International Journal of Mathematical, Engineering and Management Sciences
Vol. 6, No. 5, 1331-1344, 2021
https://doi.org/10.33889/IJMEMS.2021.6.5.080

Bi-Objective Reliability-Cost Interactive Optimization Model for Series-Parallel System

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(Received on May 5, 2021; Accepted on August 5, 2021)

Abstract
The paper aims are to determine the bi-objective reliability-cost problem of a series-parallel system by employing an interactive approach. Multi-objective optimization is a design methodology that optimizes a combination of objective functions orderly and concurrently. The fuzzy membership functions have been designated to settle the contrary nature of the objectives. Based on these functions and the moment of the objectives in the form of the weight vector, a crisp optimization design is formed. Lastly, the inherited problem is determined with the aid of the PSO (Particle Swarm Optimization) algorithm and confronted with the genetic algorithm. The solution resembling the various choices of the decision-makers towards the evaluation of their decision are listed. A decision-maker can pick an immeasurable one according to his requirement to reach at the aspired goal.

Keywords- Reliability-cost optimization, Fuzzy membership function, Series-parallel system, Uncertainty, PSO.

1. Introduction

In the current era of global conflict and faster transmission times, it has grown essential for all production practices to manage appropriately during their anticipated life span. Any damaging consequences of the reckless performance of such tools or systems guide to the request for reliability analysis. However, to prevail competitively and to present appropriate and specific aids the firms are testing loyalty and maintainability issues as a part of a corporate examination to raise the property of the outcomes and assistance given. In a common design, the regularity and component endurance are expected to be irregular variables and the operation execution is arranged by applying the probability theory. Regrettably, this axiom is not suitable in a wide array of situations, i.e., the proverbial reliability theory does not normally give salubrious experience to the practitioners due to the direction of being capable to manipulate only quantitative information. To address it, a theory of fuzzy set (Zadeh, 1965) has been incorporated with assigning a degree of membership to each element.

Huang et al. (2004) addressed a posbist fault tree analysis, based on posbist reliability theory, for predicting and diagnosing failures and assessing the reliability and safety of systems. Gupta et al. (2005), Rani et al. (2014) introduced a rule to compute reliability, availability and MTBF of the industrial system. Leung et al. (2011) recommended design for a couple cold standby repairable system with one repairman and restoration priority. Sharma et al. (2008) discussed a basic structure for risk and reliability analysis. Garg et al. (2014) introduced multi-objective reliability optimization problem (MOROP) under interval environment. Taghipour et al. (2011) intended a method for investigating the statistical maintenance data for a general infusion pump from a Canadian General Hospital with censoring and missing information. Niwas and Garg (2018) examined the reliability by using the cost-free warranty policy. Yusuf et al. (2020a) presented a model to analyze the reliability of series system with standby components. Also, Yusuf et al. (2020b) discussed a model for active parallel homogenous client by computing the reliability and hence did a performance analysis by computing availability, mean time to failure and probability of busy period of repairman. Waziri et al. (2020) investigated the performance of the series-parallel system with non-uniform failure rate and hence studied some characteristics of the age replacement with minimal repair. Bhattacharyee et al. (2020) discussed the reliability-redundancy allocation problem under the imprecise environments with chance constraint. The obtained model is solved with the help of the soft computing techniques. Waziri (2021) presented a discounted discrete schedule replacement model for a unit. Tyagi et al. (2021) characterized the hybrid renewable energy system's reliability by using supplementary variable technique. Kumar et al. (2021) evaluated the reliability of the bridge system using the universal generating function technique. Bhandari et al. (2021) analyze the reliability of the complex system with triangular fuzzy number and Weibull distribution.

To resolve the reliability optimization problems, several investigators have revealed different optimization methods which include, approximate, exact, heuristics, and multi-objective optimization methods. In the aspects of the optimization model, Bellman and Zadeh (1970) initially creates the model under the fuzzy environment. After this pioneering work, a vast quantity of articles trading with fuzzy optimization problems has arrived out. A multi-objective formulation to maximize system safety and depreciate operation cost has been characterized by Sakawa (1978) using surrogate worth trade methods. Kuo and Prasad (2000) gave some method for the reliability optimization of the system. Zimmermann (1978) inducted the purpose of fuzzy theory to optimization. Huang (1997) presented a fuzzy multi-objective decision-making problem with reliability and cost objectives. Huang et al. (2005) formed an intelligent interactive multi-objective Optimization Method for reliability design. Minguez et al. (2005) presented a method for sensitivity analysis to determine the rate of change of the cost and reliability indices due to data changes. Lei et al. (2005) introduced a proposal for safety evaluation of power administration systems by merging the fuzzy number with the network-equivalent approach. Kala (2008) described the two basic approaches for analyzing the combined statistical and epistemic uncertainty.

Also, in the literature, researchers have implemented the particle swarm optimization (PSO) algorithm (Kennedy and Eberhart, 1995) to solve the reliability optimization model. For example, Zavala et al. (2005) proposed a PSO-based algorithm, to solve a bi-objective redundant reliability problem. Chen (2006) presented a penalty-guided PSO while (Yeh et al., 2010) presented "Monte Carlo simulation" approach to solve the reliability optimization models. Garg (2014), Kang and Kwak (2009) presented the "maximum entropy principle" while (Kundu and Islam, 2019) presented a fuzzy goal programming method for reliability optimization problems. Garg and Sharma (2013) presented a multi-objective optimization model for reliability problems.

In the knowledge of studied research and to discuss the effective solution, the reliability optimization model has been formed to create the reliable components of a system to maximize their profit and length. For it, an interactive bi-objective reliability-cost optimization model has been prepared under the fuzzy features based on past failure and repair data, for optimization of system performance. For this, we organize a reliability-cost problem of some series-parallel system. The numerous choices of the decision making towards the evaluation of their choice are practiced and hence presented their corresponding solution. The addressed approach is illustrated through a case study of one system of a urea fertilizer plant.

The rest of the paper is structured as. Section 2 presents the reliability-cost MOOP model in the fuzzy environment. In Section 3, an interactive approach is presented to solve the optimization model. The presented approach has been illustrated with a case study in Section 4. Finally, a conclusion is reported in Section 5.

2. Reliability Optimization Model

The problem of system reliability may be established as a typical non-linear programming problem with non-linear cost-functions.

2.1 Formulation of Reliability-Cost Model

Consider a system comprised of n components whose system reliability is denoted by $\mathcal{R}_s$. Let R_i be the reliability of the i^{th} component for $i = 1, 2, \dots, n$. Then the system reliability ($\mathcal{R}_s$) can be expressed with R_i's as:

$$\mathcal{R}_s(R_1, R_2, \dots, R_n) = \begin{cases} \prod_{i=1}^{n} R_i & ; \ for\ series\ system \\ 1 - \prod_{i=1}^{n} (1 - R_i) & ; for\ parallel\ system \\ or\ combination\ of\ series\ and\ parallel\ system \end{cases} \tag{1}$$

Furthermore, it is manifest that the cost of the component, expressed by $\mathcal{C}_i$, is monotonically growing with reliability. Based on the i^{th} components reliability cost is $C_i(R_i)$, the system cost $\mathcal{C}_s$ is stated as:

$$\mathcal{C}_s(R_1, R_2, \dots, R_n) = \sum_{i=1}^{n} C_i(R_i) \tag{2}$$

where $C_i(R_i) = a_i \log\left(\frac{1}{1-R_i}\right) + b_i$, $C_i > 0$, a_i and b_i are the constants associated with the system cost which depends on the intrinsic property of the system/component for $i = 1, 2, \dots, n$.

In reliability optimization problems, plant personnel frequently need to reduce the cost and maximize the reliability together. To handle it, a MOOP is formulated as below:

$$Maximize:\ \mathcal{R}_s(R_1, R_2, \dots, R_n) = \begin{cases} \prod_{i=1}^{n} R_i & ; \ for\ series\ system \\ 1 - \prod_{i=1}^{n} (1 - R_i) & ; for\ parallel\ system \\ or\ combination\ of\ series\ and\ parallel\ system \end{cases}$$

$$Minimize:\ \mathcal{C}_s(R_1, R_2, \dots, R_n) = \sum_{i=1}^{n} \left\{ a_i \log\left(\frac{1}{1-R_i}\right) + b_i \right\}$$

Subject to $R_{i,min} \le R_i \le 1$; $\mathcal{R}_{s,min} \le \mathcal{R}_s \le 1$ for $i = 1, 2, \dots, n.$ (3)

2.2 Multi-Objective Optimization Problem

A MOOP is concerned with mathematical optimization problem which involve more than one objective function to be optimized. Generally, MOOP can be formulated as follows:

$$\text{Max/min } f = \left(f_1(x), f_2(x), \dots, f_Q(x)\right),$$
$$\text{Subject to } \mathcal{A}_p(x) \le 0 \;\; ; \;\; p = 1,2,\dots,P,$$
$$\mathcal{B}_k \le 0 \;\; ; \;\; k = 1,2,\dots,K,$$
$$x_r^l \le x_r \le x_r^u \;\; ; \;\; r = 1,2,\dots,n.$$

Here $f_1(x), f_2(x), \dots, f_Q(x)$ are individual objective functions and $x = (x_1, x_2, \dots, x_n) \in \mathcal{R}^n$ is a vector of decision variables. The MOOP is designed to find a feasible $x \in \mathcal{R}^n$ which gives optimal value of objective function $f = \left(f_1(x), f_2(x), \dots, f_Q(x)\right)$. In MOOP, no single feasible solution exists which optimizes all objective functions simultaneously. Therefore, it leads us to find the best compromise solution, also known as pareto optimal solution (Deb, 2011) and is defined as follows:

Definition 1. (Deb, 2011) A feasible solution $x^* \in \mathcal{R}^n$ is said to be pareto optimal, if it satisfies the following conditions:

1) There does not exist any $x \in \mathcal{R}^n$ satisfying $f_q(x) \ge f_q(x^*)$ $\left(f_q(x) \le f_q(x^*)\right)$ $\forall\, q = 1,2,\dots,Q$ in case of maximization (minimization) problem.
2) There exists at least one objective function f_q satisfying $f_q(x^*) > f_q(x)$ $\left(f_q(x^*) < f_q(x)\right)$ in case of maximization (minimization) problem for $q \in \{1, 2, \dots, Q\}$.

Further, for solving a constrained optimization problem, penalty functions are used. Penalty function replaces a constrained optimization problem to an unconstrained optimization problem whose solution ideally converge to the solution of the original constrained problem. In order to formulate penalty function, consider the following optimization problem:

Max / Min $\mathcal{F}(x)$

subject to $\mathcal{A}_p(x) \le 0$; $p = 1,2,\dots,P$ (4)

2.3 Optimization Model in Fuzzy Environment

Traditionally, it was expected that all the data information to reach the system execution are identified as exact. However, in the current world, conjectures always play a vital part in the operation of any system and its components. Thus, to become more adaptable and versatile to the process, the model (3) can be described with fuzzy numbers as:

$Max\ \{\mathcal{R}_s, -\mathcal{C}_s\}$

Subject to $R_{i,min} \le R_i \le 1$;

$\mathcal{R}_{s,min} \le \mathcal{R}_s \le 1 \;\; for \;\; i = 1,2,\dots,n$ (5)

2.4 Particle Swarm Optimization (PSO)

PSO is one of the most successful and widely used algorithms utilized in solving non-linear optimization problems. The concept of PSO was introduced by Kennedy and Eberhart (1995) and it was inspired by the flocking and schooling patterns of birds and fish. PSO is based on the communication and interaction i.e., members (particles) of the population (swarm) exchange information among them. It is a method that optimizes the problem by iteratively trying to improve a solution with respect to given measure of quality. During movement, each particle modifies its best position (local best) in accordance with its own previous best position and the best position of its neighbor particles (global best). PSO begins with a swarm of particles whose positions are initial solutions and velocities are arbitrarily given in the search space. In view of the local best and the global best data, the particles keep on updating their velocities and positions.

Each particle updates its position and velocity, in each iteration, using the following equation:

$$vel_v^{(t+1)} = w^{(t)} vel_v^{(t)} + c_1 r_{v1}^{(t)} \left(P_v^{(t)} - x_v^{(t)}\right) + c_2 r_{v2}^{(t)} \left(P_g^{(t)} - x_v^{(t)}\right) \quad (6)$$

$$x_v^{(t+1)} = x_v^{(t)} + vel_v^{(t+1)} \quad (7)$$

In the above Eq. (6), $v = 1,2, \ldots, N$, where N is the size of population, $vel_v^{(t+1)}$ is the velocity of the v-th particle in $(t+1)^{th}$ iteration, $w^{(t)}$ is the inertia weight, c_1 and c_2 are positive constants known as cognitive and social parameters respectively, r_{v1}^{t} and $r_{v2}^{(t)}$ are random parameters belonging to [0,1] used during iteration number t. Also, $P_v^{(t)}$ represents the best position of the particle achieved so far and $P_g^{(t)}$ represents the particle having smallest/largest fitness value (objective function value), in accordance with min/max problem, acquired till iteration t. A brief description of a pseudo code of PSO for maximizing a function f is given in Algorithm 1.

Algorithm 1: Pseudo Code of the PSO

1. Initialization. For each of the N particles
   (a) Initialize the positions $x_v^{(0)}$ \$ and velocities $vel_v^{(0)}$ $\forall\ v = 1,2, \ldots, N$.
   (b) Initialize the particle's best position to its initial position i.e., $P_v^{(0)} = x_v^{(0)}$.
   (c) Find the fitness value corresponding to each particle and if $f\left(x_k^{(0)}\right) \geq f\left(x_v^{(0)}\right)$ for some $k \in \{1,2, \ldots, N\}$\$ then, initialize the global best position to $x_k^{(0)}$ i.e., $P_g^{(0)} = x_k^{(0)}$.

2. Repeat the following steps, until a stopping criterion is met-
   (a) Update the particle velocity using the Eq. (6).
   (b) Update the particle position using the Eq. (7).
   (c) Evaluate the fitness value of each particle i.e., $f\left(x_v^{(t+1)}\right)$.
   (d) If $f\left(x_v^{(t+1)}\right) \geq f\left(P_v^{\{(t)\}}\right)$ then, update personal best $P_v^{(t+1)} = x_v^{(t+1)}$.
   (e) If $f\left(x_k^{(t+1)}\right) \geq f\left(P_g^{(t)}\right)$ for some $k \in \{1,2, \ldots, N\}$ then, update global best $P_g^{(t+1)} = x_k^{(t+1)}$.

3. At the end of the iterative process, optimal solution is obtained.

3. Interactive Methods for Solving MOROP

Traditionally, it is assumed that the coefficient involved in the reliability model are exact in nature. However, in modern life, it is not expected to get individual information due to the existence of ambiguities. Thus, due to this unfinished and uncertain information, mathematical representations are viewed under the fuzzy conditions.

In a fuzzy MOOP, an optimal solution that concurrently optimizes all the objects is infrequently possible. In such circumstances, one often measures to explore the most stable potential solution in the appearance of rough, or imprecise erudition which is as close to the decision maker's (DM's) expectations. Exploration for such a pleasant solution necessitates working the multi-objective fuzzy optimization problem in an interactive way wherein the DM is originally required to define his or her choices. Based on given choices, the dilemma is answered and the DM is provided with a reasonable solution. If the DM is happy with this answer the problem stops there, unless, invited to alter their choices in the knowledge of the earlier received outcomes. This iterative procedure is resumed till a satisfying solution is arrived which is close to DM's expectations.

The steps of the suggested algorithm are organized as follows:

Step 1: Obtain the Ideal Values: Solve the optimization problem (3) as a single objective, for obtaining the ideal solutions $\mathcal{R}_i^*$, by taking one objective at a time under a given set of constraints. Based on these ideal solutions, the pay-off matrix is formulated as:

$$\begin{bmatrix} \mathcal{R}_s(\mathcal{R}_1^*) & \mathcal{C}_s(\mathcal{R}_1^*) \\ \mathcal{R}_s(\mathcal{R}_2^*) & \mathcal{C}_s(\mathcal{R}_2^*) \end{bmatrix}.$$

From this matrix, the lower and upper bound for each objective is calculated as:

$$\mathcal{R}_s^l = \min\{\mathcal{R}_s(\mathcal{R}_1^*), \mathcal{R}_s(\mathcal{R}_2^*)\} \quad ; \quad \mathcal{R}_s^u = \max\{\mathcal{R}_s(\mathcal{R}_1^*), \mathcal{R}_s(\mathcal{R}_2^*)\}.$$

and

$$\mathcal{C}_s^l = \min\{\mathcal{C}_s(\mathcal{R}_1^*), \mathcal{C}_s(\mathcal{R}_2^*)\} \quad ; \quad \mathcal{C}_s^u = \max\{\mathcal{C}_s(\mathcal{R}_1^*), \mathcal{C}_s(\mathcal{R}_2^*)\}.$$

Step 2: Establishing the Fuzzy Goals:

Let $\widetilde{f_1}$ and $\widetilde{f_2}$ be the fuzzy region of satisfaction of system reliability ($\mathcal{R}_s$) and system cost ($\mathcal{C}_s$) respectively and $\mu_{\mathcal{R}_s}$ and $\mu_{\mathcal{C}_s}$ be their corresponding membership functions. Then the fuzzy objective asserted by a designer can be quantified by extorting corresponding linear membership functions, using the minimal and maximal feasible values of each objective as obtained during Step 1, and are defined as:

For maximization goal ($\mathcal{R}_s$)

$$\mu_{\mathcal{R}_s}(x) = \begin{cases} 1 & ; \ \mathcal{R}_s(x) \geq \mathcal{R}_s^u \\ \frac{\mathcal{R}_s(x) - \mathcal{R}_s^l}{\mathcal{R}_s^u - \mathcal{R}_s^l} & ; \ \mathcal{R}_s^l \leq \mathcal{R}_s(x) \leq \mathcal{R}_s^u \\ 0 & ; \ \mathcal{R}_s(x) \leq \mathcal{R}_s^l \end{cases} \tag{8}$$

Here, $\mu_{\mathcal{R}_s}(x)$ is strictly monotonically increasing function of $\mathcal{R}_s(x)$.

For minimization goal ($\mathcal{C}_s$)

$$\mu_{\mathcal{C}_s}(x) = \begin{cases} 1 & ; \ \mathcal{C}_s(x) \leq \mathcal{C}_s^l \\ \frac{\mathcal{C}_s^u - \mathcal{C}_s(x)}{\mathcal{C}_s^u - \mathcal{C}_s^l} & ; \ \mathcal{C}_s^l \leq \mathcal{C}_s(x) \leq \mathcal{C}_s^u \\ 0 & ; \ \mathcal{C}_s(x) \geq \mathcal{C}_s^u \end{cases} \tag{9}$$

Here, $\mu_{\mathcal{C}_s}(x)$ is strictly monotonically decreasing function of $\mathcal{C}_s(x)$.

Step 3: Crisp Optimization Problem:
Using the obtained $\mu_{\mathcal{R}_s}$ and $\mu_{\mathcal{C}_s}$, and weight vector w_1 and w_2, called the objective weights, Huang (1997) defined the problem as:

$$\mu_D = \min\left(\min\left(1, \frac{\mu_{\mathcal{R}_s}}{w_1}\right), \min\left(1, \frac{\mu_{\mathcal{C}_s}}{w_2}\right)\right) \quad (10)$$

Hence, an optimization model reduces to

$$Maximize : \mu_D(x) = \min\left(min\left(1, \frac{\mu_{\mathcal{R}_s}(x)}{w_1}\right), min\left(1, \frac{\mu_{\mathcal{C}_s}(x)}{w_2}\right)\right) \quad (11)$$

Subject to $x_k^l \leq x_k \leq x_k^u$; $k = 1,2, \dots, K$
$w_t \in [0,1]$, $t = 1,2$

where w_t represents the t^{th} objective weight suggested by DM, x is the vector of decision variables, x_k^l and x_k^u are the lower and upper bounds of decision vector x_k, respectively. The obtained optimization problem is solved with PSO algorithm.

Step 4: Select the DM Choice: If the DM is unpleased with the optimal results obtained through Step 3 then they can alter their choices as $W = [w_1, w_2]$. Based on these updated choices, Step 3 will be executed again and so on. The process is terminated once DM is happy with the results.

4. System Description

The field study is carried in the National Fertilizer Limited plant situated at Panipat, near Delhi, India having urea production capacity ranging from 1500-2000 metric tons per day. A fertilizer industry is a large, complex and repairable engineering unit which is a combination of ammonia and urea plant (Garg and Sharma, 2012). An ammonia plant is composed of large number of operating systems mainly shell gasification and carbon recovery, desulphurization, CO shift conversion, CO_2 cooling, CO_2 removal and ammonia synthesis system arranged in series, parallel or the combination of both. In the ammonia production process, the Low Sulphur High Carbon (LSHC) oil and oxygen mixed with steam enters the shell gasification and carbon recovery system in which carbon and sulphur are removed from this mixed gas. This gas enters the desulphurization system in which H_2S and sulphur compounds are absorbed by cold methanol. This gas free from H_2S and sulphur is heated and then sent to CO shift conversion system, where the reaction takes place. The CO contents are reduced by two stage shift converters. This gas mixture is first cooled and then sent to decarbonation system for CO_2 cooling and CO_2 removal. The CO_2 is absorbed in to the CO_2 generation subsystem of CO_2 removal system. The CO_2 gas is collected and sent to urea plant. The gas mixture free from CO_2 sent to the ammonia synthesis system where the nitrogen is added to this gas to make the hydrogen and nitrogen ratio as 3:1. The pressure and temperature of this gas is raised by ammonia synthesis compressor and further by hot heat exchanger. This gas then enters the ammonia converter in which ammonia gas is produced by the reaction of hydrogen and nitrogen. This ammonia gas is collected in a tank and the supplied to urea plant for the production of urea. The raw materials for urea production are ammonia and carbon dioxide. The urea plant is composed of “urea synthesis”, “urea decomposition”, “urea crystallization”, and “urea prilling system”. In this process the ammonia and CO_2 enters the urea synthesis reactor. The reactants from urea synthesis reactor enter the urea decomposer in which urea is separated from reactants. These are further sent to urea crystallizer in which the urea solution is concentrated and crystallized. The urea crystals are separated by centrifuge and conveyed pneumatically to the top of urea prilling system. In this system urea crystals are melted, sprayed through distributers and fall down in urea prilling tower against the ascending air allowing getting prilled on the way. The prilled urea is collected at the bottom of urea prilling system and sent to bagging section. Among this system, the most important part is the prilling system. In this system, urea crystals are melted, sprayed through distributors and fall in urea prilling tower against the ascending air allowing getting prilled on the way. The prilled urea is collected at the bottom of the urea prilling system and sent to the bagging section.

4.1 Prilling and Carbon Recovery Unit

In brief, the various components in series configuration and their associated subsystems of this unit are described as follows:

- Subsystem (U_j) consists of four units viz- cyclone (U_1), screw conveyor (U_2), meltor (U_3) and strainer (U_4). Failure of any one unit causes complete failure of the system.
- Subsystem (V), consists of ten distributors operating simultaneously with one standby. Failure of any one unit does not affect the system availability. Complete failure take place only when more than one failure occurs.
- The belt conveyor (W), consists of one unit and is employed for carrying the product to trommer. Failure of belt conveyor leads to huge accumulation of urea, blocking the flow path of prilled urea. Hence failure of subsystem (W) would affect the working of the system.

The systematic diagram of the working components of the system is shown in Figure 1.

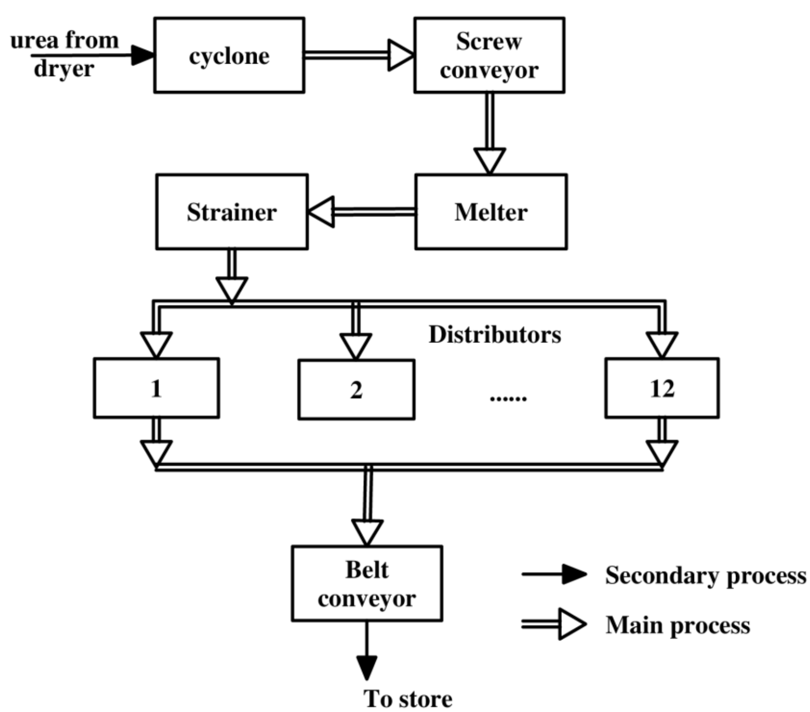

Figure 1. Systematic diagram of prilling and carbon recovery unit.

4.2 Mathematical Model of the System

The considered system consists of major six components, whose λ_i ("failure rates") and τ_i ("repair times") are recorded from the logbooks and are summarized in Table 1.

Table 1. Input data for prilling and carbon recovery unit.

Component	**Failure rate** (λ_i) (hrs^{-1})	**Repair time** (τ_i) **(hrs)**
Cyclone $(i = 1)$	0.0035	1.5
Screw conveyor $(i = 2)$	0.0105	2.0
Meltor $(i = 3)$	0.00131	1.35
Strainer $(i = 4)$	0.0051	1.45
Distributors $(i = 5,6, \ldots ,16)$	0.0027	1.0
Belt conveyor $(i = 17)$	0.0104	2.5

Based on these components and the information, a MOOP of system reliability $(\mathcal{R}_s)$ and cost $(\mathcal{C}_s)$, is formulated under the uncertain environment as:

Maximize $\mathcal{R}_s = \exp(-\lambda_s t)$

$$Minimize \quad \mathcal{C}_s = \sum_{i=1}^{17} \left\{ a_i \log\left(\frac{1}{1 - \exp(-\lambda_i t)}\right) + b_i \right\}$$

Subject to $(1 - s)x_k \leq x_k \leq (1 + s)x_k$

$k = 1,2, \ldots ,34$

$x = [\lambda_1, \lambda_2, \ldots \ldots \lambda_{17}, \tau_1, \tau_2, \ldots , \tau_{17}]$

$\lambda_5 = \lambda_6 = \cdots = \lambda_{16}$

$\tau_5 = \tau_6 = \cdots = \tau_{16}$

$s = 0.15$ (considered uncertainty level)

where λ_s is the system failure rate expressed as

$$\lambda_s = \sum_{i=1}^{4} \lambda_i + \prod_{i=5}^{16} \lambda_i \left[\sum_{i=5}^{16} \prod_{\substack{j=5 \\ i \neq j}}^{16} \tau_j \right] + \lambda_{17} \quad (12)$$

The values of the parameters a_i and b_i are tabulated in Table 2.

Table 2. Values of a_i's and b_i's for prilling and carbon recovery unit.

Component →	Cyclone $(i = 1)$	Screw conveyor $(i = 2)$	Meltor $(i = 3)$	Strainer $(i = 4)$	Distributors $(i = 5,6, \ldots ,16)$	Belt conveyor $(i = 17)$
a_i	7.5	10	8.75	6.54	3.53	5.5
b_i	50	70	65	50	30	50

4.3 Solution Procedure

The steps of the suggested method are performed here to obtain the optimal solution of the model defined in the above section.

Step 1: Solve the model for $\mathcal{R}_s$ and $\mathcal{C}_s$ individually and hence a pay off matrix is formed as:

$$\begin{matrix} \mathcal{R}_s & \mathcal{C}_s \end{matrix}$$
$$\begin{bmatrix} 0.72674108 & 930.103195 \\ 0.73484183 & 906.239993 \end{bmatrix}.$$

Based on this payoff matrix, we get the ideal values of the objective functions are $\mathcal{R}_s^l = 0.72674108$, $\mathcal{R}_s^u = 0.73484183$, $\mathcal{C}_s^l = 906.239993$ and $\mathcal{C}_s^u = 930.103195$.

Step 2: A linear membership function corresponding to $\mathcal{R}_s$ and $\mathcal{C}_s$ is constructed as:

$$\mu_{\mathcal{R}_s}(x) = \begin{cases} 1 & ; \ \mathcal{R}_s(x) \geq \ 0.73484183 \\ \dfrac{\mathcal{R}_s(x) - 0.72674108}{0.73484183 - 0.72674108} & ; \ 0.72674108 \leq \mathcal{R}_s(x) \leq 0.73484183 \\ 0 & ; \mathcal{R}_s(x) \leq 0.72674108 \end{cases} \quad (13)$$

and

$$\mathcal{C}_s(x) = \begin{cases} 1 & ; \ \mathcal{C}_s(x) \leq 906.239993 \\ \dfrac{930.103195 - \mathcal{C}_s(x)}{930.103195 - 906.239993} & ; \ 906.239993 \leq \mathcal{C}_s(x) \leq 930.103195 \\ 0 & ; \ \mathcal{C}_s(x) \geq 930.103195 \end{cases} \quad (14)$$

Step 3: By using Eqs. (13), (14) and the importance factor $W = [w_1, w_2]$, the crisp optimization model is formulated as:

$$Maximize \ : \ \mu_D(x) = \min\left(\min\left(1, \frac{\alpha_1}{w_1}\right), \min\left(1, \frac{\alpha_2}{w_2}\right)\right)$$

Subject to $\alpha_1 = \mu_{\mathcal{R}_s}(x); \quad \alpha_2 = \mu_{\mathcal{C}_s}(x)$ (15)

$$(1 - s)x_k \leq x_k \leq (1 + s)x_k,$$

$$k = 1,2, \ldots ,34.$$

$$x = [\lambda_1, \lambda_2, \ldots \ldots \lambda_{17}, \tau_1, \tau_2, \ldots, \tau_{17}],$$

$$\lambda_5 = \lambda_6 = \cdots = \lambda_{16}.$$

$$\tau_5 = \tau_6 = \cdots = \tau_{16}.$$

$s = 0.15$ (considered uncertainty level).

Step 4: PSO has been employed to resolve the equation (15) with the initial preference as $W1 = [1, 1]$. In this iteration, $W1 = [1,1]$ means that DM reward equal consideration towards each objective. Consequences corresponding to the first iteration are dispensed in Table 3 under the iteration I column, which is: $(\mathcal{R}_s, \mathcal{C}_s)$ = (0.73515995, 909.333116) with membership value $(\mu_{\mathcal{R}_s}, \mu_{\mathcal{C}_s})$ = (1.00, 0.87038). If DM is not happy with this outcome or amenable to identify other possible solutions, keeping this result in view, then DM changed their own preferences from $W1$ =

$[1, 1]$ to $W2 = [1, 0.5]$. In other words, DM needs to spend two times more regard to the reliability objective than the cost objective. The result of this iteration is: $(\mathcal{R}_s, \mathcal{C}_s)$ = (0.73874358, 915.249094) with membership value $(\mu_{\mathcal{R}_s}, \mu_{\mathcal{C}_s})$ = (1, 0.62246). The process is iterated until DM is fully compensated. In this way, DM received various potential solutions for different achievement levels. The process is terminated after V emphasis (it may proceed more till DM is happy). The outcome of iteration V is: $(\mathcal{R}_s, \mathcal{C}_s)$ = (0.73076532, 908.937000) with membership value $(\mu_{\mathcal{R}_s}, \mu_{\mathcal{C}_s})$ = (0.49683, 0.88698). This result confirms that 49.683% success for reliability and 88.698% for the cost of respective fuzzy goals. A correlation of the results recorded in the table with the outcomes obtained by GA. Thus, for various choices recommended by DMs, the best values of systems' reliability and cost are managed. The optimum design parameters of design variables corresponding to optimum values are also shortened in Table 3.

Table 3. Solution of the reliability optimization problem of prilling and carbon recovery unit.

Iteration		**I**		**II**		**III**		**IV**		**V**	
		Decision makers preferences									
	w_1	1		1.0		0.8		0.2		0.5	
	w_2	1		0.5		0.2		0.8		1.0	
		Optimal Solutions									
		GA	PSO	GA	PSO	GA	PSO	GA	PSO	GA	PSO
	$\mu_{\mathcal{R}_s}$	0.89836501	1	1	1	1	1	0.30343729	0.33043175	0.76236645	0.49683581
	$\mu_{\mathcal{C}_s}$	0.84388590	0.87038105	0.66231054	0.62246889	0.45202323	0.54777748	0.80550901	0.83076796	0.86331950	0.88698052
	$\mathcal{R}_s$	0.73401851	0.73515995	0.73504385	0.73874358	0.74208385	0.74609841	0.72919915	0.72941782	0.73291682	0.73076582
	$\mathcal{C}_s$	909.965376	909.333116	914.298345	915.249094	919.316473	917.031471	910.881171	910.278412	909.501628	908.937000
		Decision variables corresponding to main components of the system									
Cyclone	λ_i	0.00383864	0.00402419	0.00346138	0.00309557	0.00313603	0.00321235	0.00342517	0.00389626	0.00396206	0.00402500
	τ_i	1.72239805	1.65735947	1.39354434	1.52287456	1.57598926	1.70677541	1.47997715	1.33345304	1.38763749	1.27500000
Screw conveyor	λ_i	0.01081259	0.01056475	0.01001962	0.00965047	0.01014786	0.00920898	0.01067895	0.01130749	0.01140976	0.01207500
	τ_i	2.15992312	2.26868575	1.80571304	2.23738527	1.71434659	2.16249796	1.74572794	2.16410617	1.95936347	1.70000000
Meltor	λ_i	0.00147316	0.00149069	0.00116191	0.00122187	0.00142269	0.00131407	0.00149837	0.00134386	0.00148765	0.00150650
	τ_i	1.29539596	1.55195752	1.52278438	1.50935279	1.22012463	1.41049030	1.27103504	1.18178729	1.35420941	1.14750000
Strainer	λ_i	0.00531248	0.00565568	0.00506771	0.00521986	0.00566511	0.00559143	0.00447018	0.00562367	0.00527566	0.00491972
	τ_i	1.53413042	1.33489413	1.60721157	1.58299586	1.53420780	1.59419958	1.43362065	1.40471074	1.43047767	1.23250000
Distributors	λ_i	0.00308255	0.00310191	0.00298851	0.00295963	0.00258511	0.00281316	0.00308159	0.00305671	0.00308215	0.00310500
	τ_i	0.86076815	1.11660468	0.97243514	0.89969987	0.89862251	1.03822924	0.98088386	0.99996760	1.06832319	0.85000000
Belt conveyor	λ_i	0.00948524	0.00903141	0.01107190	0.01109267	0.00945762	0.00996295	0.01150817	0.00937957	0.00893717	0.00884000
	τ_i	2.55353519	2.60284494	2.54068251	2.17901956	2.68707552	2.15273268	2.41597052	2.39040962	2.76655273	2.12500000

5. Conclusions

The main contribution of the work is summarized as:

(i) Reliability-cost analysis is one of the most significant in order to improve the performance of the system. Traditionally, the analysis has been done based on the collective information without quantify the uncertainties in the data. As a result, a system analyst may get some unreliable result. To handle it, a fuzzy MOOP model is formulated for series-parallel system.

(ii) A conflicting nature between the different objectives are resolved by defining the membership functions corresponding to each objective. For it, firstly a pay-off matrix has been computed and hence based on it, the bounds of the considered objectives are defined.

(iii) To solve the optimization model, an algorithm has been presented in which different objective functions are aggregated with the help of DM attribute weights as per their preferences. Based on it, a crisp optimization model is formulated and hence solved it with the PSO algorithm. The optimal design parameters corresponding to each component of the system is reported and compared their results with the GA results.

(iv) From the computed results, based on the choices of the DM, we can conclude that a person can analyze the impact of the parameters on to the system performance. For instance, by paying equal attentions to reliability and cost objective, we get the satisfaction degree of the $\mathcal{R}_s$ and $\mathcal{C}_s$ as 1.00 and 0.87038 respectively. If DM wants to give the impact of more reliability than cost then their results are listed under the preference order [1, 0.5] with satisfaction degree as 1 and 0.62246 for reliability and cost respectively. Similarly, for other preferences, a DM may analyze their influences and their optimal design parameters can be selected (Table 3).

(v) Based on the reported parameters and the optimal cost and reliability of the system in Table 3, a decision-maker can pick an excellent one according to his necessary to reach the desired goal.

In the future, we will extend the approach under the different fuzzy environment to handle the uncertainties and solve different problems such as inventory, supply chain, etc.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors. The authors would like to thank the editor and anonymous reviewers for their comments that help improve the quality of this work.

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