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Reliability Modeling of a Food Industrial System with two Types of Repair Persons wherein Demand is Season Dependent

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Abstract
The present study pertains to developing a reliability model concerning a food industrial system that runs throughout the month except for Sundays. There are two types of repair persons engaged with the system, one known as operator while another is fitter. The operator is responsible for minor repairs, while the fitter is responsible for major repairs. It is also noticed that the operator is the first person who attends the failed unit in case of a major failure. If the operator is found incapable of repairing the unit within some patience time, then the fitter is called. While gathering the actual month-wise data of the plant, two types of seasons, namely normal and festival seasons, have been recorded. The festival season is from July to November, while the normal season is from December to June, based on consumer demand. During the months of the normal season, the system is shut down for a few days as the demand gets accomplished before the month ends. However, during the festival season, the system has to be kept operational throughout the month to fulfill the high demand for the product. The semi-Markov process and regenerative point technique are used to assess the reliability measures, namely mean time to system failure (MTSF), system availability in both seasons, expected busy period of the repair persons, and expected downtime of the system. The overall profitability of the system is also demonstrated by Graphical interpretation.

Keywords- Festival season, Patience time, Regenerative point technique, MTSF, Expected downtime.

1. Introduction

At the beginning of human civilization, food was the only prime need for their survival. They were proficient in meeting this essential requirement of food directly from natural resources. The ancient people did not grow crops or reared meat animals for food but consumed fresh food in raw form from natural resources. Furthermore, they did not cook, process, preserve, or store the raw food for the off-season. With the lapse of time, people started to grow food crops, reared meat animals, and also practiced and learned to preserve the food for eating in the off-season. They initiated the cooking of food before consumption at the domestic level. However, with the advancement of civilization, there have been several changes in food patterns as per the availability of food, needs, and awareness of consumers and their economic status.

In the present modern era, busy schedule and over-engagement of people have entirely changed their lifestyles and food pattern. They need a variety of convenient food that is ready-to-eat or even ready-to-serve. Hence, processed foods have become popular among consumers and are in great demand globally. Thus, the food processing industry has become one of the essential industries in the world.

Food processing industries are becoming very complex owing to automation and miniaturization on account of technological advancement for producing quality food items as demanded by the target population.

Besides this, these industries are required to keep the complex systems in proper working order and maintain strict hygienic and regulatory standards. Undoubtedly, modern systems work appropriately, but the chances of their failures cannot be ignored entirely as too high or too low moisture, temperature, pressure, etc., inappropriate contents, and sanitation are some of the reasons for the failure of modern food processing units. Hence, there is a dire need to investigate the reliability of food industrial systems. This study will also assist in knowing the proper techniques to improve the availability and profit of such complex systems.

The industry and the users are always concerned with the satisfactory performance of a system or product, and both want high reliability and availability. To increase the availability and profit of the system, many researchers have incorporated the concept of patience time (Deterministic/Random) and two types of repair persons (regular and expert) in their studies. The patience time is defined as the maximum time a regular repair person can have in order to attempt the repair of the failed unit. The patience time is treated as random patience time when it is arbitrarily distributed and is treated as deterministic when it has a constant value. After the patience time is completed, the expert person repairs the failed unit. Generally, a regularly appointed repair person is instantly available and can repair minor faults of the system within the patience time but is incapable of rectifying the more complicated faults. On the contrary, it is assumed that the expert can rectify any fault. To improve the performance of the two-unit standby redundant system, Goyal and Murari (1984) have employed "patience time" as the upper limit to the repair time of the regular repairman in their study. Authors have also explored the optimum patience time for enhancing system profit. Murari et al. (1985) further extended their work to the warm standby system and developed a rule for the profitable utilization of the services of the expert person. Kumar et al. (1996) gave three models for a two-unit cold standby system by including the concept of patience time and instruction time. The authors have utilized the regenerative point technique and have compared the profit in three developed models. Several research workers (Mahmoud et al., 1994; Taneja and Naveen, 2003; Rashad et al., 2009; Mokaddis et al., 2009; Bieth et al., 2010; Andalib and Sarkar, 2021; Andalib and Sarkar, 2022) have considered patience time either random or deterministic in their research papers with different types of systems, such as single unit systems and standby systems.

The Markov process is a stochastic process that exhibits the memoryless property. The semi-Markov process is a generalized Markov process where the holding time in each state is random. The semi-Markov process has been employed by numerous researchers in developing various reliability models. The theory of the semi-Markov process, its properties, and applications can be seen in Korolyuk et al. (1974) and Csenki (1994). The regenerative point technique has been practiced as a most effective tool for researchers for a long time. The main emphasis on the regenerative point technique was given by Srinivasan and Gopalan (1973). Much research on system reliability measures has been done using the semi-Markov process and regenerative point technique. Murari et al. (1985), Goel et al. (1985), Gopalan and Muralidhar (1991), Tuteja and Taneja (1993), Siwach et al. (2001), El-Said and El-Sherbeny (2006), Rizwan et al. (2010), Batra and Taneja (2019), Sheetal et al. (2019), Taj et al. (2020), Sultan and Moshref (2021) and Bhatti and Kakkar (2022) have worked on the reliability analysis of the systems using regenerative point technique.

Aggarwal and Malik (2020) have analyzed the profit of a repairable cold standby system, where the server is free to take a rest between repairs. Kumar et al. (2021) have further studied the reliability of a cold standby system via the regenerative point technique by incorporating the concept of refreshment. In the developed model, the server is given refreshment to work efficiently. The job of the server is to inspect the unit before repair or replacement. Wang et al. (2021) evaluated the reliability measures of a warm standby repairable system with two non-identical units using the Markov renewal method. In the developed model, they have given priority to the use of unit 1. Kakkar et al. (2022) have recently investigated the performance of a three-unit redundant system involved in the paper industry using the regenerative point technique. Authors have incorporated inspection policy into their model and assumed correlated failure and repair time. Malhotra and Taneja (2013, 2014, 2015) studied the effect of varying demand on system reliability and profit. These studies are further based on the regenerative point technique.

Ahmad et al. (2011) have done configurational modeling and developed a stochastic model for a complex repairable system in a cold-drink manufacturing plant by employing the regenerative point technique approach. Tewari and Kumar (2016) employed a probabilistic approach to discuss the availability analysis of the rice milling system. The supplementary variable technique was used by Kumar and Kadyan (2018) to study the performance of a distillery plant. Kumari et al. (2019a) employed the supplementary variable method to study the profit analysis of the butter-oil making system of a milk plant. Kumari et al. (2019b) also used the same technique to upgrade the performance of skimmed milk making system that ensures the smooth functioning of the whole milk plant. Barak et al. (2021) have employed a regenerative point graphical technique to study the reliability measures of milk plant.

The review of available literature in reference to food industrial systems regarding reliability analysis indicated that these systems have not conceptualized the idea of the demand-based seasons (festival/normal), which is also equally important as the other parameters of food industrial systems as far as the reliability and cost-benefit analysis are concerned. In the purview of this concept, we, therefore, in this study introduce the two types of seasons depending upon the demand, which are normal and festival seasons. Also, we considered patience time together with two types of repair persons - one regularly appointed repair person who, as usual, is known as the operator and the other expert repair person generally known as the fitter. The operator is always present with the system with the well-known fact that this person is incapable of performing some complex repairs within some patience time. In that case, to reduce the failure time of the system, the fitter is asked to repair it. Basically, the fitter is responsible for repairs in the case, either on completion of the patience time or on system failure, whichever happens, earlier. In festival season, it is assumed that for any failure in the system, the fitter is available immediately to repair as the manufacturers cannot take any risk to fulfill the demand. So, there is no concept of patience time during festival season. Patience time is only incorporated in the normal season.

Compared to other reliability studies of food industrial systems, the present work is not based on a configurational model but has covered two critical ideas: festival season and patience time. The originality of this work lies in the fact that no such mathematical model has earlier been developed in the literature for the reliability analysis of food industrial system using the semi-Markov process and regenerative point technique by considering two demand-based seasons and two differently skilled repair persons with the concept of random patience time. In order to understand the various important parameters involved in the food industry, one food manufacturing plant in Haryana, India is visited. The same visit inspires the situation considered in the present stochastic model. In this study, the semi-Markov process and regenerative point technique are used to assess different reliability measures of the system, such as MTSF, availability of the system in festival season, availability of the system in normal season, busy periods of the operator and the fitter, and expected downtime of the system in the normal season. The proposed reliability model can assist food production managers in ensuring system performance and maximizing company profit. Moreover, such assessments can support the manufacturing plants for the fixation of the price of the products manufactured by them and validate the maximum downtime that a system can afford for a reasonable profit.

2. Model Description and Assumptions

A single-unit food processing system is undertaken in this model, which works throughout the month, but the time required for the production is demand based, i.e., either demand will be of the type that can be fulfilled by running the plant throughout the month or will be of the kind that can be fulfilled within few days before the completion of the month. These two categorizations are termed Festival and Normal seasons, respectively. In the proposed model, there are six states of the given system. These states are discussed in Table 1 and presented in the state transition diagram in Figure 1.

Table 1. Notations concerning all states of the system.

O_f(State 0)	Operative state in festival season
O_n (State 2)	Operative state in normal season
F_{ro}(State 3)	Failed state under repair by operator
F_{rf} (State 1 and state 4)	Failed state under repair by fitter
D_n (State 5)	Down state of the system in normal season

The assumptions are mentioned as under:

(a) In the normal season, it is assumed that, on failure, the operator tries to repair the failed unit first. The fitter is intimated if the operator fails to repair the system within the patience time.
(b) In festival season, it is assumed that as a failure occurs, immediately the operator presents with the system intimates to the fitter to come to repair the failed unit.
(c) The system detects failures within a negligible time without any inspection.
(d) Fitter takes insignificant time to attend the complaint.
(e) The repair persons (operator and fitter) never harm the system.
(f) The failure time, the time taken by the system to transit to down state/up state (in a normal season), and the time for the change of seasons follow exponential distributions, whereas repair times and patience time follow the general distributions.
(g) The unit works like a new one following repair.
(h) All random variables are mutually independent.

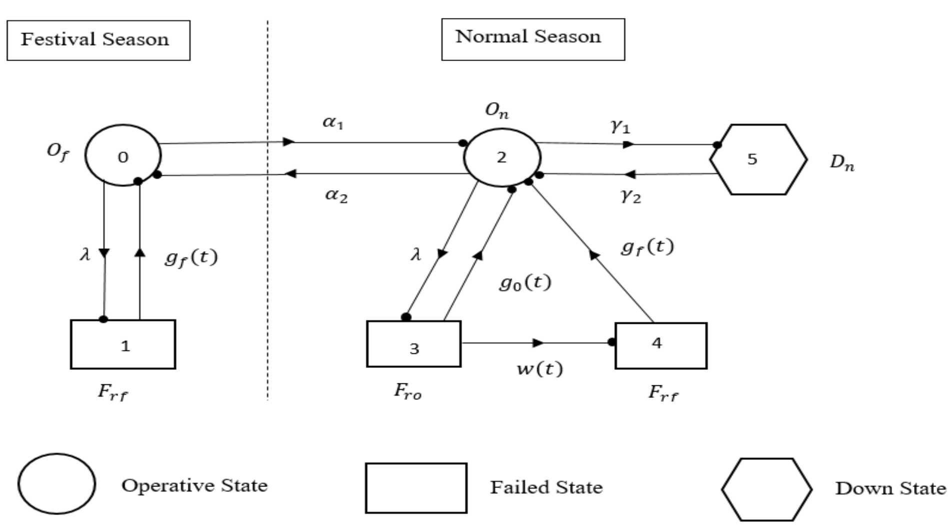

Figure 1. State transition diagram.

3. Notations

The various notations involved in this model are as follows:

λ	Constant failure rate of the system
$g_f(t)/G_f(t)$	pdf/cdf of time taken by fitter for repair
$g_o(t)/G_o(t)$	pdf/cdf of time taken by operator for repair
$w(t)/W(t)$	pdf/cdf of patience time
γ_1/γ_2	Transition rate from up/down state to down/up state in normal season
α_1, α_2	Respective rate of change of season from festival to normal and normal to festival
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of transition time from state 'i' to state 'j'
$\phi_i(t)$	cdf of first passage time from a regenerative state 'i' to a failed one
A_0^n/A_0^f	Steady state availability of the system in the normal/festival season
B_0^o/B_0^f	Busy period of the operator/fitter for repair
DT_0	Expected downtime of the system
$M_i(t)$	Probability pertaining to the system which is up initially in regenerative state 'i' and is up at instant t without transiting to any other regenerative state
$W_i(t)$	Probability concerning the engagement of the repair person who is busy in the repair of the system initially in regenerative state 'i' and is involved at time t without transiting to any other regenerative state
m_{ij}	The unconditional mean time taken by the system to visit any regenerative state 'j' provided the time is recorded from the entrance into state 'i'
μ_i	Mean sojourn time or the expected time taken by the system in regenerative state 'i' before visiting any other state
*/©	Sign to denote Laplace transform / Laplace convolution
**/Ⓢ	Sign to denote the Laplace Stieltjes transform/ Laplace Stieltjes convolution

4. Mathematical Analysis of Model

All the states of the considered system are regenerative in nature, as the time points of entry in these states are the regeneration points. Let $T_n(T_0 = 0)$ be the time of n^{th} state transition and X_n be the state of the system at a time ${T_n}^+$. Here, we have

$$P\{X_{n+1} = j, T_{n+1} - T_n \le t \mid X_0, X_1, \dots, X_n, T_0, T_1 \dots, T_n\} = P\{X_{n+1} = k, T_{n+1} - T_n \le t \mid X_n\} \quad (1)$$

for all $n = 0, 1, 2, \ldots$ and $j \in \{0,1,2,3,4,5\}, t \ge 0$. The sequence of two-dimensional random variables $\{(X_n, T_n): n = 0, 1, 2, \ldots\}$ constitutes a Markov renewal process and continuous parameter stochastic process $\{N(t),\ t \ge 0\}$, where $N(t) = X_n,\ t_n < t < t_{n+1}$ forms a semi-Markov process with a semi-Markov kernel

$$Q_{ij}(t) = P\{X_{n+1} = j, T_{n+1} - T_n \le t \mid X_n = i\}. \quad (2)$$

Using probabilistic arguments, we have obtained the following transition probabilities:

$$q_{01}(t) = \lambda\,.\,e^{-(\lambda+\alpha_1)t} \quad (3)$$
$$q_{02}(t) = \alpha_1.\,e^{-(\lambda+\alpha_1)t} \quad (4)$$
$$q_{10}(t) = g_f(t) \quad (5)$$
$$q_{20}(t) = \alpha_2\ .\,e^{-(\lambda+\alpha_2+\gamma_1)t} \quad (6)$$

$$q_{23}(t) = \lambda \,.\, e^{-(\lambda+\alpha_2+\gamma_1)t} \quad (7)$$
$$q_{25}(t) = \gamma_1 \,.\, e^{-(\lambda+\alpha_2+\gamma_1)t} \quad (8)$$
$$q_{32}(t) = g_0(t).\overline{W(t)} = E_{32}(t)\ (say) \quad (9)$$
$$q_{34}(t) = w(t).\overline{G_0(t)} = E_{34}(t)\ (say) \quad (10)$$
$$q_{42}(t) = g_f(t) \quad (11)$$
$$q_{52}(t) = \gamma_2 . e^{-\gamma_2 t}. \quad (12)$$

Further, we have evaluated the non-zero elements $p'_{ij}s$, where $p_{ij} = \lim_{s\to 0} q_{ij}{}^*(s)$. The obtained $p_{ij}{}'s$ are:

$$p_{01} = \frac{\lambda}{\lambda+\alpha_1} \quad (13)$$
$$p_{02} = \frac{\alpha_1}{\lambda+\alpha_1} \quad (14)$$
$$p_{10} = 1 \quad (15)$$
$$p_{20} = \frac{\alpha_2}{\lambda+\alpha_2+\gamma_1} \quad (16)$$
$$p_{23} = \frac{\lambda}{\lambda+\alpha_2+\gamma_1} \quad (17)$$
$$p_{25} = \frac{\gamma_1}{\lambda+\alpha_2+\gamma_1} \quad (18)$$
$$p_{32} = E_{32}^*(0) \quad (19)$$
$$p_{34} = E_{34}^*(0) \quad (20)$$
$$p_{42} = 1 \quad (21)$$
$$p_{52} = 1 \quad (22)$$

From these, we can conclude that

$$p_{01} + p_{02} = 1 \quad (23)$$
$$p_{20} + p_{23} + p_{25} = 1 \quad (24)$$

and

$$p_{32} + p_{34} = 1. \quad (25)$$

The mean sojourn times ($\mu_i{}'s$) for the states 0 to 5 are computed as:

$$\mu_0 = \frac{1}{\lambda+\alpha_1} \quad (26)$$
$$\mu_1 = -g_f^{*'}(0) \quad (27)$$
$$\mu_2 = \frac{1}{\lambda+\alpha_2+\gamma_1} \quad (28)$$
$$\mu_3 = \int_0^\infty \overline{G_0(t)}\,.\,\overline{W(t)}.\,dt = \int_0^\infty E_1(t)\,.\,dt \qquad \text{where, } E_1(t) = \overline{G_0(t)}.\overline{W(t)} \quad (29)$$
$$\mu_4 = -g_f^{*'}(0) \quad (30)$$
$$\mu_5 = \frac{1}{\gamma_2} \quad (31)$$

The unconditional mean times ($m_{ij}'s$) are computed as:

$$m_{01} = \frac{\lambda}{(\lambda+\alpha_1)^2} \tag{32}$$
$$m_{02} = \frac{\alpha_1}{(\lambda+\alpha_1)^2} \tag{33}$$
$$m_{10} = -g_f^{*'}(0) \tag{34}$$
$$m_{20} = \frac{\alpha_2}{(\lambda+\alpha_2+\gamma_1)^2} \tag{35}$$
$$m_{23} = \frac{\lambda}{(\lambda+\alpha_2+\gamma_1)^2} \tag{36}$$
$$m_{25} = \frac{\gamma_1}{(\lambda+\alpha_2+\gamma_1)^2} \tag{37}$$
$$m_{32} = -E_{32}^{*'}(0) \tag{38}$$
$$m_{34} = -E_{34}^{*'}(0) \tag{39}$$
$$m_{42} = -g_f^{*'}(0) \tag{40}$$
$$m_{52} = \frac{1}{\gamma_2} \tag{41}$$

From the above computations, the sum of unconditional mean times is obtained as follows:

$$m_{01} + m_{02} = \mu_0 \tag{42}$$
$$m_{10} = \mu_1 \tag{43}$$
$$m_{20} + m_{23} + m_{25} = \mu_2 \tag{44}$$
$$m_{32} + m_{34} = \int_0^\infty t.[E_{32}(t) + E_{34}(t)].dt = K_3\ (say) \tag{45}$$
$$m_{42} = \mu_4 = \mu_1 \tag{46}$$
$$m_{52} = \mu_5 \tag{47}$$

The various reliability measures have been studied and evaluated in the following subsections.

4.1 Mean Time to System Failure (MTSF)

Failure of a system is a natural thing. However, sudden failure sometime harms a system to a large extent. So, to avoid sudden failure, we have to find out some measure representing the system's lifetime, and that is the MTSF. The MTSF is the expected time the system is in working condition prior to complete failure. The failed states are taken as the absorbing states for assessing the MTSF. Thus, the iterative relations for $\phi_i(t)$ will be

$$\phi_0(t) = Q_{01}(t) + Q_{02}(t)Ⓢ\phi_2(t) \tag{48}$$
$$\phi_2(t) = Q_{20}(t)Ⓢ\phi_0(t) + Q_{23}(t) + Q_{25}(t)Ⓢ\phi_5(t) \tag{49}$$
$$\phi_5(t) = Q_{52}(t)Ⓢ\phi_2(t). \tag{50}$$

Taking Laplace Stieltjes transform of the equations (48-50) mentioned above, we have

$$\phi_0^{**}(s) = \frac{N_1(s)}{D_1(s)} \tag{51}$$

where,

$$N_1(s) = \begin{vmatrix} q_{01}^*(s) & -q_{02}^*(s) & 0 \\ q_{23}^*(s) & 1 & -q_{25}^*(s) \\ 0 & -q_{52}^*(s) & 1 \end{vmatrix} \quad (52)$$

and

$$D_1(s) = \begin{vmatrix} 1 & -q_{02}^*(s) & 0 \\ -q_{20}^*(s) & 1 & -q_{25}^*(s) \\ 0 & -q_{52}^*(s) & 1 \end{vmatrix} \quad (53)$$

Thus, the MTSF, when the system starts from the state '0' is expressed as:

$$\text{MTSF} = \lim_{s \to 0} \frac{(1-\phi_0^{**}(s))}{s} = \frac{N_1}{D_1} \quad (54)$$

where,

$$N_1 = (1 - p_{25}).\mu_0 + p_{02}.(\mu_2 + p_{25}.\mu_5) \quad (55)$$

and

$$D_1 = p_{23} + p_{20}.p_{01}. \quad (56)$$

4.2 Availability

High availability is necessary for a system, as it ensures continuous operations or uptime for an extended period, resulting in a maximum profit for the manufacturers. Here two cases arise:

4.2.1 Availability in Festival Season

For the system availability during the festival season, $A_i^f(t)$ represents the probability of the system being in the up state in festival season at the instant 't', provided that the system went in the regenerative state 'i' at $t = 0$. Thus, the iterative relations will be:

$$A_0^f(t) = M_0(t) + q_{01}(t)©A_1^f(t) + q_{02}(t)©A_2^f(t) \quad (57)$$

$$A_1^f(t) = q_{10}(t)©A_0^f(t) \quad (58)$$

$$A_2^f(t) = q_{20}(t)©A_0^f(t) + q_{23}(t)©A_3^f(t) + q_{25}(t)©A_5^f(t) \quad (59)$$

$$A_3^f(t) = q_{32}(t)©A_2^f(t) + q_{34}(t)©A_4^f(t) \quad (60)$$

$$A_4^f(t) = q_{42}(t)©A_2^f(t) \quad (61)$$

$$A_5^f(t) = q_{52}(t)©A_2^f(t) \quad (62)$$

where,

$$M_0(t) = e^{-(\lambda+\alpha_1)t}. \quad (63)$$

Solving the above equations (57-62) with the aid of the Laplace transform, the steady-state availability in festival season is evaluated as

$$A_0^f = \lim_{s\to 0}\left[s.\frac{N_{21}(s)}{D_{21}(s)}\right] = \frac{N_{21}}{D_{21}} \tag{64}$$

where,

$$N_{21}(s) = \begin{vmatrix} M_0^*(s) & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{65}$$

and

$$D_{21}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{66}$$

which gives

$$N_{21} = \mu_0.p_{20} \tag{67}$$

and

$$D_{21} = p_{20}.\mu_0 + p_{01}.p_{20}.\mu_1 + p_{02}.\mu_2 + p_{23}.p_{02}.K_3 + p_{34}.p_{23}.p_{02}.\mu_4 + p_{25}.p_{02}.\mu_5. \tag{68}$$

4.2.2 Availability in Normal season

For the normal season, $A_i^n(t)$ represents the probability of the system being in the up state in normal season at the instant 't', provided that the system went in the regenerative state 'i' at $t = 0$. Accordingly, the iterative relations will be:

$$A_0^n(t) = q_{01}(t)©A_1^n(t) + q_{02}(t)©A_2^n(t) \tag{69}$$

$$A_1^n(t) = q_{10}(t)©A_0^n(t) \tag{70}$$

$$A_2^n(t) = M_2(t) + q_{20}(t)©A_0^n(t) + q_{23}(t)©A_3^n(t) + q_{25}(t)©A_5^n(t) \tag{71}$$

$$A_3^n(t) = q_{32}(t)©A_2^n(t) + q_{34}(t)©A_4^n(t) \tag{72}$$

$$A_4^n(t) = q_{42}(t)©A_2^n(t) \tag{73}$$

$$A_5^n(t) = q_{52}(t)©A_2^n(t) \tag{74}$$

where,

$$M_2(t) = e^{-(\lambda+\gamma_1+\alpha_2)t}. \tag{75}$$

Using the Laplace transform, the steady-state availability in normal season from equations (69-74), is attained as

$$A_0^n = \lim_{s\to 0}\left[s.\frac{N_{22}(s)}{D_{22}(s)}\right] = \frac{N_{22}}{D_{22}} \tag{76}$$

where,

$$N_{22}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ M_2^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{77}$$

and

$$D_{22}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{78}$$

which gives

$$N_{22} = \mu_2.p_{02} \tag{79}$$

and

$$D_{22} = p_{20}.\mu_0 + p_{01}.p_{20}.\mu_1 + p_{02}.\mu_2 + p_{23}.p_{02}.K_3 + p_{34}.p_{23}.p_{02}.\mu_4 + p_{25}.p_{02}.\mu_5. \tag{80}$$

4.3 Busy Period Analysis

The repair person is usually responsible for the repair and upkeep of the industrial plant he/she works in by solving various issues with the production equipment, such as mechanical issues, electrical issues, etc. In the present study of the food industrial system, we have considered the two types of repair persons, known as operator and fitter. Their busy periods for repairing the system have been studied by considering two cases.

4.3.1 Busy Period Analysis of Operator

To determine the busy period of the operator with the system, we have the following iterative relations for $B_i^o(t)$, where $B_i^o(t)$ represents the probability of full engagement of the operator at an instant *'t'*, provided that the system moved in regenerative state '*i*' at *t* = 0.

$$B_0^o(t) = q_{01}(t)©B_1^o(t) + q_{02}(t)©B_2^o(t) \tag{81}$$

$$B_1^o(t) = q_{10}(t)©B_0^o(t) \tag{82}$$

$$B_2^o(t) = q_{20}(t)©B_0^o(t) + q_{23}(t)©B_3^o(t) + q_{25}(t)©B_5^o(t) \tag{83}$$

$$B_3^o(t) = W_3(t) + q_{32}(t)©B_2^o(t) + q_{34}(t)©B_4^o(t) \tag{84}$$

$$B_4^o(t) = q_{42}(t)©B_2^o(t) \tag{85}$$

$$B_5^o(t) = q_{52}(t)©B_2^o(t) \tag{86}$$

where,

$$W_3(t) = \overline{W(t)}.\overline{G_0(t)}. \tag{87}$$

The total segment of the time during which the system is being repaired by the operator, i.e., the busy period of the operator, is obtained by solving the equations (81-86) for the steady state following the Laplace transform as mentioned below

$$B_0^o = \lim_{s\to 0}\left[s.\frac{N_{31}(s)}{D_{31}(s)}\right] = \frac{N_{31}}{D_{31}} \tag{88}$$

where,

$$N_{31}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ W_3^*(s) & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{89}$$

and

$$D_{31}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{90}$$

which gives

$$N_{31} = \mu_3.p_{23}.p_{02} \tag{91}$$

and

$$D_{31} = p_{20}.\mu_0 + p_{01}.p_{20}.\mu_1 + p_{02}.\mu_2 + p_{23}.p_{02}.K_3 + p_{34}.p_{23}.p_{02}.\mu_4 + p_{25}.p_{02}.\mu_5. \tag{92}$$

4.3.2 Busy Period Analysis of Fitter

For the busy period of the fitter with the system, the following iterative relations can be obtained where $B_i^f(t)$ represents the probability due to full engagement of the fitter at an instant 't', provided when the system went in regenerative state 'i' at t = 0.

$$B_0^f(t) = q_{01}(t)©B_1^f(t) + q_{02}(t)©B_2^f(t) \tag{93}$$

$$B_1^f(t) = W_1(t) + q_{10}(t)©B_0^f(t) \tag{94}$$

$$B_2^f(t) = q_{20}(t)©B_0^f(t) + q_{23}(t)©B_3^f(t) + q_{25}(t)©B_5^f(t) \tag{95}$$

$$B_3^f(t) = q_{32}(t)©B_2^f(t) + q_{34}(t)©B_4^f(t) \tag{96}$$

$$B_4^f(t) = W_4(t) + q_{42}(t)©B_2^f(t) \tag{97}$$

$$B_5^f(t) = q_{52}(t)©B_2^f(t) \tag{98}$$

where,
$$W_1(t) = \overline{G_f(t)} \tag{99}$$

and
$$W_4(t) = \overline{G_f(t)}. \tag{100}$$

By having Laplace transform of equations (93-98) and followed by their solution for steady state, the total segment of the time during which the system is being repaired by the fitter, i.e., the busy period of the fitter, is obtained as

$$B_0^f = \lim_{s\to 0}\left[s.\frac{N_{32}(s)}{D_{32}(s)}\right] = \frac{N_{32}}{D_{32}} \tag{101}$$

where,
$$N_{32}(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ W_1^*(s) & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ W_4^*(s) & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{102}$$

and
$$D_{32}(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{103}$$

which gives
$$N_{32} = \mu_4.(p_{01}.p_{20} + p_{23}.p_{34}.p_{02}) \tag{104}$$

and
$$D_{32} = p_{20}.\mu_0 + p_{01}.p_{20}.\mu_1 + p_{02}.\mu_2 + p_{23}.p_{02}.K_3 + p_{34}.p_{23}.p_{02}.\mu_4 + p_{25}.p_{02}.\mu_5. \tag{105}$$

4.4 Expected Downtime of the System

A system needs to be put in down state on account of unavailability of raw material, inappropriate moisture, failure of the main unit, and for many more reasons. There are seven months under consideration in the normal season. In any specific month out of seven, it is generally found that the production meets the demand before the completion of the month, so the system needs to be put in a down state for the next few days until the new month starts. To find out the expected time in which the system is in the down state in normal season, let $DT_i(t)$ be the probability of the system being down at an instant 't', provided that the system moved in the regenerative state 'i' at t = 0. Thus, the iterative relations will be:

$$DT_0(t) = q_{01}(t)©DT_1(t) + q_{02}(t)©DT_2(t) \tag{106}$$

$$DT_1(t) = q_{10}(t)©DT_0(t) \tag{107}$$

$$DT_2(t) = q_{20}(t)©DT_0(t) + q_{23}(t)©DT_3(t) + q_{25}(t)©DT_5(t) \tag{108}$$

$$DT_3(t) = q_{32}(t)©DT_2(t) + q_{34}(t)©DT_4(t) \tag{109}$$

$$DT_4(t) = q_{42}(t)©DT_2(t) \tag{110}$$

$$DT_5(t) = M_5(t) + q_{52}(t)©DT_2(t) \tag{111}$$

where,

$$M_5(t) = e^{-\gamma_2 t}. \tag{112}$$

By having Laplace transform these equations (106-111) and followed by solutions thereof for steady state, the total segment of the time during which the system is in a down state can be evaluated as

$$DT_0 = \lim_{s \to 0}\left[s.\frac{N_4(s)}{D_4(s)}\right] = \frac{N_4}{D_4} \tag{113}$$

where,

$$N_4(s) = \begin{vmatrix} 0 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ M_5^*(s) & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{114}$$

and

$$D_4(s) = \begin{vmatrix} 1 & -q_{01}^*(s) & -q_{02}^*(s) & 0 & 0 & 0 \\ -q_{10}^*(s) & 1 & 0 & 0 & 0 & 0 \\ -q_{20}^*(s) & 0 & 1 & -q_{23}^*(s) & 0 & -q_{25}^*(s) \\ 0 & 0 & -q_{32}^*(s) & 1 & -q_{34}^*(s) & 0 \\ 0 & 0 & -q_{42}^*(s) & 0 & 1 & 0 \\ 0 & 0 & -q_{52}^*(s) & 0 & 0 & 1 \end{vmatrix} \tag{115}$$

which gives

$$N_4 = \mu_5.p_{25}.p_{02} \tag{116}$$

and

$$D_4 = p_{20}.\mu_0 + p_{01}.p_{20}.\mu_1 + p_{02}.\mu_2 + p_{23}.p_{02}.K_3 + p_{34}.p_{23}.p_{02}.\mu_4 + p_{25}.p_{02}.\mu_5. \tag{117}$$

4.5 Profit Analysis

Profit is necessary for the survival as well as the growth of any industrial unit. If the plant does not make enough profit, it will not survive in the ever-increasing competitive world. So, reliability analysis, an effective and beneficial tool to provide profitable strategies, is necessary. The following expression can assess the profit

$$\text{Profit} = \left(C_{01}.A_0^f + C_{02}.A_0^n\right) - \left(C_{ro}.B_0^o + C_{rf}.B_0^f + C_d.DT_0\right) \tag{118}$$

where, $C_{01}/\ C_{02}$is the revenue per unit up time of the system in festival/normal season,C_{ro}/C_{rf} is the cost per unit time for which the system is under repair by the operator/fitter and C_d is loss per unit time during which the system remains down. Profit assessed by the system will be helpful for finding the cut-off points of the rates involved in the model, which is graphically interpreted in the coming section.

4.6 Numerical Calculation

The performance of the present system has been evaluated by considering the patience time and repair time of the operator and fitter as exponentially distributed with parameters β, δ_1 and δ_2, respectively. Correspondingly, we have $p_{32}=\frac{\delta_1}{\delta_1+\beta}$, $p_{34}=\frac{\beta}{\delta_1+\beta}$, $\mu_1=\frac{1}{\delta_2}$, $\mu_3=\frac{1}{\delta_1+\beta}$ and $\mu_4=\frac{1}{\delta_2}$. As per discussion and collected data from the food industrial plant, various rates and involved costs have been estimated as $\lambda=0.01555$, α_1 =0.00027, $\alpha_2=0.00019$, $\gamma_1=0.00181$, $\gamma_2=0.01041$, $\beta=4$, δ_1 =5, δ_2 =1.75438, C_{01} =160000, C_{02} =106000, $C_{ro}=$ 1600, C_{rf} =2120 and C_d =24000. Using these values, we have obtained various reliability measures of the system as $MTSF=64.52328045$, $A_0^f=0.342207672$, $A_0^n=0.554973055$, $B_0^o=0.00095887$, $B_0^f=0.005220692$ and $DT_0=0.096493874$. Different graphs are plotted by giving some numerical values to the concerned parameters in the next section. Although there have been many parameters considered in the present model which have an impact on the profit of the system yet rate of going to the down state of the system (γ_1) affect the profit to a large extent. So, here we have focused mainly on the parameter γ_1 to study the nature of the reliability measures and the profit of the system.

5. Result and Discussion

Numerical calculation (section 4.6) reveals that the considered model has more availability in the normal season than in the festival season, as per the taken parameters. The notable difference in the availabilities of the system in two seasons is 0.212765383. Also, the obtained values of the busy period show that the fitter is 0.004261822 hrs. busier than the operator.

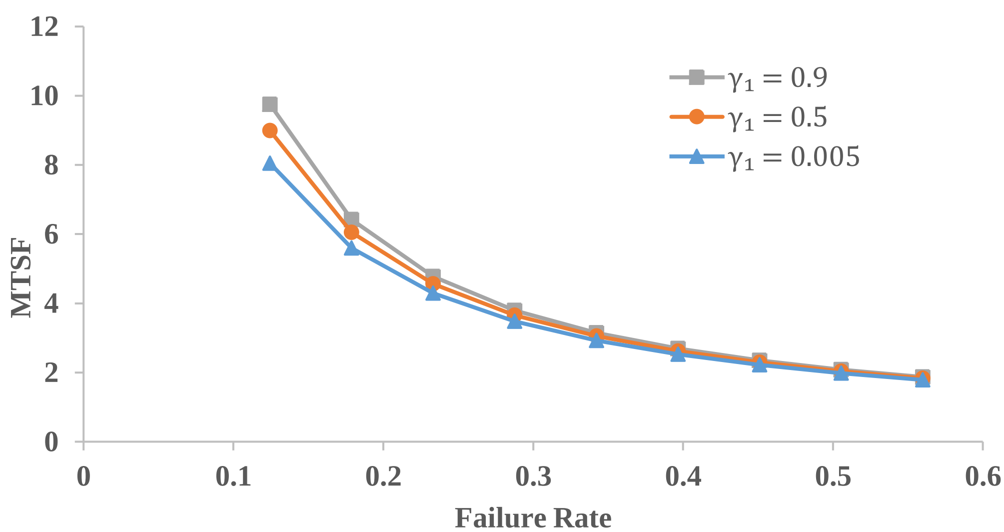

Figure 2. Effects of failure rate on MTSF with varying levels of γ_1.

MTSF, availability in normal as well as in festival season, and profit incurred to the system in both seasons have been interpreted graphically. It can be seen from Figure 2 that MTSF diminishes with the rise in the failure rate of the system. Corresponding to an initial increase in the system failure rate from 0.1 to 0.3, we have a sharp decrease in the MTSF. This figure also reveals that MTSF is greater for larger values of γ_1.

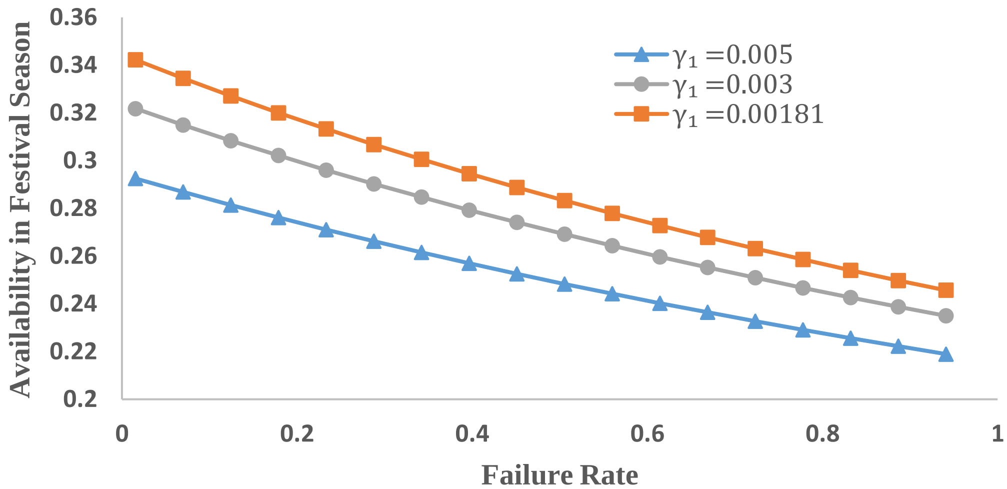

Figure 3. Effect of failure rate on availability of the system in the festival season with varying levels of γ_1.

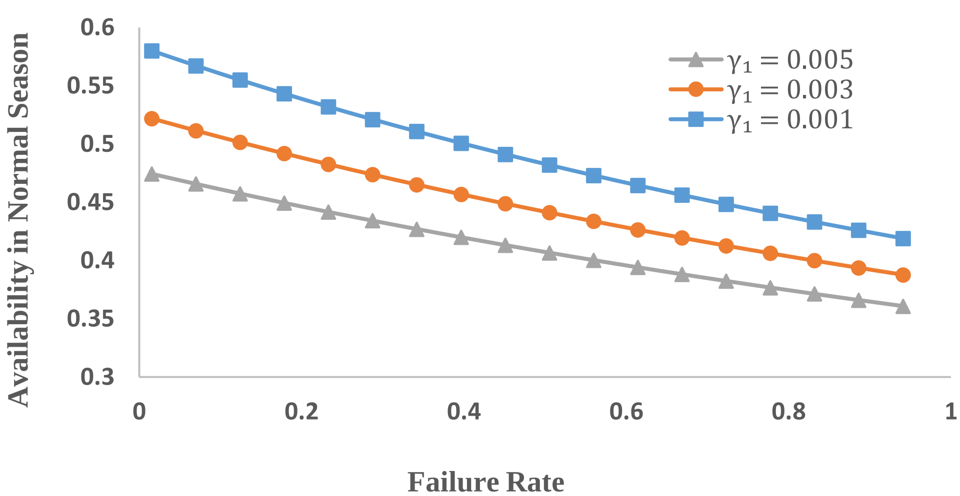

Figure 4. Effect of failure rate on availability of the system in the normal season with varying levels of γ_1.

We can conclude from the figures (Figures 3 and 4) that the availability in the festival, as well as the normal season, declines with the rise in the levels of failure rate as well as owing to the rise in the levels of γ_1 from 0.00181 to 0.005 and 0.001 to 0.005, respectively. For lower values of failure rates λ, the decrease in the transition rate γ_1 results in a good increase in system availability. Figures 3 and 4 further confirm that the system has more availability in the normal season than in the festival season. Regarding profitability, cut-off points for various parameters have been obtained, which can be seen in Figure 5 and Figure 6.

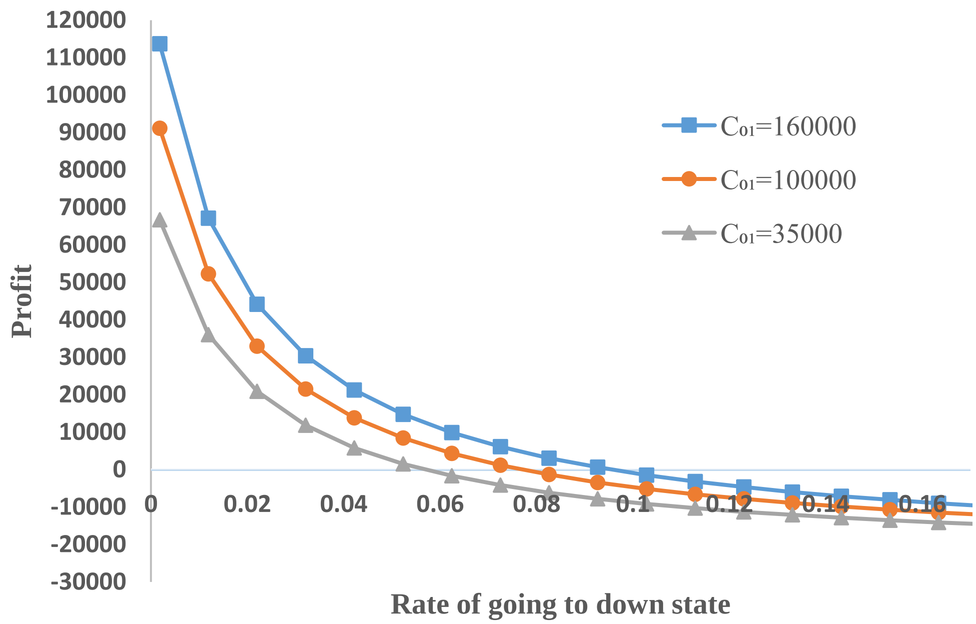

Figure 5. Profit v/s Rate of going to down state for different values of C_{01}.

For overall profit evaluation, we have taken parameters as $\lambda = 0.01555$, $\alpha_1 = 0.00027$, $\alpha_2 = 0.00019$, $\gamma_2 = 0.01041$, $\beta = 4$, $\delta_1 = 5$, δ_2 =1.75438, C_{ro} =1600, C_{rf} =2120 and C_d =24000. The impact of revenues earned per unit time in both seasons on system profit has been explored by varying one of them at a time. Figures 5 and 6 depict that the profit declines with the rise in the transition rate γ_1. It can also be noticed from these figures that profit is high for the higher values of revenue cost per unit time. To study the effect of revenue on overall profit, firstly, we have assumed C_{02} =106000 as constant and varied revenue C_{01} as 160000, 100000, and 35000. It can be observed from Figure 5 that corresponding to revenue C_{01} as 160000, 100000, and 35000, the profit is +ve or zero or -ve when γ_1< or = or > 0.093, 0.073 and 0.055, respectively. This indicates that for $C_{01} = 160000$, the established model would be beneficial only if γ_1 is less than 0.093. In a similar manner, for $C_{01} = 100000$ and $C_{01} = 35000$, transition rate γ_1 should be less than 0.073 and 0.055, respectively.

Similarly, the impact of earned revenue on overall profit is studied by assuming $C_{01} = 106000$ as constant and varying revenue C_{02} as 100000, 50000, and 25000. It can be seen from Figure 6 that corresponding to revenue C_{02} as 100000, 50000, and 25000, the profit is +ve or zero or -ve when γ_1< or = or > 0.093, 0.064 and 0.054, respectively. This again shows that for the system's benefit, for $C_{02} = 100000$, we should have a transition rate γ_1 less than 0.093. Similarly, corresponding to $C_{02} = 50000$ and $C_{02} = 25000$, transition rate γ_1 should be smaller than 0.064 and 0.0654. This indicates that any decrease in revenue cost strongly urges to have reduced transition rate γ_1, as otherwise, the system will suffer loss.

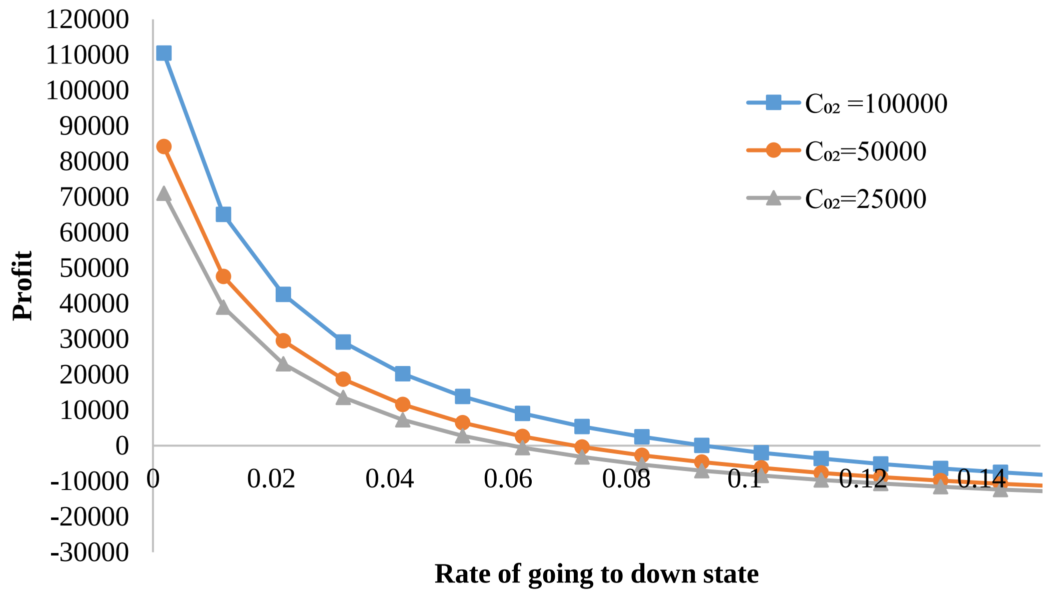

Figure 6. Profit v/s Rate of going to down state for different values of C_{02}.

6. Conclusion

In the present study, we have developed a reliability model for food industrial system via a semi-Markov process and regenerative point technique by incorporating the notion of two demand-based seasons along with two repair persons and patience time. The various reliability measures have been evaluated and explored by plotting different graphs. In the proposed model, MTSF and system availability decrease with increase in the system failure rate. The considered one-unit system has high availability in the normal season compared to the festival season. It has been observed that increase in the transition rate γ_1 results in an increase in MTSF, but at the same, it results in a decrease in system availability. The system profit analysis indicates that the parameter γ_1 is very crucial for making the system profitable. An optimum value of transition rate γ_1 along with a lower failure rate, λ, can increase system availability, MTSF, and profit. Food industries can easily adopt this study for evaluating various reliability measures. The developed model can guide manufacturing plants in suitably choosing the parameter γ_1 for a reasonable profit. The proposed model can be made more practical by considering proper maintenance and inspection strategies so that the system's complete failure can be minimized to a large extent. The study can be further strengthened by using the optimization method to find the optimal transition rate γ_1. In the future, one could develop a more advanced model by considering partial failure in the system.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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