0.33. This demonstrates that compromise solution obtained by our proposed approach is more optimal than the GA Matlab tool. To show the effectiveness, the same is plotted in the Figure 3. Therefore, the proposed approach can be used to solve any fuzzy fractional optimization problem efficiently. Overall, the proposed strategy is better suited to problems involving fractional structures.

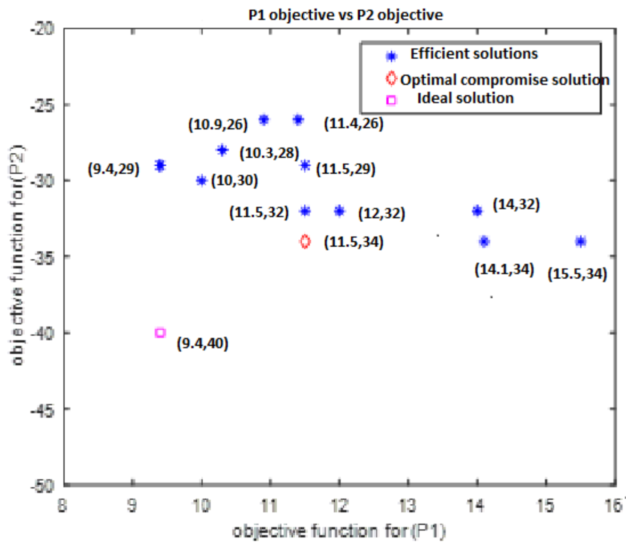

Figure 2. Solutions of BOAP obtained by genetic approach.

Table 9. Comparison of problem (P) results to GA Matlab tool.

α **values**	**Different approaches**	
	Genetic approach	**GA Matlab tool software**
$\alpha = 0$	$\frac{11.5}{34}$	$\frac{11.4}{26.31}$
$\alpha = 0.5$	$\frac{8.4}{28}$	$\frac{7.5}{18.51}$
$\alpha = 1$	$\frac{4.4}{20}$	$\frac{3.6}{11.05}$

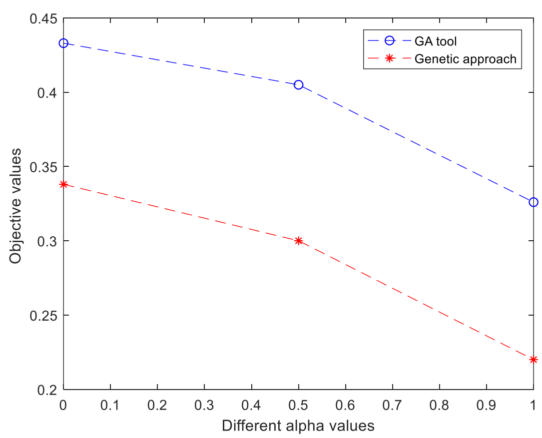

Figure 3. Results of P at different alpha levels are represented graphically.

6. Conclusions

FFAP is closely related to the present way of life, the range and locations of activity, and the commodities and services that will be available for consumption. The investigation of fractional objectives is required since they prove to be a more effective tool for gauging system efficiency. The FFAP is now converted into a deterministic problem by applying the α - cut of the fuzzy linear membership function to each parameter and then solved using a GA procedure. The efficiency of the GA process has been illustrated through a numerical example. The effectiveness of the proposed approach is compared to a GA Matlab Tool. Figure 3 represents a comparison between the proposed approach and GA Matlab tool, which shows the effectiveness of our proposed technique over the GA tool. The proposed approach that produces a set of all efficient/non-efficient solutions and an optimal compromise solution to FAP with fuzzy parameters can assist the society with logistics challenges. If necessary, we can generalize this approach over the nonlinear membership function such as exponential membership function, hyperbolic membership function and sensitivity anlaysis or extended to solve multi-objective optimization problems with large value parameters.

Conflict of Interest

There are no conflicting interests declared by the authors.

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