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Application of a Novel Interval-Valued Intuitionistic Fuzzy Correlation Coefficient to the Supplier Selection Problem

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Abstract
The connection between two or more factors plays a key role in many choices in industry where data often lacks clarity or completeness. Classical correlation coefficients, however, fail to fully capture a complete measure of the delicate balance between membership and non-membership information in fuzzy data. Existing interval-valued intuitionistic fuzzy measures also show inconsistency in handling negative relationships, complementary patterns, and structured linguistic variability, and thus underscore the need for a more generalized and reliable correlation framework to manage uncertainty effectively. In this paper, we propose a generalized correlation coefficient specifically formulated for interval-valued intuitionistic fuzzy data. It uniquely represents uncertainty through interval-based membership and non-membership degrees. The proposed coefficient gives a better picture of unclear relationships. The performance of the coefficient is illustrated through numerical examples and through its application in a multicriteria group decision-making problem focused on supplier selection. A detailed comparative study with existing methods is conducted to highlight the superiority of the proposed approach in terms of robustness and reliability. A sensitivity analysis is also performed to examine the impact of an adjustable parameter in the proposed measure. It reveals that the proposed measure remains consistent and produces dependable results even under varying decision conditions.

Keywords- Interval-valued intuitionistic fuzzy set, Generalized correlation coefficient, MCGDM, Supplier selection.

1. Introduction

The correlation coefficient is a statistical measure that is most frequently used in research for understanding how two variables move in relation to each other. It is a significant tool in sciences, where finding the relationships among variables is essential for predicting the behaviour of the system (Pearson, 1895; Benesty et al., 2009). The correlation coefficient quantifies the strength and direction of relationships between variables, if these relationships are positive, negative, or negligible. This makes it especially useful for researchers who need to simplify complex data interactions into clear, actionable insights. The values of correlation coefficients, typically range from -1 to $+1$. A value that is very close to $+1$ will indicate a strong positive association, where variables increase together. On the other hand, a value that is very close to -1 will point to a strong inverse relationship, in which one variable decreases while the other increases (Chan, 2003). The values close to zero suggest a weak or no linear relationship between the variables.

Zadeh (1965) introduced the concept of fuzzy sets aimed to solve the problem of ambiguity that is inherent in a system. Atanassov (1986) extended this idea by introducing intuitionistic fuzzy sets (IFSs) with a bipolar treatment of ambiguity/vagueness in a system. Extension of fuzzy sets and IFSs to interval-valued fuzzy sets (IVFSs) (Zadeh, 1975) and interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov and Gargov, 1989), respectively, represent membership and non-membership as intervals. These changes have led to an even broader range of options for dealing with imprecise information. The concept of the correlation coefficient within these contexts has gained significant attention in recent literature. Dumitrescu (1978) introduced the idea of fuzzy correlation coefficient that is similar to conventional statistical correlation coefficient. Chiang and Lin (1999) proposed a fuzzy correlation coefficient receiving its value in $[-1, 1]$. Gerstenkorn and Manko (1991) pioneered the introduction of a correlation measure and correlation coefficient for IFSs within finite spaces. For more studies on correlation measures based on intuitionistic fuzzy data (IFD), we can refer to (Hung, 2001; Xu, 2006; Wei et al., 2011b; Liu et al., 2015; Garg and Kumar, 2018; Thao et al., 2019; Ejegwa, 2020; Ejegwa and Onyeke, 2020; Ejegwa et al., 2024; Ejegwa et al., 2025). Bustince and Burillo (1995) introduced a correlation measure specifically for IVIFSs. Hong (1998) extended the study of Bustince and Burillo (1995) for IVIFSs within general probability spaces. However, a significant drawback of the correlation coefficients developed by Bustince and Burillo (1995) and Hong (1998) is that they consider only positive correlations and ignore negative correlations. Hung (2001) further extended the correlation coefficient for IVIFSs to overcome the limitation identified in Bustince and Burillo (1995) and Hong (1998). Incorporating the element of hesitation, Park et al. (2009a) and Park et al. (2009b) refined Bustince and Burillo (1995) correlation and investigated multi-attribute group decision making (MAGDM) problems. Zeng and Wang (2011) also presented a method for determining the correlation coefficient of IVIFSs. Liu et al. (2015) suggested a new method to assess the correlation between IFSs. Their work also extended traditional statistical measures, such as deviation, variance, and covariance for IFSs, and established a correlation coefficient that remains bounded within the interval $[-1,1]$. Thao (2018) and Thao et al. (2019) derived correlation coefficients based on the variance and covariance between two IFSs.

The correlation values are very useful in fuzzy and extended fuzzy contexts and have been used in a wide range of applications. For example, Huang and Guo (2019) utilized the correlation coefficient of IFSs in medical diagnosis and clustering analysis. Augustine (2021) used the intuitionistic fuzzy correlation coefficient to multi-criteria decision-making (MCDM). Numerous other studies (Singh and Lalotra, 2018, 2019; Ganie et al., 2020; Singh et al., 2020; Ejegwa et al., 2023) have explored the use of correlation coefficients in different disciplines such as clustering, pattern recognition, medical diagnosis, and MAGDM, in frameworks like IFSs, hesitant fuzzy sets, hesitant fuzzy soft sets, and picture fuzzy sets. These works highlight the versatility of correlation measures in handling various types of fuzzy information.

As to the use of correlation measures in real-life scenarios, which is very extensive, this study deals with a supplier selection problem under the IVIF framework. Supplier selection is an important process in which a company chooses vendors who are trustworthy, affordable, and able to provide good-quality products, on-time delivery, and proper service. In this case, selection of the right supplier is crucial to build long-term and mutually beneficial relationships. A significant part of a company’s budget is spent on its suppliers. Therefore, choosing the appropriate supplier is important for reducing risks, getting good value, and meeting long-term goals (Taherdoost and Brard, 2019; Nezhad et al., 2024). But the selection process is not always straightforward because decision-makers often deal with unclear or incomplete information. To handle this uncertainty, many researchers use fuzzy set theory, which provides a practical way to describe and analyze vague or imprecise judgments (Naqvi and Amin, 2021; Demir, 2024).

1.1 Motivation and Contribution of the Study

Based on the literature review, we have identified the following issues:

- Most of the existing correlation coefficients that assume values in the interval $[-1, 1]$ yield negative values for dissimilar sets, but these values do not reflect a perfectly negative correlation, even when sets are dissimilar. (See Example 3.1)
- Existing non-parametric IVIFS correlation coefficients become indecisive in certain situations. However, a parametric version, by suitable tuning of the parameter, resolves this issue and also minimizes the error. (See Example 5.1)
- Although correlation coefficients are commonly applied across diverse fields, their use in multi-criteria group decision-making (MCGDM) for IVIFSs remains largely unexplored.

These limitations serve as the primary motivation for developing a novel, parametric correlation coefficient for IVIFSs with a range of $[-1,1]$. The proposed correlation coefficient is able to distinguish positive as well as negative correlations more efficiently; thus, it is a significant point of the paper that the existing methods do not reveal this feature. Their usage in decision-making problems is hence extended. To highlight the necessity and impact of the proposed method, it is essential to check how existing measures perform with respect to the criteria that characterize the robustness of correlation coefficients for IVIFSs.

The following points provide a logical framework for analyzing the performance of existing measures:
P1: The correlation coefficient is a good measure of negative relationships between IVIFSs.
P2: The correlation coefficient is 1 if and only if the two IVIFSs are identical.
P3: The correlation coefficient between any two IVIFSs is -1 if and only if the two IVIFSs are a perfect complement of each other.
P4: The correlation coefficient is able to effectively recognize structured linguistic variables.

Table 1 summarizes how existing methods satisfy (or fail to satisfy) these properties and highlights their strengths and limitations.

Table 1. Analysis of limitations across existing IVIF correlation coefficients.

IVIF correlation coefficient	P1	P2	P3	P4
$\mathbb{K}_{BB}$ (Bustince and Burillo, 1995)	×	✓	×	×
$\mathbb{K}_{HO}$ (Hong, 1998)	×	✓	×	×
$\mathbb{K}_{Hu}$ (Hung, 2001)	✓	✓	✓	×
$\mathbb{K}_{Xu}$ (Xu, 2006)	×	×	×	×
$\mathbb{K}_{PA}$ (Park et al., 2009a)	×	✓	×	×
$\mathbb{K}_{WE}$ (Wei et al., 2011b)	×	✓	×	×
$\mathbb{K}_{Pa}$ (Park et al., 2009b)	✓	✓	×	✓
$\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	×	✓	×	×
$\mathbb{K}_{LI}$ (Liu et al., 2015)	×	✓	×	×
$\mathbb{K}_{TH}$ (Thao, 2018)	✓	✓	×	×
$\mathbb{K}_{TMF}$ (Thao et al., 2019)	✓	✓	×	×

As there is a need for a novel correlation coefficient to overcome these limitations, it is also important to justify why a generalized formulation is required.

***What is the necessity of a generalized correlation coefficient*?**

Most of the existing correlation measures do not include any adjustable parameters. Because of this, they cannot adapt well to different types of data or varying decision-making situations. Introducing a generalized correlation coefficient with a parameter α helps to overcome this rigidity. The parameter allows the measure to adjust its sensitivity depending on the nature of the IVIF information. This flexibility enables the coefficient to detect both strong and weak relationships more accurately. Thus, making it applicable practically in all types of fuzzy decision-making problems.

The novel contribution of this paper is as follows:

- We propose a new generalized (parametric) correlation coefficient for IVIFSs. It helps to explore greater flexibility for assessing correlations within IVIF data.
- The proposed method has been applied to an MCGDM problem.
- A comparative analysis has been made to buttress the credibility of the proposed method.

The content of this paper is organized as follows: Section 2 lays out the basic ideas. Section 3 presents a newly generalized correlation coefficient for IVIFSs. In Section 4, the focus is on illustrating the application of the proposed correlation coefficient in MCGDM. This section also presents a comparative evaluation to emphasize the benefits of the proposed approach. Section 5 presents a comparative analysis based on structured linguistic variables supported by sensitivity analysis of parameter α and the measure of error. Finally, Sections 6 and 7, respectively, provide the discussion and conclusion of the study by summarizing the key findings and recommendations for future research directions. The framework of the study is illustrated in **Figure 1**.

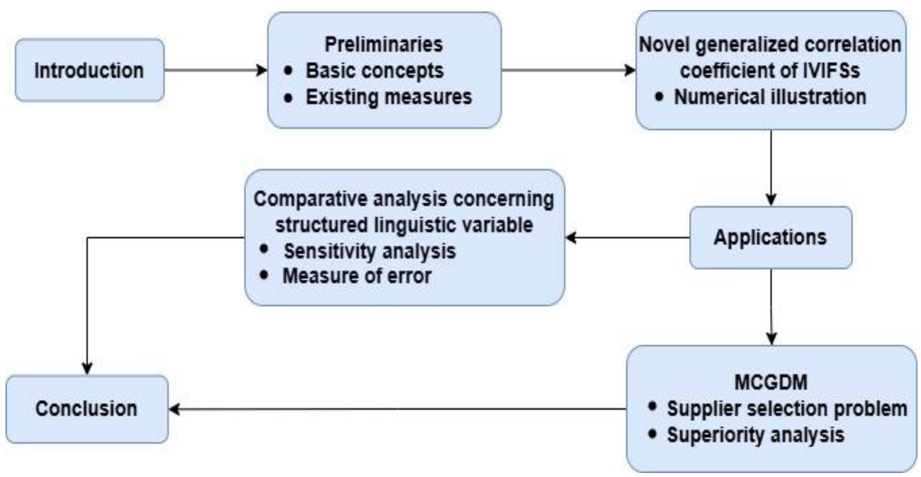

Figure 1. Framework of the study.

2. Preliminaries

This section recalls fundamental concepts and revisits significant existing correlation coefficients. These concepts form the basis for the methods discussed in this paper and support the comparative analysis carried out in the later sections.

2.1 Some Basic Concepts of IVIFSs

Definition 1: (Atanassov, 1986) Let $U = \{x_1, x_2, \dots, x_n\}$ be a universal set. Then an intuitionistic fuzzy set X in U is defined as

$$X = \{(x_i, \mu_X(x_i), \nu_X(x_i)) \mid x_i \in U, i = 1,2,3, \dots, n\}$$

where, $\mu_X: U \to [0,1]$ and $\nu_X: U \to [0,1]$ are called the membership and non-membership functions for X, respectively, and satisfy the condition

$$0 \le \mu_X(x_i) + \nu_X(x_i) \le 1.$$

In addition, $\pi_X(x_i) = 1 - \mu_X(x_i) - \nu_X(x_i)$ is called the hesitancy degree of x_i to $\forall x_i \in U$.

Definition 2: (Atanassov and Gargov, 1989) Let $U = \{x_1, x_2, \dots, x_n\}$ be a universal set. Then an interval-valued intuitionistic fuzzy set X in U is defined as

$$X = \{(x_i, \mu_X(x_i), \nu_X(x_i)): x_i \in U\}.$$

where, $\mu_X(x_i) = [\mu_X^l(x_i), \mu_X^u(x_i)]$ and $\nu_X(x_i) = [\nu_X^l(x_i), \nu_X^u(x_i)]$ are interval membership and non-membership of x_i in X, respectively. Furthermore, $\mu_X: U \to [0,1]$ and $\nu_X: U \to [0,1]$ are membership and non-membership functions, respectively, which satisfy the conditions $0 \le \mu_X^l(x_i) + \nu_X^l(x_i) \le 1$ and $0 \le \mu_X^u(x_i) + \nu_X^u(x_i) \le 1$.

In addition, $[\pi_X^l(x_i), \pi_X^u(x_i)] = [1 - \mu_X^u(x_i) - \nu_X^u(x_i), 1 - \mu_X^l(x_i) - \nu_X^l(x_i)]$ is called the interval-valued intuitionistic index of x_i in X, which is also called the hesitancy degree of x_i to X.

Definition 3: (Bustince and Burillo, 1995) Let $U = \{x_1, x_2, \dots, x_n\}$ be a universal set such that the cardinality of U is $n < \infty$. Then for each IVIFS $X = \{(x, [\mu_X^l(x), \mu_X^u(x)], [\nu_X^l(x), \nu_X^u(x)]): x \in U\}$, an informational energy of an IVIFS is defined as

$$E_{IVIFS}(X) = \sum_{i=1}^{n} \frac{{\mu_X^l}^2(x_i) + {\mu_X^u}^2(x_i) + {\nu_X^l}^2(x_i) + {\nu_X^u}^2(x_i)}{2} \tag{1}$$

and $E_{IVIFS}(X)$ satisfies the following properties:
E_1: $E_{IVIFS}(X) = 0$ if and only if $\mu_X(x) = \nu_X(x) = 0$ for all $x \in U$,
E_2: $E_{IVIFS}(X) = E_{IVIFS}(X_C)$ for all X in IVIFSs,
E_3: $E_{IVIFS}(X) \le n$ for all X in IVIFSs,
E_4: If $X \le Y$, then $E_{IVIFS}(X) \le E_{IVIFS}(Y)$.

Definition 4: (Bustince and Burillo, 1995) Let $U = \{x_1, x_2, \dots, x_n\}$ be a universal set. Let X and Y be two IVIFSs in U, then the correlation of IVIFSs is defined as

$$C_{IVIFS}(X, Y) = \frac{1}{2} \sum_{i=1}^{n} \left(\mu_X^l(x_i)\mu_Y^l(x_i) + \mu_X^u(x_i)\mu_Y^u(x_i) + \nu_X^l(x_i)\nu_Y^l(x_i) + \nu_X^u(x_i)\nu_Y^u(x_i) \right) \tag{2}$$

and $C_{IVIFS}(X, Y)$ satisfies the following properties:
C_1: $C_{IVIFS}(X, X) = E_{IVIFS}(X)$,
C_2: $C_{IVIFS}(X, Y) = C_{IVIFS}(Y, X)$.

Definition 5: (Bustince and Burillo, 1995) Let $U = \{x_1, x_2, \dots, x_n\}$ be a universal set. Let X and Y be two IVIFSs in U, then the correlation coefficient of IVIFSs is defined as

$$\mathbb{K}_{IVIFS}(X, Y) = \frac{C_{IVIFS}(X, Y)}{\sqrt{E_{IVIFS}(X) E_{IVIFS}(Y)}} \tag{3}$$

and $\mathbb{K}_{IVIFS}(X, Y)$ satisfies the following properties:
CC_1: If $X = Y$, then $\mathbb{K}_{IVIFS}(X, Y) = 1$,
CC_2: $\mathbb{K}_{IVIFS}(X, Y) = \mathbb{K}_{IVIFS}(Y, X)$,

$CC_3: 0 \leq \mathbb{K}_{IVIFS}(X,Y) \leq 1$.

Definition 6: Let $IVIFS(U) = \{X_1, X_2, \dots, X_p\}$ be a collection of IVIFSs. Then generalized correlation efficiency of any IVIFSs X_k $(k = 1,2,\dots,p)$ is defined as

$$C_{IVIFS}^{\alpha}(X_k) = \frac{\sum_{l=1}^{p} \mathbb{K}_{IVIFS}^{\alpha}(X_k, X_l)}{p-1}, k \neq l, l = 1,2,\dots,p; \alpha \in \mathbb{R} \quad (4)$$

Definition 7: Let $IVIFS(U) = \{X_1, X_2, \dots, X_p\}$ be a collection of IVIFSs. Then normalized correlation efficiency of any IVIFSs X_k $(k = 1,2,\dots,p)$ is defined as

$$\mathbb{N}_{IVIFS}(X_k) = \frac{C_{IVIFS}^{\alpha}(X_k)}{\sum_{l=1}^{p} C_{IVIFS}^{\alpha}(X_l)}, \alpha \in \mathbb{R} \quad (5)$$

such that $\sum_{k=1}^{p} \mathbb{N}_{IVIFS}(X_k) = 1$.

2.2 Existing Correlation Coefficient

This section provides an overview of the correlation coefficients for IVIFSs introduced by various researchers.
Let X and Y be two IVIFSs in $U = \{x_1, x_2, \dots, x_n\}$.

(i) $$\mathbb{K}_{BB}(X,Y) = \frac{C_{IVIFS}(X,Y)}{\sqrt{E_{IVIFS}(X)E_{IVIFS}(Y)}} = \frac{\frac{1}{2}\sum_{i=1}^{n}\left(\mu_X^l(x_i)\mu_Y^l(x_i)+\mu_X^u(x_i)\mu_Y^u(x_i)+\nu_X^l(x_i)\nu_Y^l(x_i)+\nu_X^u(x_i)\nu_Y^u(x_i)\right)}{\sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\nu_X^l}^2(x_i)+{\nu_X^u}^2(x_i)\right)\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_Y^l}^2(x_i)+{\mu_Y^u}^2(x_i)+{\nu_Y^l}^2(x_i)+{\nu_Y^u}^2(x_i)\right)}}$$
(Bustince and Burillo, 1995).

(ii) $$\mathbb{K}_{HO}(X,Y) = \frac{\frac{1}{2}\int\left(\mu_X^l(x_i)\mu_Y^l(x_i)+\mu_X^u(x_i)\mu_Y^u(x_i)+\nu_X^l(x_i)\nu_Y^l(x_i)+\nu_X^u(x_i)\nu_Y^u(x_i)\right)dP}{\sqrt{\left(\frac{1}{2}\int\left({\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\nu_X^l}^2(x_i)+{\nu_X^u}^2(x_i)\right)dP\left(\frac{1}{2}\int\left({\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\nu_X^l}^2(x_i)+{\nu_X^u}^2(x_i)\right)dP\right)\right)}}$$ (Hong, 1998).

where, P is the probability.

(iii) $\mathbb{K}_{Hu}(X,Y) = \frac{1}{2}((\mathbb{K}_{Hu})_1 + (\mathbb{K}_{Hu})_2)$ (Hung, 2001).
where, $(\mathbb{K}_{Hu})_1 = \frac{\sum_{i=1}^{n}(m_X-\bar{m}_X)(m_Y-\bar{m}_Y)}{\sqrt{\sum_{i=1}^{n}(m_X-\bar{m}_X)^2\sum_{i=1}^{n}(m_Y-\bar{m}_Y)^2}}$ and $(\mathbb{K}_{Hu})_2 = \frac{\sum_{i=1}^{n}(\eta_X-\bar{\eta}_X)(\eta_Y-\bar{\eta}_Y)}{\sqrt{\sum_{i=1}^{n}(\eta_X-\bar{\eta}_X)^2\sum_{i=1}^{n}(\eta_Y-\bar{\eta}_Y)^2}}$.

Here, m_X, m_Y and η_X,η_Y are middle points of membership and non-membership intervals, respectively and $\bar{m}_X, \bar{m}_Y$ and $\bar{\eta}_X, \bar{\eta}_Y$ are average membership and non-membership respectively.

(iv) $\mathbb{K}_{Xu}(X,Y) = \frac{1}{4n}\sum_{i=1}^{n}\left[\frac{\Delta\mu_{min}^l+\Delta\mu_{max}^l}{\Delta\mu_i^l+\Delta\mu_{max}^l} + \frac{\Delta\mu_{min}^u+\Delta\mu_{max}^u}{\Delta\mu_i^u+\Delta\mu_{max}^u} + \frac{\Delta\upsilon_{min}^l+\Delta\upsilon_{max}^l}{\Delta\upsilon_i^l+\Delta\upsilon_{max}^l} + \frac{\Delta\upsilon_{min}^u+\Delta\upsilon_{max}^u}{\Delta\upsilon_i^u+\Delta\upsilon_{max}^u}\right]$ (Xu, 2006).

where, $\Delta\mu_i^l = \left|\mu_X^l(x_i) - \mu_Y^l(x_i)\right|, \Delta\upsilon_i^l = \left|\upsilon_X^l(x_i) - \upsilon_Y^l(x_i)\right|,\ \Delta\mu_i^u = |\mu_X^u(x_i) - \mu_Y^u(x_i)|$,
$\Delta\upsilon_i^u = |\upsilon_X^u(x_i) - \upsilon_Y^u(x_i)|, \Delta\mu_{min}^l = \min_i\left\{\left|\mu_X^l(x_i) - \mu_Y^l(x_i)\right|\right\}, \Delta\mu_{min}^u = \min_i\{|\mu_X^u(x_i) - \mu_Y^u(x_i)|\}$,

$\Delta\mu_{max}^{l} = \max_i\{|\mu_X^l(x_i) - \mu_Y^l(x_i)|\}, \Delta\mu_{max}^{u} = \max_i\{|\mu_X^u(x_i) - \mu_Y^u(x_i)|\}$,

$\Delta\nu_{min}^{l} = \min_i\{|\upsilon_X^l(x_i) - \upsilon_Y^l(x_i)|\}, \Delta\nu_{min}^{u} = \min_i\{|\upsilon_X^u(x_i) - \upsilon_Y^u(x_i)|\}$,

$\Delta\nu_{max}^{l} = \max_i\{|\nu_X^l(x_i) - \nu_Y^l(x_i)|\}, \ \Delta\nu_{max}^{u} = \max_i\{|\nu_X^u(x_i) - \nu_Y^u(x_i)|\}$.

(v) $\mathbb{K}_{PA}(X,Y) = \frac{\frac{1}{2}\sum_{i=1}^{n}\left(\mu_X^l(x_i)\mu_Y^l(x_i)+\mu_X^u(x_i)\mu_Y^u(x_i)+\nu_X^l(x_i)\nu_Y^l(x_i)+\nu_X^u(x_i)\nu_Y^u(x_i)+\pi_X^l(x_i)\pi_Y^l(x_i)+\pi_X^u(x_i)\pi_Y^u(x_i)\right)}{\sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\upsilon_X^l}^2(x_i)+{\upsilon_X^u}^2(x_i)+{\pi_X^l}^2(x_i)+{\pi_X^u}^2(x_i)\right)\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_Y^l}^2(x_i)+{\mu_Y^u}^2(x_i)+{\upsilon_Y^l}^2(x_i)+{\upsilon_Y^u}^2(x_i)+{\pi_Y^l}^2(x_i)+{\pi_Y^u}^2(x_i)\right)}}$

(Park et al., 2009a).

(vi) $\mathbb{K}_{WE}(X,Y) = \frac{\frac{1}{2}\sum_{i=1}^{n}\left(\mu_X^l(x_i)\mu_Y^l(x_i)+\mu_X^u(x_i)\mu_Y^u(x_i)+\nu_X^l(x_i)\nu_Y^l(x_i)+\nu_X^u(x_i)\nu_Y^u(x_i)\right)}{\sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\upsilon_X^l}^2(x_i)+{\upsilon_X^u}^2(x_i)\right)\left(\frac{1}{2}\sum_{i=1}^{n}{\mu_Y^l}^2(x_i)+{\mu_Y^u}^2(x_i)+{\upsilon_Y^l}^2(x_i)+{\upsilon_Y^u}^2(x_i)\right)}}$ (Wei et al., 2011b).

(vii) $\mathbb{K}_{Pa}(X,Y) = \frac{1}{3}((\mathbb{K}_{Pa})_1 + (\mathbb{K}_{Pa})_2 + (\mathbb{K}_{Pa})_3)$ (Park et al., 2009b).

where,

$(\mathbb{K}_{Pa})_1 = \frac{\sum_{i=1}^{n}(m_X(x_i)-\overline{m}_X)(m_Y(x_i)-\overline{m}_Y)}{\sqrt{\sum_{i=1}^{n}(m_X(x_i)-\overline{m}_X)^2\sum_{i=1}^{n}(m_Y(x_i)-\overline{m}_Y)^2}}$, $(\mathbb{K}_{Pa})_2 = \frac{\sum_{i=1}^{n}(\eta_X(x_i)-\overline{\eta}_X)(\eta_Y(x_i)-\overline{\eta}_Y)}{\sqrt{\sum_{i=1}^{n}(\eta_X(x_i)-\overline{\eta}_X)^2\sum_{i=1}^{n}(\eta_Y(x_i)-\overline{\eta}_Y)^2}}$ and

$(\mathbb{K}_{Pa})_3 = \frac{\sum_{i=1}^{n}(\pi_X(x_i)-\overline{\pi}_X)(\pi_Y(x_i)-\overline{\pi}_Y)}{\sqrt{\sum_{i=1}^{n}(\pi_X(x_i)-\overline{\pi}_X)^2\sum_{i=1}^{n}(\pi_Y(x_i)-\overline{\pi}_Y)^2}}$.

Here m_X, m_Y, η_X, η_Y and π_X, π_Y are middle points of membership, non-membership and hesitation intervals, respectively and $\overline{m}_X, \overline{m}_Y$ and $\overline{\eta}_X, \overline{\eta}_Y$ are average membership and non-membership.

(viii) $\mathbb{K}_{ZW}(X,Y) = \frac{\frac{1}{2n}\sum_{i=1}^{n}\left(\mu_X^l(x_i)\mu_Y^l(x_i)+\mu_X^u(x_i)\mu_Y^u(x_i)+\nu_X^l(x_i)\nu_Y^l(x_i)+\nu_X^u(x_i)\nu_Y^u(x_i)+\pi_X^l(x_i)\pi_Y^l(x_i)+\pi_X^u(x_i)\pi_Y^u(x_i)\right)}{\sqrt{\left(\frac{1}{2n}\sum_{i=1}^{n}{\mu_X^l}^2(x_i)+{\mu_X^u}^2(x_i)+{\upsilon_X^l}^2(x_i)+{\upsilon_X^u}^2(x_i)+{\pi_X^l}^2(x_i)+{\pi_X^u}^2(x_i)\right)\left(\frac{1}{2n}\sum_{i=1}^{n}{\mu_Y^l}^2(x_i)+{\mu_Y^u}^2(x_i)+{\upsilon_Y^l}^2(x_i)+{\upsilon_Y^u}^2(x_i)+{\pi_Y^l}^2(x_i)+{\pi_Y^u}^2(x_i)\right)}}$

(Zeng and Wang, 2011).

(ix) $\mathbb{K}_{LI}(X,Y) = \frac{\sum_{i=1}^{n}d(X_i,\bar{X})d(Y_i,\bar{Y})}{\sqrt{\sum_{i=1}^{n}\left(d(X_i,\bar{X})\right)^2\sum_{i=1}^{n}\left(d(Y_i,\bar{Y})\right)^2}}$ (Liu et al., 2015).

where, $d(X,Y) = \frac{\mu_X^l-\mu_X^u+\upsilon_X^l-\upsilon_X^u}{2} - \frac{\mu_Y^l-\mu_Y^u+\upsilon_Y^l-\upsilon_Y^u}{2}$,

$\bar{X} = E(X) = (\bar{\mu}_X, \bar{\nu}_X) = \left(\frac{1}{n}\sum_{i=1}^{n}\mu_X(x_i), \frac{1}{n}\sum_{i=1}^{n}\nu_X(x_i)\right)$. Similarly, for Y and $\bar{Y}$.

(x) $\mathbb{K}_{TH}(X,Y) = \mathbb{K}_{TH}(X,Y) = \frac{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_X(x_i)-\bar{\mu}_X)(\mu_Y(x_i)-\bar{\mu}_Y)+(\upsilon_X(x_i)-\bar{\upsilon}_X)(\upsilon_Y-\bar{\upsilon}_Y)+d_i(X)d_i(Y)\right)}{\sqrt{\left\{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_X(x_i)-\bar{\mu}_X)^2+(\upsilon_X(x_i)-\bar{\upsilon}_X)^2+d_i^2(X)\right)\right\}\left\{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_Y(x_i)-\bar{\mu}_Y)^2+(\upsilon_Y(x_i)-\bar{\upsilon}_Y)^2+d_i^2(Y)\right)\right\}}}$

where, $d_i(X) = (\mu_X(x_i) - \bar{\mu}_X) - (\upsilon_X - \bar{\upsilon}_X)$, $d_i(Y) = (\mu_Y(x_i) - \bar{\mu}_Y) - (\upsilon_Y(x_i) - \bar{\upsilon}_Y)$,

$\bar{\mu}_X = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\mu_X^l(x_i) + \frac{1}{n}\sum_{i=1}^{n}\mu_X^u(x_i)\right)$, $\bar{\upsilon}_X = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\upsilon_X^l(x_i) + \frac{1}{n}\sum_{i=1}^{n}\upsilon_X^u(x_i)\right)$.

Similarly, for $\bar{\mu}_Y$ and $\bar{\upsilon}_Y$ (Thao, 2018).

(xi) $\mathbb{K}_{TMF}(X,Y) = \frac{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_X(x_i)-\bar{\mu}_X)(\mu_Y(x_i)-\bar{\mu}_Y)+(\upsilon_X(x_i)-\bar{\upsilon}_X)(\upsilon_Y(x_i)-\bar{\upsilon}_Y)\right)}{\sqrt{\left\{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_X(x_i)-\bar{\mu}_X)^2+(\upsilon_X(x_i)-\bar{\upsilon}_X)^2\right)\right\}\left\{\frac{1}{n-1}\sum_{i=1}^{n}\left((\mu_Y(x_i)-\bar{\mu}_Y)^2+(\upsilon_Y(x_i)-\bar{\upsilon}_Y)^2\right)\right\}}}$

where, $\bar{\mu}_X = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\mu_X^l(x_i) + \frac{1}{n}\sum_{i=1}^{n}\mu_X^u(x_i)\right)$, $\bar{\upsilon}_X = \frac{1}{2}\left(\frac{1}{n}\sum_{i=1}^{n}\upsilon_X^l(x_i) + \frac{1}{n}\sum_{i=1}^{n}\upsilon_X^u(x_i)\right)$.

Similarly, for $\bar{\mu}_Y$ and $\bar{\upsilon}_Y$ (Thao et al., 2019).

The aforementioned correlation coefficients have contributed to extending traditional correlation ideas into the IVIF setting. However, many of these measures still fall short of consistently meeting the key requirements of an ideal correlation coefficient.

Four fundamental properties (P1–P4) outlined in the introduction were used to assess the new correlation coefficient's effectiveness. These properties define the essential expectations for an effective IVIF correlation coefficient. A comparative summary of existing approaches with respect to these properties is presented in **Table 1**. The analysis shows that most of the existing formulations are unable to characterize negative dependencies, identify complementary IVIFSs, and interpreting the structured linguistic variables properly.

These limitations highlight the need for an improved formulation. The next section introduces a new correlation coefficient designed to overcome these limitations and provide a more stable solution for IVIF situations.

3. Generalized Correlation Coefficient for IVIFSs

This section introduces a new generalized correlation coefficient for IVIFSs, removing certain pitfalls of the existing methods.

Let X and Y be two IVIFSs in $U = \{x_1, x_2, \dots, x_n\}$, then we define the following correlation coefficient between X and Y.

$\mathbb{K}^{\alpha}(X,Y) = \frac{C_{IVIFS}(X,Y)}{\max\{E_{IVIFS}(X), E_{IVIFS}(Y)\}}$

$$= \frac{\frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{c}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\frac{\alpha}{2}}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\end{array}\right]}{max\left\{\begin{array}{c}\left(\frac{1}{2}\sum_{i=1}^{n}\left(\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\alpha}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\alpha}+\left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\alpha}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\alpha}\right)\right),\\ \left(\frac{1}{2}\sum_{i=1}^{n}\left(\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\alpha}+\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\alpha}+\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\alpha}+\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\right)\right)\end{array}\right\}} \tag{6}$$

where, $\alpha > 0$ is a real number, $\bar{\mu}_X^l(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_X^l(x_i)$, $\bar{\mu}_X^u(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_X^u(x_i)$, $\bar{\nu}_X^l(x_i) = \frac{1}{n}\sum_{i=1}^{n}\nu_X^l(x_i)$, $\bar{\nu}_X^u(x_i) = \frac{1}{n}\sum_{i=1}^{n}\nu_X^u(x_i)$, $\bar{\mu}_Y^l(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_Y^l(x_i)$, $\bar{\mu}_Y^u(x_i) = \frac{1}{n}\sum_{i=1}^{n}\mu_Y^u(x_i)$, $\bar{\nu}_Y^l(x_i) = \frac{1}{n}\sum_{i=1}^{n}\nu_Y^l(x_i)$ and $\bar{\nu}_Y^u(x_i) = \frac{1}{n}$.

We now verify that the proposed correlation coefficient satisfies the key properties.

Theorem 3.1 Let X and Y are two IVIFSs. Then
1) If $X = Y$ then $\mathbb{K}^{\alpha}(X, Y) = 1$.
2) $\mathbb{K}^{\alpha}(X, Y) = \mathbb{K}^{\alpha}(Y, X)$.
3) $-1 \leq \mathbb{K}^{\alpha}(X, Y) \leq 1$.

***Proof*:**
1) It is trivial.

$$\mathbb{K}^{\alpha}(X, Y) = \frac{C_{IVIFS}(X,Y)}{\max\{E_{IVIFS}(X),E_{IVIFS}(Y)\}}$$

$$= \frac{\frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{c}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\frac{\alpha}{2}}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\end{array}\right]}{max\left\{\begin{array}{c}\left(\frac{1}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\alpha}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\alpha}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\alpha}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\alpha}\end{array}\right)\right),\\ \left(\frac{1}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\alpha}+\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\alpha}+\\ \left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\alpha}+\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\end{array}\right)\right)\end{array}\right\}}$$

$$= \frac{\frac{1}{2}\sum_{i=1}^{n}\left[\begin{array}{c}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\frac{\alpha}{2}}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\end{array}\right]}{max\left\{\begin{array}{c}\left(\frac{1}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\alpha}+\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\alpha}+\\ \left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\alpha}+\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\end{array}\right)\right),\\ \left(\frac{1}{2}\sum_{i=1}^{n}\left(\begin{array}{c}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\alpha}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\alpha}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\alpha}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\alpha}\end{array}\right)\right)\end{array}\right\}}$$

$= \mathbb{K}^{\alpha}(Y, X)$.

Thus, $\mathbb{K}^{\alpha}(X, Y) = \mathbb{K}^{\alpha}(Y, X)$.

3) Let $a_i = \left(\mu_X^l(x_i) - \bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}$,

$b_i = \left(\mu_Y^l(x_i) - \bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}}.$

Using the Cauchy–Schwarz inequality, we have

$|\sum_{i=1}^{n} a_i b_i| \leq \sqrt{\sum_{i=1}^{n} a_i^2}\sqrt{\sum_{i=1}^{n} b_i^2}.$

Applying this inequality to all terms, we get

$$\frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}\left(\mu_X^l(x_i) - \bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^l(x_i) - \bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}} + \\ \left(\mu_X^u(x_i) - \bar{\mu}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^u(x_i) - \bar{\mu}_Y^u(x_i)\right)^{\frac{\alpha}{2}} + \\ \left(\upsilon_X^l(x_i) - \bar{\upsilon}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^l(x_i) - \bar{\upsilon}_Y^l(x_i)\right)^{\frac{\alpha}{2}} + \\ \left(\upsilon_X^u(x_i) - \bar{\upsilon}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^u(x_i) - \bar{\upsilon}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\end{bmatrix} \leq \left\{\sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_X^l(x_i) - \bar{\mu}_X^l(x_i)\right)^{\alpha} + \\ \left(\mu_X^u(x_i) - \bar{\mu}_X^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^l(x_i) - \bar{\upsilon}_X^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^u(x_i) - \bar{\upsilon}_X^u(x_i)\right)^{\alpha}\end{pmatrix}\right)} \times \sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_Y^l(x_i) - \bar{\mu}_Y^l(x_i)\right)^{\alpha} + \\ \left(\mu_Y^u(x_i) - \bar{\mu}_Y^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^l(x_i) - \bar{\upsilon}_Y^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^u(x_i) - \bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\end{pmatrix}\right)}\right\}.$$

Now, $$\left\{\sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_X^l(x_i) - \bar{\mu}_X^l(x_i)\right)^{\alpha} + \\ \left(\mu_X^u(x_i) - \bar{\mu}_X^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^l(x_i) - \bar{\upsilon}_X^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^u(x_i) - \bar{\upsilon}_X^u(x_i)\right)^{\alpha}\end{pmatrix}\right)} \times \sqrt{\left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_Y^l(x_i) - \bar{\mu}_Y^l(x_i)\right)^{\alpha} + \\ \left(\mu_Y^u(x_i) - \bar{\mu}_Y^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^l(x_i) - \bar{\upsilon}_Y^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^u(x_i) - \bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\end{pmatrix}\right)}\right\} \leq max\left\{\begin{matrix}\left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_X^l(x_i) - \bar{\mu}_X^l(x_i)\right)^{\alpha} + \\ \left(\mu_X^u(x_i) - \bar{\mu}_X^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^l(x_i) - \bar{\upsilon}_X^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_X^u(x_i) - \bar{\upsilon}_X^u(x_i)\right)^{\alpha}\end{pmatrix}\right), \\ \left(\frac{1}{2}\sum_{i=1}^{n}\begin{pmatrix}\left(\mu_Y^l(x_i) - \bar{\mu}_Y^l(x_i)\right)^{\alpha} + \\ \left(\mu_Y^u(x_i) - \bar{\mu}_Y^u(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^l(x_i) - \bar{\upsilon}_Y^l(x_i)\right)^{\alpha} + \\ \left(\upsilon_Y^u(x_i) - \bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\end{pmatrix}\right)\end{matrix}\right\}.$$

Combining all these above results, we have

$$|\mathbb{K}^{\alpha}(X,Y)| == \frac{\frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\frac{\alpha}{2}}+\\ \left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\frac{\alpha}{2}}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\frac{\alpha}{2}}\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\frac{\alpha}{2}}\end{bmatrix}}{max\begin{Bmatrix}\left(\frac{1}{2}\sum_{i=1}^{n}\left(\left(\mu_X^l(x_i)-\bar{\mu}_X^l(x_i)\right)^{\alpha}+\left(\mu_X^u(x_i)-\bar{\mu}_X^u(x_i)\right)^{\alpha}+\left(\upsilon_X^l(x_i)-\bar{\upsilon}_X^l(x_i)\right)^{\alpha}+\left(\upsilon_X^u(x_i)-\bar{\upsilon}_X^u(x_i)\right)^{\alpha}\right)\right),\\ \left(\frac{1}{2}\sum_{i=1}^{n}\left(\left(\mu_Y^l(x_i)-\bar{\mu}_Y^l(x_i)\right)^{\alpha}+\left(\mu_Y^u(x_i)-\bar{\mu}_Y^u(x_i)\right)^{\alpha}+\left(\upsilon_Y^l(x_i)-\bar{\upsilon}_Y^l(x_i)\right)^{\alpha}+\left(\upsilon_Y^u(x_i)-\bar{\upsilon}_Y^u(x_i)\right)^{\alpha}\right)\right)\end{Bmatrix}} \leq 1.$$

Thus, $-1 \leq \mathbb{K}^{\alpha}(X,Y) \leq 1$.

3.1 Numerical Illustration

We provide an example with different IVIFSs to show how the proposed generalized correlation coefficient outperforms existing approaches.

Example 3.1 Suppose $\mathcal{A}$ and $\mathcal{B}$ are IVIFSs in $X = \{x_1, x_2, \dots, x_n\}$, where $\mathcal{A} = \left\{\langle\frac{\{[0.3,0.35],[0.6,0.65]\}}{x_1}\rangle, \langle\frac{\{[0.4,0.45],[0.5,0.55]\}}{x_2}\rangle, \langle\frac{\{[0.2,0.25],[0.7,0.75]\}}{x_3}\rangle\right\}$ and

$\mathcal{B} = \left\{\langle\frac{\{[0.7,0.75],[0.2,0.25]\}}{x_1}\rangle, \langle\frac{\{[0.6,0.65],[0.3,0.35]\}}{x_2}\rangle, \langle\frac{\{[0.8,0.85],[0.1,0.15]\}}{x_3}\rangle\right\}$.

Clearly, $\mathcal{A}$ and $\mathcal{B}$ is perfectly negative since $\mathcal{A} \neq \mathcal{B}$, $\mathcal{A} \nsubseteq \mathcal{B}$ and $\mathcal{B} \nsubseteq \mathcal{A}$.

Table 2. Correlation coefficient values between $\mathcal{A}$ and $\mathcal{B}$.

Methods	$\mathbb{K}_{BB}$ (Bustince and Burillo, 1995), $\mathbb{K}_{HO}$ (Hong, 1998), $\mathbb{K}_{WE}$ (Wei et al., 2011b)	$\mathbb{K}_{PA}$ (Park et al., 2009a), $\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	$\mathbb{K}_{Pa}$ (Park et al., 2009b)	$\mathbb{K}_{LI}$ (Liu et al., 2015)	$\mathbb{K}_{TH}$ (Thao, 2018)	$\mathbb{K}_{TMF}$ (Thao et al., 2019)	Proposed method ($\alpha \geq 2$)
Values	0.6629	0.6659	−0.9699	0	−0.3086	0.2177	−1

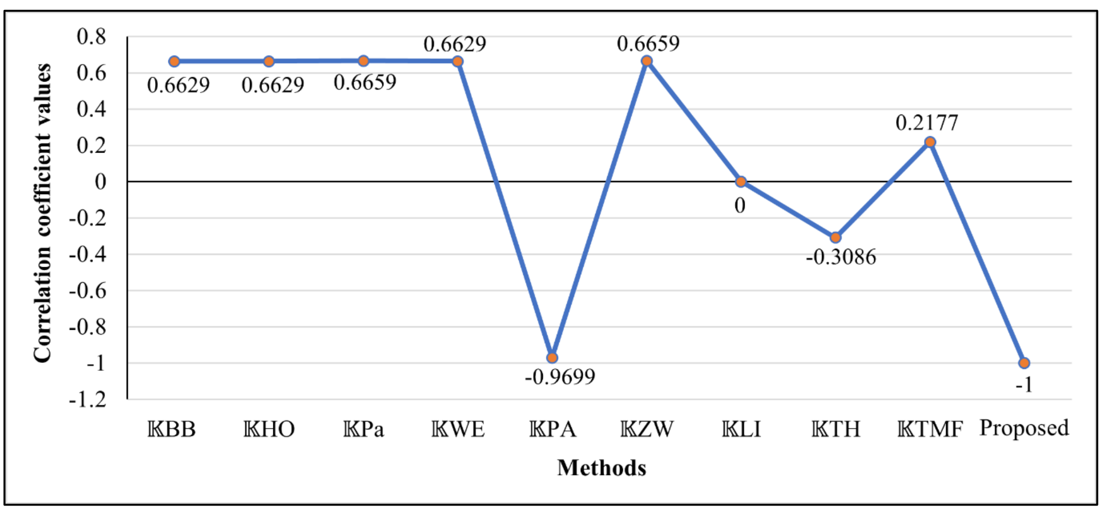

Figure 2. Graphical representation of correlation coefficient values for Example 3.1.

From **Table 2** and **Figure 2**, we see that the proposed method is the only one to give a correlation value of -1, indicating that $\mathcal{A}$ and $\mathcal{B}$ are perfectly negative correlated. This result aligns with their evident dissimilarity and shows that the proposed generalized correlation coefficient is capable of capturing negative dependence between IVIFSs. Hence, it can be concluded that the proposed method demonstrates greater reliability and accuracy than most of the existing methods. So, the proposed method established its superiority in evaluating correlations among IVIFSs.

The next section demonstrates the practical usefulness of the proposed coefficient by applying it to an MCGDM problem.

4. Application of the Proposed Generalized Correlation Coefficient in MCGDM

MCGDM is a widely used approach for addressing complex decision-making scenarios where several criteria must be considered at the same time and different decision-makers contribute their views. The proposed correlation coefficient helps in this process by measuring how criteria or alternatives relate to each other when the available data are given as IVIF values.

To formalize the MCGDM framework, let us define ϕ as the set of alternatives, represented as $\phi = \{\phi_1, \phi_2, \dots, \phi_m\}$ and ξ as the set of criteria, represented as $\xi = \{\xi_1, \xi_2, \dots, \xi_n\}$. Each criterion is associated with a weight ω_j, where, $0 \le \omega_j \le 1$ and $\sum_{j=1}^{n} \omega_j = 1$. Suppose $D = \{D_1, D_2, \dots, D_p\}$ represent the set of domain experts where each domain expert $D_k; k = 1,2, \dots, p$ provides their respecting rating. The flowchart for the MCGDM method is as shown in **Figure 3**.

Algorithm

Step 1: Formation of decision matrices

A group of decision-makers $D_k; k = 1,2, \dots, p$ provide their evaluations on the alternatives using IVIFSs $a_{ij}^k = \{[\mu_{ij}^l, \mu_{ij}^u], [\nu_{ij}^l, \nu_{ij}^u]\}$, $k = 1,2, \dots, p$; $i = 1,2, \dots, m$; $j = 1,2, \dots, n$ and each decision-maker assigns interval-valued intuitionistic fuzzy values (IVIFVs) to indicate the relative importance of each criterion with respect to the alternative values. Based on these values, p-decision matrices $(D_1, D_2, \dots, D_p)$ are constructed.

Step 2: Computation of the correlation coefficient of decision matrices

The correlation coefficient $\mathbb{K}_{IVIFS}^{\alpha}(D_k, D_l); k, l = 1,2, \dots, p$ as defined in Definition 5 is computed for each pair of IVIFVs. This correlation coefficient quantifies the degree of association between the evaluations provided by different decision-makers using their respective IVIFSs.

Step 3: Computation of correlation efficiency and normalized correlation efficiency

Using Definition 6 and Definition 7, correlation efficiency $C_{IVIFS}^{\alpha}(D_k)$ and normalized correlation efficiency $\mathbb{N}_{IVIFS}^{\alpha}(D_k)$ for each pair of IVIFVs D_k are calculated. The normalized correlation efficiency values are used as weight vector $\delta = \{\delta_1, \delta_2, \dots, \delta_p\}$, where each δ_k corresponds to the weight assigned to decision-maker D_k.

Step 4: Computation of the aggregated interval-valued intuitionistic fuzzy decision matrix

The aggregation of the fuzzy rating a_{ij} of alternatives ϕ_i with respect to criteria ξ_j is done using the symmetric interval-valued intuitionistic fuzzy weighted averaging (SIVIFWA) operator (Liao et al., 2014). This operator combines the individual ratings given by the decision-makers into one aggregated value while taking into consideration their weights. The aggregated value can be expressed as follows:

$$ℓ_{ij} = SIVIFWA\left(a_{ij}^{1}, a_{ij}^{2}, \dots, a_{ij}^{p}\right) =$$

$$\left\{\left[\frac{\prod_{k=1}^{p}\left(\left(\left(\mu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}{\prod_{k=1}^{p}\left(\left(\left(\mu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}+\prod_{k=1}^{p}\left(1-\left(\left(\mu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}, \frac{\prod_{k=1}^{p}\left(\left(\left(\mu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}{\prod_{k=1}^{p}\left(\left(\left(\mu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}+\prod_{k=1}^{p}\left(1-\left(\left(\mu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}\right],\right.$$

$$\left.\left[\frac{\prod_{k=1}^{p}\left(\left(\left(\nu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}{\prod_{k=1}^{p}\left(\left(\left(\nu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}+\prod_{k=1}^{p}\left(1-\left(\left(\nu_{ij}^{l}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}, \frac{\prod_{k=1}^{p}\left(\left(\left(\nu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}{\prod_{k=1}^{p}\left(\left(\left(\nu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}+\prod_{k=1}^{p}\left(1-\left(\left(\nu_{ij}^{u}\right)^{k}\right)^{\frac{\alpha}{2}}\right)^{\delta_k}}\right]\right\} \quad (7)$$

where, $i = 1,2, \dots, m; j = 1,2, \dots, n; k = 1,2, \dots, p.$

The aggregated interval-valued intuitionistic fuzzy decision matrix is

$$\mathcal{B} = \begin{bmatrix} ℓ_{11} & ℓ_{12} & \cdots & ℓ_{1n} \\ ℓ_{21} & ℓ_{22} & \dots & ℓ_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ ℓ_{m1} & ℓ_{m2} & \cdots & ℓ_{mn} \end{bmatrix},$$

where, $ℓ_{ij} = \left\{\left[\mu_{ij}'^{l}, \mu_{ij}'^{u}\right], \left[\nu_{ij}'^{l}, \nu_{ij}'^{u}\right]\right\}; i = 1,2, \dots, m; j = 1,2, \dots, n.$

Step 5: Determine objective weights for criteria using the entropy method

In order to find the objective weights of attributes, the entropy value $\mathcal{E}_j$ is calculated for each criterion using the data from the aggregated decision matrix $\mathcal{B}$. The entropy value $(\mathcal{E}_j)$ of matrix $\mathcal{B}$ can be computed as follows (Gao and Wei, 2012):

$$\mathcal{E}_j = \mathcal{E}(\xi_j) = \frac{\min\left\{\sum_{i=1}^{m}\left(2-\mu_{ij}'^{l}-\mu_{ij}'^{u}\right), \sum_{i=1}^{m}\left(2-\nu_{ij}'^{l}-\nu_{ij}'^{u}\right)\right\}}{\max\left\{\sum_{i=1}^{m}\left(2-\mu_{ij}'^{l}-\mu_{ij}'^{u}\right), \sum_{i=1}^{m}\left(2-\nu_{ij}'^{l}-\nu_{ij}'^{u}\right)\right\}} \quad (8)$$

After entropy is calculated, the degree of divergence $(\mathcal{DD})_j$ for each criterion is calculated using the following formula:

$$(\mathcal{D}_{\mathcal{D}})_j = 1 - \mathcal{E}_j \quad (9)$$

The final objective weight w_j for each criterion is calculated using the degree of divergence values $(\mathcal{DD})_j$ as follows:

$$w_j = \frac{(\mathcal{D}_{\mathcal{D}})_j}{\sum_{i=1}^{n}(\mathcal{D}_{\mathcal{D}})_j}, j = 1,2, \dots, n \quad (10)$$

These objective weights obtained from the entropy and divergence measures reflect the significance of each criterion in the decision-making process.

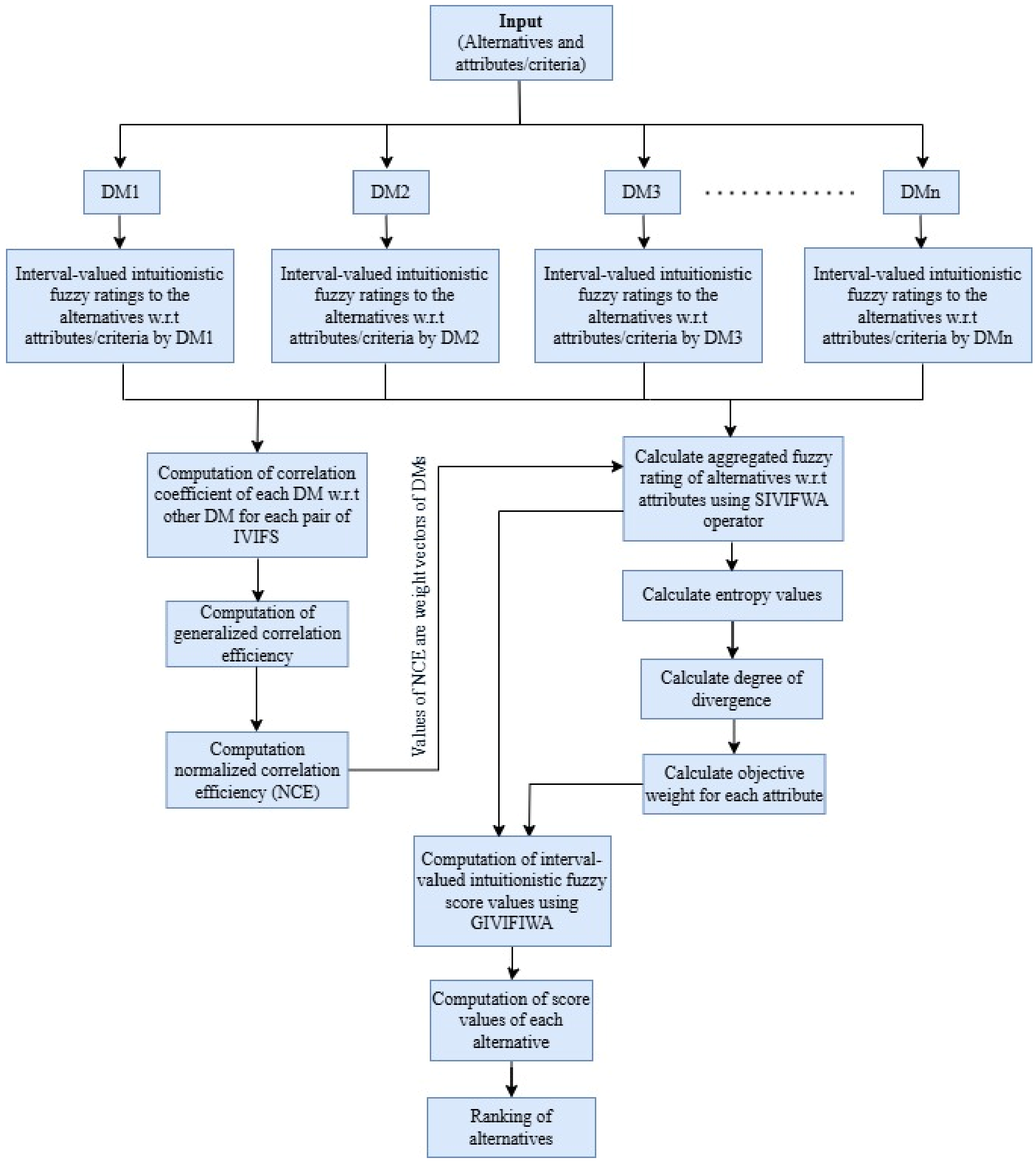

Figure 3. Flowchart of the proposed algorithm for solving the MCGDM problem.

Step 6: Construction of interval-valued intuitionistic fuzzy score values

Once the aggregated interval-valued intuitionistic fuzzy decision matrix has been constructed and the criterion weights are known, the interval-valued intuitionistic fuzzy score values corresponding to each alternative are calculated. These values are represented as

$$\mathcal{V}_i = \{[(\mu_i''^l), (\mu_i''^u)], [(\nu_i''^l), (\nu_i''^u)]\}.$$

for each alternative ϕ_i $(i = 1,2, \dots, m)$. To compute these values, the generalized interval-valued intuitionistic fuzzy interactive weighted averaging (GIVIFIWA) (Garg, 2016) operator is used. The interval-valued intuitionistic fuzzy score values are calculated as:

$$\mathcal{V}_i = GIVIFIWA(\mathscr{b}_{i1}, \mathscr{b}_{i2}, \dots, \mathscr{b}_{in}) = \{[(\mu_i''^l), (\mu_i''^u)], [(\nu_i''^l), (\nu_i''^u)]\} \quad (11)$$

where,

$$(\mu_i^{\prime\prime l}) = \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - (\nu_{ij}^{\prime l})\right)^{\lambda} + \left(1 - \left((\mu_{ij}^{\prime l}) + (\nu_{ij}^{\prime l})\right)\right)^{\lambda}\right)^{w_j} + \prod_{j=1}^{n}\left(1 - \left((\mu_{ij}^{\prime l}) + (\nu_{ij}^{\prime l})\right)\right)^{\lambda w_j}\right)^{1/\lambda} - \prod_{j=1}^{n}\left(1 - \left((\mu_{ij}^{\prime l}) + (\nu_{ij}^{\prime l})\right)^{w_j}\right),$$

$$(\mu_i^{\prime\prime u}) = \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - (\nu_{ij}^{\prime u})\right)^{\lambda} + \left(1 - \left((\mu_{ij}^{\prime u}) + (\nu_{ij}^{\prime u})_{ij}\right)\right)^{\lambda}\right)^{w_j} + \prod_{j=1}^{n}\left(1 - \left((\mu_{ij}^{\prime u}) + (\nu_{ij}^{\prime u})\right)\right)^{\lambda w_j}\right)^{1/\lambda} - \prod_{j=1}^{n}\left(1 - \left((\mu_{ij}^{\prime u}) + (\nu_{ij}^{\prime u})\right)^{w_j}\right),$$

$$(\nu_i^{\prime\prime l}) = 1 - \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - (\nu_{ij}^{\prime l})\right)^{\lambda} + \left(1 - \left((\mu_{ij}^{\prime l}) + (\nu_{ij}^{\prime l})\right)\right)^{\lambda}\right)^{w_j} + \prod_{j=1}^{n}\left(1 - (\mu_{ij}^{\prime l}) + (\nu_{ij}^{\prime l})\right)^{\lambda w_j}\right)^{1/\lambda}, \text{ and}$$

$$(\nu_i^{\prime\prime u}) = 1 - \left(1 - \prod_{j=1}^{n}\left(1 - \left(1 - (\nu_{ij}^{\prime u})\right)^{\lambda} + \left(1 - \left((\mu_{ij}^{\prime u}) + (\nu_{ij}^{\prime u})\right)\right)^{\lambda}\right)^{w_j} + \prod_{j=1}^{n}\left(1 - (\mu_{ij}^{\prime u}) + (\nu_{ij}^{\prime u})\right)^{\lambda w_j}\right)^{1/\lambda}.$$

where, $\lambda > 0$ is a real number.

Step 7: Computation of score values

The score values for each alternative ϕ_i $(i = 1,2,\ldots,m)$ are computed using the following formula (Garg, 2016):

$$\mathcal{S}_i = \frac{(\mu_i^{\prime\prime l}) + (\mu_i^{\prime\prime u}) + (\mu_i^{\prime\prime l})\left(1 - (\mu_i^{\prime\prime l}) - (\nu_i^{\prime\prime l})\right) + (\mu_i^{\prime\prime u})\left(1 - (\mu_i^{\prime\prime u}) - (\nu_i^{\prime\prime u})\right)}{2} \tag{12}$$

Step 8: Ranking the alternatives

After score values $\mathcal{S}_i$ for each alternative ϕ_i have been calculated, the alternatives are arranged in decreasing order of their score values. The alternative with the highest score is considered the most preferable alternative while those with lower scores follow in decreasing order of preference. The ranking of alternatives thus facilitates the selection of the optimal alternative that best matches the specified criteria and the decision-makers' evaluations.

4.1 Supplier Selection Problem

In order to demonstrate how the proposed method can be practically applied, we have taken a supplier selection problem to explain as our case study. A worldwide company is deciding which supplier will be best to supply their new manufacturing plant and has narrowed down its options to five potential suppliers. There are five suppliers for the company to choose from. Supplier A (ϕ_1), Supplier B (ϕ_2), Supplier C (ϕ_3), Supplier D (ϕ_4), and Supplier E (ϕ_5) are the five suppliers. The decision-makers will assess these suppliers based on four key criteria: Cost of acquisition (ξ_1), Proximity to existing supply chains and customers (ξ_2), Availability of skilled workforce (ξ_3), Environmental impact and Compliance with regulations (ξ_4).

A panel of three experts from different domains - finance (D_1), logistics (D_2), and environmental sustainability (D_3) will contribute to their assessments to ensure a thorough evaluation. Each expert will evaluate the suppliers using IVIFSs to reflect the uncertainty and fuzziness that are natural to the decision-making process.

The data used in this study is created artificially for experimental purposes. It allows for a controlled verification of the suggested method without the need for a real industry dataset. This ensures that the focus remains on demonstrating the methodology's applicability and robustness.

Figure 4 presents various elements of a supplier selection problem.

We address this problem through the proposed MCGDM approach (Step 1 to Step 8).

Step 1: Formation of decision matrices

The ratings given by three decision makers (DMs) ($D_k, k = 1,2,3$) for the five suppliers based on the specified criteria are shown in **Tables 3, 4** and **5**.

Table 3. The ratings assigned by decision maker 1 to the suppliers.

Alternatives /Criteria	ξ_1	ξ_2	ξ_3	ξ_4
ϕ_1	{[0.3,0.4], [0.2,0.3]}	{[0.4,0.5], [0.2,0.4]}	{[0.5,0.6], [0.1,0.2]}	{[0.2,0.3], [0.1,0.2]}
ϕ_2	{[0.4,0.6], [0.1,0.2]}	{[0.3,0.7], [0.1,0.2]}	{[0.3,0.6], [0.2,0.3]}	{[0.2,0.3], [0.1,0.2]}
ϕ_3	{[0.4,0.5], [0.3,0.4]}	{[0.2,0.6], [0.1,0.3]}	{[0.4,0.5], [0.3,0.4]}	{[0.2,0.6], [0.1,0.3]}
ϕ_4	{[0.6,0.8], [0.1,0.2]}	{[0.5,0.6], [0.1,0.2]}	{[0.4,0.6], [0.1,0.2]}	{[0.3,0.4], [0.2,0.3]}
ϕ_5	{[0.6,0.7], [0.2,0.3]}	{[0.2,0.6], [0.1,0.3]}	{[0.3,0.7], [0.1,0.2]}	{[0.4,0.8], [0.1,0.2]}

Table 4. The ratings assigned by decision maker 2 to the suppliers.

Alternatives /Criteria	ξ_1	ξ_2	ξ_3	ξ_4
ϕ_1	{[0.6,0.7], [0.2,0.3]}	{[0.2,0.6], [0.1,0.3]}	{[0.3,0.6], [0.2,0.3]}	{[0.3,0.4], [0.2,0.3]}
ϕ_2	{[0.3,0.4], [0.2,0.3]}	{[0.5,0.6], [0.1,0.2]}	{[0.2,0.6], [0.1,0.3]}	{[0.4,0.8], [0.1,0.2]}
ϕ_3	{[0.4,0.5], [0.3,0.4]}	{[0.4,0.5], [0.2,0.4]}	{[0.4,0.6], [0.1,0.2]}	{[0.5,0.6], [0.1,0.2]}
ϕ_4	{[0.4,0.6], [0.1,0.2]}	{[0.2,0.6], [0.1,0.3]}	{[0.5,0.6], [0.1,0.2]}	{[0.2,0.3], [0.1,0.2]}
ϕ_5	{[0.6,0.8], [0.1,0.2]}	{[0.3,0.7], [0.1,0.2]}	{[0.4,0.5], [0.3,0.4]}	{[0.2,0.3], [0.1,0.2]}

Table 5. The ratings assigned by decision maker 3 to the suppliers.

Alternatives /Criteria	ξ_1	ξ_2	ξ_3	ξ_4
ϕ_1	{[0.3,0.4], [0.2,0.3]}	{[0.6,0.8], [0.1,0.2]}	{[0.3,0.6], [0.2,0.3]}	{[0.2,0.3], [0.1,0.2]}
ϕ_2	{[0.5,0.8], [0.1,0.2]}	{[0.2,0.6], [0.1,0.3]}	{[0.4,0.8], [0.1,0.2]}	{[0.2,0.6], [0.1,0.3]}
ϕ_3	{[0.4,0.8], [0.1,0.2]}	{[0.2,0.3], [0.1,0.2]}	{[0.4,0.5], [0.1,0.4]}	{[0.6,0.7], [0.2,0.3]}
ϕ_4	{[0.2,0.6], [0.1,0.3]}	{[0.3,0.7], [0.1,0.2]}	{[0.5,0.6], [0.1,0.2]}	{[0.6,0.8], [0.1,0.2]}
ϕ_5	{[0.5,0.8], [0.1,0.2]}	{[0.6,0.7], [0.2,0.3]}	{[0.5,0.8], [0.1,0.2]}	{[0.3,0.6], [0.2,0.3]}

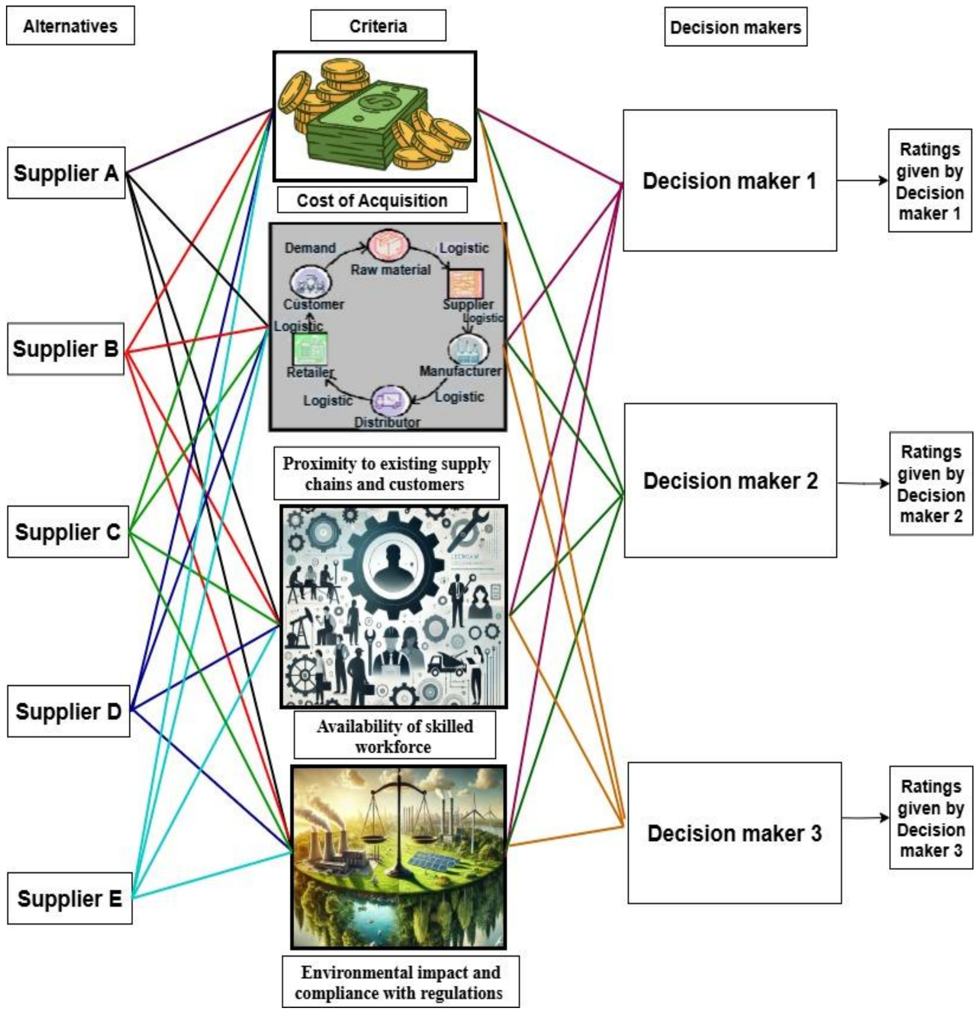

Figure 4. A schematic illustration of the framework addressing the supplier selection problem.

Step 2: Computation of correlation coefficients of decision matrices

Using Definition 5, the correlation coefficient $\mathbb{K}^{\alpha}_{IVIFS}(D_k, D_l); k, l = 1,2,3$ for each pair of IVIFVs is shown in **Table 6**.

Table 6. Correlation coefficients of decision matrices.

	DM_1	DM_2	DM_3
DM_1	1	0.0085	−0.3000
DM_2	0.0085	1	0.1695
DM_3	−0.3000	0.1695	1

Step 3: Computation of correlation efficiency and normalized correlation efficiency

Using Definition 6 and Definition 7, the correlation efficiency $C^{\alpha}_{IVIFS}(D_k)$ and normalized correlation efficiency $\mathbb{N}^{\alpha}_{IVIFS}(D_k)$ for each pair of IVIFVs D_k is as shown in **Table 7.**

Table 7. Generalized correlation efficiency and normalized correlation efficiency of IVIFVs.

	Generalized correlation efficiency	Normalized correlation efficiency
DM_1	−0.1458	1.1947
DM_2	0.0890	−0.7295
DM_3	−0.0653	0.5348

Remark: The normalized correlation efficiency of IVIFSs represents the degree of linear association between two IVIFSs. In contrast to standard correlation measures, this efficiency can take on negative values, signifying an inverse correlation between the sets. A negative value indicates that as one IVIFS increases, the other tends to decrease, and vice versa. This ability to express both directions of association is essential when the interaction between sets is not always positive, allowing the measure to represent a wider range of real-world relationships.

An important point to note is that the sum of the normalized correlation efficiency values for any pair of IVIFSs should always equal to 1. This property ensures that the total degree of correlation, whether positive or negative, is properly balanced, reflecting the complete relationship between the sets being analyzed.

Step 4: Computation of aggregated interval-valued intuitionistic fuzzy decision matrix
Using Equation (7), the aggregated interval-valued intuitionistic fuzzy decision matrix is obtained in **Table 8**.

Table 8. Aggregated interval-valued intuitionistic fuzzy decision matrix.

Alternatives /Criteria	ξ_1	ξ_2	ξ_3	ξ_4
ϕ_1	{[0.15,0.21], [0.20,0.30]}	{[0.68,0.61], [0.23,0.35]}	{[0.54,0.6], [0.09,0.18]}	{[0.14,0.24], [0.06,0.14]}
ϕ_2	{[0.53,0.82], [0.06,0.14]}	{[0.15,0.72], [0.10,0.25]}	{[0.45,0.72], [0.23,0.24]}	{[0.11,0.14], [0.10,0.25]}
ϕ_3	{[0.40,0.68], [0.17,0.28]}	{[0.11,0.51], [0.06,0.19]}	{[0.40,0.43], [0.36,0.58]}	{[0.19,0.66], [0.15,0.39]}
ϕ_4	{[0.51,0.83], [0.10,0.25]}	{[0.64,0.66], [0.10,0.14]}	{[0.38,0.60], [0.10,0.20]}	{[0.55,0.71], [0.23,0.32]}
ϕ_5	{[0.55,0.68], [0.23,0.32]}	{[0.31,0.58], [0.15,0.39]}	{[0.33,0.85], [0.04,0.11]}	{[0.52,0.92], [0.15,0.25]}

Step 5: Determine objective weights for attributes using the entropy method
Using the data from **Table 8**, compute the entropy values first using Equation (8). Next, calculate $(\mathcal{D}_\mathcal{D})_j$ and w_j by applying Equation (9) and Equation (10). The results of these calculations are presented in **Table 9**.

Table 9. Objective weights.

	ξ_1	ξ_2	ξ_3	ξ_4
$\mathcal{E}_j$	0.5851	0.6283	0.5977	0.7305
$(\mathcal{D}_\mathcal{D})_j$	0.4149	0.5640	0.4904	0.6409
w_j	0.2845	0.2549	0.2758	0.1848

Step 6: Construction of interval-valued intuitionistic fuzzy score values
Using Equation (11), the interval-valued intuitionistic fuzzy score values are obtained as

$$\mathcal{V}_1 = \{[0.4383, 0.6689], [0.2055, 0.3310]\},$$
$$\mathcal{V}_2 = \{[0.3548, 0.7552], [0.1354, 0.2448]\},$$
$$\mathcal{V}_3 = \{[0.2973, 0.5780], [0.2315, 0.4220]\},$$
$$\mathcal{V}_4 = \{[0.5236, 0.7245], [0.1326, 0.2755]\},$$
$$\mathcal{V}_5 = \{[0.4300, 0.7868], [0.1647, 0.2132]\}.$$

Step 7: Calculation of score values

The score values for each alternative ϕ_i $(i = 1,2, \dots ,5)$ are computed using Equation (12) and are given as

$$\mathcal{S}_1 = 0.6317,$$
$$\mathcal{S}_2 = 0.6455,$$
$$\mathcal{S}_3 = 0.5077,$$
$$\mathcal{S}_4 = 0.7141,$$
$$\mathcal{S}_5 = 0.6955.$$

Step 8: Ranking the Alternatives

After computing the score function $\mathcal{S}_i$ $(i = 1,2, \dots ,6)$ for each alternative ψ_i $(i = 1,2, \dots ,6)$, ranking is as follows:

$$\phi_4 > \phi_5 > \phi_2 > \phi_1 > \phi_3.$$

Thus, ϕ_4 is the best alternative. So, Supplier D is the best supplier for a new manufacturing plant.

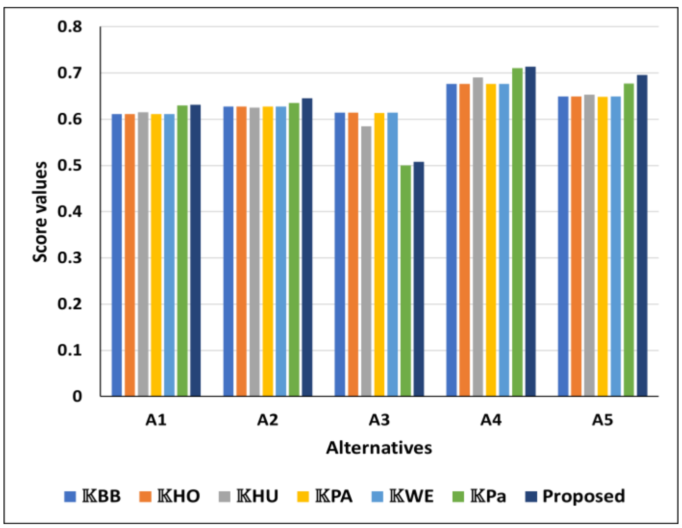

Figure 5. Comparison of score values obtained from different methods.

The score values obtained from the different methods are shown in **Figure 5**. A closer look at the figure shows that, for most of the alternatives, the proposed measure produces noticeably higher scores than the others. This pattern suggests that the proposed approach performs more reliably and gives stronger outcomes in comparison with the existing methods, demonstrating its overall advantage in a wide range of cases.

4.2 Superiority Analysis

To demonstrate the strength of the proposed methodology, a comparative evaluation is carried out using the concept of the degree of confidence (Luo and Zhang, 2024).

The degree of confidence is defined as

$$DoC = \sum_{i=1, i \neq i_0}^{n} \left| \mathcal{S}_i - \mathcal{S}_{i_0} \right| \tag{13}$$

where, $\mathcal{S}_{i_0}$ is the score of the best alternative.

An approach that shows a greater level of confidence is considered superior. It delivers more reliable and stable outcomes compared to the alternative approaches.

The comparative results for Example 4.1 are summarized in **Table 10** and shown in **Figure 6**.

Table 10. Comparison of the result rankings obtained through various methods.

Method	Ranking	Best alternative	Degree of confidence
$\mathbb{K}_{BB}$(Bustince and Burillo, 1995)	$\phi_4 > \phi_5 > \phi_2 > \phi_3 > \phi_1$	ϕ_4	0.2026
$\mathbb{K}_{HO}$ (Hong, 1998)	$\phi_4 > \phi_5 > \phi_2 > \phi_3 > \phi_1$	ϕ_4	0.2026
$\mathbb{K}_{Hu}$ (Hung, 2001)	$\phi_4 > \phi_5 > \phi_2 > \phi_1 > \phi_3$	ϕ_4	0.2838
$\mathbb{K}_{PA}$ (Park et al., 2009a)	$\phi_4 > \phi_5 > \phi_2 > \phi_3 > \phi_1$	ϕ_4	0.2048
$\mathbb{K}_{WE}$ (Wei et al., 2011b)	$\phi_4 > \phi_5 > \phi_2 > \phi_3 > \phi_1$	ϕ_4	0.2026
$\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	$\phi_4 > \phi_5 > \phi_2 > \phi_3 > \phi_1$	ϕ_4	0.2048
Proposed method	$\phi_4 > \phi_5 > \phi_2 > \phi_1 > \phi_3$	ϕ_4	0.3760

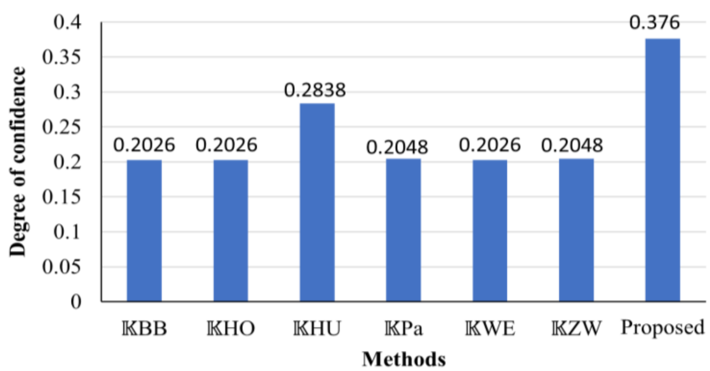

Figure 6. Comparison of degree of confidence obtained from different measures.

It is very clear from **Table 10** and **Figure 6** that the proposed method is the one that attained the greatest degree of confidence by most of the existing methods. Therefore, it indicates its comparative superiority and shows its potential to separate and evaluate different alternatives to a higher degree of accuracy and consistency.

5. Comparative Analysis Concerning Structured Linguistic Variables

Zadeh (1972) introduced the idea of the structured linguistic variables, which are generally called linguistic hedges. The idea has become a primary tool in finding the answer to countless problems in the real world. These linguistic variables make it possible to inculcate human-like reasoning into computational frameworks by capturing terms such as "very similar", "somewhat similar", or "not very similar". What they do is to provide a very efficient way for human beings to go from their subjective judgment to an objective numerical analysis, thus complex information becomes easily accessible.

Along with correlation coefficients, linguistic variables significantly influence both the clarity and the accuracy of relationship analysis. Correlation coefficients, that indicate how strong and in which direction the relationships between variables are, benefit from the addition of linguistic hedges by more accurately representing subtle human perceptions of similarity or dissimilarity.

Linguistic variables can be described with the help of modifiers as those defined in Equation (14) (Dymova and Sevastjanov, 2016). These modifiers represent varying degrees of magnitude or intensity for linguistic terms including High (H), Very High (VH), Very-Very High (VVH) and More or Less High (MLH). We can consider A as (H), $A^{\frac{1}{2}}$ as (MLH), A^2 as (VH) and A^4 as (VVH).

The generalized modifier is presented as follows:

$$A^{\lambda} = \left\{\left[\left(\mu_A^l(x_i)\right)^{\lambda}, \left(\mu_A^u(x_i)\right)^{\lambda}\right], \left[1-\left(1-\nu_A^l(x_i)\right)^{\lambda}, 1-\left(1-\nu_A^u(x_i)\right)^{\lambda}\right] \middle| x_i \in X, i = 1,2,3, \dots, n\right\} \quad (14)$$

where, $\lambda > 0$.

The linguistic variable (MLH) demonstrates greater similarity to the linguistic variable (H) compared to (VH) and (VVH) (Singh and Singh, 2025). Using mathematical language, this concept can be elaborated to formally define the expressions corresponding to inequalities (15) to (18), (19) to (22) and (23) to (26).

The intended hierarchy for the correlation measure is given by the set of inequalities (15) to (18).

$$\mathbb{K}(MLH, H) > \mathbb{K}(MLH, VH) > \mathbb{K}(MLH, VVH) \quad (15)$$

$$\mathbb{K}(H, MLH) > \mathbb{K}(H, VH) > \mathbb{K}(H, VVH) \quad (16)$$

$$\mathbb{K}(VH, H) > \mathbb{K}(VH, VVH) > \mathbb{K}(VH, MLH) \quad (17)$$

$$\mathbb{K}(VVH, VH) > \mathbb{K}(VVH, H) > \mathbb{K}(VVH, MLH) \quad (18)$$

The intended hierarchy for the similarity measure is given by the set of inequalities (19) to (22).

$$S(MLH, H) > S(MLH, VH) > S(MLH, VVH) \quad (19)$$

$$S(H, MLH) > S(H, VH) > S(H, VVH) \quad (20)$$

$$S(VH, H) > S(VH, VVH) > S(VH, MLH) \quad (21)$$

$$S(VVH, VH) > S(VVH, H) > S(VVH, MLH) \quad (22)$$

The set of inequalities (23) to (26) establishes the intended hierarchy for the distance/dissimilarity measure.

$$D(MLH, H) < D(MLH, VH) < D(MLH, VVH) \quad (23)$$

$$D(H, MLH) < D(H, VH) < D(H, VVH) \quad (24)$$

$$D(VH, H) < D(VH, VVH) < D(VH, MLH) \quad (25)$$

$$D(VVH, VH) < D(VVH, H) < D(VVH, MLH) \quad (26)$$

Now, a numerical example is presented to assess the performance and effectiveness of the proposed measures in comparison with the existing approaches.

Example 5.1 Consider an IVIFS A in the universal set $X = \{x_1, x_2, \dots, x_n\}$ given as:

$$A = \left\{\begin{array}{c} (x_1, \{[0.2,0.5], [0.1,0.3]\}), \\ (x_2, \{[0.3,0.6], [0.2,0.35]\}), \\ (x_3, \{[0.4,0.7], [0.15,0.2]\}) \end{array}\right\},$$

Using the fuzzy set modifier defined in Equation (14), we calculate the IVIFSs $A^{\frac{1}{2}}$, A^2 and A^4 as follows

$$A^{\frac{1}{2}} = \left\{\begin{array}{l} (x_1, \{[0.45,0.71], [0.05,0.16]\}), \\ (x_2, \{[0.55,0.81], [0.13,0.19]\}), \\ (x_3, \{[0.67,0.84], [0.08,0.13]\}) \end{array}\right\} \quad (27)$$

$$A^2 = \begin{Bmatrix} (x_1, \{[0.04,0.25],[0.19,0.51]\}), \\ (x_2, \{[0.09,0.42],[0.44,0.58]\}), \\ (x_3, \{[0.20,0.49],[0.28,0.44]\}) \end{Bmatrix} \tag{28}$$

$$A^4 = \begin{Bmatrix} \left(x_1, \begin{Bmatrix}[0.0016,0.0625], \\ [0.3439,0.7599]\end{Bmatrix}\right), \\ \left(x_2, \begin{Bmatrix}[0.0081,0.1785], \\ [0.6836,0.8215]\end{Bmatrix}\right), \\ \left(x_3, \begin{Bmatrix}[0.0410,0.2401], \\ [0.4780,0.6836]\end{Bmatrix}\right) \end{Bmatrix} \tag{29}$$

We apply both the proposed and existing measures to calculate the correlation/similarity/distance values between various pairs of linguistic variables H, VH, VVH and MLH. The results obtained using these measures are summarized in **Table 11**.

Table 11. Calculated values for pairs of linguistic variables utilizing different comparison methods.

Correlation/Similarity/Distance methods	Computed values between different pairs of linguistic variables			
$\mathbb{K}_{BB}$ (Bustince and Burillo, 1995), $\mathbb{K}_{HO}$ (Hong, 1998)	MLH	H	VH	VVH
MLH	1	0.9529	0.6788	0.3410
H	0.9529	1	0.8645	0.5871
VH	**0.6788**	**0.8645**	1	**0.9086**
VVH	0.3410	0.5871	0.9086	1
$\mathbb{K}_{Hu}$ (Hung, 2001)	MLH	H	VH	VVH
MLH	1	0.9999	0.9990	0.9941
H	0.9999	1	0.9996	0.9955
VH	**0.9990**	**0.9996**	1	**0.9979**
VVH	0.9941	0.9955	0.9979	1
$\mathbb{K}_{PA}$ (Park et al., 2009a)	MLH	H	VH	VVH
MLH	1	0.9356	0.7146	0.4482
H	0.9356	1	0.9093	0.6972
VH	**0.7146**	**0.9093**	1	**0.9172**
VVH	0.4482	0.6972	0.9172	1
$\mathbb{K}_{WE}$(Wei et al., 2011b), $\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	MLH	H	VH	VVH
MLH	1	0.9529	0.6788	0.3410
H	0.9529	1	0.8645	0.5871
VH	**0.6788**	**0.8645**	1	**0.9086**
VVH	0.3410	0.5871	0.9086	1
$\mathbb{K}_{LI}$ (Liu et al., 2015)	MLH	H	VH	VVH
MLH	1	0.9851	0.9562	0.8721
H	0.9851	1	0.9795	0.8954
VH	**0.9562**	**0.9795**	1	**0.9414**
VVH	0.8721	0.8954	0.9414	1
$\mathbb{K}_{TH}$ (Thao, 2018)	MLH	H	VH	VVH
MLH	1	0.9063	0.4907	0.0010
H	0.9063	1	0.8016	0.3691
VH	**0.4907**	**0.8016**	1	**0.8407**
VVH	0.0010	0.3691	0.8407	1
$\mathbb{K}_{TMF}$ (Thao et al., 2019)	MLH	H	VH	VVH
MLH	1	0.9410	0.7113	0.4066
H	0.9410	1	0.8975	0.6415
VH	**0.7113**	**0.8975**	1	**0.9035**
VVH	0.4066	0.6415	0.9035	1
S_S (Singh, 2012)	MLH	H	VH	VVH
MLH	1	0.9616	0.6722	0.3201
H	0.9616	1	0.8496	0.5677
VH	**0.6722**	**0.8496**	1	**0.9166**
VVH	0.3201	0.5677	0.9166	1

Table 11 continued…

S_{DS} (Dhivya and Sridevi, 2018)	MLH	H	VH	VVH
MLH	1	0.8784	0.7380	0.5815
H	0.8784	1	0.8623	0.7153
VH	**0.7380**	**0.8623**	1	**0.8649**
VVH	0.5815	0.7153	0.8649	1
S_{We}(Wei et al., 2011a)	MLH	H	VH	VVH
MLH	1	0.7422	0.5075	0.3157
H	0.7422	1	0.6804	0.4540
VH	**0.5075**	**0.6804**	1	**0.6867**
VVH	0.3157	0.4540	0.6867	1
S_Y (Ye, 2013)	MLH	H	VH	VVH
MLH	1	0.9354	0.7172	0.4510
H	0.9354	1	0.9100	0.6932
VH	**0.7172**	**0.9100**	1	**0.9168**
VVH	0.4510	0.6932	0.9168	1
S_{HL}(Hu and Li, 2013)	MLH	H	VH	VVH
MLH	1	0.8449	0.6503	0.4583
H	0.8449	1	0.8054	0.6134
VH	**0.6503**	**0.8054**	1	**0.8080**
VVH	0.4583	0.6134	0.8080	1
S_{MC} (Meng, 2016)	MLH	H	VH	VVH
MLH	1	0.7201	0.4538	0.2790
H	0.7201	1	0.6682	0.4325
VH	**0.4538**	**0.6682**	1	**0.6700**
VVH	0.2790	0.4325	0.6700	1
S_{Ae} (Alolaiyan et al., 2024)	MLH	H	VH	VVH
MLH	1	0.7266	0.4260	0.1189
H	0.7266	1	0.6780	0.3654
VH	**0.4260**	**0.6780**	1	**0.6799**
VVH	0.1189	0.3654	0.6799	1
D_{Ze} (Zhou et al., 2016)	MLH	H	VH	VVH
MLH	0	0.0488	0.1113	0.1716
H	0.0488	0	0.0625	0.1228
VH	**0.1113**	**0.0625**	0	**0.0603**
VVH	0.1716	0.1228	0.0603	0
D_{Fe} (Fares et al., 2019)	MLH	H	VH	VVH
MLH	0	0.1371	0.3616	0.6215
H	0.1371	0	0.2595	0.5723
VH	**0.3616**	**0.2595**	0	**0.4453**
VVH	0.6215	0.5723	0.4453	0
D_{Qe} (Qin et al., 2023)	MLH	H	VH	VVH
MLH	0	0.1992	0.4353	0.6357
H	0.1992	0	0.2577	0.4814
VH	**0.4353**	**0.2577**	0	**0.2577**
VVH	0.6357	0.4814	0.2577	0
D_{AO} (Ohlan, 2022)	MLH	H	VH	VVH
MLH	0	0.0756	0.3884	0.9610
H	0.0756	0	0.1169	0.4701
VH	**0.3884**	**0.1169**	0	**0.1119**
VVH	0.9610	0.4701	0.1119	0
D_{Ve} (Vishnukumar et al., 2024)	MLH	H	VH	VVH
MLH	0	0.2858	0.4933	0.6239
H	**0.2858**	0	**0.2258**	**0.3917**
VH	**0.4933**	**0.2258**	0	**0.1917**
VVH	0.6239	0.3917	0.1917	0
Proposed ($\alpha = 2$)	MLH	H	VH	VVH
MLH	1	0.7382	0.5818	0.4122
H	0.7382	1	0.8479	0.6661
VH	**0.5818**	**0.8479**	1	**0.9127**
VVH	0.4122	0.6661	0.9127	1

Table 11 continued…

Proposed ($\alpha = 6$)	MLH	H	VH	VVH
MLH	1	0.5703	0.1922	0.0249
H	0.5703	1	0.5318	0.1237
VH	0.1922	0.5318	1	0.4488
VVH	0.0249	0.1237	0.4488	1
Proposed ($\alpha = 10$)	MLH	H	VH	VVH
MLH	1	0.4537	0.0600	0.0015
H	0.4537	1	0.3236	0.0231
VH	0.0600	0.3236	1	0.2314
VVH	0.0015	0.0231	0.2314	1

The **bold** values in a row indicate the incorrect results.

Table 12. The outcomes of the comparison measures are derived from **Table 11**.

Measure	Inequalities failed	Reason for failure
$\mathbb{K}_{BB}$ (Bustince and Burillo, 1995), $\mathbb{K}_{HO}$ (Hong, 1998)	(15)	$0.8645 \ngtr 0.9086 > 0.6788$
$\mathbb{K}_{Hu}$ (Hung, 2001)	(17)	$0.9996 > 0.9979 \ngtr 0.9990$
$\mathbb{K}_{PA}$ (Park et al., 2009a)	(17)	$0.9093 \ngtr 0.9172 > 0.7146$
$\mathbb{K}_{WE}$ (Wei et al., 2011b), $\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	(17)	$0.8645 \ngtr 0.9086 > 0.6788$
$\mathbb{K}_{LI}$ (Liu et al., 2015)	(17)	$0.9795 > 0.9414 \ngtr 0.9562$
$\mathbb{K}_{TH}$ (Thao, 2018)	(17)	$0.8016 \ngtr 0.8407 > 0.4907$
$\mathbb{K}_{TMF}$ (Thao et al., 2019)	(17)	$0.8975 \ngtr 0.9035 > 0.7113$
S_S (Singh, 2012)	(17)	$0.8496 \ngtr 0.9166 > 0.6722$
S_{DS} (Dhivya and Sridevi, 2018)	(17)	$0.8683 \ngtr 0.8649 > 0.7380$
S_{We} (Wei et al., 2011a)	(17)	$0.6804 \ngtr 0.6867 > 0.5075$
S_Y (Ye, 2013)	(17)	$0.9100 \ngtr 0.9168 > 0.7172$
S_{HL} (Hu and Li, 2013)	(17)	$0.8054 \ngtr 0.8080 > 0.6503$
S_{MC} (Meng, 2016)	(17)	$0.6682 \ngtr 0.6700 > 0.4538$
S_{Ae} (Alolaiyan et al., 2024)	(17)	$0.4260 > 0.6780 \ngtr 0.6799$
D_{Ze} (Zhou et al., 2016)	(25)	$0.0625 \nless 0.0603 < 0.1113$
D_{Fe} (Fares et al., 2019)	(25)	$0.2595 < 0.4453 \nless 0.3616$
D_{Qe} (Qin et al., 2023)	(25)	$0.4353 < 0.2577 \nless 0.2577$
D_{AO} (Ohlan, 2022)	(25)	$0.3884 < 0.1169 \nless 0.1119$
D_{Ve} (Vishnukumar et al., 2024)	(25)	$0.4933 < 0.2258 \nless 0.1917$
Proposed ($\alpha = 2$)	(17)	$0.8479 \ngtr 0.9127 > 0.5818$
Proposed ($\alpha = 6$)	None	None
Proposed ($\alpha = 10$)	None	None

The comparative analysis from **Table 12** illustrates the limitations of existing measures. All fail to satisfy the given inequalities and cannot differentiate effectively in critical scenarios. The proposed measure shows certain limitations at $\alpha = 2$, but it proves to be highly effective and consistent when an optimal value of α is selected. This demonstrates that the method becomes more dependable and accurate than traditional measures under challenging conditions.

The proposed measure exhibits considerable improvements when a proper value of α is chosen. It fulfills all the required inequalities and has strong differentiation capabilities. This makes it suitable for handling complex fuzzy data and ensuring reliable decision-making. Overall, these findings confirm the usefulness of the proposed approach and highlight its advantage over existing similarity measures.

5.1 Sensitivity Analysis of Parameter α

In order to examine the stability and discrimination power of the proposed correlation coefficient, a detailed sensitivity analysis with respect to the parameter α was performed. **Table 12** provides the outcomes of several correlation, similarity and distance measures, highlighting their failure to satisfy the essential inequality conditions (15), (17) and (25) which are crucial for effective discrimination of structured linguistic variables.

The analysis shows that:

- For lower values of parameter, such as $\alpha = 2$, the measure proposed fails to fully satisfy inequality (17), in the same way as other existing methods. This suggests that at a lower value of α, the proposed correlation coefficient cannot be used for the classification of the linguistic variables.
- As α increases (e.g., $\alpha = 6$ and $\alpha = 10$), the proposed measure not only distinguishes the different structured linguistic variables but also satisfies all the required inequalities. This indicates that the proposed generalized correlation coefficient is sensitive towards the parameter α. This justifies the applicability of the generalized correlation coefficient in real-life scenarios.

This analysis highlights the tuning function of parameter α. By changing the value of α, decision-makers can enhance or relax the discriminative strength of the correlation measure depending on the degree of closeness of the linguistic variables.

Hence, the sensitivity analysis serves as a clear indication of the importance of selecting an appropriate value of α. This gives the proposed measure the ability to be at its best when compared with the classical measures which do not have the flexibility feature introduced by the parameter, thus making it a strong and adaptable approach in complex decision-making environments.

The sensitivity analysis insights the importance of the parameter α in enhancing the responsiveness of the correlation coefficient. It improved the distinction among alternatives in decision-making scenarios. Following this observation, it becomes essential to assess not only the sensitivity but also the reliability of the proposed method. For this purpose, we incorporate the measure of error- originally proposed by Muthukumar and Krishnan (2016) for intuitionistic fuzzy soft sets as an additional evaluation metric.

5.1.1 Measure of Error

The measure of error provides the extent of variation or inconsistency among the alternatives by analyzing the relationship between the chosen optimal alternative and all other alternative pairs. In this study, we extend this concept to the framework of IVIFSs by substituting conventional similarity measures with correlation coefficients, which more effectively capture the level of association between the alternatives. Let $\mathbb{K}(A_i, A_j)$ denote the correlation coefficient between two IVIFSs A_i and A_j, and $\mathbb{K}(A_t, A_r)$ denote the highest value of correlation coefficient between any two IVIFSs A_t and A_r. Then, the measure of error for the IVIFS-based method is defined as

$$M_E = \mathbb{K}(A_t, A_r) + \frac{1}{\sum_{(i,j)\neq(t,r)}\left(\mathbb{K}(A_t, A_r) - \mathbb{K}(A_i, A_j)\right)} \quad (30)$$

For Example 5.1, let $A_1 = A^{1/2}, A_2 = A, A_3 = A^2$ and $A_4 = A^4$. The correlation coefficient values between two IVIFSs A_i and A_j for different values of parameter α are shown in **Table 13**. **Table 13** summarizes the computed pairwise correlation coefficients for six different values of α, illustrating how the relationships among alternatives evolve as the sensitivity parameter α changes.

Table 13. Correlation coefficient values for different values of parameter α.

	$\alpha = 2$	$\alpha = 4$	$\alpha = 6$	$\alpha = 10$	$\alpha = 12$	$\alpha = 14$
$\mathbb{K}(A_1, A_2)$	0.7382	0.6379	0.5703	0.4537	0.4021	0.3550
$\mathbb{K}(A_1, A_3)$	0.5818	0.3388	0.1922	0.0600	0.0335	0.0187
$\mathbb{K}(A_1, A_4)$	0.4122	0.1039	0.0249	0.0015	0.0004	0.0001
$\mathbb{K}(A_2, A_3)$	0.8479	0.6828	0.5318	0.3236	0.2578	0.2085
$\mathbb{K}(A_2, A_4)$	0.6661	0.2921	0.1237	0.0231	0.0102	0.0046
$\mathbb{K}(A_3, A_4)$	0.9127	0.6458	0.4488	0.2314	0.1705	0.1267

Based on these results, the corresponding values of the measure of error (M_E) using Equation (30) for different values of α are computed and summarized in **Table 14**.

Table 14. Calculated values of the measure of error for the proposed method (Example 5.1).

Values of α	M_E
2	1.6718
4	1.3994
6	1.2239
10	1.0676
12	1.0523
14	1.0610

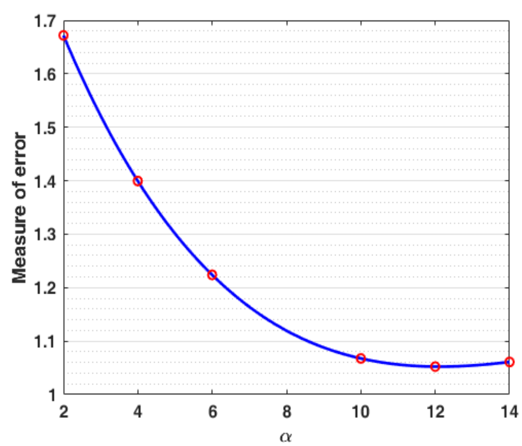

Figure 7. Measure of error for different values of parameter α.

Table 14 and **Figure 7** show that as the parameter α increases, the corresponding measure of error M_E decreases. As a result of this behavior, one may infer that an increase in the value of parameter α makes the model stronger since it can better discriminate among alternatives resulting in a lower error when identifying the most appropriate option. However, after a certain point (e.g., after $\alpha = 12$), the measure of error begins to increase, highlighting that the selection of an appropriate value for α is crucial.

In other words, the method is becoming more stable and accurate in finding the best alternative as the α increases.

5.1.2 Comparative Evaluation Based on Measure of Error

A comparative analysis based on the M_E was carried out to rigorously evaluate the computational performance and reliability of the proposed correlation coefficient.

Table 15 and **Figure 8** provide a comparative assessment of the M_E corresponding to different existing correlation coefficient methods, which have been evaluated using Example 5.1.

Table 15. Calculated values of the measure of error for the various existing methods (Example 5.1).

Values of α	M_E
$\mathbb{K}_{BB}$ (Bustince and Burillo, 1995), $\mathbb{K}_{HO}$ (Hong, 1998)	1.6752
$\mathbb{K}_{Hu}$ (Hung, 2001)	75.627
$\mathbb{K}_{PA}$ (Park et al., 2009a)	1.9442
$\mathbb{K}_{WE}$ (Wei et al., 2011b), $\mathbb{K}_{ZW}$ (Zeng and Wang, 2011)	1.6752
$\mathbb{K}_{LI}$ (Liu et al., 2015)	4.5451
$\mathbb{K}_{TH}$ (Thao, 2018)	1.3993
$\mathbb{K}_{TMF}$ (Thao et al., 2019)	1.8147
S_S (Singh, 2012)	1.6365
S_{DS} (Dhivya and Sridevi, 2018)	2.4657
S_{We} (Wei et al., 2011a)	1.6797
S_Y (Ye, 2013)	1.9467
S_{HL} (Hu and Li, 2013)	1.9696
S_{MC} (Meng, 2016)	1.6317
S_{Ae} (Alolaiyan et al., 2024)	1.4593
Proposed method ($\alpha = 12$)	1.0523

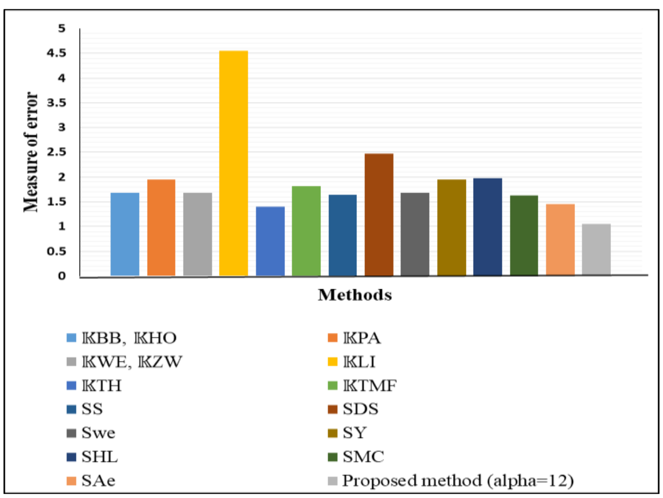

Figure 8. Measure of error for various methods.

Key observations from **Table 15** and **Figure 8**:

- Measure $\mathbb{K}_{Hu}$ (Hung, 2001), generate a very large error value of 75.627 which shows that it performs poorly and lacks stability in this particular environment.
- Measures $\mathbb{K}_{LI}$ (Liu et al., 2015) and S_{DS} (Dhivya and Sridevi, 2018) also output significantly high error values (above 4.5 and 2.4, respectively). These results lead to the conclusion that the existing approaches are inconsistent when taking into account structured linguistic information.

- Although a few methods show moderate performance, none of them can be at the level of the proposed method in terms of precision and error minimization.
- The proposed method (at $\alpha = 12$) is the one that produces the lowest M_E value of 1.0523, thereby confirming its high computational accuracy and reliability.

6. Discussion

This study presents a novel generalized correlation coefficient for IVIFSs, that can go a long way in resolving the problems typical of fuzzy decision-making environments, such as ambiguity, inverse relationships, and nuanced preference modelling. The key results and implications of this work are as follows:

- **Accurate representation of dissimilarity:** One of the most prominent features of the proposed measure is its ability to attain a correlation value of -1 for completely dissimilar IVIFSs. Many existing approaches fail to achieve this. Thus, making this property both meaningful and essential for accurate modelling.
- **Enhanced modelling in MCGDM applications:** This measure has been used to solve a real-world supplier selection problem, thus proving its capability in MCGDM. The use of normalized correlation efficiency is a very efficient tool for the assignment of both positive and negative expert weights, thus making the representation of the conflicting views more accurate.
- **Strong differentiation via confidence evaluation:** The method leads to very high degrees of confidence almost all the time, and thus it can be used for making exact distinctions between alternatives. This provides a foundation for its use in ranking, selection, and other decision-based evaluations.
- **Enhanced handling of structured linguistic terms:** In cases where slight differences in the linguistic variables (e.g., MLH, H, VH, VVH) are considered, the proposed coefficient better distinguishes between variables than the existing measures. Although a few limitations are observed at lower parameter settings, optimal parameter tuning significantly improves accuracy and interpretability.
- **Sensitivity analysis:** The sensitivity analysis revealed that the parameter α was the main factor that influenced the measure's behavior. Its performance was constantly increasing as the value of α increases and reaches its best performance at $\alpha = 12$ for the test case considered (see **Table 14**). Beyond this value, accuracy begins to decline, which highlights the importance of identifying an optimal α.
- **Validated by Measure of Error analysis:** The measure of error analysis is an additional source that validates the robustness of the proposed approach. At $\alpha = 12$, the method attains the smallest error value, $M_E = 1.0523$, outperforming all comparable methods and demonstrating superior computational reliability.

7. Conclusion

This paper introduces a generalized correlation coefficient for IVIFSs that addresses the issues of existing methods. The new coefficient more accurately reflects both positive and negative relationships. It captures linguistic vagueness more delicately and offers strong computational efficiency. Due to these qualities, the measure is better suited for real decision-making scenarios involving incomplete or imprecise information.

While the parameter α is one of the major strengths, its best selection is still very much dependent on user's judgment, indicating that adaptive or data-driven strategies for choosing α could enhance usability to a greater extent. Besides, the method has the following limitations:

(i) Attribute interdependencies: The incorporation of objective weights notwithstanding, the current formulation does not consider the interdependencies of attributes, which may have an impact on the results in complex scenarios.

(ii) Scalability: The method has been shown to work efficiently for small and medium-sized problems, however, it's not clear how it will behave with every large dataset.
(iii) Performance under extreme uncertainty: Additional studies should be carried out to confirm the robustness of the proposed correlation coefficient in cases of very uncertain or highly variable IVIF information.

Future research might also consider automated selection of α, use different kinds of data that represent attribute relationships, and develop the method for other fuzzy environments. Moreover, this approach can be used on big data sets from real-world datasets, which will further validate its practical relevance.

Besides that, delving into new functionalities of these coefficients in complicated cases and their complex scenarios, such as machine learning or optimization techniques, may have a significant impact on their power to deal with uncertainty and facilitating informed decisions across various domains. Information from recent studies on cosine similarity measures for IFSs (Ahemen et al., 2024) could also be an idea-generating source for the development of generalized cosine-based correlation coefficients for IVIFSs, providing a promising direction for future research.

Conflict of Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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AI Disclosure
The author(s) declare that no assistance is taken from generative AI to write this article.

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