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Solution of Fisher Kolmogorov Petrovsky Equation Driven via Haar Scale-3 Wavelet Collocation Method

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Abstract
The design of the proposed study is to examine the presentation of a novel numerical techniques based on Scale-3 Haar wavelets for a kind of reaction-diffusion system i.e., Fisher KPP (Kolmogorov Petrovsky Piskunove) Equation. Haar scale-3 wavelets are employed to space and time derivatives approximation involved in the system. The collocation approach is applied with space and time variables discretization to construct an implicit and explicit numerical scheme for the reaction-diffusion system. We have used various numerical problems containing non-linearity and different source term to inquest the exactness, efficiency and authenticity of the proposed numerical strategy. In addition, the obtained results are graphically displayed and systematized. Even with a small number of collocation Points, we attain accuracy using the presented technique.

Keywords- Fisher KPP; Haar Scale-3; Collocation points; Reaction-diffusion system.

1. Introduction
Fisher (1937) and Kolmogorov et al. (1937) independently scrutinized the Fisher-Kolmogorov-Petrovsky Piskunove equation in 1937, which is now known as the Fisher equation. This equation can be used in a variety of scientific and technical domains (Franak, 1969; Arora and Kumar, 2020; Dhwan et al., 2014; Canosa, 2015). Many studies have been conducted in order to find a relevant explanation and stereotype of this equation (Man et al., 2019; Branco et al., 2007; Dhwan et al., 2013, Dhwan et al., 2021). We looked at one stereotype of this equation, which is known as the component reaction-diffusion equation collectively. Traveling wave fronts are present in many reaction-diffusion equations and play a vital role in the understanding the concepts of physical, biological, and physical events (Wawa and Gorguis, 2004; Kaur and Wazwaz, 2021). Reaction-diffusion systems are mathematical models that describe how the congregation of one or more materials scattered in space changes as a result of two processes: general chemical reactions, which involved transformation of substances into one another, and diffusion, in which substances open up over a surface in space. Reaction diffusion systems (Franak-Kameneetiskii, 1969) are commonly used in chemistry. On the other hand, the system can be used to express non-chemical dynamical processes. For a single substance's concentration in a single spatial dimension, the most basic reaction-diffusion equation. The equation reflects pure diffusion if the response term is removed, and it becomes a parabolic partial equation in one spatial dimension if the thermal diffusivity term substitutes the diffusion term D. The KPP Fisher equation with isentropic is used to explain native agitation in advective environments (Gu et al., 2015).

The second-order nonlinear problem considered in this paper is

$$\frac{\partial\varphi}{\partial t} - D\,\frac{\partial^2\varphi}{\partial x^2} = r\varphi(1-\varphi^{\alpha}), \forall\,(x,t) \in [0,1] \times [0,T] \tag{1}$$

is a kind of reaction-diffusion system.

Subjected to the certain boundary condition

$$\varphi\,(x, 0) = \eta_1(x),\ \varphi_t(x, 0) = \eta_2(x)\ \forall\, x \in [0,1] \tag{2}$$

And then subjected to the initial defined condition of the form

$$\varphi(0, t) = \lambda_1(t),\ \varphi(1, t) = \lambda_2(t)\ , \forall\ t \in [0,T] \tag{3}$$

Here, $\eta_1(x)$, $\eta_2(x)$, $\lambda_1(t)$, $\lambda_2(t)$, g (x, t) are the given functions and φ(x, t) is the function whose value is determined.

The results of the research on mathematical features and the literature discussion of the Fisher equation have been published in a number of papers. The Fisher equation was well-summarized by Larson (1978) and Kawahara and Tanaka. (1983). The Fisher equation is studied analytically using the Haar scale-3 collocation method. However, numerical solutions to the Fisher equation were not published in the literature until 1974. Following that, several scholars looked into numerical solutions to the Fisher equation. The numerical system used in Evans et al. (1989) is highly complex, and it produces unanticipated higher regulated frequency oscillations that must be percolate out at each time step. The Fisher equation was then quantitatively solved by Parekh and Puri (1990) and Twizell et al. (1990). For the Fisher equation, (Mickens, 1994; Tang and Weber, 1991) proposed the optimum finite difference technique. Mittal and Jain (2013) suggested a numerical scheme for solving the Fisher KPP reaction-diffusion equation (Maan et al. 2019) using refitted cubic B-splines collocated across countable elements. The goal of this work is to demonstrate how the collocation approach may be used to identify the smallest possible computational error. To improve accuracy, exactness, and authenticity, we used the small collocation point approach. We discovered that this method is highly accurate and simple to apply without causing a jumble of computations (Jiwari et al., 2012; Kaur et al., 2013; Arora et al., 2018), and we can observe from the results that it is closer to the exact solution.

2. Haar Scale-3 and Their Integrals

The 3-scale Haar wavelet integral methodology is used to solve a non-linear partial differential equation which are of second order. In which the differential equations maximum derivative is bloated into scale-3 Haar wavelets and the derivatives of low order are graded by integrating the differential equations. The Haar scale-3 wavelet is more accurate and converges faster than the Haar scale-2 wavelet. Using the impertinent condition in wavelets, every quad integrable function defined on the interval [0, 1] may be simply expressed in terms of the uncountable sum of Haar wavelet series.

$$f(x) \approx c_1\varphi_1(x) + \sum_{even\ index\ i\geq 2}^{\infty} c_i{\Psi_i}^{1}(x) + \sum_{odd\ index\ i\geq 3}^{\infty} c_i{\Psi_i}^{2}(x) \tag{4}$$

Here φ_1, ${\Psi_i}^{(1)}$ ${\Psi_i}^{(2)}$, *are given by*

$$\varphi_i(t) = \begin{cases} 1; & A_1 \leq t \leq B_1 \\ 0; & elsewhere \end{cases} \quad for\ i = 1 \tag{5}$$

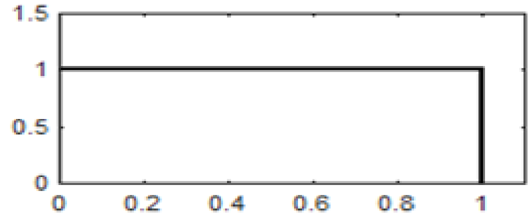

Figure 1. Haar scale 3 function.

$$\Psi_i^{(1)}(t) = \frac{1}{\sqrt{2}} \begin{cases} -1; p_1(i) \leq t < p_2(i) \\ 2; p_2 \leq t < p_3 \\ -1; p_3 \leq t < p_4 \\ 0 \quad elsewhere \end{cases} \qquad for\ i = 2, 4, \ldots 3p - 1 \tag{6}$$

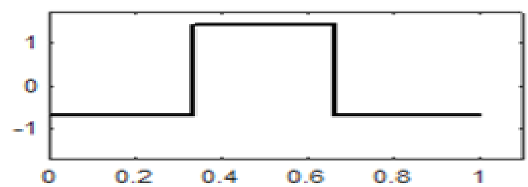

Figure 2. Haar wavelet $\boldsymbol{\psi^1(t)}$ with dilation 3.

$$\Psi_i^{(2)}(t) = \sqrt{\frac{3}{2}} \begin{cases} 1; \quad p_1(i) \leq t \ < p_2(i) \\ 0; \ p_2(i) \leq \ t \ < \ p_3(i) \\ -1; \ p_3(i) \leq \ t \ < \ p_4(i) \\ 0 \quad elsewhere \end{cases} \qquad for\ i 3, 6, \ldots 3p \tag{7}$$

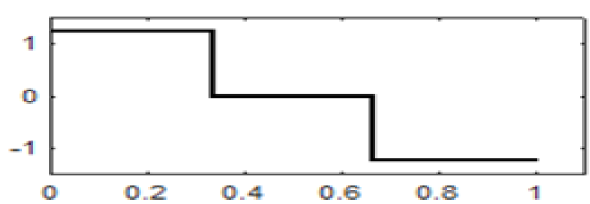

Figure 3. Haar wavelet $\boldsymbol{\psi^2(t)}$ with dilation 3.

where,

$$p_1(i) = A_1 + (B_1 - A_1)\frac{k}{m}, \tag{8}$$

$$p_2(i) = A_1 + (B_1 - A_1)\frac{k+\frac{1}{3}}{m} \tag{9}$$

$$p_3(\mathrm{i}) = A_1 + (B_1 - A_1)\frac{\mathrm{k}+\frac{2}{3}}{\mathrm{m}}, \tag{10}$$

$$p_4(i) = A_1 + (B_1 - A_1)\frac{k+1}{m} \tag{11}$$

m is defined as 3^j (j = 0, 1, 2...) and k = 0,1, 2…m-1 is a translation parameter. If i = 1 we will get scaling function $\varphi_1(x)$ which defined in equation (5) and shows in Figure -1 for [a, b] = [0, 1]. In case i > 1 the index i is calculated according to i = m + 2k +1. If i is even then we will consider $\Psi_i^{(1)}$ and if i is odd then we will consider $\Psi_i^{(2)}$. To find the solution we have done with procedure of the defined differential equation (DE) with any order. We need to integrate Scale-3 Haar wavelets i.e., we have applied on the integral

$$\varphi_{1,1}(\mathrm{t}) = \int_0^x \varphi_1(\mathrm{t})\,\mathrm{dt} = \begin{cases} t\,; & [A_1, B_1) \\ 0; & elsewhere \end{cases} \tag{12}$$

$$\Psi_{i,1}^{(1)}(t) = \int_0^x \Psi_i^{(1)}\,dt = \frac{1}{\sqrt{2}}\begin{cases} p(i) - t & ; \quad p(i) \le t < q(i) \\ 2t - 3\,q(i) + p(i) & ; \quad q(i) \le t < r(i) \\ p(i) + 3\,r(i) - 3\,q(i) - t & ; \quad r(i) \le t < s(i) \end{cases} \tag{13}$$

$$\Psi_{i,1}^{(2)}(t) = \int_0^x \Psi_i^{(2)}\,dt = \sqrt{\frac{3}{2}}\begin{cases} t - p(i) & ; \quad p(i) \le t < q(i) \\ q(i) - p(i) & ; \quad q(i) \le t < r(i) \\ r(i) + q(i) - p(i) - t & ; \quad r(i) \le t < s(i) \end{cases} \tag{14}$$

Moreover, we introduce

$$\varphi_{1,s+1}(t) = \int_0^x \varphi_{1,s}(t)\,dt \tag{15}$$

$$\Psi_{1,s+1}^{(1)}(t) = \int_0^x \Psi_{1,s}^{(1)}(t)\,dt \tag{16}$$

$$\Psi_{1,s+1}^{(2)}(t) = \int_0^x \Psi_{1,s}^{(2)}(t)\,dt \tag{17}$$

Which can explicitly be written as

$$\varphi_{i,s+1}(x) = \begin{cases} \frac{t^{s+1}}{(s+1)!} & ; \quad [A_1, B_1) \\ 0 & ; \quad Elsewhere \end{cases} \tag{18}$$

$$\Psi_{i,s+1}^{(1)}(t) = \frac{1}{\sqrt{2}}\begin{cases} 0 & ; \quad 0 \le t < p_1(i) \\ \frac{-[t-p_1(i)]^{s+1}}{(s+1)!} & ; \; p_1(i) \le t < p_2(i) \\ \frac{3\,[t-p_2(i)]^{s+1} - [t-p_1(i)]^{s+1}}{(s+1)!} & ; \; p_2(i) \le t < p_3(i) \\ \frac{3\,[t-p_2(i)]^{s+1} - 3\,[t-p_3(i)]^{s+1} - [t-p_1(i)]^{s+1}}{(s+1)!} & ; \; p_3(i) \le t < p_4(i) \\ \frac{3\,[t-p_2(i)]^{s+1} - 3[t-p_3(i)]^{s+1} - [t-p_1(i)]^{s+1} + [t-p_4(i)]^{s+1}}{(s+1)!} & ; \; p_4(i) \le t < 1 \end{cases} \tag{19}$$

$$\Psi_{1,s+1}{}^{(2)}(t) = \sqrt{\frac{3}{2}}\begin{cases} 0 & ;\ 0 \le t < p_1(i) \\ \dfrac{[t-p_1(i)]^{s+1}}{(s+1)!} & ;\ p_1(i) \le t < p_2(i) \\ \dfrac{[t-p_1(i)]^{s+1} - [t-p_2(i)]^{s+1}}{(s+1)!} & ;\ p_2(i) \le t < p_3(i) \\ \dfrac{[t-p_1(i)]^{s+1} - [t-p_2(i)]^{s+1} - [t-p_3(i)]^{s+1}}{(s+1)!} & ;\ p_3(i) \le t < p_4(i) \\ \dfrac{[t-p_1(i)]^{s+1} - [t-p_2(i)]^{s+1} - [t-p_3(i)]^{s+1} + [t-p_4(i)]^{s+1}}{(s+1)!} & ;\ p_4(i) \le t < 1 \end{cases} \quad (20)$$

3. Approximation of Space and Time Derivatives

The second-order problem considered is

$$\frac{\partial\varphi}{\partial t} - D\frac{\partial^2\varphi}{\partial x^2} - r\varphi(1-\varphi^{\alpha}) = f(x,t),\ \forall\ (x,t) \in [0,1]\times[0,T] \quad (21)$$

Subjected to the boundary condition

$$\varphi(x,0) = \eta_1(x),\ \varphi_t(x,0) = \eta_2(x)\ \forall\ x \in [0,1] \quad (22)$$

And we have given the defined initial condition of the form

$$\varphi(0,t) = \lambda_1(t),\ \varphi(1,t) = \lambda_2(t) \quad (23)$$

Here $\eta_1(x)$, $\eta_2(x)$, $\lambda_1(t)$, $\lambda_2(t)$, f (x, t) are the known functions and φ(x, t) is the function whose value is determined. We describe the discretization process of the equations above in the subsequent sections.

4. Quasilinearization Technique

The quasi-linearization approach is a generalized form of Newton– Raphson method for linearizing differential equations with non-linear term. It quadratically converges to the exact value. If there is any convergence at all, and it is monotonic.

The main idea to use this technique is based upon the fact there is no analytic method to solve many non-linear equations on the society demand we have the need to find the solution of these equation. In equation (21) φ is a non-linear term then we have to use the below recurrence relation

$$\varphi''_{r+1} = \psi(\varphi'_r, \varphi_r) + (\varphi_{r+1} - \varphi_r)\psi_{\varphi}(\varphi'_r, \varphi_r) + (\varphi'_{r+1} - \varphi'_r)\psi'_{\varphi}(\varphi'_r, \varphi_r) \quad (24)$$

Where ψ is a non-linear function of $\varphi^{n-1}{}_r$, $\varphi^{n-2}{}_r$, $\varphi^{n-3}{}_r$ ... φ'_r, φ_r and φ_r will be known value at each step which will be used to calculate φ_{r+1}. Now we have to rewrite the fisher equation using

$$\varphi^{\alpha+1}{}_{r+1} = \varphi^{\alpha+1}{}_r + (\varphi_{r+1} - \varphi_r)(\alpha+1)\varphi^{\alpha}{}_r$$
$$\varphi^{\alpha+1}{}_{r+1} = \varphi^{\alpha+1}{}_r + (\alpha+1)\varphi^{\alpha}{}_r\varphi_{r+1} - (\alpha+1)\varphi^{\alpha+1}{}_r$$
$$\varphi^{\alpha+1}{}_{r+1} = (\alpha+1)\varphi^{\alpha}{}_r\varphi_{r+1} - \alpha\varphi^{\alpha+1}{}_r \quad (25)$$

5. Approximation by Haar Wavelets

The Method of solution variable based on 2D Haar scale-3 wavelets are discretized with the assist with the help of 2D scale-3 Haar wavelets can be demonstrate below

$$\varphi_{xxt}(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}H_i(x)H_l(t) \quad (26)$$

Integrating the equation (26) equation w.r.t x with in the domain from 0 to x the above expression leads to

$$\varphi_{xt}(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}Q_{1,i}(\mathrm{x})H_l(t) + \varphi_{x,t}(0,t). \quad (27)$$

Now integrating and defined the above expression (27) w.r.t x with in the defined domain from 0 to 1 we get

$$\varphi_{xt}(0,t) = [\varphi_t(1,t) - \varphi_t(0,t)] - \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}Q_{2,i}(1)H_l(t) \quad (28)$$

Put the value of $\varphi_{xt}(0,t)$ from equation (28) into equation (27) we get

$$\varphi_{xt}(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}Q_{1,i}(\mathrm{x})\,H_l(t) + [\varphi_t(1,t) - \varphi_t(0,t)] - \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}Q_{2,i}(1)H_l(t)$$

$$\varphi_{xt}(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{1,i}(\mathrm{x}) - Q_{2,i}(1)]\,H_l(t) + [\varphi_t(1,t) - \varphi_t(0,t)] \quad (29)$$

Again, integrating and defined the expression with respect to x within the limits of x we have

$$\varphi_t(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{2,i}(\mathrm{x}) - \mathrm{x}\,Q_{2,i}(1)]\,H_l(t) + x\varphi_t(1,t) + (1-x)\varphi_t(0,t) \quad (30)$$

Again, integrating with respect to t and taking limits 0 to t we have

$$\varphi(\mathrm{x,t}) = \sum_{\mathrm{i=1}}^{3\mathrm{p}}\sum_{\mathrm{l=1}}^{3\mathrm{p}} \mathrm{a_{il}}[\mathrm{Q_{2,i}(x)} - \mathrm{x}\,\mathrm{Q_{2,i}}(1)]\,\mathrm{Q_{1,l}}(\mathrm{t}) + \mathrm{x}[\varphi(1,\mathrm{t}) - \varphi(1{,}0)] + (1 - \mathrm{x})[\varphi(0,\mathrm{t}) - \varphi(0{,}0)] + \varphi(x,0) \quad (31)$$

Differentiating with respect to x two times we get

$$\varphi_{xx}(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}H_i(x)Q_{1,l}(t) + \varphi_{xx}(x,0) \quad (32)$$

Equation (21) becomes

$$\sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{2,i}(\mathrm{x}) - \mathrm{x}\,Q_{2,i}(1)]\,H_l(t) + x\varphi_t(1,t) + (1-x)\varphi_t(0,t) - \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}H_i(x)Q_{1,l}(t) + \varphi_{xx} + [r(\alpha+1)]\varphi_{r+1}\Big[\sum_{\mathrm{i=1}}^{3\mathrm{p}}\sum_{\mathrm{l=1}}^{3\mathrm{p}} \mathrm{a_{il}}[\mathrm{Q_{2,i}(x)} - \mathrm{x}\,\mathrm{Q_{2,i}}(1)]\,\mathrm{Q_{1,l}}(\mathrm{t}) + \mathrm{x}[\varphi(1,\mathrm{t}) - \varphi(1{,}0)] + (1 - \mathrm{x})[\varphi(0,\mathrm{t}) - \varphi(0{,}0)] + \varphi(x,0)\Big]^{\alpha} \quad (33)$$

The above expression diminished to algebraic equation system and further it gets diminished to the following defined system of 4D arrays

$$A_{3p\times 3p}R_{3p\times 3p\times 3p\times 3p} = F_{3p} \times F_{3p} \quad (34)$$

Further the above array system can be diminished to the well-defined matrix form system.

$$a_{il} = b_\gamma \; and \; F_{rs} = G_\delta.$$

Then, for the defined different values of n = 1,2, 3, ..., the value of the matrix can be determined sequentially by utilizing the Thomas technique in a MATLAB application to solve the system of equations The aforementioned matrix structure can be used to restore the original wavelet coefficient a_{il}

6. Numerical Examples

The MATLAB computer language was used to do numerical calculations and to generate graphical outputs. A discrete form of the Haar scale-3 wavelet series is required to determine the numerical solution of a second order partial differential equation using Haar scale-3 wavelet. As a result, at the initial level of resolution j = 1, the collocation points at the point of discontinuity approach given in equations (24) and (25) is used to pick collocation points of the Haar Scale-3 wavelet matrix. The efficacy of the present scheme was tested by analyzing the solutions of three issues acquired by the defined scheme and calculating absolute defined errors to explain the applicability of the defined scheme for the fisher KPP equation.

6.1 Numerical Problem No – 1

In the first numerical problem we will consider the following defined nonlinear fisher equation in the form of

$$\frac{\partial\varphi}{\partial t} - D\,\frac{\partial^2\varphi}{\partial x^2} - r\varphi(1-\varphi^{\alpha}) = f(x,t), \forall\,(x,t) \in [0,1] \times [0,T] \tag{35}$$

Subjected to the boundary condition φ (x, 0) = x^2exp $(2x)$, φ_t(x, 0) = 0 $\forall\, x \in [0,1]$ (36)

And the given defined initial condition which is of the form φ(0, t) = 0, φ(1, t) = exp(2) $(1+t^2)$ (37)

With source term $f(x,t) = \frac{2x^2 t^{2-\alpha}\exp(2x)}{2-\alpha!} - 2(1+x^2)(1+4x+2x^2)\exp(2x) - [(1+t^2)x^2\exp(2x)][1-\left((1+x^2)x^2\exp(2x)\right)^3]$ (38)

From our literature for this problem the exact solution is

$$\varphi(x,t) = (1+t^2)x^2\exp\,(2x) \tag{39}$$

We proposed the following numerical solution using current approximate scheme for $\alpha = 3$

$$\varphi(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{2,i}(x) - x\,Q_{2,i}(1)]\,Q_{l,l}(t) + x[\varphi_t(1,t) - \eta_2(0)] + (1-x)[\varphi_t(0,t) - \eta_2(1)] + \eta_2(x) \tag{40}$$

Table 1. Comparison of results achieved for the numerical problem – 1 with exact solution.

X	T	Approximate Solution	Exact Solution	Absolute Error
0.055555555556	0.055555555556	0.417876453203341	0.41780711332124	9.61121e-09
0.166666666667	0.166666666667	1.28794093283068	1.28709707463046	0.35994e-09
0.277777777778	0.277777777778	2.21899323395838	2.21899805721951	1.53498e-08
0.388888888889	0.388888888889	3.25004202132713	3.25006036991267	2.60169e-09
0.500000000000	0.500000000000	4.43682925637935	4.43689877494923	3.60256e-07
0.611111111111	0.611111111111	5.85815536718128	5.85812658878215	4.58101e-09
0.722222222222	0.722222222222	7.62473041786039	7.62474915967709	5.57368e-08
0.833333333333	0.833333333333	9.89337747709111	9.89332648194420	6.59044e-06
0.944444444444	0.944444444444	12.8956130336936	12.8956573832036	7.76671e-06

Table 2. $L_2 and L_\infty$ errors at different value of J for the numerical problem – 1.

Level of Resolution	J=1	J=2	J=3
$L_2 - error\ (HS3WM)$	3.10554e-07	4.3468e-08	4.5213e-09
$L_2 - error$ [5]	1.57634e-04	3.2680e-03	1.2470e-02
$L_\infty - error\ (HS3WM)$	12.39008e-07	12.3454e-09	11.3478e-09

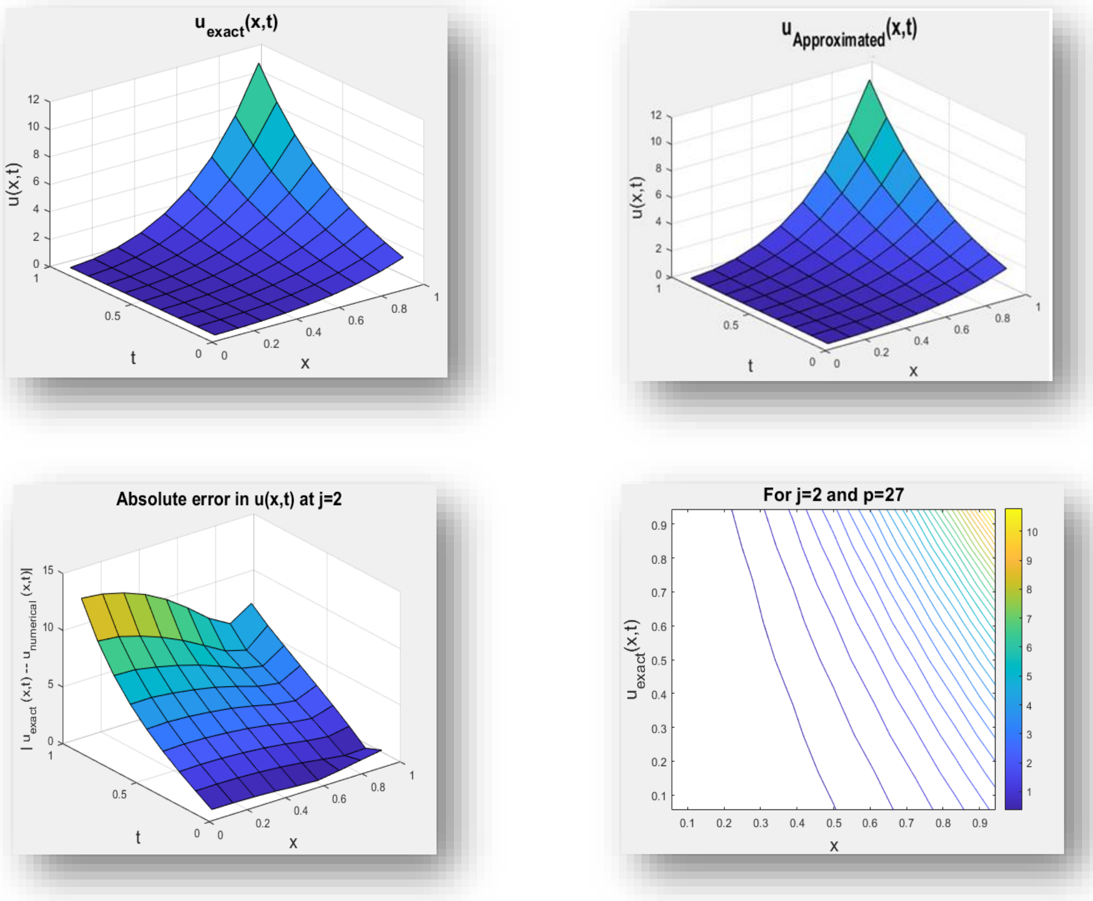

Figure 4. Graphical representation of numerical problem-1.

The graphical illustration of approximate and exact solutions for Numerical Problem-1 is shown in Figure 4. Figure 4 illustrates that the exact and numerical or approximated results for J=1 are compatible. Table 1 shows the comparison of approximate and exact answers obtained by the suggested scheme. For various values of J=1, 2, and 3, the data in tabular form show that solutions are compatible with each other. Table 2 shows the results about the errors like $L_2 and\ L_\infty\ for\ J = 1, 2, 3$.

6.2 Numerical Problem No – 2

In the second numerical problem we will consider the following defined nonlinear fisher equation in the form of

$$\frac{\partial\varphi}{\partial t} - D\frac{\partial^2\varphi}{\partial x^2} - r\varphi(1-\varphi^\alpha) = f(x,t), \forall\ (x,t) \in [0,1] \times [0,T] \quad (41)$$

Subjected to the boundary condition φ (x, 0) = 0, φ_t(x, 0) = 0 $\forall\ x \in [0,1]$ (42)

and the given defined initial condition which is of the form φ(0, t) = 0, φ(1, t) = 0 (43)

with source term $f(x,t) = \frac{24\ t^{4-\alpha}}{4-\alpha!}\sin(2\pi x) + 4\pi^2 t^4 \sin(2\pi x) - (t^4\sin(2\pi x))(1 - t^4 \sin(2\pi x))$ (44)

From our literature for this problem the exact solution is

$$\varphi(x,t) = t^4 \sin(2\pi x) \tag{45}$$

We proposed the following numerical solution using current approximate scheme for $\alpha = 2$

$$\varphi(x,t) = \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{2,i}(x) - x\,Q_{2,i}(1)]\,Q_{l,l}(t) + x[\varphi_t(1,t) - \eta_2(0)] + (1 - x)[\varphi_t(0,t) - \eta_2(1)] + \eta_2(x) \tag{46}$$

Table 3. Comparison of results achieved for the first numerical problem – 2 with exact solution.

X	T	Approximate Solution	Exact Solution	Absolute Error
0.055555555556	0.055555555556	0.00269101128159127	0.00218330012495588	3.1038e-06
0.166666666667	0.166666666667	0.00827184039774775	0.00827185479381887	7.8848e-06
0.277777777778	0.277777777778	0.0109865996338657	0.0109865992158141	8.9765e-06
0.388888888889	0.388888888889	0.00931484071050670	0.0093148736237544	5.8422e-06
0.500000000000	0.500000000000	0.00719018837841011	0.0202803569492581	1.8204e-12
0.611111111111	0.611111111111	0.00552816638664849	0.0055224327562679	5.8422e-06
0.722222222222	0.722222222222	0.00342178026447360	0.0034217885544573	8.9765e-06
0.833333333333	0.833333333333	0.00223160489049629	0.0022316029818305	7.8848e-06
0.944444444444	0.944444444444	0.00100232135898179	0.00187240745397664	3.1038e-06

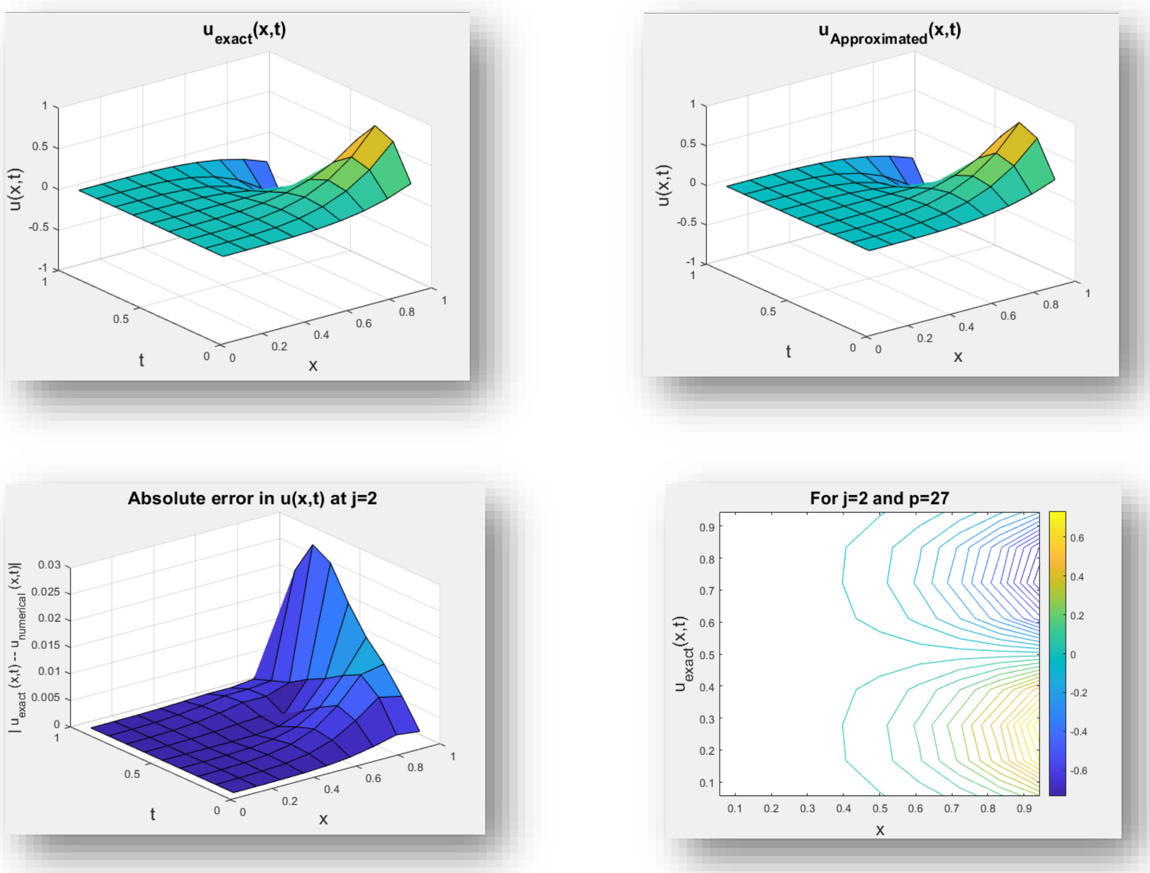

Figure 5. Graphical representation of numerical problem-2.

Table 4. $L_2 and L_\infty$ errors at different value of J for the numerical problem –2.

Level of Resolution	J=1	J=2	J=3
$L_2 - error\ (HS3WM)$	7.9947e-03	3.0745e-04	3.414e-05
$L_2 - error[5]$	5.5679e-02	2.4689e-03	2.4589e-04
$L_\infty - error\ (HS3WM)$	8.2567e-05	3.2181e-06	3.7312e-07

The graphical impression of approximate and exact solutions for Problem-2 is shown in Figure-5. Figure 5 illustrates that the exact and numerical or approximated results for J=1 are compatible. Table 1 shows the comparison of exact and approximate answers obtained by the suggested scheme. For J=1, 2, 3, the data in tabular form show that solutions are compatible with each other. Table 2 shows the results about errors like $L_2 and\ L_\infty\ for\ J = 1, 2, 3$.

6.3 Numerical Problem No – 3

In the third numerical problem we will consider the following defined nonlinear fisher equation in the form of

$$\frac{\partial\varphi}{\partial t} - D\,\frac{\partial^2\varphi}{\partial x^2} - r\varphi(1-\varphi^\alpha) = f(x,t), \forall\ (x,t) \in\ [0,1]\ \times\ [0,T] \quad (47)$$

Subjected to the boundary condition $\varphi\ (x, 0)\ =\ \frac{1}{(1+e^x)^2}\ ,\ \varphi_t(x, 0)\ =\ \frac{10e^x}{(1+e^x)^3}\quad \forall\ x\ \in\ [0, 1]$ (48)

And the given defined initial condition which is of the form $\varphi(0, t) = \frac{1}{(1+e^{-5t})^2}\ , \varphi(1, t) = \frac{1}{(1+e^{1-5t})^2}$ (49)

With source term $f(x, t) = 0$ (50)

From our literature for this problem the exact solution is

$$\varphi(x, t) = \frac{1}{(1+e^{x-5t})^2} \quad (51)$$

We proposed the following numerical solution using current approximate scheme for $\alpha = 2$

$$\varphi(x,t)\ =\ \sum_{i=1}^{3p}\sum_{l=1}^{3p} a_{il}[Q_{2,i}(x) -\ x\,Q_{2,i}(1)]\,Q_{l,l}\,(t)\ +\ x[\ \varphi_t(1,t)\ - \eta_2(0)] + (1\ -\ x)[\varphi_t(0,t) - \eta_2(1)]\ +\ \eta_2(x) \quad (52)$$

Table 5. Comparison of results achieved for the first numerical problem – 3 with exact solution.

X	T	Approximate Solution	Exact Solution	Absolute Error
0.055555555556	0.055555555556	0.00124940881137586	0.00135421346565645	1.5682e-04
0.166666666667	0.166666666667	0.00351558633128479	0.00393211060598164	4.1547e-04
0.277777777778	0.277777777778	0.00521038845416599	0.00612687996071948	5.5498e-04
0.388888888889	0.388888888889	0.00616236668082215	0.00770308583205059	5.6449e-04
0.500000000000	0.500000000000	0.00632785557416271	0.00845584180826309	4.6741e-04
0.611111111111	0.611111111111	0.00575720248508072	0.00823317696930737	3.0785e-04
0.722222222222	0.722222222222	0.00456821236001204	0.00696551700147097	1.4075e-04
0.833333333333	0.833333333333	0.00292270666434732	0.00470429064739020	2.1708e-05
0.944444444444	0.944444444444	0.00100397127139898	0.00167331203289156	1.2061e-05

Table 6. $L_2 and L_\infty$ errors at different value of J for the numerical problem –3.

Level of Resolution	J=1	J=2	J=3
$L_2 - error\ (HS3WM)$	8.7115e-04	8.7051e-05	8.7042e-06
$L_2 - error[5]$	7.3468e-04	4.4567e-04	3.4789e-05
$L_\infty - error\ (HS3WM)$	1.5548e-04	1.5517e-04	1.5564e-05

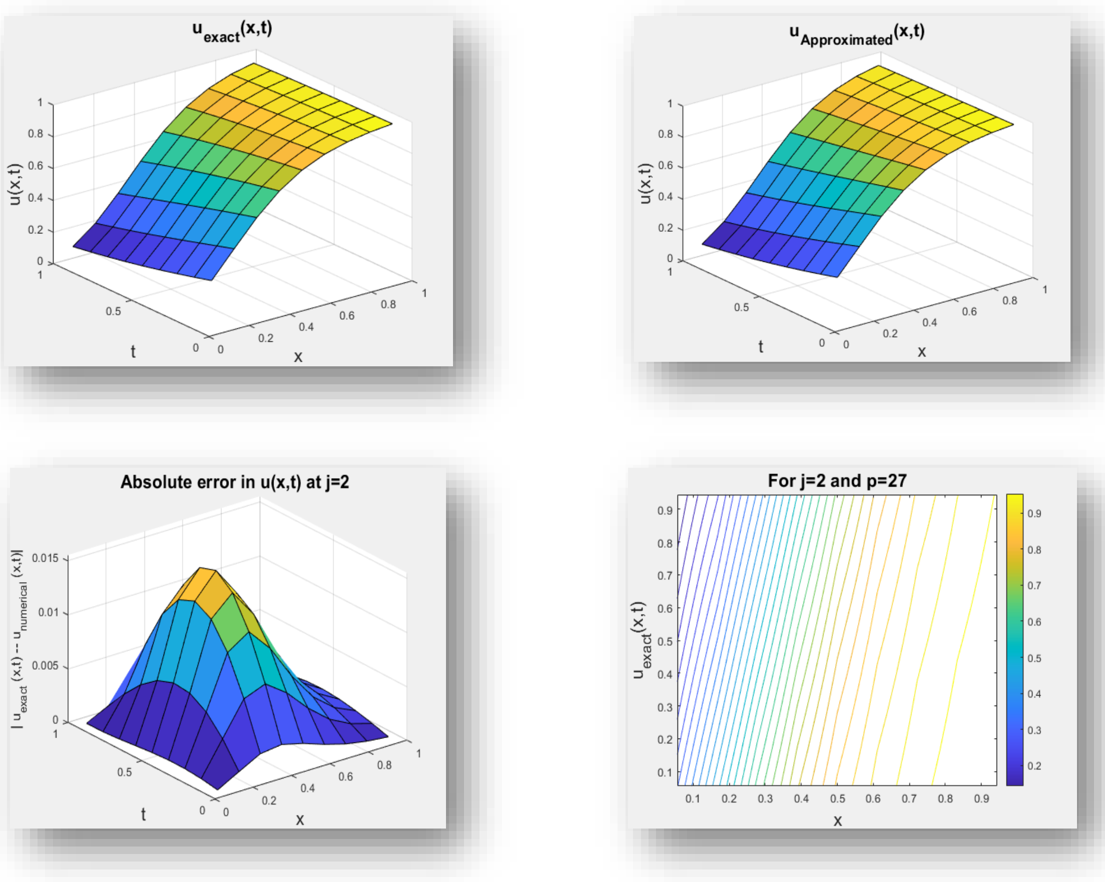

Figure 6. graphical representation of numerical problem-3.

The graphical impression of approximate and exact solutions with absolute error for Numerical Problem-3 is shown in Figure 6. Figure 6 illustrates that the exact and numerical or approximated results for J=1 are compatible. Table 1 shows the comparison between the approximate and exact answers obtained by the suggested scheme. For various values of J=1, 2, and 3, the data in tabular form show that solutions are compatible and reliable with each other. Table 2 shows the results for $L_2 and\ L_\infty\ errors\ for\ J = 1, 2, 3$.

7. Conclusion

The major purpose of the above research is to discuss and evaluate a new 3-scale Haar wavelet-based technique for second order partial differential equations. The collocation approach was used to describe numerical strategies for Fisher type equations in this study. For a wide variety of time and space approximation numerical results, the numerical results achieved by the suggested approach are fairly adequate. The computed results are in great agreement with the exact answer, as can be shown. The advantage of our method is that it appears to be easily expanded to tackle a variety of issues involving more mechanical as well as physical, or biological effects, such as response, linear diffusion, nonlinear convection and dispersion, with minor adjustments. A second-order collocation strategy based on the collocation method has been created to solve Fisher's reaction diffusion problem. Furthermore, the strategy we have proposed yields positive outcomes. The results were compared to those obtained using other approaches. The method's implementation is straightforward and easier than current approaches. The following are some of the benefits of the Haar wavelet-based technique.

- Even with a limited number of collocation points, good accuracy is achieved.
- There are few computing expenses, and the method is simple to implement in computers.
- When compared to other methods, dealing with boundary conditions is a breeze.

We also remark that, with appropriate adjustments, the new 3-scale Haar wavelet-based approach described here can be simply applied to similar challenges.

Conflict of Interest

The author(s) have no conflicts of interest to report.

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