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Cost Optimization using Genetic Algorithm in Customers Intolerance Markovian Model with Working Vacation and Multiple Working Breakdowns

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Abstract
In this paper, we consider a single server queueing model with working vacation and multiple working breakdowns. When on vacation, the server works at a different pace. Disturbances occur in the server due to multiple breakdowns. In working breakdowns server works at a different rate. During the time of interruption caused by working vacation and working breakdowns, the main server can find many implementations in operation. Both the server's lifespan and the time it takes to repair it are considered to be exponentially dispersed. Also, we have considered balking and reneging behaviours of customers. The stationary queue length distribution is computed using a matrix-analytic technique. Using Genetic Algorithm (GA) we optimize the cost function. The predicted length of a busy period, the expected length of a working vacation period, the expected length of a working breakdown period, the mean waiting time, and the average delay are all established. We compute numerical results to verify the analytical point of view. The effect of individual parameters is investigated using sensitivity analysis.

Keywords- Working breakdowns, Matrix-analytic method, Working vacation, Balking; Reneging, Genetic algorithm.

1. Introduction

With the poliferation of artificial intelligence, Nature-Inspired Algorithm (NIAs) are gradually getting prominence in the current era. This is because of their learning and adaptation capability from the nature. To address complex problems, scientists study how nature behaves in various contexts. The NIAs are based on physics, biology, and ethology concepts. They employ stochastic components, which include random variables.

Many real-world applications require the optimization of specific goals such as cost reduction, energy consumption reduction, performance, efficiency, and sustainability maximisation. Genetic Algorithm (GA) is one of the various NIAs available. GA is a method of optimization that employs search to solve problems with a large solution space. It implements a natural selection process with the purpose of generating superior solutions. GA maintains a chromosomal population. A comprehensive solution to the problem is represented by a chromosome. Chromosomes provide solutions to problems in the search space, which are rated using a fitness function. Malik et al. (2021) have used genetic algorithm for a Geo/G/I retrial model cost inspection. Jain and Jain (2022) have considered a server based retrial queueing system with breakdown and optimized the cost function using GA.

A queueing system may fail unexpectedly, and the server may be unable to provide service until the system is fixed. In telecommunications, construction, manufacturing systems, and in the hotel industry, queueing models have been implemented where interruptions occur due to vacations and breakdowns. After a certain amount of time, the server can go into a working vacation mode. During this time, secondary tasks can be done by the server. Because of their various use, queueing models with service interruption have received a lot of attention.

When a breakdown occurs in a system then the server gives service at a slower rate. It effects the queue length which can be controlled by introducing a perfect model. Several scholars have looked on queueing models that include working vacations and working breakdowns. A queueing paradigm MAP/PH/1 with working vacation was examined by Chakravarthy and Ozkar (2016). They introduced crowd sourcing, which is beneficial in fields such as healthcare, environmental science and in business. Li and Liu (2017) considered a GI/M/1 queue in which the interarrival periods are an independently dispersed, identically distributed sequence of random variables with a general distribution function. Shoukry et al. (2018) has been considered a single server queueing model in which breakdown is introduced. It is applicable in the repairing of ATM machine. Ahuja et al. (2022) investigated a single server queueing model that is unreliable, as well as multiple stage service and functioning vacation.

Ayyappan and Deepa (2018) noticed a batch arrival bulk service queue. The server has taken several vacations till at least one consumer is in the queue. The steady-state instance was obtained using the additional variable technique. Kalyanaraman and Sundaramoorthy (2019) have considered a Markovian queueing system with one server. In this paper they have observed three states busy, repair and working vacation in a single server Markovian queueing system. Majid and Manoharan (2019) were assigned to the M/M/1 queue while on leave. During working vacation, if there are at least N customers waiting in the system, the server resumes normal operations. Otherwise, the vacation continues. In order to obtain the stationary queue length distributions, the matrix geometric technique was used. Under reactive and scheduled maintenance, Liu et al. (2020) looked at a single server queueing architecture with a specific service interruption discipline. The service will be suspended when customers feedback is negative and reactive maintenance may be activated. If the response is good, it will not be implemented. The steady state probability was calculated using a matrix analytic method.

Many industries use queueing systems with balking and reneging, such as patients and inventory systems for perishable items are two examples. In many situations, the loss of earnings due to balking and reneging can be limitless. Customers' vexing behaviours should be factored into queueing system research, in order to accurately mimic real-world scenarios. Kumar and Sharma (2017) have introduced retention of reneging and balking behaviours of customers in their queueing model. To find time-dependent state probabilities, the probability generating function technique is employed. Som and Kumar (2017) looked at a Markovian model where two servers are heterogeneous and finite capacity. It has been observed that reverse balking and reneging behaviour have an effect. The stationary system size probability is calculated using iterative approaches.

Many writers have studied broad bulk service and vacations extensively in queueing theory. Due to server vacations and outages, the queueing paradigm becomes more realistic for dealing with real-life scenarios. The "General Bulk Service Rule" has been implemented by Ayyappan and Nirmala (2018) with two repair phases. A multiserver model was proposed by Kuaban et al. (2020). In a queueing model of finite capacity with balking, they established the application of linked reneging in health care. Kadi et al. (2020) has been observed a queueing system where balking and reneging behaviours of the customers are considered. The techniques utilized were based on probability generating functions. In the queueing model, Chakravarthy et al. (2020) studied a backup server, which has numerous practical uses. They used a phase type distribution to represent the service times. Panta et al. (2021) observed a multi-server queueing model where reneging behaviour of customer is observed in a fuzzy condition.

Choudhary et al. (2021) discovered a MAP/PH/1 queueing model with service rate deterioration. Laxmi et al. (2021) have analysed an infinite buffer single server queueing model. Working vacations, working breakdowns, balking, and reneging behaviours have been observed. We noticed in a single server queueing model, the impact of balking and reneging taking working vacation and multiple working breakdowns states in this article, which had never been seen before.

Computer systems, networks, industrial systems, and a number of other systems that may undergo a sudden breakdown can all benefit from our proposed paradigm. Assume that one switch fails before another in a system with several switches. It needs to be repaired, however it can also be used with other switches. The server is initially serving customers during a normal busy period, but it may be forced to shut down owing to a technical failure and may go for repair. In section 2 we have described the model, in section 3 queue size distribution is covered, governing equations and analysis in section 4, in section 5 we described analysis, GA is described in section 6, performance measures in section 7, cost analysis in section 8, sensitivity analysis in section 9, in section 10 we have given the conclusion and references is given in section 11.

2. Model

We consider a single server queueing model with working vacation state, normal busy state, multiple working breakdowns and repair states. We can use this model in scheduling programs on computer. We can consider a scenario from cash deposit section in a bank where large number of customers waiting in a queue, the server can take vacation.

Vacations are used for a variety of reasons, including personal needs and work-related activities. It is usual for service providers to fail in the service industry. In a working breakdown condition, the server can continue to provide service with minor interruptions and repairs. Customers are greeted using the Poisson method, and service is delivered in accordance with their arrivals. The time it takes to service a customer has an exponential distribution. Customers can also refuse to use the system or renege at any time in any state. The probability of balking and reneging can be utilized to predict the amount of business lost.

Let $X(t) = V$, when the server is in the state of working vacation
$= B$, when the server is in the state of normal busy
$= D$, when the server is in the state of multiple working breakdown

Notations

State	**Arrival rate**	**Servicing rate**	
Working vacation	λ_{v_n}	μ_{v_n}	$n = 0, 1, \dots\dots.\infty$
Busy	λ_{b_n}	μ_{b_n}	$n = 0, 1, \dots\dots..\infty$
Multiple working breakdowns	$\lambda_{d_n}^{i}$	$\mu_{d_n}^{i}$	$n = 0, 1, \dots\dots.\infty$ $i = 1,2, \dots\dots l$

Other parameters are

Vacation duration is exponentially distributed with mean	$\frac{1}{\Psi}$
Rate of repairing is	$\propto$

Rate of breakdown is	β
Breakdown from one state to another is with probability	$br_i,\ 1 \le i \le l$
Repairing from one state to another is with probability	$r_{in},\ 1 \le i \le l$ and $1 \le n < \infty$
The impatience timer when the server on vacation is exponentially distributed with parameter η_0	T_v
The impatience timer when the server on busy state is exponentially distributed with parameter η_1	T_b
The impatience timer when the server on working breakdown states are exponentially distributed with parameters η_d^1 , η_d^2 , η_d^3,……..η_d^l.	T_d
Reneged customers are more likely to exit the system	γ
Remain in the queue with a high chance of success	$\bar{\gamma} = 1 - \gamma$
With a high chance, a customer will choose to join the queue	θ
The probability of balking	$\bar{\theta} = 1 - \theta$

3. The Distribution of Queue Size

The system states define as $0, 1, i$, where 0 denotes the vacation state, 1 denotes the busy state and i denotes the multiple breakdown and repair states.

In vacation state probability vectors are $P_V(0,n)$ where $n = 0, 1, 2, \ldots \infty$
In busy state probability vectors are $P_B(1,n)$ where $n = 1, 2, \ldots \infty$
In breakdown and repair state vectors are $P_D(i,n)$ where $1 \le i \le l$ and $n = 1, 2, 3, \ldots \infty$

4. Governing Equations

The following equations show the flow of transitions for the different states of the system which can be balanced in order to construct steady state equations. From figure 1 we get,

$$(\lambda_{b_0} + \lambda_{v_0}) P_V(0,0) = (\mu_{v_0} + \eta_0\gamma) P_V(0,1) + (\mu_{b_0} + \eta_1\gamma) P_B(1,1) \quad (1)$$

$$[\theta\lambda_{v_1} + (\mu_{v_0} + \eta_0\gamma) + \Psi] P_V(0,1) = \lambda_{v_0} P_V(0,0) + (\mu_{v_1} + 2\eta_0\gamma) P_V(0,2) \quad (2)$$

$$[\theta\lambda_{v_n} + (\mu_{v_{n-1}} + n\eta_0\gamma) + \Psi] P_V(0,n) = \theta\lambda_{v_{n-1}} P_V(0,n-1) + \{\mu_{v_n} + (n+1)\eta_0\gamma\} P_V(0,n+1),\ n \ge 2 \quad (3)$$

$$[\theta\lambda_{b_1} + (\mu_{b_0} + \eta_1\gamma) + \beta b_{r_1}) P_B(1,1) = (\mu_{b_1} + 2\eta_1\Upsilon) P_B(1,2) + \Psi P_V(0,1) + \lambda_{b_0} P_V(0,0) + \propto \sum_{i=1}^{l} r_i P_D(i,1),\quad i = 1,2,\ldots\ldots l \quad (4)$$

$$[\theta\lambda_{b_n} + (\mu_{b_{n-1}} + n\eta_1\gamma) + \beta b_{r_1}) P_B(1,n) = (\mu_{b_n} + (n+1)\eta_1\Upsilon) P_B(1,n+1) + \Psi P_V(0,n) + \theta\lambda_{b_{n-1}} P_B(1,n-1) + \propto \sum_{i=1}^{l} r_{in} P_D(i,n),\quad n \ge 2 \quad (5)$$

$$(\theta\lambda_{d_1}^1 + \propto r_{11} + \beta b_{r_2}) P_D(1,1) = \beta b_{r_1} P_B(1,1) + (\mu_{d_1}^1 + 2\eta_d^1\gamma) P_D(1,2) \quad (6)$$

$$(\theta\lambda_{d_1}^i + \propto r_{i1} + \beta b_{r_{i+1}}) P_D(i,1) = \beta b_{r_i} P_D(i-1,1) + (\mu_{d_1}^i + 2\eta_d^i\gamma) P_D(i,2),\quad i = 2,3,\ldots\ldots\infty \quad (7)$$

$$[\theta\lambda_{d_n}^1 + \beta b_{r_2} + (\mu_{d_{n-1}}^1 + n\eta_d^1\gamma) + \propto r_{1n}] P_D(1,n) = \theta\lambda_{d_{n-1}}^1 P_D(1,n-1) + [\mu_{d_n}^1 + (n+1)\eta_d^1\gamma] P_D(1,n+1) + \beta b_{r_1} P_B(1,n),\ n = 2,3,\ldots\ldots\infty \quad (8)$$

$$[\theta\lambda_{d_n}^i + \beta b_{r_{i+1}} + (\mu_{d_{n-1}}^i + n\eta_d^i\gamma) + \propto r_{in}] P_D(i,n) = \theta\lambda_{d_{n-1}}^i P_D(i,n-1) + [\mu_{d_n}^i + (n+1)\eta_d^i\gamma] P_D(i,n+1) + \beta b_{r_i} P_D(i-1,n),\ n = 2,3,\ldots\ldots\infty,\quad 2 \le i \le l \quad (9)$$

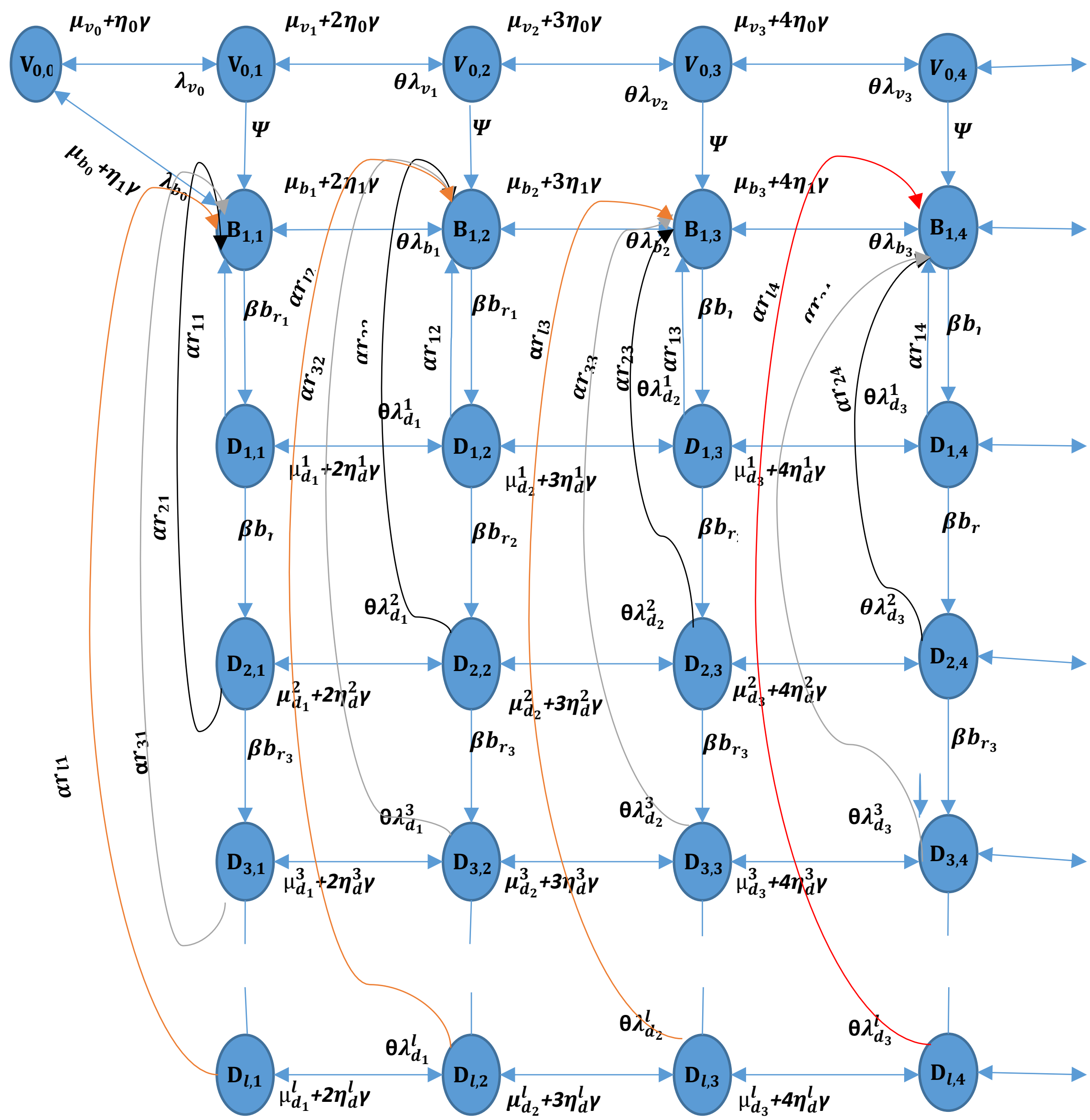

Figure 1. State transition diagram with probability vector.

Figure 1 shows the transition for different states and the states define the customers arrival, servicing along with balking and reneging behaviour. In working vacation state denoted by the vector $P_v(0,n), n = 0,1, \dots\dots \infty$. In normal busy state denoted by the vector $P_B(1,n), n = 1, \dots\dots \infty$ and in working breakdown and repair state denoted by the vector $P_D(i,n), 1 \le i \le l$ and $n = 1,2,3, \dots\dots\dots \infty$.

The generator matrix and submatrices can be calculated using the transition rate diagram and the matrix-analytic approach as follows:

$$Q = \begin{bmatrix} B_{00} & A_{00} & C_{00} & & & & & & & \\ B_{01} & A_{01} & C_{01} & & & & & & & \\ B_{02} & & A_{02} & C_{02} & & & & & & \\ & & B_1 & A_1 & C_1 & & & & & \\ & \ddots & B_2 & \ddots & A_2 & C_2 & & & & \\ & \ddots & \ddots & \ddots & & A_3 & C_3 & & & \\ & & & & & & A_4 & \ddots & \ddots & \\ & & & & & & \ddots & \ddots & \ddots & \ddots \\ & & & & & & & \ddots & \ddots & \ddots \\ & & & & & & & & & A_l \end{bmatrix}$$ where $B_{00} = \left[-\left(\lambda_{b_0} + \lambda_{v_0}\right)\ \right]$,

$A_{00} = \left[\lambda_{v_0} \quad 0 \quad 0 \quad 0 \ldots\ldots 0\right]_{(1,n)}$; $C_{00} = \left[\lambda_{b_0} \quad 0 \quad 0 \quad 0 \ldots\ldots 0\right]_{(1,n)}$

$$B_{01} = \begin{bmatrix} \mu_{v_0} + \eta_0\gamma \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}_{(n,1)} ; \qquad B_{02} = \begin{bmatrix} \mu_{b_0} + \eta_1\gamma \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}_{(n,1)}$$

$$A_{01} = \begin{cases} n, n+1; & \theta\lambda_{v_n} \\ nn, & -(\theta\lambda_{v_n} + \Psi + \mu_{v_{n-1}} + n\eta_0\gamma) \\ n+1, n; & \mu_{v_n} + (n+1)\eta_0\gamma \end{cases}$$

where, $n = 1, 2, \ldots \ldots \infty$

$$A_{02} = \begin{cases} n, n+1\ ; & \theta\lambda_{b_n} \\ nn\ ; & -(\theta\lambda_{b_n} + \mu_{b_{n-1}} + n\eta_1\gamma + \beta b_{r_1}\ ) \\ n+1, n\ ; & \mu_{b_n} + (n+1)\eta_1\gamma \end{cases}$$

where, $n = 1, 2, \ldots \ldots \infty$

$$A_i = \begin{cases} n, n+1\ ; & \theta\lambda^i_{d_n} \\ (n+1), (n+1); & -[\theta\lambda^i_{d_{n+1}} + \propto r_{i,n+1} + \mu^i_{d_n} + (n+1)\eta^i_d\gamma + \beta b_{r_{i+1}}], \qquad n \geq 1,\ 1 \leq i \leq l \\ (n+1), n\ ; & \mu^i_{d_n} + (n+1)\eta^i_d\gamma \end{cases}$$

a_{11} is the element of first row and first column of each A_i [i.e. A_1, $A_2, \ldots \ldots \ldots A_l$]
$a_{11} = -(\theta\lambda^i_{d_n} + \propto r_{in} + \beta b_{n+i}\ )$, $n = 1$, $1 \leq i \leq l$

$C_{01} = diag\ [\Psi]_{(n,\ n)}$; $C_{02} = diag\ \left[\beta b_{r_1}\right]_{(n,n)}$; $C_i = diag\ \left[\beta b_{r_{i+1}}\right]_{(n,n)}$
$B_i = diag\ [\propto r_{in}]_{nXn}$ where $n \geq 1$, $1 \leq i \leq l$

The dimension of the matrix A_{01}, A_{02}, A_i, B_i is an (n, n) matrices and the dimension of Q is $[\ (l+2)n+1, (l+2)n+1\ ]$, $i = 1, 2, 3, \cdots\cdots\cdots l$.

5. Analysis

Let $\overline{\Pi} = (\overline{\pi_{00}}, \overline{\pi_{01}}, \overline{\pi_{02}}, \overline{\pi_1}, \overline{\pi_2}, \cdots\cdots\cdots, \overline{\pi_n})$ be the steady state probability vectors of generating matrix. T = B_{01} + A_{01} + C_{01}. The QBD process with generator matrix Q is stable, if and only if, the drift rate to the left is larger than rate of drift to the right, i.e., $\pi C_{01} e < \pi B_{01} e$. The equations $\overline{\Pi} Q = \overline{\mathbf{0}}$ and $\overline{\Pi} e = \overline{1}$ are satisfied by $\overline{\Pi}$ where e is a column vector that has all entries as '**1**'.

We find the steady state vector explicitly. We derive the following equations after doing the algorithm given below:

Step 1: Input parameters
Step 2: Array with all matrices
Step 3: Using matrix-analytic method
Step 4: Consider $\overline{\Pi} e = \overline{1}$ then find $\overline{\pi_{00}}$
Step 5: Verify the steady state condition

From $\overline{\Pi} Q = \overline{\mathbf{0}}$ we obtain

$$\overline{\pi_{00}} B_{00} + \overline{\pi_{01}} B_{01} + \overline{\pi_{02}} B_{02} = \overline{\mathbf{0}} \quad (10)$$

$$\overline{\pi_{00}} A_{00} + \overline{\pi_{01}} A_{01} = \overline{\mathbf{0}} \quad (11)$$

$$\overline{\pi_{00}} C_{00} + \overline{\pi_{01}} C_{01} + \overline{\pi_{02}} A_{02} + \overline{\pi_1} B_1 + \overline{\pi_2} B_2 + \cdots\cdots + \overline{\pi_n} B_n = \overline{\mathbf{0}} \quad (12)$$

$$\overline{\pi_{02}} C_{02} + \overline{\pi_1} A_1 = \overline{\mathbf{0}} \quad (13)$$

$$\overline{\pi_n} C_n + \overline{\pi_{n+1}} A_{n+1} = \overline{\mathbf{0}} \quad (14)$$

where, $n = 1, 2, 3, \ldots \ldots .. l - 1$

From above equations we obtain after substitutions

$$\overline{\pi_{01}} = -(\overline{\pi_{00}})(A_{00})(A_{01})^{-1} \quad (15)$$

$$\overline{\pi_{02}} = (\overline{\pi_{00}})(B_{02})^{-1}[(A_{00})(A_{01})^{-1} B_{01} - B_{00}] \quad (16)$$

$$\overline{\pi_1} = -(\overline{\pi_{00}})(B_{02})^{-1}(A_1)^{-1} C_{02}[(A_{00})(A_{01})^{-1} B_{01} - B_{00}] \quad (17)$$

$$\overline{\pi_n} = (-1)^n (\overline{\pi_{00}})(C_{n-1}.C_{n-2} \cdots\cdots C_1)(A_n^{-1}.A_{n-1}^{-1}. \cdots\cdots A_1^{-1}) B_{02}{}^{-1}.\Theta \quad (18)$$

where, $C_{02}\,[(A_{00})(A_{01})^{-1} B_{01} - B_{00}] = \Theta$.

From $\overline{\Pi} e = \overline{1}$ we get

$$\overline{\pi_{00}} = [1 - (A_{00})(A_{01})^{-1} + (B_{02})^{-1}\{(A_{00})(A_{01})^{-1} B_{01} - B_{00}\} - (B_{02})^{-1}(A_1)^{-1}.\Theta + C_1 A_2{}^{-1}(B_{02})^{-1}(A_1)^{-1}.\Theta \cdots\cdots\cdots (-1)^n C_{n-1} C_{n-2} C_{n-3} \cdots\cdots C_1 (A_n^{-1})(A_{n-1}^{-1}) \cdots\cdots (A_1^{-1})(B_{02}^{-1}).\Theta]^{-1}.$$

$\lim_{n\to\infty} (p)^{(n)} = \overline{\Pi}$ where the state probability distribution at step n. Also $\overline{\Pi} P = \overline{\Pi}$.

6. Genetic Algorithm

We have done cost analysis using Genetic Algorithm. It shows how the cost function is affected by various parameters such as repair and breakdown rates.

We consider a population of a size of our choice of chromosomes with 7 decision factors. The cost function is dependent on seven choice factors, which are genes.

(i) Create a population of size N. Each individual is referred to as a chromosome. Each chromosome consists of 7 genes which are randomly initialised.
(ii) The fitness is determined using the expected cost function to get the ideal cost value. A python function is used to calculate the value. The chromosome with the highest value is the fittest. It

expresses chromosome fitness as the inverse of the cost value, indicating that the chromosome with the lowest cost value is the fittest.

(iii) Roulette wheel selection and mutation are used for generating the next generations.

(iv) We take 200 as the number of generations. The steps from 1 to 3 are repeated for those many times.

We have used Continuous Gen Alg Solver from general Python package to simulate the GA.

7. Performance Measures

The performance measure has been constructed using probability vectors in different states as obtained from section 5.

- In working vacation state, the probability of the server is obtained as

$$P(V) = \sum_{n=0}^{\infty} P_V(0,n) \tag{19}$$

- In busy state the probability of the server is obtained as

$$P(B) = \sum_{n=1}^{\infty} P_B\,(1,n) \tag{20}$$

- In working breakdown state the probability of the server is obtained as

$$P(D) = \sum_{n=1}^{\infty} P_D(i,n) \quad \text{where } 1 \le i \le l \tag{21}$$

- Mean number of customers when the server is in working vacation state is obtained as

$$E(V) = \sum_{n=0}^{\infty} nP_V\,(0,n) \tag{22}$$

- When the server is in busy state, mean number of customers is obtained as

$$E(B) = \sum_{n=1}^{\infty} nP_B\,(1,n) \tag{23}$$

- When the server is in working breakdown state, mean number of customers is obtained as

$$E(D) = \sum_{n=1}^{\infty} nP_D(\,i,n) \quad \text{where } 1 \le i \le l \tag{24}$$

- In the system mean count of customers is obtained as

$$E(N) = E(V) + E(B) + E(D) \tag{25}$$

- Throughput is obtained as

$$TP = \mu_{v_n} \sum_{n=0}^{\infty} P_V\,(0,n) + \mu_{b_n} \sum_{n=1}^{\infty} P_B\,(1,n) + \mu_{d_n}^{i} \sum_{n=1}^{\infty} P_D\,(i,n)$$

where, $1 \le i \le l$ (26)

- Mean time of waiting is obtained as

$$E(W) = \frac{E(N)}{\lambda_{eff}}$$

where, $\lambda_{eff} = \sum_{n=0}^{\infty} \lambda_{v_n} P_V\,(0,n) + \sum_{n=1}^{\infty} \lambda_{b_n} P_B\,(1,n) + \sum_{n=1}^{\infty} \lambda_{d_n}^{i} P_D\,(i,n)\ [\,1 \le i \le l]$ (27)

- Mean time of delay is calculated as

$$E(D) = \frac{E(N)}{TP} \tag{28}$$

8. Cost Analysis

We consider the cost function as

$$F(\mu_v, \mu_b, \alpha\,, \beta\,, \theta, \gamma, \mu_d) = C_v.E(V) + C_b.E(B) + C_d.\alpha + C_w.\beta + C_1\theta + C_2\gamma + C_3 E(D)$$

where, C_v = the cost per unit of time for a server that is on vacation
C_b = the cost per unit of time for a server with a busy service
C_d = the cost per unit of time at the moment of the repair, multiplied by the rate $\propto$
C_w = the cost per unit of time at the point of breakdown with rate β
C_1 = the cost per unit of time with rate θ at the time of balking
C_2 = the cost per unit of time at the time of renege multiplied by the rate γ
C_3 = the cost per unit of time for a server in a state of breakdown
$E(V)$= Mean number of customers when the server is in working vacation state
$E(B)$= Mean number of customers when the server is in busy state
$E(D)$= Mean number of customers when the server is in working breakdown state

Fitness function is $chromosome[0] * E(V) + chromosome[1] * E(B) + chromosome[2] * \alpha + chromosome[3] * \beta + chromosome[4] * \theta + chromosome[5] * \gamma + chromosome[6] * E(D)$.

Using genetic algorithm given in section 5 we have found the effect of the total cost which is described in the next section.

9. Sensitivity Analysis

By adjusting the different system parameters, a thorough numerical analysis was performed to analyse the different performance indices of the series queueing system. We tested the influence of arrival rate using the MATLAB application for various parameter settings.

Table 1 shows the effect of total cost with the variations of different parameters. In Table 1 demonstrates the impact of E(V), E(B) and E(D) in optimum effect of other parameters ($\propto, \beta$, θ, γ, C_v, $C_b, C_d, C_w, C_1, C_2, C_3$) with increasing trend of cost. Figure 2-11 shows the effect of total cost with the variation of different parameters vs. fitness function using GA. In Figure 2 it is observed that the total cost increases when repairing rate $\propto$ increases. In Figure 3 it is seen that the total cost increases when breakdown rate β increases. In Figure 4 the effect of the total cost has been shown when $\propto$ and β both increases. In Figure 5 the total cost increases when the value of θ increases. Figure 6 shows the effect of the total cost with the increment of $E(V)$. Figure 7 shows that total cost decreases when γ decreases. Figure 8 shows the effect of θ taking the parameter $\propto, \beta$ fixed. In Figure 9 it is shown that the cost function decreases when $\propto, \beta$ decrease keeping θ, γ fix value. Figure 10 shows that the total cost increases when $\propto, \beta$ increase keeping θ, γ fix value. Figure 11 shows that the total cost decreases when both $\propto, \gamma$ increase. Here we keep θ, β fix value.

Table 1. Effect of total cost with the variations of different parameters.

E(V)	**E(B)**	**E(D)**	α	β	θ	γ	C_v	C_b	C_d	C_w	C_1	C_2	C_3	**Total cost**
20	25	10	1.2	0.5	2	3	11.7	9.86	9.52	10.89	9.76	10.02	11.21	**560.79**
			2.0	1.2	2	3	9.85	9.70	11.62	32.78	7.09	19.98	9.83	**564.28**
25	25	10	3.0	1.2	2.2	3.3	9.86	10.73	9.49	9.91	18.31	12.42	9.83	**627.70**
			3.5	1.5	2.2	3.3	12.63	11.01	9.19	11.63	9.50	25.21	9.88	**733.59**
			2.5	1.0	2.2	3.3	9.98	10.65	9.05	9.90	9.91	9.71	10.31	**598.62**
			2.5	1.0	2.5	3.5	10.22	9.84	18.71	60.01	11.23	9.73	9.43	**666.64**
30	20	15	4.0	2.0	2.5	3.5	10.92	10.20	8.94	15.88	18.81	9.27	10.47	**836.16**
			4.5	2.0	2.5	3.6	10.87	9.23	14.16	20.47	9.51	9.60	9.85	**821.89**

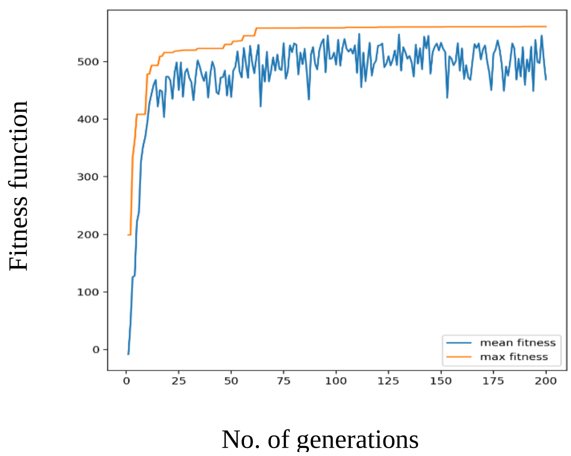

Figure 2. Effect of total cost with varying parameter α.

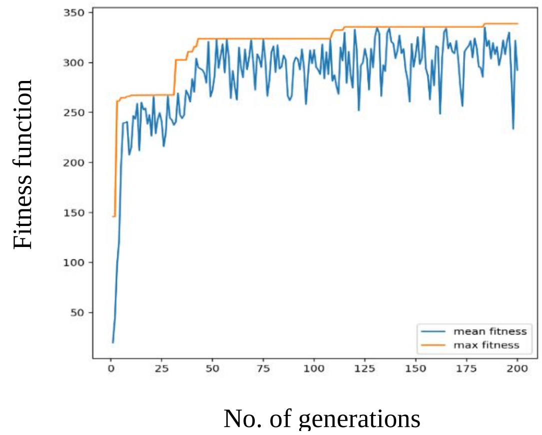

Figure 3. Effect of total cost with varying parameter β.

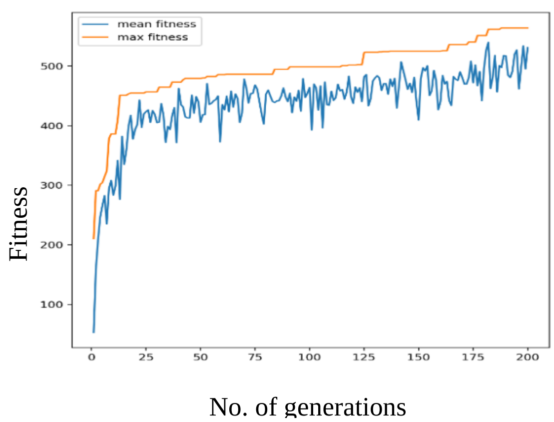

Figure 4. Effect of total cost with varying parameters α and β.

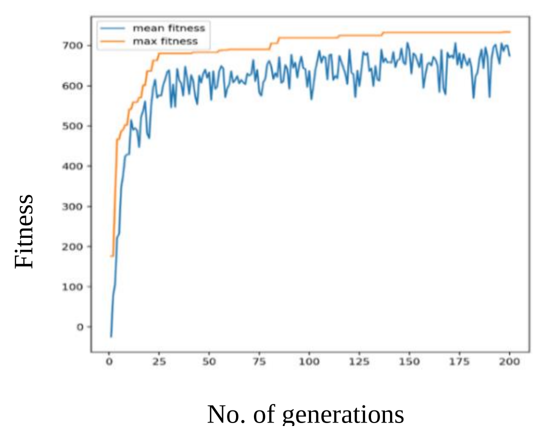

Figure 5. Effect of total cost with varying parameter θ.

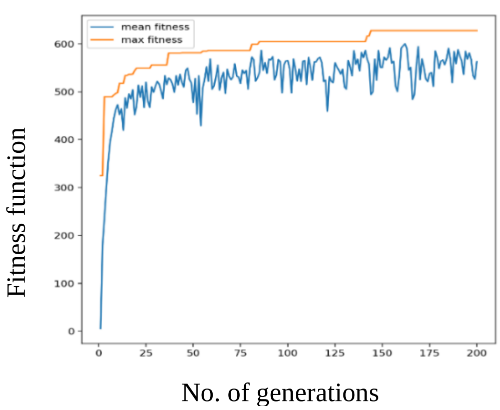

Figure 6. Effect of total cost with varying parameter E(V).

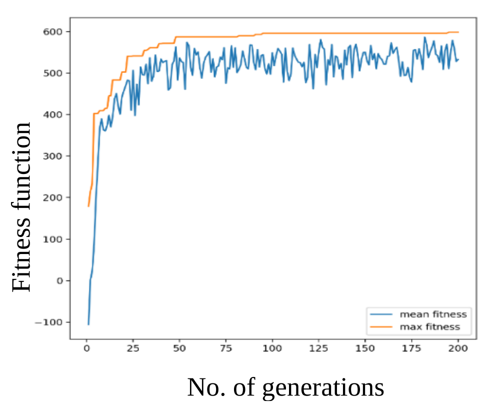

Figure 7. Effect of total cost with varying parameter γ.

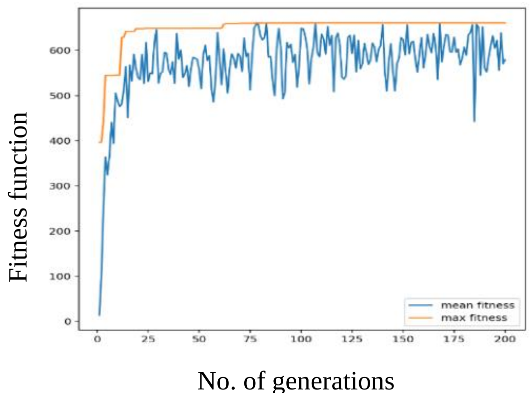

Figure 8. Effect of total cost with varying parameter θ taking $\propto, \beta$ fixed.

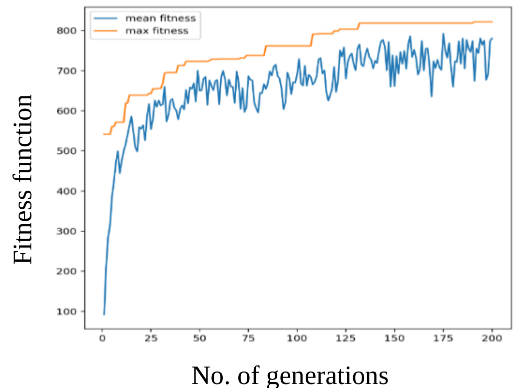

Figure 9. Effect of total cost with varying parameters $\propto, \beta$ taking θ, γ fixed.

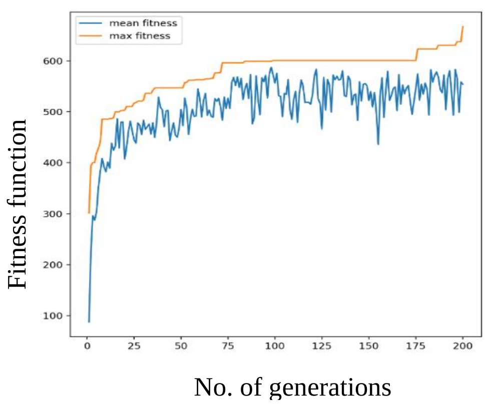

Figure 10. Effect of total cost with varying parameters $\propto, \beta$ taking θ, γ fixed.

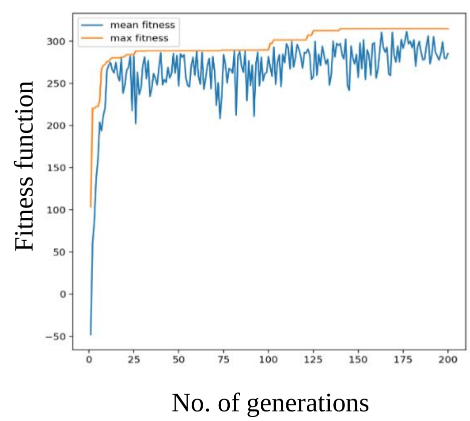

Figure 11. Effect of total cost with varying parameters $\propto, \gamma$ taking θ, β fixed.

Figures 12-17 shows the increasing trend of $E(N)$ with respect to λ. Figure 12 shows that expected queue length $E(N)$ increases when number of breakdowns (l) increases. This shows that the increment is very rapid. Figure 13 shows that expected queue length $E(N)$ increases when number of customers (n) increases. At first it is increasing slowly and after that it is increasing quickly for the large value of (λ). Figure 14 shows that the expected queue length $E(N)$ increases gradually with the increasing values of the rate of breakdown (β). Figure 15 shows the increment of the expected queue length $E(N)$ with the increasing values of the joining rate in queue (θ). For large values of λ it increases sharply. Figure 16 shows the decrement of the expected queue length $E(N)$ with the increment values of reneging rate η. It decreases leisurely for large values of λ. Figure 17 shows the decrement of the expected queue length $E(N)$ with the increment values of repairing rate α. It decreases very fast with the increasing value of α.

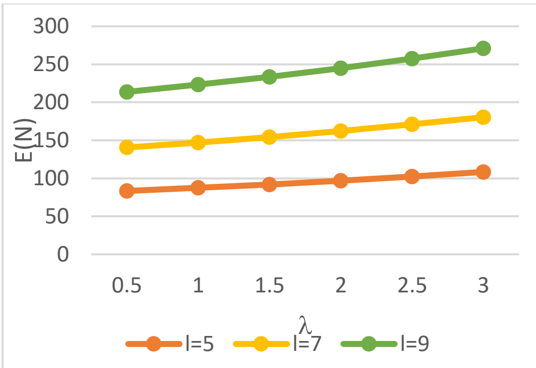

Figure 12. Variation of E(N) w.r.t. λ for different values of *l*.

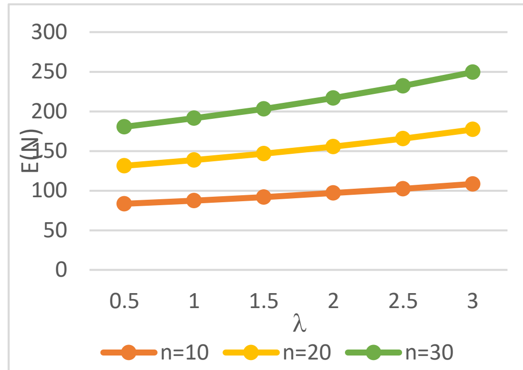

Figure 13. Variation of E(N) w.r.t. λ for different values of *n*.

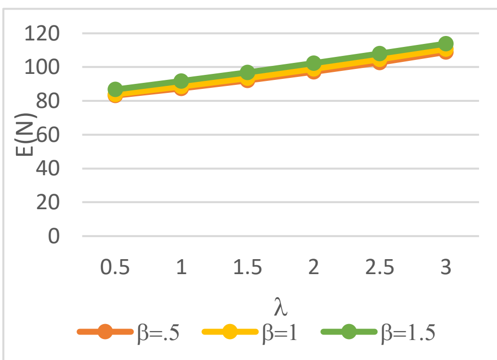

Figure 14. Variation of E(N) w.r.t. λ for different values of β.

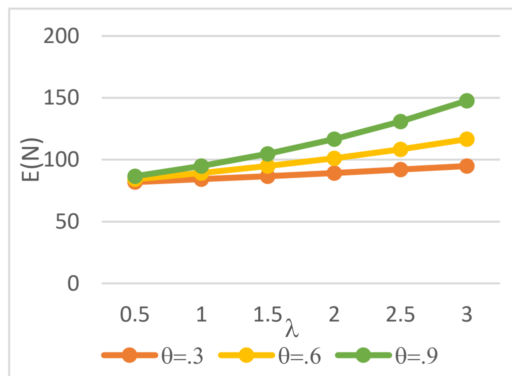

Figure 15. Variation of E(N) w.r.t. λ for different values of θ.

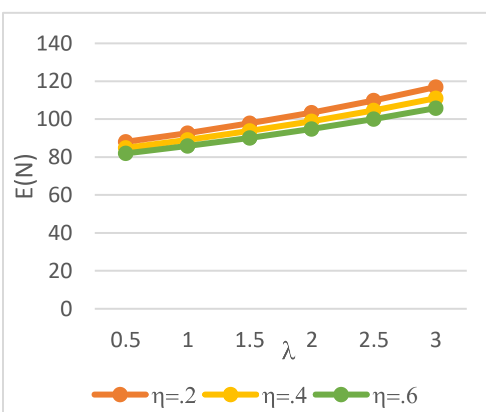

Figure 16. Variation of E(N) w.r.t. λ for different values of η.

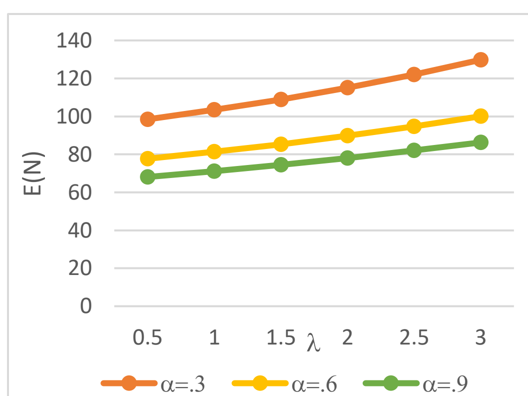

Figure 17. Variation of E(N) w.r.t. λ for different values of α.

10. Conclusions

In this paper, customers' balking and reneging behaviour are considered in a single server queueing system with working vacation. Also, multiple breakdown and repair states have been considered. Customers may balk or renege during working vacation period and breakdown states. A qualitative analysis of the model is carried out, as well as a steady state analysis is done using matrix analytical method. Numerical experiment is done to check the effects of the system parameters. We have done the sensitivity analysis which will be helpful to upgrade the system accessibility and measurements. We have got the results according to our expectations. Works on this type has not done in the past. Finally, the GA is applied to optimize the cost function. This model may be generalised to the queueing system with more than one server. We can consider a multi-server model with multiple repairmen. This extension can be explored in future.

Conflict of Interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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