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Meta-Heuristic Approaches for Optimizing Pharmaceutical Inventory Control under Two-Level Trade Credit

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Abstract
The healthcare sector relies on inventory planning to ensure the continuous availability of emergency medications while minimizing wastage and expiry. This study focuses on enhancing the total profitability by jointly optimizing order quantity and replenishment cycle time of the pharmaceutical products. A differential equation is developed to represent time-dependent demand and product expiration, and the resulting cost function of the inventory is transformed into a non-linear minimization problem. Convexity of the model is examined through graphical pairwise analysis of the decision variables. The model considers practical industry factors of the pharmaceutical sector, including the time-sensitive demand, shelf life and trade credit. The optimality of the proposed model is evaluated using conventional and metaheuristic algorithms such as CA, CSA, and RSA. Numerical experiments based on real data on credit period are collected from local and chain pharmacies, demonstrating the applicability of the proposed model. This study offers a framework for pharmaceutical inventory management, providing actionable insights to enhance operational efficiency and profitability while managing time-sensitive healthcare products.

Keywords- Expiry date, Meta-heuristic, Pharmaceuticals, Time-dependent, Two-level trade credit.

1. Introduction

Healthcare is a vital pillar of the economic system. Diagnoses, treatment, rehab, and disease prevention are all part of how healthcare supports the economy. It brings together several interconnected sectors, including pharmaceutical manufacturing of drugs and medical equipment, all of which work toward safeguarding individual and public health. Among these, the pharmaceutical sector holds an important part as it supplies life-saving medicines that many patients depend on daily and in emergency situations. Since pharmaceutical products like medicines, vaccines, and so on have a limited shelf life. This ensures timely availability and minimize waste of pharmaceutical items as careful planning and control of inventories are more essential.

Various inventory model on deterioration and perishable items was developed over time. Harris (1915) first introduced an EOQ inventory model with infinite shelf-life items like fashion goods. Umamaheswari et al. (2016) developed a perishable inventory model with a fixed lifetime under FIFO issuing policy. This policy was designed to minimize wastage by determining the optimal order quantity with an expiry date. These initial inventory models laid the foundation for the proposed inventory model. In recent years, inventory models have been applied to different perishable items, including dairy products (Miller, 1994; Keramatpour et al., 2018), livestock management (Hatibaruah and Saha, 2023; Arunadevi et al., 2025; Jayashri and Umamaheswari, 2025a), Blood (Chithraponnu and Umamaheswari, 2023; Silva Magalhães et al., 2024) and pharmaceutical items. These application-oriented inventory model helps to manage the sensitive perishable items to solve industry-specific challenges in perishable goods and resource efficiency. The pharmaceutical management is more significant than other inventory management as it directly impacts patient health.

Research on the pharmaceutical items mainly focuses on the demand pattern, as the rise and fall of demand occur during disease outbreaks. Sahoo and Panda (2024) and Dwivedi et al. (2025a) proposed a pharmaceutical inventory to minimize wastage and the timely availability of medicines by utilizing fuzzy models and hybrid evolutionary algorithms. The aim of these models focuses on the inventory operation and determining the order quantity. These lag a major gap in focusing on the financial aspect of the inventory management. Hence, the proposed model tends to solve the financial problem faced with cost-effectiveness cash flow during the pandemic situation.

The inventory management facilitates timely access to medicines, reduces wastage from expired stock, and improves overall healthcare performance. Inventory control is a key strategy for preventing unnoticed delays that could affect the treatment and elevate operational costs. To meet the health sector's needs, inventory management must be studied to maintain optimal inventory levels, minimize waste, and ensure continuous access to essential medicines. The proposed pharmaceutical inventory management model is graphically presented in **Figure 1**, which outlines the overview of the proposed study with heuristic search mechanisms that work together to maximize profit and minimize wastage.

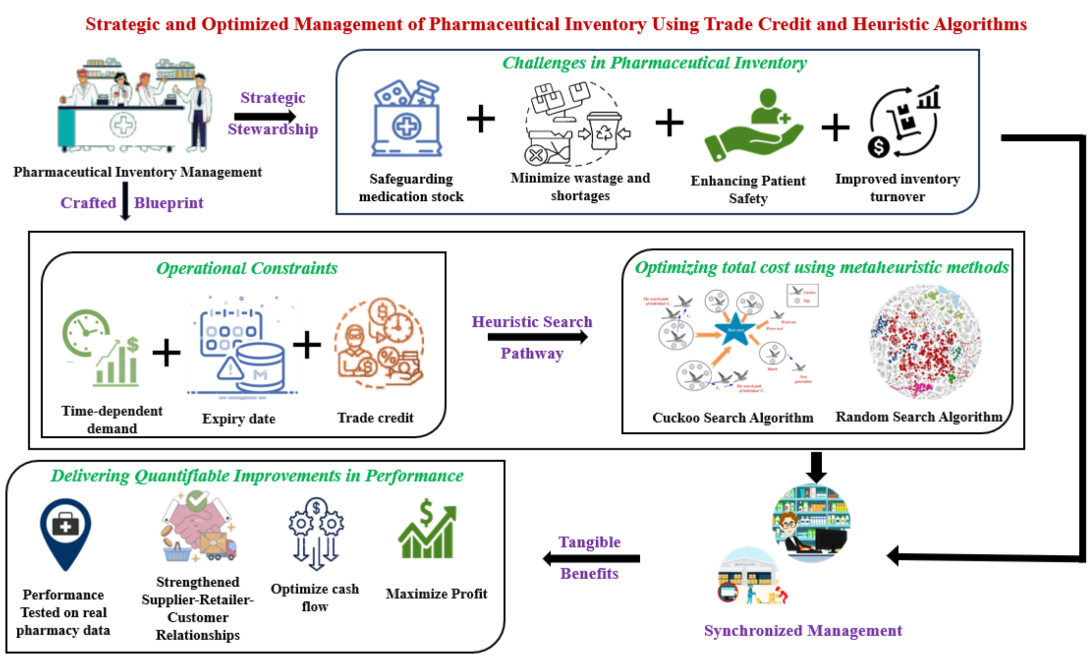

Figure 1. Conceptual framework of the pharmaceutical inventory management through trade credit and metaheuristic optimization.

A key factor in maintaining effective inventory is the EOQ (Economic Order Quantity). The order quantity of any inventory is influenced by various factors like demand, deterioration, replenishment time, and storage conditions. The EOQ model incorporates the perishability and time-sensitive nature of medications to guarantee that healthcare facilities have enough essential medicines by eliminating wastage and operational inefficiencies. Healthcare providers can improve patient care outcomes, enhance distribution reliability, and optimize costs by utilizing strategic inventory management techniques. The entrepreneurs adopt inventory strategies that ensure a balance between supply and demand, along with cost efficiency, to effectively control these demand fluctuations.

Various inventory models have been developed to show the demand fluctuations in pharmaceutical management. Time-dependent models were proposed by Uthayakumar and Karuppasamy (2017a, b); Karuppasamy and Uthayakumar (2018); Hatibaruah and Saha (2024); and Gupta et al. (2025), focusing on the variations in demand over time. Price-sensitive Inventory models were examined by Rastogi and Singh (2019) to assess how pharmaceutical pricing strategy affects the demand in pharma. Santhi and Karthikeyan (2018) studied the cubic-based demand pattern and Sharma and Srivastava (2018) and Sharma et al. (2022) examined the ramp-type demand models that describe the fluctuation in the demand pattern. Trend-based demand models are analysed by Umadevi and Umamaheswari (2023), which incorporate long-term demand shifts influenced by advancements in healthcare and changes in the population demographics. Out of all of these models, the time-dependent demand model received significant attention, as these models are best suited for predicting the demand patterns in the real world, as they account for external influences (e.g., weather patterns, epidemic outbreaks, and evolving disease profiles) affecting the pharmaceutical inventory and provide insight into the replenishments based on demand patterns.

Dwivedi et al. (2025b) developed an inventory model for pharmaceutical items that integrates the carbon emission mechanism. The study incorporated variables with price, infection rate, and preservation strategies dependent demand, which support decision-making in inventory management. Singh et al. (2025) developed an inventory model with a carbon emission mechanism for pharmaceutical products. This study highlights the importance of maintaining an inventory that helps in optimizing both economic and environmental constraints. A smoother cash flow is the result of effective operational management, which can greatly enhance inventory. The timely payment improves cash flow in inventory frameworks, which indirectly strengthens inventory by ensuring the availability and streamlining logistics. Integrating trade credit helps minimizing the shortages and delays of medicines due to financial constraints.

Inventory managers for the pharmaceutical items face two significant challenges. First, insufficient medical supplies can lead to untreated illnesses, which cause severe health risks. Second, excess storage of medicines leads to deterioration over time, which increases the financial losses. As discussed, it's crucial to determine the optimal order quantity to ensure the efficiency and reliability of pharmaceutical items in healthcare systems. Entrepreneurs are adopting trade credit as a financial tool to strengthen inventory management. Trade credit is considered a flexible payment scheme that ensures smooth business operations by offering a credit period to the buyers. It is an interest-free loan between the seller and buyers that ensures the continuous supply of goods throughout the replenishment cycle. Retailers can purchase essential medications with deferred payment, which improves the cash flow by maintaining a consistent supply of goods to satisfy the demand.

Trade credit was initially applied to suppliers and retailers, where suppliers offer a credit period to retailers. In recent times, the dual credit period has played a vital role. Where suppliers offer a credit period to the retailers and retailers extend the credit period to the customers. In this study, two-level trade credit is incorporated, where suppliers offer a credit period to the pharmacist and the pharmacist in return offers a credit period to the patients. This represents the real-life situation taking place in local pharmaceutical stores and hospitals.

The two-level trade credit effectually captures the financial interactions by offering several benefits within the pharmaceutical sector and its inventory management. It built stronger relationships between suppliers and retailers by encouraging larger order quantities. It enhances retailers' cash flow and affordability for the end customer during critical and emergency. This study not only improves cash flow and minimizes shortages but also supports the financial flexibility and sustainability of pharmaceutical systems.

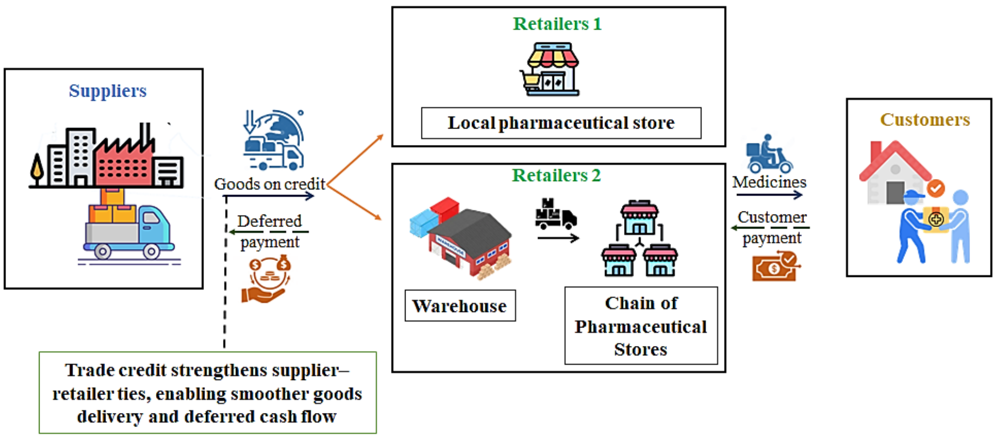

Figure 2. A schematic illustration depicts the dynamic flow of goods and financial transactions through the inventory system.

Figure 2 represents the graphical flow of goods and cash in the proposed study. This highlights the role played by suppliers, retailers, and consumers in ensuring seamless inventory management. The entrepreneurs have the ability to optimize total inventory cost by minimizing the wastage, and the availability of life-saving medications by integrating EOQ with two-level trade credit.

Traditional EOQ models required retailers to make immediate payments upon receiving orders. In the trade credit system, suppliers provide a credit period to retailers a certain time period to settle without interest. Haley and Higgins (1973) introduced trade credit in inventory management to determine the optimal order quantity. Goyal (1985) extended the traditional EOQ model by incorporating trade credit. It demonstrates that order quantity and replenishment cycle time increase as the total costs decrease. Molamohamadi et al. (2014) analysed an inventory model under trade credit to optimize the optimal order quantity of deteriorating items. The study incorporated a metaheuristic algorithm to determine optimal pricing and replenishment time to maximizing total inventory profit. Das et al. (2021) proposed an EOQ model with trade credit that exploited Taylor's series approximation for solving nonlinear optimal production problems. Sahu et al. (2017) proposed an inventory model for deteriorating items with price-dependent demand and shortage. The study integrates preservation technology to extend product usability.

In 2017, Uthayakumar and Karuppasamy (2017b) proposed a novel inventory model that introduced a trade credit framework in the pharmaceutical sector. This significant study laid the groundwork for future advancements in a pharmaceutical inventory optimization model that balances both financial and stock management. Huang (2003) first introduced a two-level trade credit into the inventory management system, where suppliers extend credit to retailers, who in turn offer credit terms to customers. In the pharmaceutical sector, this system allows suppliers to provide stock without immediate payment to the pharmacist, with payments settled before the credit period. Pharmacists, in turn extend credit-based payment options to the regular customers.

The integration of two-level trade credit in the pharmaceutical sector offers various advantages that significantly enhance financial and operational efficiency. This payment process strengthens coordination among suppliers, retailers (pharmacy/hospital) and patients, ensuring availability of essential medications and smooth cash flow. Trade credit is a key component in inventory that supports a financial mechanism that facilitates continuous stock while minimizing financial risks and maximizing profitability. This system provides payment flexibility for customers, particularly in urgent medical situations, by granting them a time period to settle the amount in hospitals. Credit period is offered to the regular customers, making healthcare more accessible and affordable. This solves the financial risks for both retailers and patients across the healthcare network. This framework enhances cash flow management, builds strong industry relationships and contributes to the long-term financial sustainability of the pharmaceutical business. The significance of this model extends beyond financial optimization and plays a vital role in enhancing healthcare accessibility and sustainability.

In **Table 1**, an overview of the existing literature with the key findings is summarized. This structure simplifies the research gaps in the pharmaceutical sector, specifying the cash flow, demand and deterioration while maintaining the inventory system.

Table 1. Key characteristics of the proposed study with a review of the existing deteriorating inventory model with trade credit.

Authors	**Inventory model**	**Pharmaceutical**	**Demand**	**Deterioration**	**Trade credit**	**Framework**
Arunadevi and Umamaheswari (2024)	EOQ	-	Price and time-dependent	Weibull	-	Retailers
Madugu et al. (2023)	EOQ	-	Time	Delayed	1-level Trade credit	Retailers
Patra et al. (2024)	EOQ	-	Power demand	With & without preservation-technology	1-level Trade credit	Vendors
Jayashri and Umamaheswari (2022)	EOQ	-	Stock dependent	Weibull	1-level Trade credit	Retailers
Das et al. (2024)	EOQ	-	Time and price-dependent	Expiry date	2-level Trade credit	Retailers
Praveen and Manoharan (2025)	EOQ	-	Selling price and freshness	Expiry date	2-level Trade credit	Retailers
Ebrahimzadeh-Afruzi and Aliahmadi (2018)	EOQ	✓	Inventory level dependent	-	Credit period contract	Retailers
Jayashri and Umamaheswari (2025b)	EOQ	-	Price	constant	2-level Trade credit	Suppliers-retailers - customers
Karuppasamy and Uthayakumar (2019)	EOQ	✓	Variable demand	-	Order size dependent	Manufacturers-wholesalers - hospital
Proposed model	**EOQ**	✓	**Time-dependent**	**Expiry date**	**2-level Trade credit**	**Suppliers - Retailers - Customers**

This research laid the groundwork for developing a holistic pharmaceutical inventory model that integrates time-dependent demand, shelf life and a two-level trade credit system. These constraints help in modelling the inventory to determine the stock management, minimizes wastage, and availability of pharmaceutical drugs to the patients. This ensures the essential drugs are available in an emergency situation and also reduces the financial stress on businesses. The model improves the efficiency of the logistics system by preventing shortages and excessive inventory. A variety of conventional and advanced optimization techniques are used in this study to obtain the optimal order quantity and replenishment time to improve inventory performance. The proposed research model is represented in **Figure 3**.

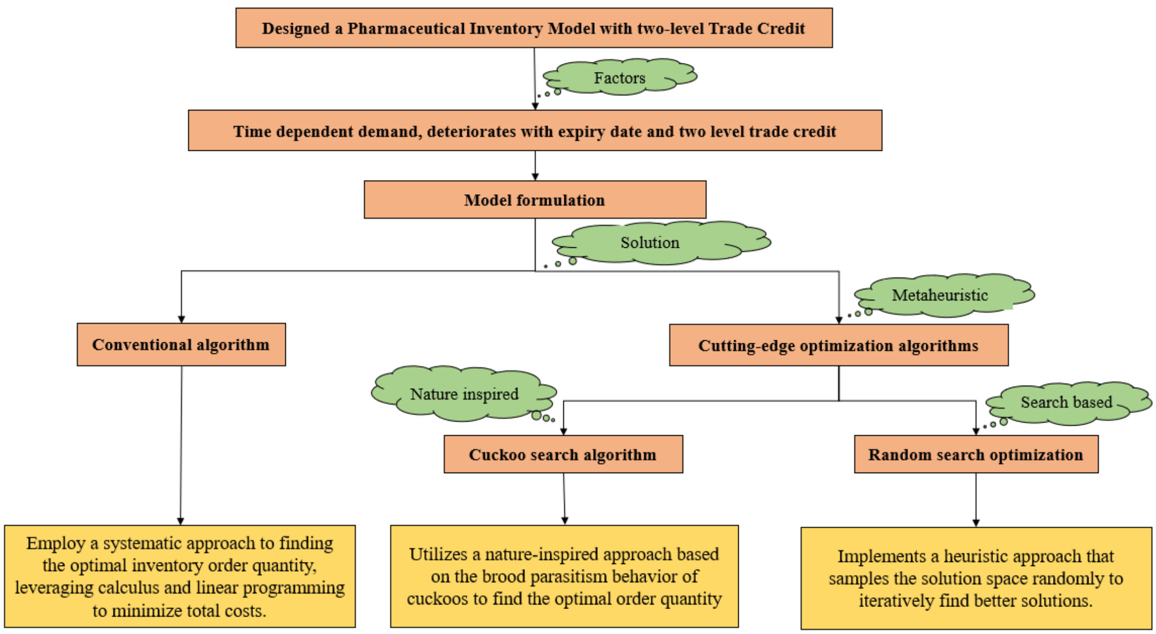

Figure 3. Layout of the proposed research model.

1.1 Research Gap

The two-level trade credit has been missing in current research on pharmaceutical inventory management as it impacts the flexible credit periods on retailers and patients during critical health event. This gap prevents the development of strategies that balance financial stability and patient care. This study focusses on determining replenishment cycles that prevent storage of pharmaceutical items, which minimize wastage and ensure the continuous availability of fresh medicines. In this study, a new inventory model based on pharmacy type and operational dynamics is developed with a two-level trade credit framework, where suppliers extend interest-free loans to pharmacist, who in turn offer credit periods to patients. The model aims to minimize inventory costs while ensuring continuous inventory by considering real-life constraints like quadratic time-dependent demand, product expiration, and permissible payment delays. Advanced metaheuristic algorithms, such as RSA and CSA, are employed in the research to achieve the best optimization solutions. This approach provides a framework for pharmaceutical inventory decision-making, with the potential to enhance operational efficiency and strengthen the healthcare distribution system.

1.2 Research Problem

Pharmaceutical inventory management plays a vital role in solving the challenging factors like the perishable nature of medicines and varying patient demand due to the pandemic and disease outbreak. If stock levels are not maintained, products may expire leading to financial losses, disrupted cash flows and negative health outcomes. This study is designed to address both supply-side and demand-side dynamics to address these complex challenges. This study aims to solve this gap by introducing a dual credit period, where suppliers extend interest-free credit periods to pharmacists, in turn pharmacists offer short-term credit to patients. Pharmacies that operate in diverse contexts, including small local drugstores to large pharmaceutical chains, can benefit greatly from this approach, as credit arrangements can greatly impact inventory replenishment strategies and financial sustainability. To better reflect real-life conditions, the model incorporates quadratic time-dependent demand along with payment delays and product expiration dates. The model aims to optimize ordering decisions by providing continuous medicine availability during the critical emergency period.

Following are the highlights of the study:

- Introduces a novel two-level trade credit that improves cash flow and ensures medicines are available during critical health emergencies in the pharmaceutical management.
- Determines optimal order quantities and replenishment cycles by considering demand variability, deterioration rates, and expiration constraints to various pharmacy types like local and chain of pharmaceutical stores.
- Supports retailers in maintaining adequate stocks in inventory and helps the customers to access necessary medicines with delayed payment.
- Employs differential equations to solve complex linear demand and expiration dynamics models to minimizing inventory costs and wastage.
- Employs metaheuristic algorithms like CSA and RSA to achieve the best inventory operations.
- Enhances inventory performance by minimizing deterioration, financial losses, and improving patient care.

The paper is organized as follows: Section 2 introduces the assumptions and notations of the proposed study. Section 3 discusses the methodology used to develop and analyse the model. Section 4 formulates the mathematical model to calculate the total inventory cost based on the cost analysis. Section 5 presents the conventional benchmark algorithm, which serves as an evaluation optimization. Section 6 provides the advanced optimization methodologies such as CSA and RSA. Section 7 presents a numerical analysis that compares the computational efficiency and accuracy of the algorithms. Section 8 outlines the model's real-world applicability and system integration. Section 9 offers a sensitivity analysis that examines how changes in key parameters impact the model's outcome. Section 9 provides a detailed sensitivity analysis that examines how changes in key parameters influence the model's outcome. Section 10 highlights the model's importance and its practical applications in the industry. Finally, Section 11 concludes that summarizes the study and outlines promising paths for future research and limitations.

2. Assumptions and Notations

2.1 Assumptions

These assumptions are used in designing the mathematical model

- The model addresses the inventory management of the single perishable items for the usable (non-expired) pharmaceutical products
- The demand for the items is generated as time-dependent,

$$D = (a + bt + ct^2).$$

where, $a > 0$ is the initial rate of demand, and b is the rate at which the demand rate increases. The parameter c changes to either positive or negative values based on the pharmaceutical demand patterns, based on seasonal variations and disease outbreaks. The demand rate is to be monotonically increasing ($c > 0$) or to initially increase and subsequently decrease ($c < 0$) based on the demand pattern.

- Pharmaceutical items deteriorate when they reach their expiration date m.

$$\theta = \left(\frac{1}{1+m-t}\right).$$

- The supplier offers a credit period of M to the retailer, who in turn extends a credit period N to customers, thereby creating a two-level trade credit that supports smooth financial transactions across the inventory.
- Holding cost per unit increases linearly over time as $(h_1 + h_2\, t)$, where h_1 is the base cost and h_2 accounts for additional expenses related to prolonged storage and product ageing.
- The rate of replenishment is infinitely instantaneous but the size is finite and the time horizon is infinite.
- Stockouts are not permitted to ensure the uninterrupted availability of essential medicines. No backlogging or partial delivery options are considered.
- Once an item deteriorates due to expiry or quality degradation during the planning horizon, it is discarded from inventory and remains unreplenished until the subsequent replenishment cycle.
- Lead time is negligible.

2.2 Notations and Descriptions

Inventory parameters

R	Replenishment at the time
D_i	The number of items deteriorates per unit of time
I_p	Interest paid per year
I_e	Interest earned per year
a, b, c	Demand parameters, $a, b, c > 0$
m	Expiry date (in years)
N	Credit period offered by retailers to their customers (in years)
M	Credit period offered by the suppliers to their retailers (in years)
$Q(t)$	Inventory level at time t
TC	Total Inventory cost per unit of time ($ per order)

Cost parameters

c	Unit cost of an item ($ per unit)
p	Unit selling price ($ per unit)
d_c	Deterioration cost per unit item ($ per unit)
O_c	Ordering cost per unit item ($ per order)
h_1, h_2	Holding cost parameters of an item ($ per unit)

Decision variable

T	Positive Inventory cycle (in years)

3. Methodology

This article presents a pharmaceutical inventory model that integrates time-dependent demand and trade credit into the products that deteriorate over time due to expiry date. The model helps in solving the key challenges faced in pharmaceutical inventory like changes in demand due to seasonal disease outbreaks and the perishable nature of medicines. This study helps in minimizes the deterioration of essential drugs due to expiration by continuously replenishing them at the optimal replenishment cycle time. The study helps inventory management to determine optimal order quantities and replenishment cycle time. Beyond optimization, the model enhances cash flow, reduces deterioration and maintains sufficient stock.

Integrating various theoretical methodologies and different practical constraints into the model to improve traditional inventory. The monitoring of inventory levels and product expiry continuously helps in restocking the goods based on demand, which prevents drug shortages. The EOQ model boosts inventory by determining the optimal order quantity and cycle time. This helps in minimizing the total cost by reducing deterioration and holding costs. In practice, the model helps in reducing storage due to expired goods which results in better healthcare facilities and cost-effective inventory management. This is a novel pharmaceutical inventory management system that addresses the challenges faced in the healthcare sector to improve the operational efficiency and availability of essential medications. **Figure 4** illustrates the flow of research methodology used in this study, which explains the step-by-step process, including model formulation, optimization, and results.

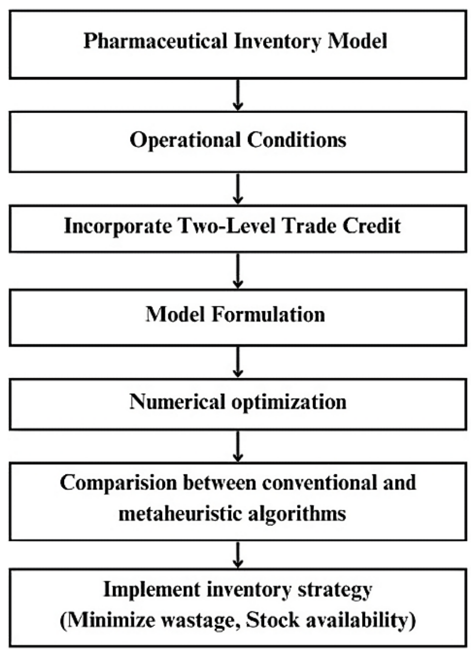

Figure 4. Methodology framework for optimizing the pharmaceutical inventory model.

4. Mathematical Formulation

The change in the inventory level of the pharmaceutical item is demonstrated in **Figure 5**. The depletion of the inventory curve is driven by the demand and deterioration of the perishable item. Consequently, the variation in inventory levels over the interval [0, T] can be mathematically modelled by the following differential equation:

$$\frac{dQ(t)}{dt} = -\theta Q(t) - D,$$

$$\frac{dQ(t)}{dt} = -\left(\frac{1}{1+m-t}\right) Q(t) - (a + bt + ct^2) \tag{1}$$

This first-order linear ODE is solved using the integrating factor method to determine I(t), with the initial boundary condition, $Q(0) = R$ and $Q(T) = 0$.

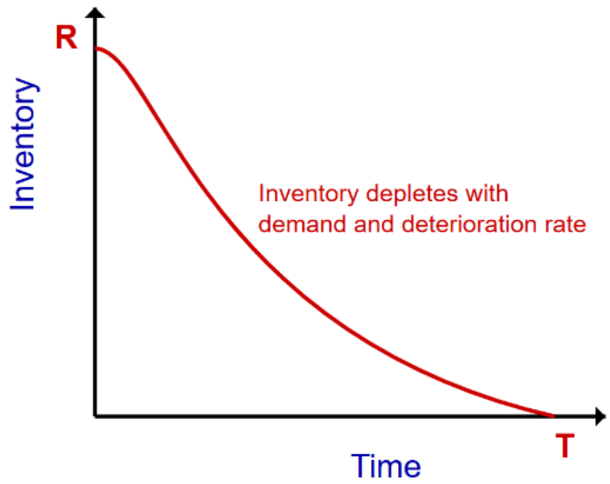

Figure 5. Graphical representation of inventory over time.

Solving Equation (1) yields

$$Q(t) = (t-m-1)\left\{\begin{array}{c}[c(1+m)^2+b(1+m)+a][\log(1+m-T)-\log(1+m-t)] \\ +\frac{c}{2}[(T-m-1)^2-(t-m-1)^2] \\ +(2c(1+m)+b)[(T-m-1)-(t-m-1)]\end{array}\right\} \quad (2)$$

In order to ensure product efficacy, minimize wastage, and maintain optimal inventory levels, the pharmaceutical company ensures cumulative demand, order quantity, and deterioration quantity over the replenishment cycle [0, T].

***Cumulative demand*:** Cumulative demand functions determine the stock levels required to meet market demand without incurring shortages by calculating the total demand over the cycle time.

$$\int_0^T D dt = \int_0^T (a+bt+ct^2)dt$$

$$= aT + \frac{bT^2}{2} + \frac{cT^3}{3} \quad (3)$$

***Order quantity*:** The order quantity represents the number of products to be replenished at the beginning of each cycle.

$$R = Q(0) = (-m-1)\left\{\begin{array}{c}[c(1+m)^2+b(1+m)+a]\log\left(\frac{1+m-T}{1+m}\right)+ \\ \frac{c}{2}[(T-m-1)^2-(-m-1)^2]+[2c(1+m)+b]T\end{array}\right\} \quad (4)$$

***Deterioration quantity*:** The deteriorated item denotes the portion of inventory that is unusable due to degradation or expiration during the cycle.

$D_i = R - D$

$$= (-m-1)\left\{\begin{matrix} [c(1+m)^2 + b(1+m) + a]\log\left(\frac{1+m-T}{1+m}\right) \\ +\frac{c}{2}[(T-m-1)^2 - (1+m)^2] \\ +[2c(1+m)+b]T - aT - \frac{bT^2}{2} - \frac{cT^3}{3} \end{matrix}\right\} \quad (5)$$

In determining the optimal solution of the pharmaceutical sector, ordering cost, deterioration cost, holding cost, interest earned and interest paid are calculated. The purchasing cost is a constant term to the total cost per unit time and therefore does not influence the optimization of the decision variable. Hence, it is excluded from the objective function without loss of generality.

- ***Ordering cost*** **(OC):** The ordering cost denotes the cost associated with pharmaceutical entrepreneurs when an order is placed. This cost helps the pharmaceutical companies to improve efficiency and meet customer demand to optimizes the total cost.

Cost of ordering over the cycle from 0 to T $= O_c$ (6)

- ***Deterioration cost*** **(DC):** Inventory deterioration cost refers to the carrying cost due to obsolescence, which signifies the expenses incurred as pharmaceutical inventory degrades in value or quality during storage. The perishable pharmaceutical items like medicines and vaccines, easily deteriorate due to expiry. The deterioration cost is critical as it influences the financial loss and quality of the pharmaceutical products to the patients.

Cost of deterioration over the cycle from 0 to T = $d_c * D_i$ (7)

- ***Holding cost*** **(HC):** Inventory holding costs refer to the cost associated with storing and managing inventory in the warehouse/pharmacy. The holding cost of the pharmaceutical goods is high, and it is based on inventory levels. By including a holding cost function that depends on time, the model allows pharmaceutical companies to make informed decisions that balance product availability with cost efficiency and minimize losses from expired or overstocked items.

Cost of holding throughout the cycle from 0 to T $= \int_0^T (h_1 + h_2 t)Q(t)dt$ (8)

- The interest rates are calculated based on the credit periods M and N as
  - i. $M > N$
  - ii. $M \leq N$

Case 1: $M > N$

There are two circumstances in this case, which are classified based on the cycle time T. These conditions are classified as $M \leq T + N$ and $M > T + N$.

Case 1.1 $M \leq T + N$

As illustrated in **Figure 6**, the retailer does not receive the full payment for the items that are sold before the credit period M. Similarly, the retailer must finance all items sold from N to M, during which the outstanding amount accrues interest payable at an annual rate of I_p. Therefore, the interest paid represents the cost incurred by the retailer for maintaining the unpaid inventory balance during this credit period.

Interest paid

$$IP_1 = cI_p \int_M^{T+N} Q(t-M)\, dt \quad (9)$$

Within the interval $[N, M]$, the retailer earns revenue from customer payments received during this time. Hence, the interest earned during the period is calculated as

***Interest earned*:**

$$IE_1 = pI_e \int_N^M [Q(0) - Q(t - N)]dt \quad (10)$$

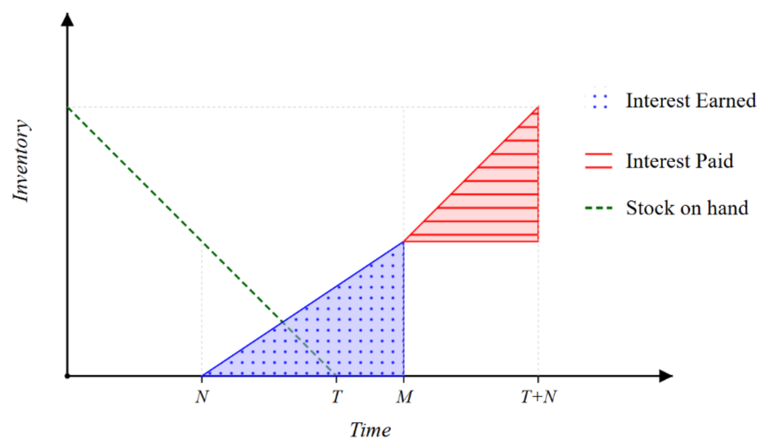

Figure 6. Interest earned and interest paid for the case $M > N \,\&\, M \le T + N$.

Total inventory cost

$TC_1 = \frac{1}{T}[OC + DC + HC + IP_1 - IE_1].$

Case 1.2: $M > T + N$

As illustrated in **Figure 7**, when $M > T + N$, the retailer receives the sales revenue for all items sold by time $T + N$ and can fully settle the cost to the supplier by the end of the credit period M. The interest paid is zero, as the retailer does not require any financing support.

Interest paid

$$IP_2 = 0 \quad (11)$$

The retailer earns interest on sales revenue on the entire sales revenue held until the supplier payment is due over the period $[T+N, M]$. The interest earned per cycle is derived by combining these two components, the amount received from the customers and the interest earned from the revenue.

Interest earned

$$IE_2 = pI_e \int_N^{T+N} [Q(0) - I(t - N)]dt + pI_e \int_{T+N}^{M} Q(0)\, dt \quad (12)$$

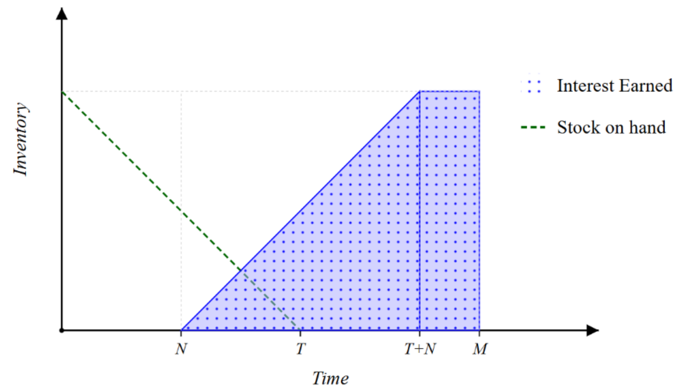

Figure 7. Interest earned and Interest paid for the case $M > N$ & $M > T + N$.

Total inventory cost

$TC_2 = \frac{1}{T}[OC + DC + HC - IE_2]$.

Case 2: M ≤ N

When $M < N$, the retailer does not earn any revenue from the customer as the payments are received after the supplier's credit period, which is illustrated in **Figure 8.**

Interest earned

$$IE_3 = 0 \tag{13}$$

During the interval $[M, N]$, the retailer must finance the entire inventory, incurring interest expenses on both the total order quantity during $(N - M)$ and the average inventory held throughout the cycle time T. Thus, the total interest paid reflects the financing cost of maintaining inventory before receiving customer payments.

Interest paid

$$IP_3 = cI_p \int_M^N Q(0)\,dt + cI_p \int_N^{T+N} Q(t-N)\,dt \tag{14}$$

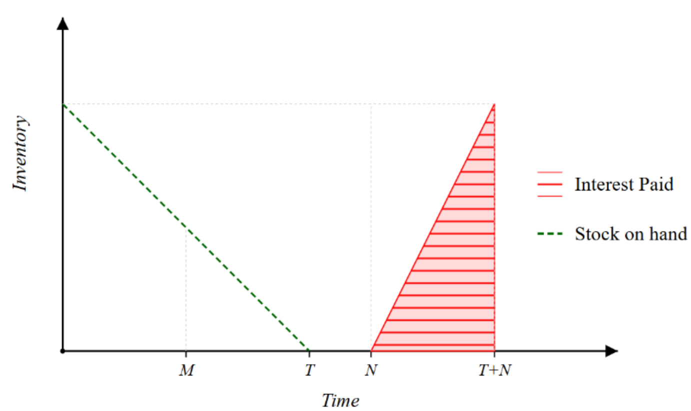

Figure 8. Interest earned and Interest paid for the case $M \leq$ N.

Total inventory cost

$TC_3 = \frac{1}{T}[OC + DC + HC + IP_3\,]$

Refer to Appendix A for the calculations of deterioration cost, holding cost, interest earned, interest paid, and total cost.

To ensure that the obtained solution corresponds to a global minimum, the convexity of the total cost function with respect to the replenishment cycle time T is analysed. A function is convex if its second derivative is non-negative over the feasible domain. In this model, we analytically verified that the second derivative of the total cost function is positive for all valid values of T, confirming that the function is strictly convex. This convexity of the solution obtained by solving the first-order optimality condition $\frac{\partial TC}{\partial T} = 0$ is globally optimal. This property justifies using efficient and reliable optimization techniques like CA, CSA, and RSA to minimize the total cost function.

5. Conventional Algorithm (CA)

The CA is the traditional optimization method designed to find the optimal solutions of an objective function by analyzing critical points. It can be applied to both single-variable and multi-variable problems using partial derivatives and second-order differential conditions. It is ideal for smooth, differentiable functions in fields of inventory management, finance, engineering and so on. CA may struggle with non-smooth, non-convex, or complex objective functions.

CA for Total Cost Minimization

Step I: Star.
Step II: Assign initial values to the parameters.
Step III: Obtain the total cost *TC* for the given model.
Step VI: Solve $\frac{\partial TC}{\partial T} = 0$ to determine all possible outcomes for critical points T_{cp} within the feasible domain of *T*.
Step V: For each critical point T_{cp}, calculate $\frac{\partial^2 TC}{\partial T^2}$. If $\frac{\partial^2 TC}{\partial T^2} > 0$ for all T_{cp}, them T_{cp} is a local minimum, then proceed to Step 6. Otherwise go to step 2.
Step VI: Check $\frac{\partial^2 TC}{\partial T^2} > 0$ for all *T* in the feasible region to confirm the global convexity of *TC(T)*. If this condition is satisfied, the solution corresponds to a global minimum.
Step VII: Assign $T^* = T_{cp}$, where $TC(T)$ is minimized.
Step VIII: Compute the minimum total cost $TC^* = TC(T^*)$.
Step IX: Report T^* and TC^* as the optimal cycle time and minimum total cost.
Step X: End.

---

6. Cutting-Edge Optimization

The cutting-edge optimization is designed by using new strategies that, inspired by nature, evolution, and swarm based optimization. Nature-inspired algorithms, which include Cuckoo Search and Bat Method, evolutionary-based techniques such as Genetic Algorithm, and swarm-based approaches like Ant Colony Optimization, Artificial Bee Algorithm, and Particle Swarm Optimization are some of the metaheuristic algorithms used in this competitive world to address optimization challenges. Metaheuristic algorithms are a key component of cutting-edge optimization that help in navigating solution spaces and determining the optimal or near-optimal solutions. In this cutting-edge optimization, both RSA and CSA play a crucial role in determining the optimal solution.

6.1 Rationale for Selecting CSA and RSA

- The CSA and the RSA were chosen to address the pharmaceutical inventory optimization problem's complex and nonlinear nature. They are both advanced metaheuristic approaches and a baseline method for the evaluation.
- The strength of CSA's global optimization capability can be demonstrated through minimal parameters and its ability to escape local optima in complex, nonlinear search spaces. CSA offers the best alternative for the GA and PSO. This will be well-suited for problems with dynamic constraints and multimodal cost functions due to its simplicity, reduced dependence on control parameters, and it gives a globally optimal solution.
- RSA is valuable for its broad and unbiased solution from the search space.

The combination of both algorithms ensures a balanced optimization approach that combines exploratory power and benchmark reliability, resulting in enhanced practical application of the model.

6.2 Cuckoo Search Algorithm (CSA)

In 2009, Yang and Deb (2009) introduced the Cuckoo Search algorithm, which is inspired by nature. This algorithm is a reflection of the clever tactics used by cuckoos when laying their eggs in the nests of host birds. This is inspired by the phenomenon of cluster behaviour of cuckoos in certain species. Yang (2010) has demonstrated an innovative algorithm that makes use of the brood parasitic behaviour observed in certain cuckoo species, which is connected to the Lévy flight behaviour commonly observed in various birds and fruit flies. **Figure 9** illustrates the flow chart of the CSA, which emphasizes the use of Lévy flights for solution updates and convergence. Molamohamadi et al. (2014) proposed an EPQ Inventory model with shortage and trade credit. This study states that the CSA offers superior solutions compared to the Genetic Algorithm in finding the optimal replenishment time, order quantity, and selling price. The traditional inventory system gives less profit than the trade credit approach. The traditional inventory system becomes less profitable for the buyer when compared to the one-level trade credit approach.

Algorithm: CSA for Total Cost Minimization

Step 1: Assign a value to the parameters
Step 2: Define the Total Cost Function
  Function total_cost(T):
    Return total_cost(T)
Step 3: Set CSA parameters
    Number of nests: 20
    Number of dimensions (T): 1
    Maximum iterations: 1000
    Discovery probability: 0.25
    Step size factor: 0.01
    Bounds for T: [0, 5]
Step 4: CS Algorithm:
Initialize nests randomly within the search space.
Evaluate initial solutions' fitness.
Identify the best nest among the initial solutions.
Repeat until max_iter:

Generate new solutions for each nest using Levy flights.
Update nests if a better solution is found.
Update the best nest.
Replace some nests by performing Levy flights.
Display the current best solution.
Step 5: Run Optimization:
Store the best solution and its fitness value.

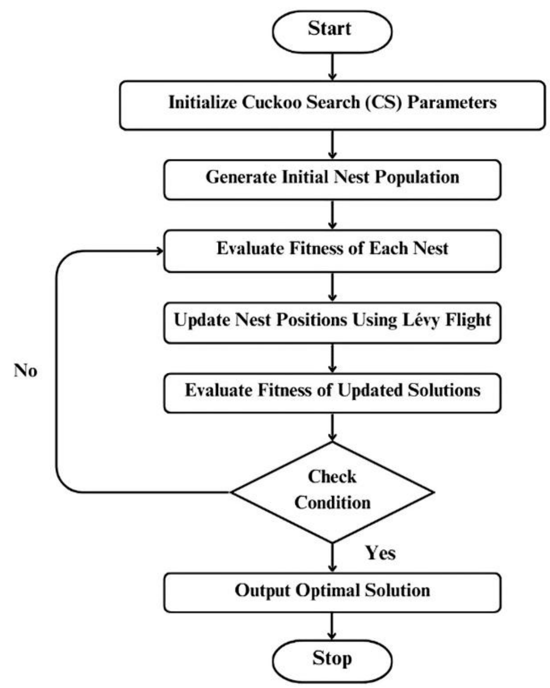

Figure 9. Flowchart of the CSA illustrating the initialization, solution generation via Lévy flights, fitness evaluation, and convergence process.

6.3 Random Search Algorithm (RSA)

The RSA is an optimization technique that explores randomly sampling solutions from the large search space. The RSA is an effective and simple optimization problem among all the advanced algorithms. Due to its large working space, it may take many iterations to find optimal or near-optimal solutions, especially in complex search spaces. **Figure 10** presents the RSM flow chart which explores the optimal solution through repeated random sampling and fitness evaluation. RSA is known for its simplicity when exploring the unknown problem domain.

Total Cost Minimization Using RSA

Step 1: Assign a value to the parameters
Step 2: Define the Total Cost Function
Function total_cost(T):

Return total_cost(T)
Step 3: Set RSA parameters
num_iterations = 1000;
T_min = 0.1;
T_max = 2;
Step 4: Initialize variables and an array to store the best and evaluated solution
Step 5: Random search loop
For i = 1 to num_iterations:
Generate a random value for T between T_min and T_max
Evaluate the objective function at the random value of T
If the evaluated value is better than the current best_value:
Update best_value to the evaluated value
Update best_T to the current T

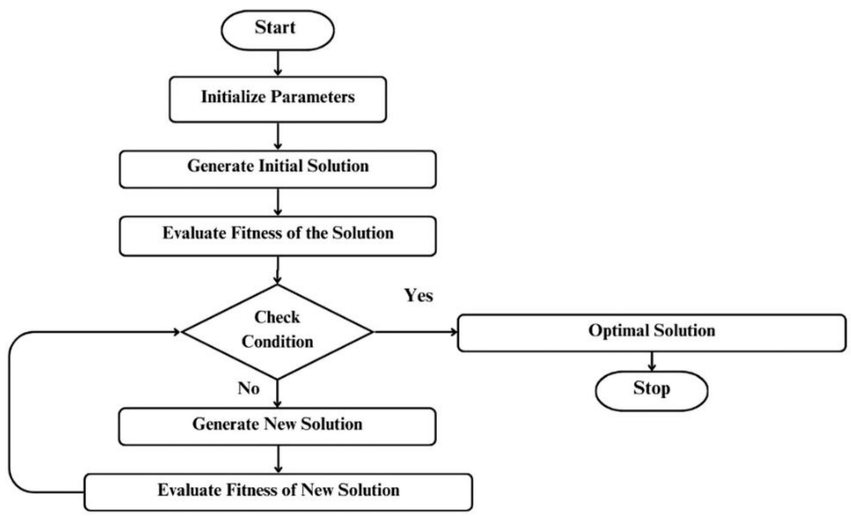

Figure 10. Flowchart of the RSA showing the iterative process of random solution generation and fitness-based selection until termination.

7. Numerical Study

The proposed model determines the optimal cycle time across all scenarios, maximizing profit by minimizing total inventory costs. Pharmaceutical companies aiming to reduce inventory cost and streamline inventory management processes by finding of optimal value. To illustrate the model, the parameter values are considered as follows: Ordering cost (O_c) = \$150, Deterioration rate (d_c) = 0.6, Demand rates (a, b, c) = 7, 6, 5 respectively, Holding costs (h_1, h_2) = \$5, \$3 respectively, Shelf life (m) = 1.5 years, Unit selling price (p) = \$150, Unit cost price (c) = \$120, Interest earned (I_e) = 13% per year, Interest paid (I_p) = 15% per year. The suppliers' credit periods M and retailers' credit N are based on real-world data obtained through direct consultations with local and chain pharmacies.

***Scenario* 1: Local pharmaceutical**

Within the local pharmaceuticals, suppliers extend the credit-based payment to pharmacists, characterized by a credit period of approximately M = 0.164, roughly 60 days. In turn, pharmacists offering a favourable credit period to their regular customers, with a credit period of around $N = 0.0192$, roughly 7 days, where M exceeds N. The computational outcomes are outlined in **Table 2**.

Table 2. Optimization solution of $M > N$ & $M \leq T + N$.

Algorithm	***M***	***N***	***T*****	***TC*****	***R*****
CA	0.164	0.0192	0.7206	333.419	9.19
CSA	0.164	0.0192	0.7205	333.419	9.18
RSA	0.164	0.0192	0.7204	333.419	9.15

* *denotes the optimal solution.*

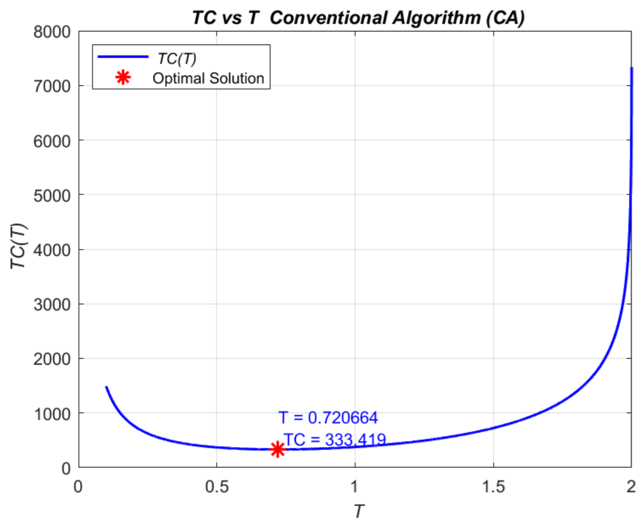

Figure 11. Graphical representation of the optimal solution obtained using the Conventional Algorithm (CA) for the case $M > N$ & $M \leq T + N$.

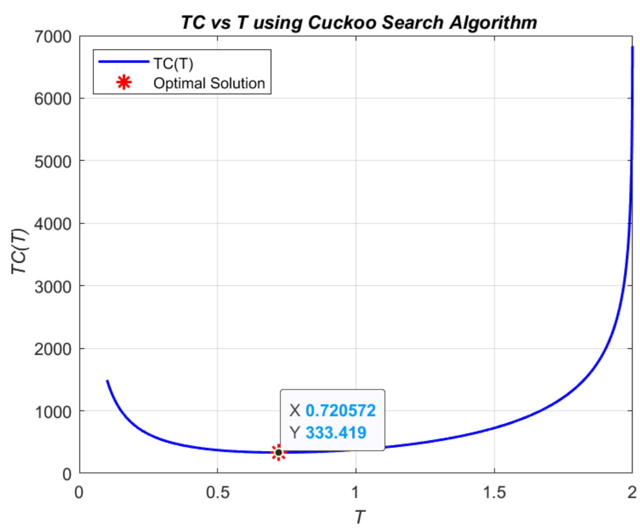

Figure 12. Graphical representation of the optimal solution obtained using the CSA for the case $M > N$ & $M \leq T + N$.

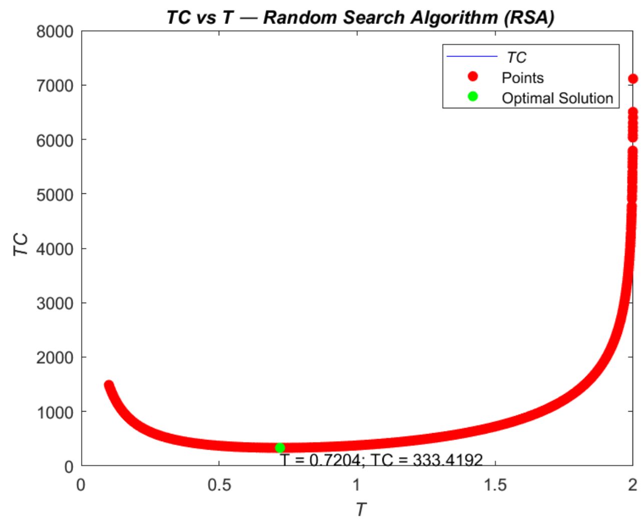

Figure 13. Graphical representation of the optimal solution obtained using the RSA for the case $M > N$ & $M \leq T + N$.

Cutting-edge Optimization and the conventional outcomes are presented in **Table 2**. The convexity of the total cost indicates that the minimum total cost occurs at T^* = 0.7204, with TC = $333.419. **Figures 11** to **13** illustrate the variation in optimal solutions between the CA, CSA and RSA. RSA has a slightly better total cost than both CA and CSA. Although the differences appear minor, they are significant in inventory sectors.

The CA determined the most suitable solution for T = 0.7206 with a minimum cost of 333.4192, in 20.96 seconds. The CSA was able to produce a highly competitive solution for a total cost of 333.4195, completing it in approximately 4.21 seconds, and consistently achieving near-optimal results across independent runs. The RSA achieved a comparable total cost of 333.4192 with a low standard deviation of 0.000144, and a lowest computational time of 3.38 seconds for 10 runs. The optimal cycle for restocking frequency for local pharmacies is approximately 0.72.

***Scenario* 2: Pharmaceutical chain**

In a pharmaceutical chain, entrepreneurs manage multiple branches and maintain a centralized warehouse for smooth inventory. Suppliers extend credit period to the pharmacists, offering a period of M = 0.019, which is roughly 7 days. In return, pharmacists offer a credit to their customers (Patient) with N = 0.082, approximately 30 days, where N exceeds M. The computational findings are presented in **Table 3**.

Table 3. Optimization solution of $M \leq N$.

Algorithm	M	N	T^*	TC^*	R^*
CA	0.0192	0.082	0.7130	354.80	9.02
CSA	0.0192	0.082	0.7130	354.80	9.02
RSA	0.0192	0.082	0.7123	354.80	9

* *denotes the optimal solution.*

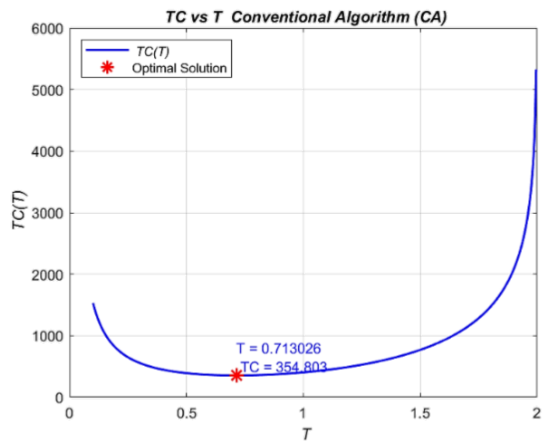

Figure 14. Graphical representation of the optimal solution obtained using the Conventional Algorithm (CA) for the case $M \leq N$.

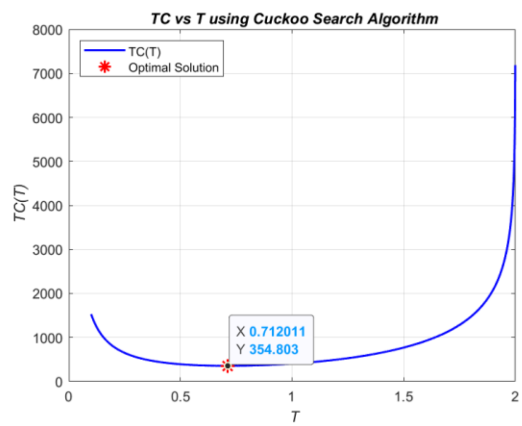

Figure 15. Graphical representation of the optimal solution obtained using the CSA for the case $M \leq N$.

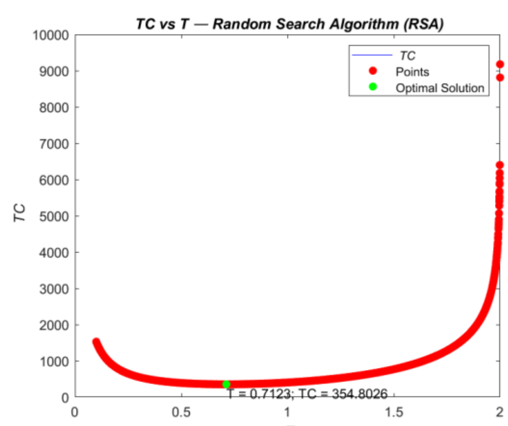

Figure 16. Graphical representation of the optimal solution obtained using the RSA for the case $M \leq N$.

Cutting-edge Optimization and the conventional outcomes are presented in **Table 3**. The convexity of the total cost shows the minimum total cost occurs at T^* = 0.7123, with TC = $354.80. **Figures 14** to **16** illustrate the variation in optimal solutions between CA, CSA and RSA. Among the methods applied, RSA delivers the most cost-effective solution under varying credit conditions. The various optimal differences appear to be small, but they are significant in inventory sectors.

The CA obtained an optimal solution T = 0.7130 with a minimum total cost of 354.8032, which took approximately 3.82 seconds to complete. The CSA proved its competitive worth by having an optimal cost of 354.8023, a slightly quicker execution time of 2.76 seconds, and consistently achieved results across runs. The RSA achieved a very stable outcome with an average cost of 354.8026, a minimal standard deviation of 0.000064 per run, and the lowest computational time of 1.60 seconds. The optimal cycle for restocking frequency of the chain of pharmaceuticals is approximately 0.71. These results demonstrate that while CA is accurate, CSA provides a good balance of accuracy and speed, and RSA is distinguished by its computational simplicity and reliability.

7.1 Managerial and Policy Implications

The findings of this study provide the following managerial insights:

- The optimal cycle time and order quantity obtained help to minimize the deterioration due to expiry and maintain an adequate inventory level to ensure the health outcomes.
- The two-level trade credit helps in managing financial flexibility by allowing retailers to purchase inventory without immediate payment and customers to have a timely credit period. This dual credit period helps both the pharmacist and the patients during emergencies or chronic treatment.
- Validation of the model with real pharmacy data helps the blooming entrepreneurs in pharmaceutical operations. This validates that the proposed framework can serve as a decision-making tool under trade credit.

These managerial insights highlight how the proposed model strengthens pharmaceutical inventory management and improves access to healthcare.

8. Real-World Applications and Operational Integration

The proposed inventory model is a practical application of the real-world pharmaceutical management suitable for both local and chain pharmacy operations. The model parameters, like demand rate, deterioration rate, holding costs, and cost of the items were adopted from established literature references to the work of Uthayakumar and Karuppasamy (2017b). The credit period data were collected from the pharmacists and inventory managers from local pharmacies and large chain pharmacies in Chennai. The collected data was pre-processed by checking, removal of outliers, and conversion into a standardized time unit (years) to ensure uniformity throughout the dataset. The pre-processed values were incorporated into the mathematical formulation with real-world data and literature-based parameters.

This proposed model is designed for pharmacy management systems typically used for inventory systems that manage stock levels, shelf life, and credit handling. These systems can supply live operational data to update model parameters helps in determining the order quantities and payment terms. Due to its flexibility and reliance on both real and literature-based data, the model is suitable for practical implementation from a small to a chain of pharmacies.

9. Sensitivity Analysis

An analysis of sensitivity is performed by changing the key parameters in this model as shown in **Figure 17** and **Table 4**.

Table 4. Sensitivity analysis.

Key parameters	% Change	Values	*T*	*TC*	% change in *TC* value
a	-20%	5.6	0.7408	317.4583	-4.78698
	-10%	6.3	0.7305	325.5029	-2.37422
	0%	7	0.7206	333.419	0
	10%	7.7	0.7108	341.2118	2.337239
	20%	8.4	0.7013	348.8848	4.63854
b	-20%	4.8	0.7366	326.8121	-1.98156
	-10%	5.4	0.7283	330.1561	-0.97862
	0%	6	0.7206	333.419	0
	10%	6.6	0.7131	336.6061	0.955884
	20%	7.2	0.7058	339.7211	1.890144
c	-20%	4	0.7313	330.4596	-0.88759
	-10%	4.5	0.7256	331.9571	-0.43846
	0%	5	0.7206	333.419	0
	10%	5.5	0.7153	334.8481	0.42862
	20%	6	0.7108	336.2455	0.847732
h_1	-20%	4	0.7294	328.6273	-1.43714
	-10%	4.5	0.7248	331.0361	-0.71469
	0%	5	0.7206	333.419	0
	10%	5.5	0.7161	335.7774	0.707338
	20%	6	0.7119	338.1115	1.407388
h_2	-20%	2.4	0.7226	332.7227	-0.20884
	-10%	2.7	0.7214	333.0717	-0.10416
	0%	3	0.7206	333.419	0
	10%	3.3	0.7195	333.7655	0.103923
	20%	3.6	0.7184	334.1104	0.207367
O_c	-20%	120	0.6709	290.3219	-12.9258
	-10%	135	0.6967	312.2551	-6.34754
	0%	150	0.7206	333.419	0
	10%	165	0.7423	353.9249	6.150189
	20%	180	0.7625	373.8579	12.12855
d_c	-20%	0.48	0.7226	329.4793	-1.18161
	-10%	0.54	0.7214	331.45	-0.59055
	0%	0.6	0.7206	333.419	0
	10%	0.66	0.7195	335.3869	0.590218
	20%	0.72	0.7184	337.3531	1.179927
p	-20%	120	0.7207	334.0873	0.200438
	-10%	135	0.7207	333.7533	0.100264
	0%	150	0.7206	333.419	0
	10%	165	0.7203	333.0852	-0.10011
	20%	180	0.7203	332.7511	-0.20032
s	-20%	96	0.7556	316.3693	-5.1136
	-10%	108	0.7374	325.078	-2.50166
	0%	120	0.7206	333.419	0
	10%	132	0.7051	341.4324	2.403402
	20%	144	0.6906	349.1504	4.718207
I_e	-20%	0.1	0.7207	334.19	0.231241
	-10%	0.11	0.7207	333.9331	0.15419
	0%	0.13	0.7206	333.419	0
	10%	0.14	0.7203	333.1623	-0.07699
	20%	0.15	0.7203	332.9053	-0.15407
I_p	-20%	0.12	0.7556	316.3693	-5.1136
	-10%	0.13	0.7431	322.2181	-3.35941
	0%	0.15	0.7206	333.419	0
	10%	0.16	0.71	338.7957	1.612596
	20%	0.18	0.6906	349.1504	4.718207

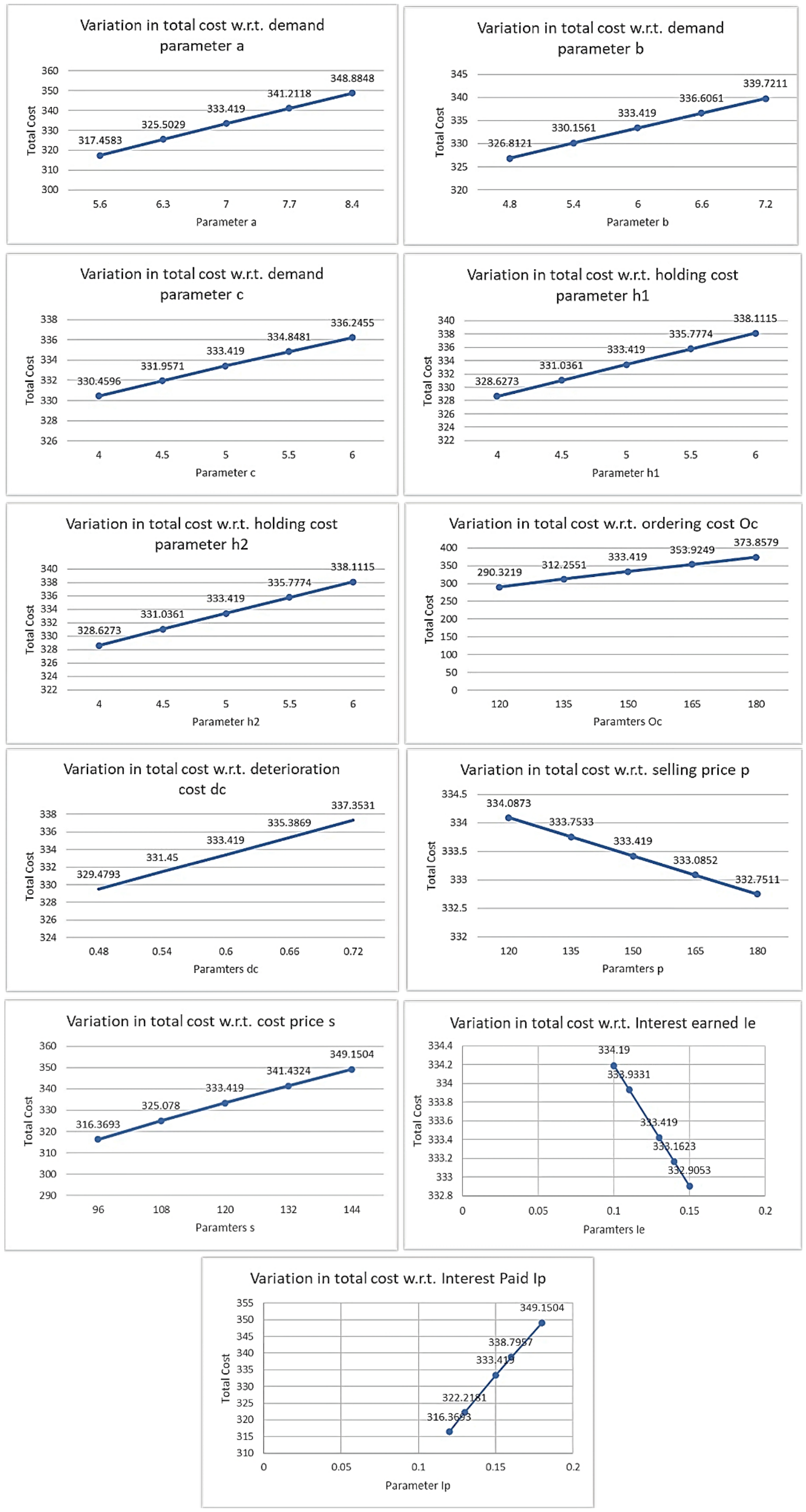

Figure 17. Graphical representation of the sensitivity analysis.

The following are some observations that can be drawn from **Table 4**:

- Total cost increases with higher values for the demand parameters (a, b), unit cost price (s), ordering cost (O_c) and interest paid (I_p).
- Total cost decreases as interest earned (I_e) and selling price (p) increase, suggesting improved cost efficiency.
- Ordering cost (O_c) has the most significant impact on total cost, followed by demand parameters (a, b), and shortage cost (s).
- Holding costs (h_1, h_2) and deterioration cost (d_c) have a moderate influence on total cost.
- Selling price (p) shows relatively minor effects on total cost.
- Replenishment cycle time (T) is mainly affected by demand parameter (a, c), other parameters have minimal impact on the replenishment cycle.

10. Significance of the Model

Pharmaceutical inventory management can be significantly improved by these constraints which address the challenges faced due to demand, expiry date of the products and determining replenishment quantity. The proposed framework incorporates time-dependent demand and trade credit, which optimizes stock levels, reduces wastage, and assures the continuous availability of medicine. The study introduces two-level trade credit that combines financial and inventory management for the highly perishable healthcare items. The inventory system's performance was improved by implementing a two-level trade credit policy, as demonstrated by the numerical analyses. This study minimizes the total inventory costs and improves cash flow in inventory management by incorporating the credit periods for both retailers and customers. The financial aspects of pharmaceutical operations are strengthened by trade credit.

Pharmaceutical retailers and suppliers can improve cash flow efficiency, reduce financial risks, and balance costs by determining the optimal replenishment time. Entrepreneurs can maintain a sustainable operation while managing inventory with the flexibility of the dual-credit framework through different structures. The model enhances decision-making by incorporating real-life situations that are specifically suited for pharmaceuticals, such as medications, vaccines, diagnostic tools, and medical devices. Demand rate helps in determining the order quantity in the healthcare distribution system in critical healthcare settings.

The model uses advanced optimization techniques like RSA and CSA to solve complex inventory challenges. The use of computational approaches ensures optimal solutions are derived in enabling efficient inventory control in vast healthcare environments. This model is an effective tool for addressing the complex pharmaceutical industries to manage inventory cost-effectively. The model's integration of trade credit policies with advanced optimization methods enhances financial support that efficiently contributes to a more sustainable inventory.

11. Conclusion

The pharmaceutical inventory model is incorporated with expiry date and two-level credit. The proposed study aims to determine the optimal replenishment time and credit period to minimize the total inventory cost. In the pharmaceutical industry, trade credit helps entrepreneurs to solve the complex distribution system easily by managing the inventory levels of the critical medications. Flexible payment enables pharmacists and related companies to maintain inventory, optimize cash flow, and satisfy customer demands. Numerical problems are analysed for both the local and the chain of pharmacy under various credit periods and cycle times. This single-item inventory model applies to all pharmaceutical items that have expiry dates. Optimisation of order quantities and replenishment cycle times provides the proposed model with financial and operational efficiency, as well as improves customers' health. The systematic inventory monitoring closely with demand not only improves inventory profit and reduces wastage but also supports Sustainable Development Goals, Good Health and Well-being and Responsible Consumption and Production.

11.1 Limitations

This model is limited by its focus on pharmaceutical inventory management, such as medications and vaccines, which have a fixed shelf life. The model is restricted to deal with a single pharmaceutical item. This single-item assumption was adopted to focus on the demand variation, deterioration behaviour, and financial interactions under a two-level trade credit system for each pharmaceutical item. In the real world, pharmaceutical inventory management involves multiple products with different shelf lives and demand patterns, which are not captured in the current formulation. The assumption that the two-level credit policy and replenishment strategies are universally effective may not align with the diverse operational conditions of all the pharmaceutical businesses. The model also does not consider external factors like market fluctuations or sudden shifts in consumer demand, which could significantly impact inventory and trade credit decisions.

11.2 Future Scope

The proposed pharmaceutical inventory model can be extended to a multi-item inventory model, considering product shelf life and priority-based stocking strategies. Future research could integrate inflationary effects and the time value of money to enhance financial decision-making in inventory decision-making. The model can be adapted for hospital and healthcare facility inventory management, addressing emergency stock levels and regulatory constraints.

Appendix A

Deterioration cost calculation

$\text{Deterioration cost } = d_c * D_i$

$$= d_c * (-m-1) * [[c(1+m)^2 + b(1+m) + a] \log\left(\frac{1+m-T}{1+m}\right) + \frac{c}{2}[(T-m-1)^2 - (1+m)^2] + [2c(1+m)+b]T - aT - \frac{bT^2}{2} - \frac{cT^3}{3}] \quad \text{(A.1)}$$

Holding cost calculation

$Holding\ cost\ = \int_0^T (h_1 + h_2 t)Q(t)dt$

$$= h_1 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(1+m-T)^2}{4} - (\frac{(1+m)^2}{4} - \frac{(1+m)^2}{2}(\log(1+m) - \log(1+m-T))] + \frac{c}{2} * [-\frac{((T-m-1)^2-(-1-m)^2)^2}{4}] + [2c(1+m)+b] * [T^2(-m-1) - \frac{T^3}{3} + (T+m+1)\frac{T^2}{2}] + h_2 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(T-m-1)^3}{9} - \frac{(-m-1)^3}{9} + \frac{m+1}{4}[(T-m-1)^2 - (-m-1)^2] - \frac{(m+1)^3}{6}[\log(1+m-T) - \log(1+m)]] + \frac{c}{2} * [-\frac{(T-m-1)^5}{5} + \frac{(-m-1)^5}{5} - \frac{m+1}{4}[(T-m-1)^4 - (-m-1)^4] + \frac{(1+m-T)^2}{3}[(T-m-1)^3 - (-m-1)^3] - \frac{(-m-1)(1+m-T)^2}{2}[(T-m-1)^2 - (-m-1)^2]] + [2c(1+m)+b] * [-\frac{T^4}{4} + \frac{(1+m+T)T^3}{3} + \frac{T^3(-m-1)}{2}]] \quad \text{(A.2)}$$

Case 1.1: $\boldsymbol{M > N}$ & $\boldsymbol{M \leq T + N}$

Interest earned

$IE_1 = pI_e \int_N^M [Q(0) - Q(t-N)]dt$

$= pI_e[[c(1+m)^2 + b(1+m) + a] * [(-m-1)T(log(1+m-T) - \log(1+m)) - \Big[\frac{(1+m+N-M)^2}{4} - \frac{(1+m)^2}{4} - \frac{(1+m+N-M)^2}{2}(\log(1+m+N-M) - \log(1+\mathrm{m}-\mathrm{T})) + \frac{(1+m)^2}{2}(\log(1+m) - \log(1+m-T))\Big]\Big] + \frac{c}{2}[T(-m-1)((T-m-1)^2 - (m-1)^2)^2 - (-\frac{((T-m-1)^2-(M-N-m-1)^2)^2}{4} + \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b + 2c(1+m)] * (T^2(-1-m) - (-\frac{NM((M-2(N+m)-2)}{2} + \frac{N^2(N-2m-2)}{2}))]$ (A.3)

Interest paid

$IP_1 = cI_p \int_M^{T+N} Q(t-M)\,dt$

$= cI_p[[c(1+m)^2 + b(1+m) + a]\,[\frac{(1+m+M-T-N)^2}{4} - \frac{(1+m)^2}{4} - \frac{(1+m+M-T-N)^2}{2}(log(1+m+M-T-N) - \log(1+m-T)) + \frac{(1+m)^2}{2}(\log(1+m) - \log(1+m-T))]] - \frac{C}{2}[\frac{((T-m-1)^2-(T+N-M-m-1)^2)^2}{4} - \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b + 2c(1+m)] * [(Mm + M^2 + M)(T+N-M) - \frac{M}{2}((T+N)^2 - M\text{^}2)]]$ (A.4)

Total inventory cost

$TC_1 = \frac{1}{T}[OC + DC + HC + IP_1 - IE_1]$

$= \frac{1}{T}\Big[O_c + d_c * (-m-1) * [[c(1+m)^2 + b(1+m) + a]\log\left(\frac{1+m-T}{1+m}\right) + \frac{c}{2}[(T-m-1)^2 - (1+m)^2] + [2c(1+m) + b]T - aT - \frac{bT^2}{2} - \frac{cT^3}{3}] + h_1 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(1+m-T)^2}{4} - (\frac{(1+m)^2}{4} - \frac{(1+m)^2}{2}(\log(1+m) - \log(1+m-T))] + \frac{c}{2} * [-\frac{\left((T-m-1)^2-(-1-m)^2\right)^2}{4}] + [2c(1+m) + b] * [T^2(-m-1) - \frac{T^3}{3} + (T+m+1)\frac{T^2}{2}] + h_2 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(T-m-1)^3}{9} - \frac{(-m-1)^3}{9} + \frac{m+1}{4}[(T-m-1)^2 - (-m-1)^2] - \frac{(m+1)^3}{6}[\log(1+m-T) - \log(1+m)]] + \frac{c}{2} * [-\frac{(T-m-1)^5}{5} + \frac{(-m-1)^5}{5} - \frac{m+1}{4}[(T-m-1)^4 - (-m-1)^4] + \frac{(1+m-T)^2}{3}[(T-m-1)^3 - (-m-1)^3] - \frac{(-m-1)(1+m-T)^2}{2}[(T-m-1)^2 - (-m-1)^2]] + [2c(1+m) + b] * [-\frac{T^4}{4} + \frac{(1+m+T)T^3}{3} + \frac{T^3(-m-1)}{2}]] - pI_e[[c(1+m)^2 + b(1+m) + a] * [(-m-1)T(log(1+m-T) - \log(1+m)) - \Big[\frac{(1+m+N-M)^2}{4} - \frac{(1+m)^2}{4} - \frac{(1+m+N-M)^2}{2}(\log(1+m+N-M) - \log(1+\mathrm{m}-\mathrm{T})) + \frac{(1+m)^2}{2}(\log(1+m) - \log(1+m-T))\Big]\Big] + \frac{c}{2}[T(-m-1)((T-m-1)^2 - (m-1)^2)^2 - (-\frac{((T-m-1)^2-(M-N-m-1)^2)^2}{4} + \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b + 2c(1+m)] * (T^2(-1-m) - (-\frac{NM((M-2(N+m)-2)}{2} + \frac{N^2(N-2m-2)}{2}))] + cI_p[[c(1+m)^2 + b(1+m) + a]\,[\frac{(1+m+M-T-N)^2}{4} - \frac{(1+m)^2}{4} - \frac{(1+m+M-T-N)^2}{2}(log(1+m+M-T-N) - \log(1+m-T)) + \frac{(1+m)^2}{2}(\log(1+$

$$m) - \log(1+m-T))]] \ -\frac{C}{2}[\frac{((T-m-1)^2-(T+N-M-m-1)^2)^2}{4} - \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b+2c(1+m)] * [(Mm + M^2+M)(T+N-M) - \frac{M}{2}((T+N)^2 - M\hat{}2\ )]] \Bigg] \quad \text{(A.5)}$$

Case 1.2: $M > N$ & $M > T+N$

Interest earned

$$IE_2 = pI_e \int_N^{T+N} [Q(0) - I(t-N)]dt + pI_e \int_{T+N}^{M} Q(0)\, dt$$

$$= pI_e[[c(1+m)^2 + b(1+m) + a] * [(-m-1)T(log(1+m-T) - log(1+m)) - \Bigg[\frac{(1+m-T)^2}{4} - \frac{(1+m)^2}{4} + \frac{(1+m)^2}{2}(log(1+m) - log(1+m-T))]\Bigg] + \frac{c}{2}[T(-m-1)((T-m-1)^2 - (m-1)^2) - \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b+2c(1+m)] * (T^2(-1-m) - (-\frac{N(T+N)(T+N-2(N+m)-2)}{2} - \frac{N^2(N+2m+2)}{2})) + \ (M-T-N) * (-m-1) * [[c(1+m)^2 + b(1+m) + a] * ((log(1+m-T) - log(1+m))) + \frac{c}{2}[(T-m-1)^2 - (m-1)^2)] + [b + 2c(1+m)] * T]] \quad \text{(A.6)}$$

Total inventory cost

$$TC_2 = \frac{1}{T}[OC + DC + HC - IE_2\ ]$$

$$= \frac{1}{T}\Bigg[O_c + d_c * (-m-1) * [[c(1+m)^2 + b(1+m) + a]\, log\left(\frac{1+m-T}{1+m}\right) + \frac{c}{2}[(T-m-1)^2 - (1+m)^2] + [2c(1+m) + b]T - aT - \frac{bT^2}{2} - \frac{cT^3}{3}] + h_1 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(1+m-T)^2}{4} - (\frac{(1+m)^2}{4} - \frac{(1+m)^2}{2}(log(1+m) - log(1+m-T))] + \frac{c}{2} * [-\frac{((T-m-1)^2-(-1-m)^2)^2}{4}] + [2c(1+m) + b] * [T^2(-m-1) - \frac{T^3}{3} + (T+m+1)\frac{T^2}{2}] + h_2 * [[c(1+m)^2 + b(1+m) + a] * [\frac{(T-m-1)^3}{9} - \frac{(-m-1)^3}{9} + \frac{m+1}{4}[(T-m-1)^2 - (-m-1)^2] - \frac{(m+1)^3}{6}[log(1+m-T) - log\,(1+m)]] + \frac{c}{2} * [-\frac{(T-m-1)^5}{5} + \frac{(-m-1)^5}{5} - \frac{m+1}{4}[(T-m-1)^4 - (-m-1)^4] + \frac{(1+m-T)^2}{3}[(T-m-1)^3 - (-m-1)^3] - \frac{(-m-1)(1+m-T)^2}{2}[(T-m-1)^2 - (-m-1)^2]] + [2c(1+m) + b] * [-\frac{T^4}{4} + \frac{(1+m+T)T^3}{3} + \frac{T^3(-m-1)}{2}]] - pI_e[[c(1+m)^2 + b(1+m) + a] * [(-m-1)T(log(1+m-T) - log(1+m)) - \Bigg[\frac{(1+m-T)^2}{4} - \frac{(1+m)^2}{4} + \frac{(1+m)^2}{2}(log(1+m) - log(1+m-T))]\Bigg] + \frac{c}{2}[T(-m-1)((T-m-1)^2 - (m-1)^2) - \frac{((T-m-1)^2-(-m-1)^2)^2}{4}] + [b + 2c(1+m)] * (T^2(-1-m) - (-\frac{N(T+N)(T+N-2(N+m)-2)}{2} - \frac{N^2(N+2m+2)}{2})) + \ (M - T - N) * (-m-1) * [[c(1+m)^2 + b(1+m) + a] * ((log(1+m-T) - log(1+m))) + \frac{c}{2}[(T-m-1)^2 - (m-1)^2)] + [b + 2c(1+m)] * T]]\Bigg] \quad \text{(A.7)}$$

Case 2: M ≤ N
Interest paid

$$IP_3 = cI_p \int_M^N Q(0)\,dt + cI_p \int_N^{T+N} Q(t-N)\,dt$$

$$= cI_p[(N-M)*(-m-1)*[[c(1+m)^2+b(1+m)+a]*((log(1+m-T)-log(1+m))+\frac{c}{2}[(T-m-1)^2-(m-1)^2]+[b+2c(1+m)]*T]+[c(1+m)^2+b(1+m)+a]\,[\frac{(1+m-T)^2}{4}-\frac{(1+m)^2}{4}+\frac{(1+m)^2}{2}(log(1+m)-log(1+m-T))]]+\frac{C}{2}[\frac{((T-m-1)^2-(-m-1)^2)^2}{4}]+[b+2c(1+m)]*[-\frac{N(T+N)(T+N-2(N+m)-2)}{2}-\frac{N^2(N+2m+2)}{2}]] \quad (A.8)$$

Total Inventory cost

$$TC_3 = \frac{1}{T}[OC + DC + HC + IP_3\,]$$

$$= \frac{1}{T}\Big[O_c + d_c*(-m-1)*[[c(1+m)^2+b(1+m)+a]\log\left(\frac{1+m-T}{1+m}\right)+\frac{c}{2}[(T-m-1)^2-(1+m)^2]+[2c(1+m)+b]T-aT-\frac{bT^2}{2}-\frac{cT^3}{3}]+h_1*[[c(1+m)^2+b(1+m)+a]*[\frac{(1+m-T)^2}{4}-(\frac{(1+m)^2}{4}-\frac{(1+m)^2}{2}(\log(1+m)-\log(1+m-T))]+\frac{c}{2}*[-\frac{\left((T-m-1)^2-(-1-m)^2\right)^2}{4}]+[2c(1+m)+b]*[T^2(-m-1)-\frac{T^3}{3}+(T+m+1)\frac{T^2}{2}]+h_2*[[c(1+m)^2+b(1+m)+a]*[\frac{(T-m-1)^3}{9}-\frac{(-m-1)^3}{9}+\frac{m+1}{4}[(T-m-1)^2-(-m-1)^2]-\frac{(m+1)^3}{6}[\log(1+m-T)-\log(1+m)]]+\frac{c}{2}*[-\frac{(T-m-1)^5}{5}+\frac{(-m-1)^5}{5}-\frac{m+1}{4}[(T-m-1)^4-(-m-1)^4]+\frac{(1+m-T)^2}{3}[(T-m-1)^3-(-m-1)^3]-\frac{(-m-1)(1+m-T)^2}{2}[(T-m-1)^2-(-m-1)^2]]+[2c(1+m)+b]*[-\frac{T^4}{4}+\frac{(1+m+T)T^3}{3}+\frac{T^3(-m-1)}{2}]]+cI_p[(N-M)*(-m-1)*[[c(1+m)^2+b(1+m)+a]*((log(1+m-T)-\log(1+m))+\frac{c}{2}[(T-m-1)^2-(m-1)^2]+[b+2c(1+m)]*T]+[c(1+m)^2+b(1+m)+a]\,[\frac{(1+m-T)^2}{4}-\frac{(1+m)^2}{4}+\frac{(1+m)^2}{2}(\log(1+m)-\log(1+m-T))]]+\frac{C}{2}[\frac{((T-m-1)^2-(-m-1)^2)^2}{4}]+[b+2c(1+m)]*[-\frac{N(T+N)(T+N-2(N+m)-2)}{2}-\frac{N^2(N+2m+2)}{2}]]\Big] \quad (A.9)$$

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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During the preparation of this work the author(s) used generative AI in order to improve the language of the article. After using this tool/service, the author(s) reviewed and edited the content as needed and take(s) full responsibility for the content of the publication.

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