International Journal of Mathematical, Engineering and Management Sciences
Vol. 10, No. 6, 1926-1952, 2025
https://doi.org/10.33889/IJMEMS.2025.10.6.089

Ram Arti Publishers

Optimizing Sustainable Two-Stage Bi-Objective Fixed-Charge 4-Dimensional Transportation Problem under Fermatean Fuzzy Environment

Elakkiya Kumar
Department of Mathematics,
School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.
E-mail: elakkiya17arun@gmail.com

Anuradha Dhanapal
Department of Mathematics,
School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu, India.
Corresponding author: anuradhadhanapal1981@gmail.com

(Received on February 24, 2025; Revised on May 9, 2025 & June 20, 2025; Accepted on June 29, 2025)

Abstract

The transportation problem is an important field in supply chain management and international trade. In transportation problem, the goods are carried from origin to final destination by using a specific mode of shipping. In reality, goods are often delivered using a variety of modes of shipping and various routes. So, by integrating supply, demand, conveyance, and route together, it becomes a Four-Dimensional Transportation Problem. The entrepreneurs often prefer two-stage transportation which will always reduce multiple payments of fixed charges and also transportation of goods by using appropriate conveyance, route, and fuel type while optimizing multiple objectives together will always preferably maximize the company's profit and simultaneously minimize the environmental pollution. Due to market fluctuations, variations in supply and so on, the problem parameters are considered as Fermatean fuzzy numbers. By consideration of all economic and environmental aspects together we focus on developing a Two-Stage Bi-Objective Fixed-Charge Four-Dimensional Transportation Problem under a Fermatean fuzzy environment. Uncertainty associated with the parameters of the developed model has been resolved by employing the (α, β)-cut technique and accuracy function. Now, the deterministic multi-objective problem is transformed into single objective problem by using the proposed Fermatean hesitant fuzzy programming technique. Then LINGO software is employed to obtain optimal compromise solution for the reduced problem. The competency of our model is clarified with a numerical illustration. The results offer managerial insights for selecting optimal routes and fuel types based on available resources, leading to sustainable development. A comparison is made with the existing method to demonstrate the efficacy of our proposed method. Lastly, sensitivity analysis and conclusions with future scopes are presented.

Keywords- Fixed charge 4D-Transportation problem, Fermatean fuzzy number, (α, β)-cut technique, Fermatean hesitant fuzzy programming method, Sustainability.

Abbreviations

TP	Transportation Problem
3D-TP	3Dimensional Transportation Problem
4D-TP	4Dimensional Transportation Problem
BOFC4D-TP	Bi-Objective Fixed-Charge 4D-Transportation Problem
SOFC4D-TP	Single Objective Fixed-Charge 4D-Transportation Problem
STP	Solid Transportation Problem
FC4D-TP	Fixed-Charge Four-Dimensional Transportation Problem
MOTP	Multi-Objective Transportation Problem
MOFC4D-TP	Multi-Objective Fixed Charge 4D-Transportation Problem
FS	Fuzzy Set
FCSTP	Fixed Charge Solid Transportation Problem
IFS	Intuitionistic Fuzzy Set
PFS	Pythagorean Fuzzy Set
MF	Membership Function

NMF	Non-Membership Function
MOFCSTP	Multi-Objective Fixed Charge Solid Transportation Problem
FFS	Fermatean Fuzzy Set
TFFN	Triangular Fermatean Fuzzy Number
IVFFS	Interval Valued Fermatean Fuzzy Set
FHFP	Fermatean Hesitant Fuzzy Programming
FHFS	Fermatean Hesitant Fuzzy Set
CNG	Compressed Natural Gas
OCS	Optimal Compromise Solution

1. Introduction

Transportation Problem (TP) is an optimization problem with significant applications. It was first introduced by Hitchcock (1941). TP is concerned with determining the amount of goods sent from sources to destinations in order to reduce total transportation costs while considering two factors: availability of supplies and demand of destination. Anuradha (2016) conducted a literature review on transportation problem. In TP, just one mode of transportation is examined. In actuality, goods are often delivered using a variety of modes of transportation such as trucks, railroads, and so on. The TP is converted into a 3Dimentional-TP (3D-TP) which allows you to select the mode of transportation. 3D-TP. Shell (1955) initially proposed 3D-TP or Solid Transportation Problem (STP). Pandian and Anuradha (2010) developed a novel method to solve STP for achieving optimal solution. Today, there are several roads linking various locations, some of which are well-maintained and others that have concerns such as traffic. When carrying fragile goods under such circumstances, route selection becomes critical in avoiding damage. Route selection makes STP more realistic. This modification converts an STP into a 4Dimentional-TP (4D-TP). Many researchers are currently focusing on 4D-TP when compared to TP or STP. Along with that, the breakable/damageable goods' breaking rates depend on the type of road surface, distance across routes, and the types of vehicles in use. To maximizes profit, this is critical for choosing the most suitable vehicle and routes for transporting breakable goods. Bera et al. (2018) expanded the TP to a 4-D structure and used the generalized reduced gradient (GRG) approach to solve the multi-item 4D-TP. Hameed and Moalla (2022) solved 4D-TP by using the particle swarm optimization algorithm to obtain the optimal compromise solution. Rani and Rizk-Allah (2024) have studied 4D-TP under a rough interval-valued environment. When an extra cost (Fixed Charge) for transportation is included in 4D-TP, the problem becomes the Fixed-Charge 4D-TP (FC4D-TP). In FC4D-TP, a fixed cost is imposed on each path whenever some amount is transferred from source to destination. The entrepreneurs will always wish to find the optimal paths that minimizes the transportation and fixed costs through meeting the supply and demand constraints of all FC4D-TP sources and destinations. Fakhrzad et al. (2019) developed an FC4D-TP and solved it by using meta-heuristic methods. Jana et al. (2019) have solved FC4-DTP for damageable multiple items. Sustainable transportation consists of transportation costs, carbon emissions, and other factors that arise during the shipment of commodities from numerous origins to numerous destinations. Traditional FC4D-TP is insufficient to regulate such situations that need balancing all the objectives simultaneously. As a result, the Multi-Objective Transportation Problem (MOTP) has been developed as well as expanded into a Multi-Objective Fixed Charge 4D-Transportation Problem (MOFC4D-TP), which included a conveyance constraint and several routes. Pandian and Anuradha (2011) developed a new method for solving bi-objective transportation problems. Anuradha et al. (2019) have solved multi-objective solid transportation problem by using fuzzy approach. Giri and Roy (2022) addressed a Four-Dimensional Fixed-Charge Transportation Problem with multiple objectives and solved it using Pythagorean hesitant fuzzy programming. Aktar et al. (2025) developed a Multi-Objective Green 4-Dimensional model and solved it using the GRG method. Distribution problems like MOFC4D-TP tend to be modelled to distribute goods over specific origins reaching determined destinations. However, distribution system concerns with multiple stages are predominant. Multiple stages of TP, consisting of moving goods from manufacturers to distribution centres and then from distribution centres to retail stores, may occur. This kind of transportation is known as two-stage transportation. Sobana and Anuradha (2019) solved two stage fuzzy transportation problem by using proposed method. Mollanoori et al. (2019) have introduced two-stage and multi-item transport in Solid step Fixed-Charge Transportation Problems and solved them by using meta-heuristic algorithms to obtain optimal solutions.

In real-life TP, the parameters are often uncertain owing to a number of reasons, including insufficient initial data information, market volatility, environmental influences, and so on. To overcome this situation, Fuzzy set theory (FST) is employed, which was first presented by Zadeh (1965). Yang et al. (2015) solved a type-2 fuzzy FCSTP by using expected, optimistic, and pessimistic values to find an optimal solution. Zhang et al. (2016) solved type-2 fuzzy fixed charge solid TP (FCSTP) by using the tabu search method to find the optimal solution. Anuradha and Sobana (2017) conducted survey on fuzzy transportation problems. Sobana and Anuradha (2018) solved Bi-objective Fuzzy Transportation Problem by using proposed method. Majumder et al. (2019) have solved MOFCSTP for multiple items with budget constraints. Jana and Jana (2020) evaluated FC4D-TP with multi-item in a fuzzy triangular and Gaussian type-2 environment by utilizing the proposed GRG method. Bera et al. (2020) developed and solved FC4D-TP in a type-2 fuzzy environment through the updated GRG algorithm. Devnath et al. (2021) proposed the reduced GRG strategy for evaluating a type-2 fuzzy two-stage FC4D-TP with many items. Devnath et al. (2022) implemented the GRG algorithm to solve fuzzy multi-item two-stage FC4D-TP. Aktar et al. (2023) solved type-2 fuzzy FC4-DTP by using the proposed GRG method. Devnath et al. (2023) updated the GRG algorithm to solve multi-item two-stage 4D-TP in a fuzzy environment. Das et al. (2020) applied type-2 fuzzy in the multi-objective green STP and solved it using fuzzy and non-fuzzy techniques. Aktar et al. (2020) developed type-2 fuzzy 4DTP for fragile items. Bera et al. (2020) analyzed a two-stage MOTP using q-fuzzy numbers with a quantity-dependent credit period policy using a genetic algorithm. Sahoo et al. (2023) solved entropy-based 4D-TP by using an uncertain vector approach. Gazi et al. (2023) developed a hexagonal fuzzy-based analytic hierarchy method and a decision-making technique. According to the FST, the value of membership function lies between 0 and 1, indicates how closely a component fits into the set. Atanassov (1999) invented the Intuitionistic fuzzy set (IFS), which is an extension of a fuzzy set. It is more useful since it provides the parameters' degree of acceptance and rejection in an IFS. This consists of a membership function (MF) and a non-membership function (NMF), with the sum of the two variables ranging from zero to one. Samanta et al. (2020) used the convex combination method to solve the multi-objective 4D-TP model to optimize time and profit by analyzing the impact of route conditions on vehicle speeds. Bind et al. (2023) developed an Intuitionistic multi-objective multi-item 4D-TP to maximize the profitability while reducing emission of carbon and travel time and solved it using an optimistic approach, a mixed approach, and a pessimistic approach. Midya et al. (2021) have solved the Intuitionistic fuzzy multi-stage MOFCSTP under the green supply chain environment by using weighted Tchebycheff metrics programming and min-max goal programming. Mondal et al. (2023) developed and solved the Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step FCSTP using an Intuitionistic fuzzy game-theoretic approach. When the sums of MF and NMF surpass one in many scenarios, to reduce such complications, the Pythagorean fuzzy set (PFS) was first developed by Yager (2013). Under relaxed circumstances, the total of the squares for both the MF and NMF must be included in the unit interval. Ghosh et al. (2022) formulated a Pythagorean hesitant fuzzy multi-objective solid transportation problem for waste management and solved it using the Pythagorean hesitant fuzzy programming method. Yu et al. (2024) solved a multi-objective two-stage in type-2 pythagorean fuzzy environment by using the Intuitionistic fuzzy programming method. Senapati and Yager (2020) developed the concept of a Fermatean fuzzy set (FFS) as an extension of PFS, in which the total of the value of MF and NMF in a cubic set must be between 0 and 1. Akram et al. (2020) have formulated a decision-making problem for an effective sanitizer for the reduction of COVID-19 by using Fermatean fuzzy environment. Sahoo (2021) developed a novel score function for the Fermatean fuzzy transportation problem and solved it by using Excel solver. Akram et al. (2023) developed a novel method for evaluating the extended multi-objective transportation problem with Fermatean fuzzy numbers. Shivani and Rani (2024) modified Vogel's approximation method for solving Fermatean fuzzy STP. Kumar and Dhanapal (2024) formulated the MOTP for two items with Fermatean fuzzy multi-choice stochastic mixed constraints and solved it by utilizing the improved global weighted sum technique. Sharma and Chaudhary (2024) solved the Fermatean hesitant fuzzy multi-objective solid transportation problem by using the fuzzy programming method. Singh et al. (2025) formulated the Fermatean fuzzy multi-objective indefinite quadratic TP with a sustainable environment.

The comparison of related literature reviews to the proposed article which is illustrated in **Table 1**, as below.

Table 1. Comparison of related literature reviews to the proposed article.

References	Objective		Four dimension	Two stages	Fermatean fuzzy parameters	Additional function		Fermatean hesitant fuzzy programming
	Single	Multi				Fixed charge	Breakable item	
Mollanoori et al. (2019)	✓			✓		✓		
Jana et al. (2019)	✓		✓				✓	
Majumder et al. (2019)		✓				✓		
Aktar et al. (2020)	✓		✓				✓	
Devnath et al. (2021)	✓		✓	✓		✓		
Ghosh et al. (2022)		✓						
Sahoo et al. (2023)	✓		✓					
Bind et al. (2023)		✓	✓			✓	✓	
Samanta et al. (2024)	✓		✓	✓			✓	
Rani and Rizk-Allah (2024)	✓		✓					
Aktar et al. (2025)		✓	✓				✓	
Proposed article		✓	✓	✓	✓	✓	✓	✓

In the above literature review, there are several research gaps in the two-stage BOFC4D-TP study, which are discussed below.

1.1 Research Gap and Contribution

This article identifies and addresses the research gaps presented in **Table 1** with the following contributions:

(a) After examining the research papers of Jana et al. (2019), Mollanoori et al. (2019), Aktar et al. (2020), Devnath et al. (2021), Sahoo et al. (2023), Rani and Rizk-Allah (2024), and Samanta et al. (2024), it has been noted that the authors of these studies have addressed only a single objective transportation problem. But in real-world situations, the entrepreneurs always wish to optimize multiple objectives together to improve their profit. So, the present work deals with this gap by constructing a model that considers multiple objectives.

(b) After evaluating the frameworks provided by Majumder et al. (2019), Mollanoori et al. (2019), and Ghosh et al. (2022), it can be noted that the authors did studies for 3D-TP. But in today's world, the transport of goods using multiple vehicles and routes plays a significant role. So, this work deals with this gap by constructing a model that considers 4D-TP by considering different vehicles and routes.

(c) After analyzing the works of Jana et al. (2019), Majumder et al. (2019), Aktar et al. (2020), Ghosh et al. (2022), Bind et al. (2023), Sahoo et al. (2023), Rani and Rizk-Allah (2024), and Aktar et al. (2025), it has been observed that the authors of these studies used only a single stage in TP. In reality, the entrepreneurs can't always distribute their goods by using a single stage. As in **Figure 2**, transportation of goods from manufacturers to warehouse (where the goods are stored) and then distributed to retailers, which will reduce multiple trips for the shipment of goods directly from manufacturers to retailers. So, the present work deals with this gap by constructing a model that considers two stage-TP. This will help the entrepreneurs to improve profit by reducing carbon emissions. In this way the entrepreneurs can contribute to sustainable development by reducing environmental pollution without compromising their profit.

(d) The literature shows that several authors, such as Jana et al. (2019), Aktar et al. (2020), Ghosh et al. (2022), Sahoo et al. (2023), Rani and Rizk-Allah (2024), Samanta et al. (2024), and Aktar et al. (2025), have worked without considering the additional payments like fixed charges. The transportation of goods in two stages will reduce the additional payments of fixed costs imposed on routes by storing goods in warehouses. So, this study utilizes fixed costs imposed on routes to capture such reality and fills this gap by constructing a model that considers fixed charges in two stages.

(e) The researchers (Jana et al., 2019; Majumder et al., 2019; Mollanoori et al., 2019; Aktar et al., 2020; Devnath et al., 2021; Ghosh et al., 2022; Sahoo et al., 2023; Bind et al., 2023; Rani and Rizk-Allah, 2024; Samanta et al., 2024; Aktar et al., 2025) handled the uncertainty in the parameters of TP by using the fuzzy, Intuitionistic fuzzy and Pythagorean fuzzy numbers; whereas, the importance of Fermatean fuzzy parameters as discussed in Section 1, FFN can provide more accurate and robust results than the Intuitionistic and the Pythagorean fuzzy numbers in dealing with the uncertainty of TP. By considering such advantages, the variables of this present study have been described as TFFN.

To fill the above-mentioned gaps, this study introduces a mathematical framework. It combines considerations of two-stage networks, multiple objectives, four dimensions, fixed charges, and FFN.

Because of the consequence of all motivations, the main contributions of our research are stated as below:

- Two-stage bi-objective fixed charge 4D-transportation problems (BOFC4D-TP) under a Fermatean fuzzy environment is formulated with the objectives of improving profit and lowering carbon emissions by choosing the appropriate vehicle, route and fuel type in each stage is a novel achievement in this field.
- A novel method, namely the Fermatean hesitant fuzzy programming method, is presented, and then, by using the proposed method, the two-stage BOFC4D-TP is transformed into the two-stage single-objective FC4-DTP (SOFC4D-TP). Through the use of the LINGO software, the reduced two-stage SOFC4D-TP is effectively solved to obtain an optimal compromise solution (OCS).

The remaining section of this article is organized in the following manner: Section 2 includes the essential definitions. Section 3 provides a mathematical formulation for the two-stage bi-objective fixed charge 4D-transportation problem. Section 4 describes the solution methodology. Section 5 provides a numerical example of the formulated model. Section 6 shows results and discussions Section 7 provides the comparative study. Section 8 includes sensitivity analysis. Section 9 shows managerial implications. Section 10 incorporates the conclusions.

2. Preliminaries

In this section**,** we provided a few essential definitions and representations that are related with the uncertainty theory associated to the proposed work are defined as follows.

2.1 Definition (Zadeh, 1965)

The Fuzzy set (FS) in the universal set X is described as $\tilde{A}=\left\{\left(x,\mu_{\tilde{A}}(x)\right):x\in X\right\}$, where $\mu_{\tilde{A}}(x):X\to[0,1]$ indicate membership function (MF) of *x*, respectively, such that

$$0\le\mu_{\tilde{A}}(x)\le 1,\ \forall x\in X.$$

2.2 Definition (Atanassov, 1986)

An Intuitionistic fuzzy set (IFS) in the universal set X is described as $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in X\}$, where $\mu_{\tilde{A}^I}(x) : X \to [0,1]$, $\nu_{\tilde{A}^I}(x) : X \to [0,1]$ represent MF and non-membership function (NMF) of x respectively, such that $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1,\ \forall x \in X$.

2.3 Definition (Yager, 2013)

For a universe of discourse X, the Pythagorean fuzzy set (PFS) is defined as $\tilde{A}^P = \{(x, \mu_{\tilde{A}^P}(x), \nu_{\tilde{A}^P}(x)) : x \in X\}$, where $\mu_{\tilde{A}^P}(x) : X \to [0,1]$, $\nu_{\tilde{A}^P}(x) : X \to [0,1]$ represent MF and NMF of x respectively, such that $0 \le \mu^2_{\tilde{A}^P}(x) + \nu^2_{\tilde{A}^P}(x) \le 1,\ \forall x \in X$.

2.4 Definition (Senapati and Yager, 2020)

Let X be a non-empty set, then the Fermatean fuzzy set (FFS) in X is defined as $\tilde{A}^F = \{(x, \mu_{\tilde{A}^F}(x), \nu_{\tilde{A}^F}(x)) : x \in X\}$, where $\mu_{\tilde{A}^F}(x) : X \to [0,1]$, $\nu_{\tilde{A}^F}(x) : X \to [0,1]$ indicate MF and NMF of x, respectively, such that $0 \le \mu^3_{\tilde{A}^F}(x) + \nu^3_{\tilde{A}^F}(x) \le 1,\ \forall x \in X$.

For the element X in the set $\tilde{A}^F$, the degree of indeterminacy $(\pi_{\tilde{A}^F}(x))$ is defined as $\pi_{\tilde{A}^F}(x) = \sqrt{1 - \mu^3_{\tilde{A}^F}(x) - \nu^3_{\tilde{A}^F}(x)}$.

From the definitions (2.2-2.4), it is understood that Fermatean fuzzy set domain $0 \le \mu^3_{\tilde{A}^F}(x) + \nu^3_{\tilde{A}^F}(x) \le 1$, completely contain the IFS domain $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$, and PFS domain $0 \le \mu^2_{\tilde{A}^P}(x) + \nu^2_{\tilde{A}^P}(x) \le 1$. This shows the need and the advantages of using Fermatean Fuzzy Sets in complex, uncertain decision-making scenarios which is visually shown in **Figure 1** below.

Figure 1 illustrates a comparison of IFS, PFS, and FFS and it shows the wide range of FFS over IFS and PFS. So, Fermatean fuzzy sets can capture more comprehensive and accurate levels of hesitation and uncertainty than other fuzzy models like Intuitionistic or Pythagorean fuzzy sets. This motivated us to choose FFS over IFS and PFS.

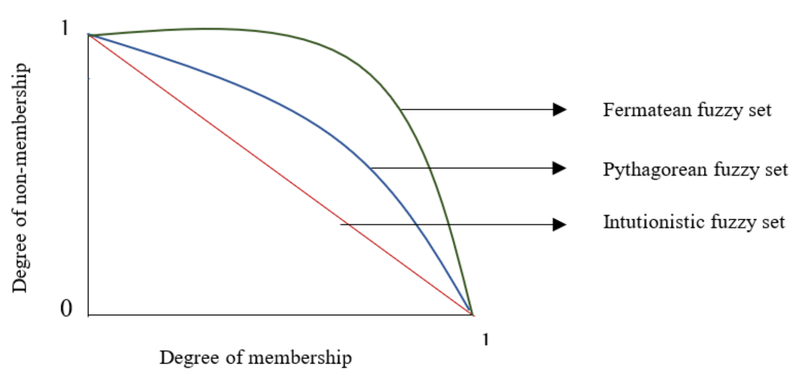

Figure 1. Comparison of IFS, PFS, and FFS.

2.5 Definition (Torra and Narukawa, 2009)

Let X be a universal set, then Hesitant fuzzy set (HFS), $\tilde{A}_h$ on X is stated as $\tilde{A}_h = \left\{x, \mu_{\tilde{A}_h} : x \in X\right\}$, where $\mu_{\tilde{A}_h}(x)$ is a set of some different values in [0,1], denoting the possible membership function of the element $x \in X$.

2.6 Definition (Kirisci, 2023)

To describe the hesitant condition of FFS, HFS is introduce in FFS which extends FFS into Fermatean Hesitant fuzzy set (FHFS). For universal set X, FHFS $\tilde{A}_h^F$ is defined as: $\tilde{A}_h^F = \left\{\left(x, \mu_{\tilde{A}_h^F}(x), \nu_{\tilde{A}_h^F}(x)\right) : x \in X\right\}$, where $\mu_{\tilde{A}_h^F}(x) : X \to [0,1]$ and $\nu_{\tilde{A}_h^F}(x) : X \to [0,1]$ are denoting the possible Fermatean hesitant membership and Fermatean hesitant non-membership function of the element $x \in X$ to the set $\tilde{A}_h^F$, respectively, with the condition $0 \le \mu^3_{\tilde{A}_h^F}(x) + \nu^3_{\tilde{A}_h^F}(x) \le 1,\ \forall x \in X.$

It has become clear that the FHFSs consist of a Fermatean hesitant membership function and a Fermatean hesitant non-membership function, resulting in a more reliable framework that allows for flexible assignment of values to each element in the domain as well as can deal with hesitancy in the situation.

2.7 Definition (Akram et al., 2023)

In Fermatean fuzzy set (FFS), a triangular Fermatean fuzzy number (TFFN) is represented as $\tilde{A}^F = \left\{(x_1, x_2, x_3); \mu_{\tilde{A}^F}, \nu_{\tilde{A}^F}\right\}$ is a FFS with the MF $\left(\mu_{\tilde{A}^F}(x)\right)$ and NMF $\left(\nu_{\tilde{A}^F}(x)\right)$ are represented as:

$$\mu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{(x - x_1)\mu_{\tilde{A}^F}}{x_2 - x_1}, x_1 \le x < x_2 \\ \mu_{\tilde{A}^F}, \quad x = x_2 \\ \dfrac{(x_3 - x)\mu_{\tilde{A}^F}}{x_2 - x_3}, x_2 < x \le x_3 \\ 0, \quad x < x_1\, or\, x > x_3 \end{cases}$$

$$\nu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{\left[x_2 - x + \nu_{\tilde{A}^F}(x - x_1)\right]}{x_2 - x_1}, x_1 \le x < x_2 \\ \nu_{\tilde{A}^F}, \quad x = x_2 \\ \dfrac{[x - x_2 + \nu_{\tilde{A}^F}(x_3 - x)]}{x_2 - x_3}, x_2 < x \le x_3 \\ 1, \quad x < x_1\, or\, x > x_3 \end{cases}$$

$\mu_{\tilde{A}^F}$ represents the value of maximum MF $\left(\mu_{\tilde{A}^F}(x)\right)$ and $\nu_{\tilde{A}^F}$ represents the value of minimum NMF $\left(\nu_{\tilde{A}^F}(x)\right)$, respectively, such that $\mu_{\tilde{A}^F} \in [0, 1], \nu_{\tilde{A}^F} \in [0, 1]$ and $0 \le \left(\mu_{\tilde{A}^F}(x)\right)^3 + \left(\nu_{\tilde{A}^F}(x)\right)^3 \le 1.$

If $\mu_{\tilde{A}^F} = 1$ and $\nu_{\tilde{A}^F} = 0$, TFFN $\tilde{A}^F$ assumes the form $\tilde{A}^F = \{(x_1, x_2, x_3); (\bar{x}_1, x_2, \bar{x}_3)\}$ whose MF $\left(\mu_{\tilde{A}^F}(x)\right)$ and NMF $\left(\nu_{\tilde{A}^F}(x)\right)$ are given below.

$$\mu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{x - x_1}{x_2 - x_1}, x_1 \le x < x_2 \\ 1, \quad x = x_2 \\ \dfrac{x_3 - x}{x_2 - x_3}, x_2 < x \le x_3 \\ 0, \quad x < x_1 \, or \, x > x_3 \end{cases}$$

$$\nu_{\tilde{A}^F}(x) = \begin{cases} \dfrac{x_2 - x}{x_2 - x_1}, x_1 \le x < x_2 \\ 0, \quad x = x_2 \\ \dfrac{x - x_2}{x_2 - x_3}, x_2 < x \le x_3 \\ 1, \quad x < x_1 \, or \, x > x_3 \end{cases}$$

where, $\bar{x}_1 \le x_1 \le x_2 \le x_3 \le \bar{x}_3$.

2.8 Definition (Kumar and Dhanapal, 2024)

(α, β)-cut of a TFFN $\tilde{A}^F = (x_1, x_2, x_3; \bar{x}_1, x_2, \bar{x}_3)$ is the set of all x with a value of MF greater than or equal to α and value of NMF less than or equal to β. This is defined as $\tilde{A}^F_{(\alpha,\beta)} = \{x : \mu_{\tilde{A}^F}(x) \ge \alpha$ and $\nu_{\tilde{A}^F}(x) \le \beta, (\alpha + \beta) \le 1 : x \in X\}$.

The α-cut of $\tilde{A}^F$ as $\left[\mu^l_{\tilde{A}^F}, \mu^u_{\tilde{A}^F}\right] = \left[x_1 + \alpha(x_2 - x_1), x_3 - \alpha(x_3 - x_2)\right]$ and β-cut of $\tilde{A}^F$ as $\left[\nu^l_{\tilde{A}^F}, \nu^u_{\tilde{A}^F}\right] = \left[x_2 - \beta(x_2 - \bar{x}_1), x_2 + \beta(\bar{x}_3 - x_2)\right]$.

2.9 Definition (Kumar and Dhanapal, 2024)

Consider a TFFN $\tilde{A}^F = (x_1, x_2, x_3; \bar{x}_1, x_2, \bar{x}_3)$. Then, the accuracy function (AF) is $\mathrm{M}(\tilde{A}^F) : \bar{X}(\tilde{A}^F) \to \mathbb{R}$, where $\bar{X}(\tilde{A}^F)$ is the set of all interval valued FFS (IVFFS) acquired from (α,β)-cut. The AF has been described as an IVFFS $\bar{X}(\tilde{A}^F) = \left\langle \left[\mu^l_{\tilde{A}^F}, \mu^u_{\tilde{A}^F}\right], \left[\nu^l_{\tilde{A}^F}, \nu^u_{\tilde{A}^F}\right] \right\rangle$ in the following manner:

$$\mathrm{M}(\tilde{A}^F) = \frac{\left(\mu^l_{\tilde{A}^F} + \mu^u_{\tilde{A}^F} + \nu^l_{\tilde{A}^F} + \nu^u_{\tilde{A}^F}\right)}{2}.$$

3. Mathematical Formulation

The following section includes notations and mathematical formulation of 2-stage BOFC4D-TP in a Fermatean fuzzy environment. The notations employed in the model are given in **Table 2** as follows.

3.1 Notations

Table 2. Notations used in the formulation of proposed model.

Description	
Variables	
i	Index of sources (i=1, 2…, I).
g	Index of warehouses (g=1, 2…, G).
j	Index of retailers (j=1, 2…, J).
p	Index that represents different shipping modes (p=1, 2…, P) from sources to warehouses.
q	Index that represents different shipping modes (q=1, 2…, Q) from warehouses to retailers.
u	Index that represents various routes from each source to each warehouse (u = 1, 2…, U).
v	Index that represents various routes from each warehouse to each retailer (v = 1, 2…, V).
Parameters	
S_r	Unit selling price at j^{th} retailer.
$\tilde{\rho}_i^F$	Unit purchase price at i^{th} source.
$\tilde{\varepsilon}_g^F$	Storage cost at g^{th} warehouse.
$\tilde{T}_{1_{igpu}}^F$	Transportation cost calculated per unit distance to ship via u^{th} route by p^{th} conveyance from i^{th} source to g^{th} warehouse.
$\tilde{T}_{2_{grqv}}^F$	Transportation cost calculated per unit distance to ship via v^{th} route by q^{th} conveyance from g^{th} warehouse to j^{th} retailer.
$d_{1_{igu}}$	Distance of u^{th} route from i^{th} source to g^{th} warehouse.
$d_{2_{grqv}}$	Distance of v^{th} route from g^{th} warehouse to j^{th} retailer.
$\tilde{a}_i^F$	Product availability at i^{th} source.
$\tilde{C}_g^F$	g^{th} warehouse holding capacity.
$\tilde{b}_j^F$	Product demand at j^{th} retailer.
$\tilde{e}_{1_p}^F$	Capacity for single p^{th} shipping modes for the shipment in the 1^{st} stage from sources to warehouses.
$\tilde{e}_{2_q}^F$	Capacity for single q^{th} shipping mode for shipment in the 2^{nd} stage from warehouses to retailers.
$\beta_{1_{igpu}}$	Breakability rate of the goods from i^{th} supplier to g^{th} warehouse by p^{th} shipping mode through u^{th} route.
$\beta_{2_{grqv}}$	Breakability rate of the goods from g^{th} warehouse to j^{th} retailer by q^{th} shipping mode through v^{th} route.
$\tilde{f}_{igpu}^1$	Amount of diesel fuel used per unit while moving from i^{th} source to g^{th} warehouse by p^{th} conveyance via u^{th} route.
$\tilde{f}_{igpu}^2$	Amount of petrol fuel used per unit while moving from i^{th} source to g^{th} warehouse by p^{th} conveyance via u^{th} route.
$\tilde{f}_{igpu}^3$	Amount of CNG fuel used per unit while moving from i^{th} source to g^{th} warehouse by p^{th} conveyance via u^{th} route.
$\tilde{f}_{grqv}^1$	Amount of diesel fuel used per unit while moving from g^{th} warehouse to j^{th} retailer by q^{th} conveyance via v^{th} route.
$\tilde{f}_{grqv}^2$	Amount of petrol fuel used per unit while moving from g^{th} warehouse to j^{th} retailer by q^{th} conveyance via v^{th} route.
$\tilde{f}_{grqv}^3$	Amount of CNG fuel used per unit while moving from g^{th} warehouse to j^{th} retailer by q^{th} conveyance via v^{th} route.
$E_{co_2}^1$	Amount of carbon emissions that diesel produces per unit.
$E_{co_2}^2$	Amount of carbon emissions that petrol produces per unit.
$E_{co_2}^3$	Amount of carbon emissions that CNG produces per unit.
$\tilde{x}^F$	TFFN defined as $\tilde{x}^F = \{(x_1, x_2, x_3); (\overline{x}_1, x_2, \overline{x}_3)\}$.
Binary variables	
$y_{1_{igpu}}$	Fixed charges that are included only when the goods are transported from i^{th} source to g^{th} warehouse using p^{th} conveyance through u^{th} route.

Table 2 continued…

$y_{2_{grqv}}$	Fixed charges which are included only when the goods are transported from g^{th} warehouse to j^{th} destination using q^{th} conveyance through v^{th} route.
ϕ	Diesel is used as fuel.
ψ	Petrol is used as fuel.
x	CNG is used as fuel.
Decision variables	
$x_{1_{igpu}}$	When the quantity of goods to be shipped from i^{th} source to g^{th} warehouse by p^{th} conveyance through u^{th} route.
$x_{2_{grqv}}$	When the quantity of goods to be shipped from g^{th} warehouse to j^{th} retailer q^{th} conveyance through v^{th} route.

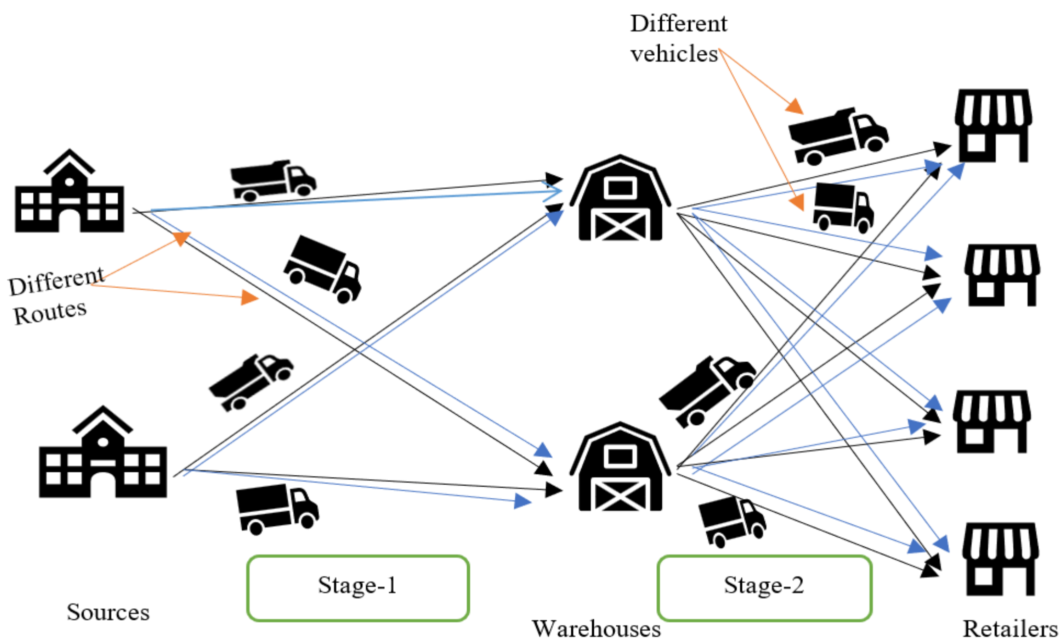

Figure 2. Pictorial representation of 2-stage BOFC4D-TP.

3.2 Formulation of Two-stage Bi-Objective Fixed Charge 4D-Transportation Problem in a Fermatean Fuzzy Environment

In BO4D-TP, the implementation of two-stage, many routes, and conveyances will reduce breakability, and multiple payments of fixed charges will improve profit. The implementation of the appropriate fuel type for the conveyance in each stage will help to reduce carbon emissions (CE) from conveyance. In real-world scenarios, the companies all over the entire globe have been under pressure to boost the profits by decreasing different expenditures as well as to minimize carbon emissions during transportation.

In this article, we developed the 2-stage BOFC4D-TP in a Fermatean fuzzy environment, where goods are transported from i^{th} source to g^{th} warehouse by means of u^{th} different types of conveyance via p^{th} route, and then stored goods in g^{th} warehouse are transported to j^{th} destination through by means of v^{th} different types of shipping mode via q^{th} route by optimizing the objectives Z_1 (maximize profit) and Z_2 (minimize carbon emissions). The pictorial representation of developed model is shown in **Figure 2** and according to this basis, the mathematical expression for 2-stage BOFC4D-TP under a Fermatean fuzzy environment (G1) is as follows:

(G1) $$Max\, Z_1 = \sum_j S_j \tilde{D}_j^F - \sum_{igpu}\{\tilde{\rho}_i^F + \tilde{T}_{1_{igpu}}^F d_{1_{igu}}\} x_{1_{igpu}} - \sum_{igpu} \tilde{h}_{1_{igpu}}^F y_{1_{igpu}} - \sum_{grqvt} \tilde{T}_{2_{gjqv}}^F d_{2_{gjv}} x_{2_{gjqvt}} - \sum_{gjqv} \tilde{h}_{2_{gjqv}}^F y_{2_{gjqv}} - \sum_g \tilde{\varepsilon}_g^F \tag{1}$$

$$Min\, Z_2 = \sum_{igpu}\{\tilde{f}_{igpu}^1 E_{co_2}^1 \phi + \tilde{f}_{igpu}^2 E_{co_2}^2 \psi + \tilde{f}_{igpu}^3 E_{co_2}^3 \chi\} d_{1_{igu}} x_{1_{igpu}} + \sum_{gjqv}\{\tilde{f}_{gjqv}^1 E_{co_2}^1 \phi + \tilde{f}_{gjqv}^2 E_{co_2}^2 \psi + \tilde{f}_{gjqv}^3 E_{co_2}^3 \chi\} d_{2_{gjv}} x_{2_{gjqv}} \tag{2}$$

Subject to,

$$\sum_{gpu} x_{1_{igpu}} \le \tilde{a}_i^F, \qquad i = 1,2,...,\mathrm{I} \tag{3}$$

$$\sum_{ipu} (1-\beta_{1_{igpu}}) x_{1_{igpu}} = \tilde{C}_g^F, \qquad \mathrm{g} = 1,2,...,\mathrm{G} \tag{4}$$

$$\sum_{jqv} x_{2_{gjqv}} \le \tilde{C}_g^F, \qquad g = 1,2,...,G \tag{5}$$

$$\sum_{gqv} (1-\beta_{2_{gjqv}}) x_{2_{gjqv}} \ge \tilde{b}_j^F, \qquad j = 1,2,...,\mathrm{J} \tag{6}$$

$$\sum_{gu} x_{1_{gjqv}} \le \tilde{e}_{1_p}^F, \qquad i = 1,2,...,\mathrm{I}, \mathrm{p} = 1,2,...,\mathrm{P} \tag{7}$$

$$\sum_{jv} x_{2_{gjqv}} \le \tilde{e}_{2_q}^F, \qquad g = 1,2,...,G, q = 1,2,...,\mathrm{Q} \tag{8}$$

$$\sum_{ipu} (1-\beta_{1_{igpu}}) x_{1_{igpu}} \ge \sum_{jqv} x_{2_{gjqv}}, \; g = 1,2,...,G \tag{9}$$

$$x_{1_{igpu}} \ge 0, i = 1,2,...,\mathrm{I}, \mathrm{g} = 1,2,...,\mathrm{G}, \mathrm{p} = 1,2,...,\mathrm{P}, \mathrm{u} = 1,2,...,\mathrm{U} \tag{10}$$

$$x_{2_{gjqv}} \ge 0, g = 1,2,...,G, j = 1,2,...,\mathrm{J}, q = 1,2,...,\mathrm{Q}, v = 1,2,...,\mathrm{V} \tag{11}$$

In model G1, Equations (1) and (2) describe two objective functions. The first equation represents the economic objective function, which aims to maximize profit, where $y_{1_{igpu}}$ represent fixed charges that are included only when the goods are transported from i^{th} source to g^{th} warehouse using p^{th} conveyance through u^{th} route. $y_{2_{grqv}}$ represent fixed charges which are included only when the goods are transported from g^{th} warehouse to j^{th} destination using q^{th} conveyance through v^{th} route. $y_{1_{igpu}}$ and $y_{2_{gjqv}}$ are binary variables for fixed charges in 1st stage and 2nd stage respectively, which are defined as follows:

$$y_{1_{igpu}} = \begin{cases} 1, \text{if } \mathrm{x}_{igpu} > 0 \\ 0, \text{otherwise} \end{cases}, \; y_{2_{gjqv}} = \begin{cases} 1, \text{if } \mathrm{x}_{gjqv} > 0 \\ 0, otherwise \end{cases}.$$

The second equation represents the environmental objective function, which aims to reduce carbon emissions where ϕ, ψ, and x in the second objective are binary variables which are defined as follows:

$$\phi = \begin{cases} 1, if\ diesel\ is\ used\ as\ fuel \\ 0, otherwise \end{cases}, \quad \psi = \begin{cases} 1, if\ petrol\ is\ used\ as\ fuel \\ 0, otherwise \end{cases}, \quad \chi = \begin{cases} 1, if\ CNG\ is\ used\ as\ fuel \\ 0, otherwise \end{cases}$$

Constraint (3) specifies that the number of products carried from every supplier must be less than or equal to total availability. Constraint (4) states that the number of unbroken products carried from source points to the warehouses should be equal to the capacity of the warehouse. Constraint (5) implies that the number of products carried from warehouse to retailer should not exceed the warehouse's capacity. Constraint (6) indicates the total amount of unbroken products at the retailer that should match the retailer's requirements. Constraints (7) and (8) show the aggregated amount transported by conveyance and route available in the first and second stages, respectively. Constraint (9) indicates that the quantity of products transported from warehouse to retailers should not exceed the quantity of products transported from suppliers to that warehouse. Constraints (10) and (11) describe the non-negative constraint of decision variables in the first and second stages, respectively.

While solving the model (G1), sometimes the presence of impreciseness in the data may violate non-negative constraints of the decision variables. So, the imprecise data in the model (G1) are converted to deterministic data through utilizing definition 2.8 and definition 2.9. Now the deterministic model (G2) is obtainable as shown below.

$$\text{(G2)}\ Max\, Z_1 = \sum_{j} S_j D_j - \sum_{igpu} \{\rho_i + T_{1_{igpu}} d_{1_{igu}}\} x_{1_{igpu}} - \sum_{igpu} h_{1_{igpu}} y_{1_{igpu}} - \sum_{gjqv} T_{2_{gjqv}} d_{2_{gjv}} x_{2_{gjqv}} - \sum_{gjqv} h_{2_{gjqv}} y_{2_{gjqv}} - \sum_{g} \varepsilon_g \tag{12}$$

$$Min\, Z_2 = \sum_{igpu} \{\mathrm{f}^1_{igpu} E^1_{co_2}\phi + \mathrm{f}^2_{igpu} E^2_{co_2}\psi + \mathrm{f}^3_{igpu} E^3_{co_2}\chi\} d_{1_{igu}} x_{1_{igpu}} + \sum_{gjqv} \{\mathrm{f}^1_{gjqv} E^1_{co_2}\phi + \mathrm{f}^2_{gjqv} E^2_{co_2}\psi + \mathrm{f}^3_{gjqv} E^3_{co_2}\chi\} d_{2_{gjv}} x_{2_{gjqv}} \tag{13}$$

Subject to,

$$\sum_{gpu} x_{1_{igpu}} \le a_i, \qquad i = 1,2,\ldots,\mathrm{I} \tag{14}$$

$$\sum_{ipu} (1-\beta_{1_{igpu}}) x_{1_{igpu}} = C_g, \qquad g = 1,2,\ldots,\mathrm{G} \tag{15}$$

$$\sum_{jqv} x_{2_{gjqv}} \le C_g, \qquad g = 1,2,\ldots,G \tag{16}$$

$$\sum_{gqv} (1-\beta_{2_{gjqv}}) x_{2_{gjqv}} \ge b_j, \qquad j = 1,2,\ldots,\mathrm{J} \tag{17}$$

$$\sum_{gu} x_{1_{grqv}} \le e_{1_p}, \qquad i = 1,2,\ldots,\mathrm{I},\ p = 1,2,\ldots,\mathrm{P} \tag{18}$$

$$\sum_{jv} x_{2_{gjqv}} \le e_{2_q}, \qquad g = 1,2,\ldots,G,\ q = 1,2,\ldots,\mathrm{Q} \tag{19}$$

$$\sum_{ipu} (1-\beta_{1_{igpu}}) x_{1_{igpu}} \ge \sum_{jqv} x_{2_{gjqv}},\ g = 1,2,\ldots,G \tag{20}$$

$$x_{1_{igpu}} \ge 0, i = 1,2,\ldots,\mathrm{I}, g = 1,2,\ldots,\mathrm{G}, p = 1,2,\ldots,\mathrm{P}, u = 1,2,\ldots,\mathrm{U} \tag{21}$$

$$x_{2_{gjqv}} \ge 0, g = 1,2,\ldots,G, j = 1,2,\ldots,\mathrm{J}, q = 1,2,\ldots,\mathrm{Q}, v = 1,2,\ldots,\mathrm{V} \tag{22}$$

The reduced deterministic bi-objective model (G2) is then turned into a single-objective model (R1) by using Fermatean hesitant fuzzy programming, and then the model (R1) has been solved by employing Lingo 18.0 software to determine the optimal compromise solution. The next section focuses on solution methodology together with the proposed transformation technique, namely Fermatean hesitant fuzzy programming, which is based on Ghosh et al. (2022) to solve two-stage MOFC4-DTP under a Fermatean fuzzy environment.

4. Solution Methodology

This section describes the solution methodology for the Fermatean fuzzy 2-stage BOFC4D-TP.

Step 1: Develop the model (G2) from the model (G1).

i. To convert TFFN parameters into IVFFS, we utilize the (α, β)-cut approach as defined in 2.8.

ii. Use the definition 2.9 to convert IVFFS obtained from step 1 (I) into an equivalent deterministic form with alpha and beta values ranging from 0 to 1.

Step 2: Now, the developed model (G2) is converted to an equivalent single-objective FC4D-TP (R1) by using our proposed Fermatean hesitant fuzzy programming in the following manner:

Fermatean Hesitant Fuzzy Programming

The following procedure is provided to convert the 2-stage BOFC4D-TP to a 2-stage SOFC4D-TP using the Fermatean Hesitant Fuzzy Programming **(**FHFP) method:

Here, a novel modified programming approach FHFP, is proposed based on Ghosh et al. (2022), in which the author introduced Pythagorean hesitant fuzzy programming method, which is a hybrid combination of the Pythagorean fuzzy set and the Hesitant fuzzy set. By considering the advantage of Fermatean fuzzy sets over Pythagorean fuzzy sets as in **Figure 1**, we have proposed a hybrid combination of the fermatean fuzzy set (Senapati and Yager, 2020) and the hesitant fuzzy set (Torra and Narukawa, 2009) based on (Ghosh et al., 2022). It is introduced to identify the optimal compromise solution of the proposed 2-stage BOFC4D-TP. The proposed transformation technique FHFP increases Fermatean hesitant MF while decreasing Fermatean hesitant NMF. Bellman and Zadeh (1970), introduced the notion of fuzzy set decision-making. The fuzzy decision set ($\tilde{D}$) on a decision-making problem is defined as the intersection of the fuzzy objective function ($\tilde{G}$) and fuzzy constraint ($\tilde{C}$). Fermatean hesitant fuzzy (FHF) decision ($\tilde{D}_h^F$) is an extension of fuzzy decision that can be represented as $\tilde{D}_h^F = \tilde{G}_h^F \cap \tilde{C}_h^F = \left\{x, \mu_h^F, \gamma_h^F\right\}$, where $\mu_h^F(x) \in \left(\mu_{G_h^F}(x) \cap \mu_{C_h^F}(x)\right)$: $\mu_h^F(x) \le \min\left\{\max\left(\mu_{G_h^F}(x) \cap \mu_{C_h^F}(x)\right)\right\}$ and $\gamma_h^F(x) \in \left(\gamma_{G_h^F}(x) \cap \gamma_{C_h^F}(x)\right)$: $\max\left\{\min\left(\gamma_{G_h^F}(x) \cap \gamma_{C_h^F}(x)\right)\right\}$. $\mu_{D_h^F}(x)$ and $\gamma_{D_h^F}(x)$ are the sets of membership values of approval and denial of the FHF solution under the FHF decision set, respectively. $\tilde{G}_h^F$, $\tilde{C}_h^F$ are the hesitant fuzzy (HF) objective function and HF constraints. By using the FHF solution standards. The step-by-step process for solving the model (G2) shown below:

Step a: Solve each deterministic objective individually by considering all constraints for finding 's' solutions set ($x_1, x_2, \dots x_s$) of each objective.

Step b: Write the payoff matrix by substituting the solution obtained from Step a into each objective function and calculate the upper bound (i.e. $U_s = \max\{z_s(\mathrm{x}_s)\}$) and lower bound (i.e. $L_s = \min\{z_s(\mathrm{x}_s)\}$) for each objective.

Step c: Using the upper and lower bound of each objective, we formulated the MF and NMF based on Ghosh et al. (2022) for each objective function for FHFN are as follows:

For minimization objective,

$$\mu_h^F = \begin{cases} 1, & if\ Z_s(\mathrm{x}) < \mathrm{L}_s \\ \alpha_s\left(\dfrac{U_s - Z_s(\mathrm{x})}{U_s - L_s}\right), & if\ L_s \le Z_s(\mathrm{x}) \le U_s \\ 0, & if\ Z_s(\mathrm{x}) > U_s \end{cases} \tag{23}$$

$$\gamma_h^F = \begin{cases} 0, & if\ Z_s(\mathrm{x}) < \mathrm{L}_s \\ \beta_s\left(\dfrac{Z_s(\mathrm{x}) - \mathrm{L}_s}{U_s - L_s}\right), & if\ L_s \le Z_s(\mathrm{x}) \le U_s \\ 1, & if\ Z_s(\mathrm{x}) > U_s \end{cases} \tag{24}$$

For maximization objective,

$$\mu_h^F = \begin{cases} 0, & if\ Z_s(\mathrm{x}) < \mathrm{L}_s \\ \alpha_s\left(\dfrac{Z_s(\mathrm{x}) - \mathrm{L}_s}{U_s - L_s}\right), & if\ L_s \le Z_s(\mathrm{x}) \le U_s \\ 1, & if\ Z_s(\mathrm{x}) > U_s \end{cases} \tag{25}$$

$$\gamma_h^F = \begin{cases} 1, & if\ Z_s(\mathrm{x}) < \mathrm{L}_s \\ \beta_s\left(\dfrac{U_s - Z_s(\mathrm{x})}{U_s - L_s}\right), & if\ L_s \le Z_s(\mathrm{x}) \le U_s \\ 0, & if\ Z_s(\mathrm{x}) > U_s \end{cases} \tag{26}$$

The values of hesitant corresponding to MF and NMF are α_s and β_s respectively lies between the interval 0 and 1. (i.e. $\alpha_s, \beta_s \in [0, 1]$).

Step d: In order to determine the maximum value of satisfaction and the minimum value of rejection, the FHFP method corresponding to two-stage BOFCTP can be defined in the subsequent model (R1) and can be represented as follows:

(R1) $$Maximize\ \frac{\sum \xi_s^3}{s} - \frac{\sum \varphi_s^3}{s} \tag{27}$$

Subject to

$$\left[\mu_h^F\left(Z_s(\mathrm{x})\right)\right]^3 \ge \xi_s^3 \tag{28}$$

$$\left[\gamma_h^F(\mathrm{Z}_s(\mathrm{x}))\right]^3 \le \varphi_s^3 \tag{29}$$

$$\xi_s^3 \ge \varphi_s^3 \tag{30}$$

$$\xi_s^3 + \varphi_s^3 \in [0,1], \xi_s^3 \in [0,1], \varphi_s^3 \in [0,1] \tag{31}$$

with respect to the constraints (14)-(22)

Here, s is the number of objectives, ξ and ϕ are the values of MF and NMF of each objective function. Now by using the model (R1), two-stage BOFC4D-TP is reduced into a two-stage SOFCTP.

Step 3: Now the reduced two-stage SOFC4-DTP (R1) obtained in step 2 is solved by using Lingo 18.0 software to determine OCS.

The strategy for solving two-stage BOFC4D-TP under a Fermatean fuzzy environment is shown in **Figure 3** below.

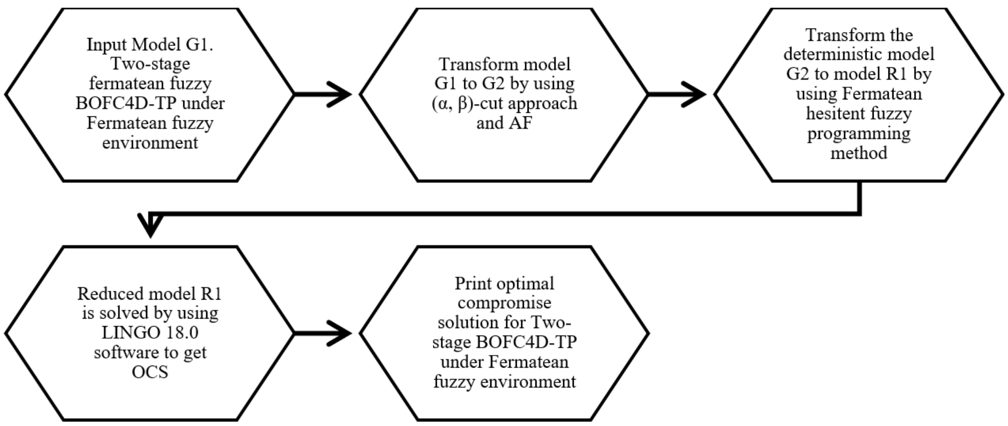

Figure 3. Flowchart for solving the two-stage BOFC4D-TP under Fermatean fuzzy environment.

5. Numerical Example

Glass bottles are produced by Hindustan Glass and Industries Limited, a prominent glassware manufacturing firm in India. The product, made of glass material, is purchased in huge numbers from two manufacturing factories (i=2) located in Chennai and Bangalore. Then, the products are kept in two warehouses (g=2) located at Vellore, Chengalpattu. Then, the products are sold at four different stores (j=4) located at Krishnagiri, Tiruvannamalai, Viluppuram, and Cuddalore. Furthermore, there are linkages between each and every manufacturing firm and warehouse, and also between warehouses and retail stores, which are made possible by two different pathways. These routes allow for two sorts of transportation modes: big trucks and medium trucks, which move breakable items through both stages. In this type of industrial scenario, fixed costs can be observed in couple of distinct ways. For highway transportation, the organization will pay a toll tax for different mode of conveyance used to convey goods from factories to customers, as well as vehicle maintenance costs. The entrepreneurs wish to reduce overall transportation costs (transportation costs and fixed costs), as well as total CO_2 emissions from various forms of transportation on different routes. We estimate the transportation cost per ton in rupees for an open route and CO_2 emissions in *gm/l*. As a result of unforeseen occurrences at the plant, road traffic, weather circumstances, and so on, the input parameters are treated as TFFN. It is critical to investigate ways to improve profits while lowering carbon emissions. Here, we use different types of fuels (diesel, petrol, and compressed natural gas (CNG)), which are explored here by changing fuel type in each stage, and the influence on carbon emissions is examined via case studies.

The following **Tables (3)** to **(8)** shows the values of all input parameters with their deterministic values, which are obtained by using step 1.

Table 3. Availability $\tilde{a}_i^F$, demand $\tilde{b}_j^F$, capacity of the warehouse $\tilde{C}_g^F$, Capacity of the conveyances in two stages $\tilde{e}_{1_p}^F$, $\tilde{e}_{2_q}^F$ holding price ε_g with its corresponding deterministic value.

$\tilde{a}_1^F = (280,300,320;260,300,340)$, $M(\tilde{a}_1^F) = 600$	$\tilde{a}_2^F = (180,200,220;160,200,240)$, $M(\tilde{a}_2^F) = 400$
$\tilde{b}_1^F = (120,150,180;100,150,200)$, $M(\tilde{b}_1^F) = 40$	$\tilde{b}_2^F = (105,115,125;95,115,140)$, $M(\tilde{b}_2^F) = 60$
$\tilde{b}_3^F = (60,80,100;40,80,120)$, $M(\tilde{b}_3^F) = 75$	$\tilde{b}_4^F = (250,270,290;220,270,310)$, $M(\tilde{b}_4^F) = 90$
$\tilde{C}_1^F = (400,900,1000;200,900,2000)$, $M(\tilde{C}_1^F) = 1600$	$\tilde{C}_2^F = (500,800,900;300,800,1000)$, $M(\tilde{C}_2^F) = 1500$
$\tilde{e}_{1_1}^F = (400,500,600;400,500,750)$, $M(\tilde{e}_{1_1}^F) = 300$	$\tilde{e}_{1_2}^F = (500,600,650;450,600,720)$, $M(\tilde{e}_{1_2}^F) = 200$
$\tilde{e}_{2_1}^F = (500,600,650;450,600,720)$, $M(\tilde{e}_{2_1}^F) = 350$	$\tilde{e}_{2_2}^F = (200,275,350;190,275,500)$, $M(\tilde{e}_{2_2}^F) = 100$
$\varepsilon_1 = (5,7,9;2,7,13)$, $M(\varepsilon_1) = 14$	$\varepsilon_2 = (6,8,12;2,8,13)$, $M(\varepsilon_2) = 17$

Table 4. Unit selling prices S_j, distance in different routes $d_{1_{igu}}$, $d_{2_{gjv}}$, rate of breakability in different routes $\beta_{1_{igpu}}$, $\beta_{2_{gjqv}}$ and amount of carbon emissions produces per unit for diesel $E_{co_2}^1$, petrol $E_{co_2}^2$ and CNG $E_{co_2}^3$.

$S_1 = 207$, $S_2 = 237$, $S_3 = 228$, $S_4 = 262$
$d_{1_{111}} = 50$, $d_{1_{121}} = 42$, $d_{1_{211}} = 40$, $d_{1_{221}} = 53$, $d_{1_{112}} = 40$, $d_{1_{122}} = 30$, $d_{1_{212}} = 35$, $d_{1_{222}} = 40$
$d_{2_{111}} = 11$, $d_{2_{121}} = 13$, $d_{2_{131}} = 8$, $d_{2_{141}} = 10$, $d_{2_{121}} = 9$, $d_{2_{221}} = 11$, $d_{2_{231}} = 12$, $d_{2_{241}} = 16$ $d_{2_{112}} = 9$, $d_{2_{122}} = 15$, $d_{2_{132}} = 6$, $d_{2_{142}} = 8$, $d_{2_{212}} = 10$, $d_{2_{222}} = 7$, $d_{2_{232}} = 6$, $d_{2_{242}} = 10$
$\beta_{1_{1111}} = 0.01$, $\beta_{1_{1121}} = 0.012$, $\beta_{1_{1211}} = 0.016$, $\beta_{1_{1221}} = 0.019$, $\beta_{1_{2111}} = 0.019$, $\beta_{1_{2121}} = 0.016$, $\beta_{1_{2211}} = 0.015$, $\beta_{1_{2221}} = 0.013$, $\beta_{1_{1112}} = 0.011$, $\beta_{1_{1122}} = 0.015$, $\beta_{1_{1212}} = 0.011$, $\beta_{1_{1222}} = 0.015$, $\beta_{1_{2112}} = 0.015$, $\beta_{1_{2122}} = 0.01$, $\beta_{1_{2212}} = 0.011$, $\beta_{1_{2222}} = 0.018$
$\beta_{2_{1111}} = 0.011$, $\beta_{2_{1121}} = 0.015$, $\beta_{2_{1211}} = 0.015$, $\beta_{2_{1221}} = 0.015$, $\beta_{2_{1311}} = 0.01$, $\beta_{2_{1321}} = 0.011$, $\beta_{2_{1411}} = 0.013$, $\beta_{2_{1421}} = 0.01$, $\beta_{2_{2111}} = 0.014$, $\beta_{2_{2121}} = 0.013$, $\beta_{2_{2211}} = 0.01$, $\beta_{2_{2221}} = 0.011$, $\beta_{2_{2311}} = 0.016$, $\beta_{2_{2321}} = 0.012$, $\beta_{2_{2411}} = 0.01$, $\beta_{2_{2421}} = 0.018$, $\beta_{2_{1112}} = 0.017$, $\beta_{2_{1122}} = 0.019$, $\beta_{2_{1212}} = 0.019$, $\beta_{2_{1222}} = 0.016$, $\beta_{2_{1312}} = 0.012$, $\beta_{2_{1322}} = 0.01$, $\beta_{2_{1412}} = 0.015$, $\beta_{2_{1422}} = 0.017$, $\beta_{2_{2112}} = 0.018$, $\beta_{2_{2122}} = 0.014$, $\beta_{2_{2212}} = 0.013$, $\beta_{2_{2222}} = 0.012$, $\beta_{2_{2312}} = 0.011$, $\beta_{2_{2322}} = 0.014$, $\beta_{2_{2412}} = 0.012$, $\beta_{2_{2422}} = 0.015$
$E_{co_2}^1 = 0.6\ gm/l$, $E_{co_2}^2 = 0.55\ gm/l$, $E_{co_2}^3 = 0.5\ gm/l$

Table 5. Transportation cost and deterministic transportation cost ($\tilde{T}_{1_{igpu}}^F$, $M(\tilde{T}_{1_{igpu}}^F)$), fixed cost and deterministic fixed cost ($\tilde{h}_{igpu}^F$, $M(\tilde{h}_{igpu}^F)$) (1st stage).

i	g	p	($\tilde{T}_{1_{igpu}}^F$, $M(\tilde{T}_{1_{igpu}}^F)$), ($\tilde{h}_{igpu}^F$, $M(\tilde{h}_{igpu}^F)$)	($\tilde{T}_{1_{igpu}}^F$, $M(\tilde{T}_{1_{igpu}}^F)$), ($\tilde{h}_{igpu}^F$, $M(\tilde{h}_{igpu}^F)$)
1	1	1	((0.05,0.07,0.09;0.04,0.07,0.10), 0.14), ((80,100,120;60,100,140), 200)	((0.03,0.05,0.09;0.01,0.05,0.11), 0.11), ((120,140,160;100,140,180), 280)
		2	((0.07,0.09,0.11;0.06,0.09,0.12), 0.18), ((60,80,100;40,80,120), 160)	((0.05,0.07,0.10;0.03,0.07,0.12), 0.145), ((120,140,160;100,140,180), 280)
	2	1	((0.04,0.06,0.08;0.03,0.06,0.09), 0.12), ((150,170,190;130,170,210), 340)	((0.04,0.07,0.11;0.02,0.07,0.13), 0.145), ((70,90,110;50,90,130), 180)
		2	((0.05,0.07,0.09;0.04,0.07,0.10), 0.14), ((150,170,190;130,170,210), 340)	((0.03,0.05,0.08;0.01,0.05,0.11), 0.105), ((80,100,120;60,100,140), 200)
2	1	1	((0.07,0.08,0.09;0.06,0.08,0.10), 0.16), ((110,130,150;90,130,170), 260)	((0.04,0.07,0.1;0.02,0.07,0.13), 0.14), ((40,60,80;20,60,100),120)
		2	(0.09,0.10,0.11;0.08,0.10,0.12), 0.2), ((30,50,70;10,50,90),100)	((0.05,0.08,0.10;0.03,0.08,0.12), 0.155), ((90,110,130;70,110,150),120)
	2	1	((0.06,0.08,0.10;0.05,0.08,0.11),0.16), ((40,60,80;20,60,100),120)	((0.02,0.04,0.07;0.01,0.04,0.08),0.085), ((120,140,160;100,140,180),280)
		2	((0.08,0.10,0.12;0.07,0.10,0.13),0.2), ((150,160,170;140,160,180),320)	((0.03,0.05,0.08;0.02,0.05,0.09),0.105), ((90,100,110;80,100,120),200)

Table 6. Transportation cost and deterministic transportation cost ($\tilde{T}^F_{2_{grqv}}$, $\mathrm{M}\left(\tilde{T}^F_{2_{grqv}}\right)$), fixed cost and deterministic fixed cost ($\tilde{h}^F_{grqv}$, $\mathrm{M}\left(\tilde{h}^F_{grqv}\right)$) (2nd stage).

g	j	q	($\tilde{T}^F_{2_{grqv}}$, $\mathrm{M}\left(\tilde{T}^F_{2_{grqv}}\right)$), ($\tilde{h}^F_{grqv}$, $\mathrm{M}\left(\tilde{h}^F_{grqv}\right)$)	($\tilde{T}^F_{2_{grqv}}$, $\mathrm{M}\left(\tilde{T}^F_{2_{grqv}}\right)$), ($\tilde{h}^F_{grqv}$, $\mathrm{M}\left(\tilde{h}^F_{grqv}\right)$)
1	1	1	((0.10,0.11,0.12;0.09,0.11,0.13), 0.22), ((110,130,150;90,130,170),260)	((0.07,0.08,0.09;0.06,0.08,0.10), 0.16), ((40,60,80;20,60,100),120)
		2	((0.11,0.12,0.13;0.10,0.12,0.14),0.24), ((80,100,120;60,100,140),200)	((0.04,0.06,0.08;0.03,0.06,0.09),0.12), ((120,140,160;100,140,180),280)
	2	1	((0.05,0.06,0.07;0.04,0.06,0.08),0.12), ((30,50,70;10,50,90),100)	((0.06,0.07,0.10;0.02,0.07,0.11),0.15), ((90,110,130;70,110,150),220)
		2	((0.07,0.08,0.09;0.06,0.08,0.10),0.16), ((60,80,100;40,80,120),160)	((0.04,0.08,0.13;0.01,0.08,0.14),0.165), ((120,140,160;100,140,180),280)
	3	1	((0.09,0.10,0.11;0.08,0.10,0.12),0.2), ((40,60,80;20,60,100),120)	((0.04,0.05,0.07;0.01,0.05,0.09),0.105), ((120,140,160;100,140,180),280)
		2	((0.11,0.12,0.13;0.10,0.12,0.14), 0.24), ((150,170,190;130,170,210),340)	((0.04,0.06,0.16;0.01,0.06,0.20), 0.16), ((120,140,160;100,140,180),180)
	4	1	((0.14,0.15,0.16;0.13,0.15,0.17),0.3), ((150,160,170;140,160,180),320)	((0.06,0.07,0.10;0.02,0.07,0.11),0.15), ((90,100,110;80,100,120),200)
		2	((0.15,0.16,0.17;0.14,0.16,0.18), 0.32), ((150,170,190;130,170,210),340)	((0.03,0.08,0.16;0.01,0.08,0.19), 0.175), ((80,100,120;60,100,140),200)
2	1	1	((0.03,0.05,0.09;0.01,0.05,0.11), 0.11), ((90,100,110;80,100,120),200)	((0.05,0.07,0.09;0.04,0.07,0.10), 0.14), ((120,140,160;100,140,180),280)
		2	((0.07,0.09,0.11;0.06,0.09,0.12), 0.18), ((90,110,130;70,110,150),220)	((0.05,0.07,0.10;0.03,0.07,0.12), 0.145), ((40,60,80;20,60,100),120)
	2	1	((0.04,0.07,0.11;0.02,0.07,0.13), 0.145), ((80,100,120;60,100,140),200)	((0.04,0.06,0.08;0.03,0.06,0.09), 0.12), ((70,90,110;50,90,130),180)
		2	((0.05,0.07,0.09;0.04,0.07,0.10), 0.14), ((120,140,160;100,140,180),280)	((0.03,0.05,0.08;0.01,0.05,0.11), 0.105), ((120,140,160;100,140,180),280)
	3	1	((0.04,0.07,0.10;0.02,0.07,0.13), 0.14), ((150,160,170;140,160,180),320)	((0.07,0.08,0.09;0.06,0.08,0.10), 0.16), ((40,60,80;20,60,100),120)
		2	((0.09,0.10,0.11;0.08,0.10,0.12),0.2), ((30,50,70;10,50,90),100)	((0.05,0.08,0.10;0.03,0.08,0.12),0.155), ((110,130,150;90,130,170),260)
	4	1	((0.02,0.04,0.07;0.01,0.04,0.08),0.085), ((150,170,190;130,170,210),340)	((0.06,0.08,0.10;0.05,0.08,0.11),0.16), ((60,80,100;40,80,120),160)
		2	((0.08,0.10,0.12;0.07,0.10,0.13),0.2), ((80,100,120;60,100,140),200)	((0.03,0.05,0.08;0.02,0.05,0.09),0.105), ((150,170,190;130,170,210),340)

Table 7. Diesel consumption rate f^1_{igpu} , petrol consumption rate f^2_{igpu} and CNG consumption rate f^3_{igpu} (1st stage) with corresponding deterministic value.

i	g	p	u	(f^1_{igpu} , $M(f^1_{igpu})$)	(f^2_{igpu} , $M(f^2_{igpu})$)	(f^3_{igpu} , $M(f^3_{igpu})$)
1	1	1	1	((0.5,0.7,0.9;0.3,0.7,1), 1.4)	((0.3,0.5,0.7;0.1,0.7,0.8), 1)	((0.2,0.3,0.4;0.1,0.3,0.5), 0.6)
			2	((1.5,1.8,2.2;1.1,1.8,2.5), 3.65)	((1.0,1.2,1.4;0.8,1.2,1.6), 2.4)	((0.4,0.5,0.6;0.2,0.5,0.9),1)
		2	1	((1.1,1.4,1.6;0.9,1.4,1.8), 2.75)	((1.0,1.3,1.5;0.8,1.3,1.7),2.55)	((0.7,0.9,1.0;0.5,0.9,1.1),1.75)
			2	((0.4,0.6,0.9;0.1,0.6,1.1), 1.25)	((0.4,0.7,0.9;0.2,0.7,1.1),1.35)	((0.2,0.3,0.4;0.1,0.3,0.5),0.6)
	2	1	1	((1.2,1.3,1.5;1.1,1.3,1.6), 2.65)	((1.4,1.5,1.6;1.2,1.5,1.7),3)	((0.6,0.8,0.9;0.4,0.8,1.0),1.55)
			2	((1.9,2.2,2.5;1.7,2.2,2.7), 4.4)	((1.2,1.4,1.5;0.8,1.4,1.6), 2.75)	((0.7,0.9,1.0;0.5,0.9,1.2), 1.75)
		2	1	((0.6,0.8,0.9;0.4,0.8,1.1), 1.55)	((1.1,1.3,1.4;0.7,1.3,1.5), 2.55)	((0.6,0.8,0.9;0.4,0.8,1.2), 1.55)
			2	((0.9,1.2,1.5;0.7,1.2,1.8),2.4)	((1.2,1.6,1.8;0.8,1.6,1.9),3.1)	((0.6,0.7,0.8;0.5,0.7,1.0),1.4)
2	1	1	1	((1.3,1.7,2;1,1.7,2.3), 3.35)	((1.1,1.2,1.4;0.5,1.2,1.6), 2.45)	((0.6,0.8,0.9;0.4,0.8,1.1), 1.55)
			2	((0.8,1.1,1.4;0.5,1.1,1.7),2.2)	((0.2,0.3,0.4;0.1,0.3,0.5), 0.6)	((0.2,0.3,0.4;0.1,0.3,0.5), 0.6)
		2	1	((0.4,0.7,0.9;0.2,0.7,1.1),1.35)	((0.3,0.4,0.5;0.1,0.4,0.6),0.8)	((0.2,0.3,0.4;0.1,0.3,0.5), 0.6)
			2	((0.4,0.6,0.9;0.2,0.6,1.1),1.25)	((2.0,2.5,3.0;1.5,2.5,3.5),5)	((0.8,1.0,1.2;0.6,1.0,1.4),2)
	2	1	1	((1.5,1.9,2.2;1.3,1.9,2.5),3.75)	((2.0,3.0,4.0;1.0,3.0,4.5),6)	((0.9,1.0,1.2;0.6,1.0,1.4),2.05)
			2	((2.5,2.8,3.2;2.1,2.8,3.5),5.65)	((0.7,1.0,1.2;0.5,1.0,1.4),1.95)	((0.6,0.9,1.0;0.4,0.9,1.1),1.7)
		2	1	((1.3,1.6,1.9;1,1.6,2.2),3.2)	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)
			2	((3.2,3.6,4;2.8,3.6,4.3),7.2)	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)

Table 8. Diesel consumption rate f^1_{grqv}, Petrol consumption rate f^2_{grqv} and CNG consumption rate f^3_{grqv} (2nd stage) with corresponding deterministic value.

g	j	q	v	$(f^1_{grqv}, M(f^1_{grqv}))$	$(f^2_{grqv}, M(f^2_{grqv}))$	$(f^3_{grqv}, M(f^3_{grqv}))$
1	1	1	1	((1.6,2.0,2.4;1.3,2.0,2.7),4)	((1.0,1.5,2.0;0.5,1.5,2.5),3)	((0.5,0.6,0.7;0.3,0.6,0.8),1.2)
			2	((0.3,0.4,0.5;0.1,0.4,0.6),0.8)	((0.9,1.0,1.2;0.6,1.0,1.4),2.05)	(0.6,0.8,0.9;0.3,0.8,1.0),1.55)
		2	1	((0.8,1.2,1.4;0.6,1.2,1.6),2.3)	((0.3,0.6,0.9;0.1,0.6,1.2),1.2)	((0.2,0.4,0.6;0.1,0.4,0.8),0.8)
			2	((0.4,0.6,0.8;0.2,0.6,1.2),1.2)	((1.3,1.4,1.5;1.2,1.3,1.6),2.8)	((0.9,1.0,1.1;0.6,1.0,1.2),2)
	2	1	1	((0.5,0.8,1.3;0.2,0.8,1.5),1.7)	((1.1,1.3,1.5;0.8,1.3,1.8),2.6)	((0.4,0.6,0.9;0.3,0.6,1.0),1.25)
			2	((2.7,3.1,3.7;2.3,3.1,4.1),6.3)	((1.5,1.6,1.8;1.1,1.6,1.9),3.25)	((0.8,0.9,1.2;0.7,0.9,1.4),1.9)
		2	1	((2.0,2.4,2.9;1.7,2.4,3.3),4.85)	((1.1,1.4,1.6;0.5,1.4,1.8),2.75)	((0.5,0.8,0.9;0.3,0.8,1.1),1.5)
			2	((3.5,4.0,4.5;3.0,4.0,4.9),8)	((1.3,1.4,1.5;1.1,1.4,1.6),2.8)	((0.7,0.9,1.1;0.5,0.9,1.2),1.8)
	3	1	1	((0.7,1.0,1.3;0.5,1.0,1.6),2)	((1.5,1.8,2.0;1.2,1.8,2.2),3.55)	((0.8,0.9,1.1;0.7,0.9,1.3),1.85)
			2	((0.8,1.1,1.5;0.6,1.1,1.7),2.25)	((0.6,1.0,1.2;0.4,1.0,1.6),1.9)	((0.4,0.6,0.8;0.2,0.6,1.0),1.2)
		2	1	((0.7,1.0,1.3;0.5,1.1,1.6),2)	((0.4,0.7,1.0;0.1,0.7,1.3),1.4)	((0.3,0.4,0.5;0.2,0.4,0.6),0.8)
			2	((1.7,1.9,2.2;1.5,1.7,2.5),3.85)	((1.5,2.0,2.5;1.0,2.0,3.0),4)	((0.6,0.8,1.0;0.4,0.8,1.2),1.6)
	4	1	1	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)	((0.5,0.9,1.0;0.3,0.9,1.2),1.65)	((0.4,0.6,0.9;0.3,0.6,1.2),1.25)
			2	((3.1,3.9,4.4;2.5,3.9,5.0),7.65)	((1.0,1.5,2.0;0.5,1.5,2.5),3)	((0.8,0.9,1.0;0.7,0.9,1.2),1.8)
		2	1	((0.2,0.3,0.6;0.1,0.3,0.7),0.7)	((2.5,3.0,3.5;2.0,3.0,4.0),6)	((0.5,0.8,1.0;0.4,0.8,1.2),1.55)
			2	((2.4,3.0,3.7;1.9,3.0,4.1),6.05)	((2.0,2.5,3.0;1.5,2.5,3.5),5)	((0.8,1.0,1.2;0.6,1.0,1.4),2)
2	1	1	1	((1.65,2.05,2.45;1.35,2.05,2.75),4.1)	((1.05,1.55,2.05;0.55,1.55,2.55),3.1)	((0.55,0.65,0.75;0.35,0.65,0.85),1.3)
			2	((0.35,0.45,0.55;0.15,0.45,0.65),0.9)	((0.95,1.05,1.25;0.65,1.05,1.45),2.15)	((0.65,0.85,0.95;0.35,0.85,1.05),1.65)
		2	1	((0.85,1.25,1.45;0.65,1.25,1.65),2.4)	((0.35,0.65,0.95;0.15,0.65,1.25),1.3)	((0.25,0.45,0.65;0.15,0.45,0.85),0.9)
			2	((0.45,0.65,0.85;0.25,0.65,1.25),1.3)	((1.35,1.45,1.55;1.25,1.35,1.65),2.9)	((0.95,1.05,1.15;0.65,1.05,1.25),2.1)
	2	1	1	((0.55,0.85,1.35;0.25,0.85,1.55),1.8)	((1.15,1.35,1.55;0.85,1.35,1.85),2.7)	((0.45,0.65,0.95;0.35,0.65,1.05),1.35)
			2	((2.75,3.15,3.75;2.35,3.15,4.15),6.4)	((1.55,1.65,1.85;1.15,1.65,1.95),3.35)	((0.85,0.95,1.25;0.75,0.95,1.45),2)
		2	1	((2.05,2.45,2.95;1.75,2.45,3.35),4.95)	((1.15,1.45,1.65;0.55,1.45,1.85),2.025)	((0.55,0.85,0.95;0.35,0.85,1.15),1.6)
			2	((3.55,4.05,4.55;3.05,4.05,4.95),8.1)	((1.35,1.45,1.55;1.15,1.45,1.65),2.9)	((0.75,0.95,1.15;0.55,0.95,1.25),1.9)
	3	1	1	((0.75,1.05,1.35;0.55,1.05,1.65),2)	((1.55,1.85,2.05;1.25,1.85,2.25),3.65)	((0.85,0.95,1.15;0.75,0.95,1.35),1.475)
			2	((0.85,1.15,1.55;0.65,1.15,1.75),2.35)	((1.65,1.05,1.25;0.45,1.05,1.65),2)	((0.45,0.65,0.85;0.25,0.65,1.05),1.3)
		2	1	((0.75,1.05,1.35;0.55,1.05,1.65),2.1)	((0.45,0.75,1.0;0.15,0.75,1.35),1.475)	((0.35,0.45,0.55;0.25,0.45,0.65),0.9)
			2	((1.75,1.95,2.25;1.55,1.75,2.55),3.95)	((1.55,2.05,2.55;1.05,2.05,3.05),4.1)	((0.65,0.85,1.05;0.45,0.85,1.25),1.7)
	4	1	1	((0.25,0.35,0.65;0.15,0.35,0.75),0.8)	((0.55,0.95,1.05;0.35,0.95,1.25),1.75)	((0.45,0.65,0.95;0.35,0.65,1.25),1.35)
			2	((3.15,3.95,4.45;2.55,3.95,5.05),7.75)	((1.05,1.55,2.05;0.55,1.55,2.55),3.1)	((0.85,0.95,1.05;0.75,0.95,1.25),1.9)
		2	1	((0.25,0.35,0.65;0.15,0.35,0.75),0.8)	((2.55,3.05,3.55;2.05,3.05,4.05),6.1)	((0.55,0.85,1.05;0.45,0.85,1.25),1.65)
			2	((2.45,3.05,3.75;1.95,3.05,4.15),6.15)	((2.05,2.55,3.05;1.55,2.55,3.55),5.1)	((0.85,1.05,1.25;0.65,1.05,1.45),2.1)

Now, the two-stage BOFC4-DTP is obtained in deterministic form (G2) by using step 1. If we solve two-stage BOFC4-DTP directly, the deterministic bi-objective problem is too complex and time-consuming. By step 2, to simplify the computing procedures and to reduce time, we convert two-stage BOFC4-DTP into a two-stage SOFC4-DTP. Although the literature has numerous methods for reducing a two-stage BOFC4-DTP into a two-stage SOFC4-DTP. In this study, we employed the Fermatean Hesitant Fuzzy Programming method for the transformation because it is easy to implement with less computational effort. In order to start applying the Fermatean Hesitant Fuzzy Programming method in the two-stage BOFC4-DTP, by step a, one objective should be selected at a time subject to all constraints and solved using Lingo 18.0 software for ideal solutions. Then, by step b, by substituting the obtained solution in step a, we determined the payoff matrices, upper and lower bounds for each objective. Here, we considered two-stage BOFC4-DTP in nine cases by varying the fuel type (diesel, petrol, and CNG) in both stages. The payoff matrices, the upper and lower bounds of each objective of cases 1-9, are given below in **Table 9**.

Then, by step c and step d, construct the mathematical model R2 (for case 1) using the Fermatean Hesitant Fuzzy Programming method (R1) in the following way:

$$\text{(R2)}\ Maximize\ \frac{\xi_1^3+\xi_2^3}{2}-\frac{\varphi_1^3+\varphi_2^3}{2}$$

$$\text{Subject to}\ \left((0.5)\left(\frac{Z_1(\mathrm{x})-39603.45}{40303.57-39603.45}\right)\right)^3 \geq \xi_1^3,$$

$$\left((0.5)\left(\frac{22327.31 - Z_2(\mathrm{x})}{22327.31 - 17226.57}\right)\right)^3 \geq \xi_2^3.$$

$$\left((0.5)\left(\frac{40303.57 - Z_1(\mathrm{x})}{40303.57 - 39603.45}\right)\right)^3 \leq \varphi_1^3,$$

$$\left((0.5)\left(\frac{Z_2(\mathrm{x}) - 17226.57}{22327.31 - 17226.57}\right)\right)^3 \leq \varphi_2^3.$$

$$\xi_s^3 \geq \varphi_s^3,$$

$$\xi_s^3 + \varphi_s^3 \in [0,1], \xi_s^3 \in [0,1], \varphi_s^3 \in [0,1], \mathrm{s} = 1,2.$$

with respect to the constraints (14)-(22),

where, ξ and φ are the values of MF and NMF of each objective function. The values of α_s and β_s values lie between 0 and 1, here we choose α_s as 0.5 and β_s as 0.5.

Table 9. Payoff matrices, the upper and lower bound of each objective of cases 1-9.

	Case 1 (Diesel in first and second stage)		**Case 2** (Diesel in first stage and petrol in second stage)		**Case 3** (Diesel in first stage and CNG in second stage)	
	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2
x_1	40303.57(U_1)	22327.31(U_2)	40303.57(U_1)	20678.06(U_2)	40303.57(U_1)	17429.91(U_2)
x_2	39603.45(L_1)	17226.57(L_2)	39655.48(L_1)	16607.08(L_2)	39650.05(L_1)	16350.42(L_2)
	Case 4 (Petrol in first stage and diesel in second stage)		**Case 5** (Petrol in first and second stage)		**Case 6** (Petrol in first stage and CNG in second stage)	
x_1	40303.57(U_1)	24184.50(U_2)	40303.57(U_1)	22535.25(U_2)	40303.57(U_1)	19287.11(U_2)
x_2	37108.17(L_1)	9911.349(L_2)	37160.20(L_1)	9291.863(L_2)	37154.77(L_1)	9035.201(L_2)
	Case 7 (CNG in first stage and diesel in second stage)		**Case 8** (CNG in first stage and petrol in second stage)		**Case 9** (CNG in first and second stage)	
x_1	40303.57(U_1)	14257.95(U_2)	40303.57(U_1)	12608.71(U_2)	40303.57(U_1)	9360.560(U_2)
x_2	37954.70(L_1)	8550.772(L_2)	38006.73(L_1)	7931.286(L_2)	38001.30(L_1)	7674.623(L_2)

Now, the two-stage BOFC4-DTP is reduced to the two-stage SOFC4-DTP. Then by step 3, the reduced two-stage SOFC4-DTP is solved by using Lingo 18.0 software. Then the obtained optimal compromise solutions for case 1 of reduced problem are $Z_1 = 38730.46$, $Z_2 = 18000.28$ and the corresponding optimal transported quantities are $x_{1211} = 106.0285$, $x_{1222} = 200$, $y_{2211} = 60.6060$, $y_{2411} = 90.9090$, $y_{2112} = 40.7332$, $y_{2312} = 75.83418$, whereas the other values of decision variables are equal to zero. In a similar manner, we can solve other cases (cases 2-9) to OCS. The OCS and its allocations of all cases are shown in the **Table 10**.

6. Results and Discussions

The OCS obtained by solving all the cases (1-9) are given in **Table 10** as follows.

Table 10. Optimal compromise solutions of cases 1-9.

Fuel type		Max Z_1	Min Z_2	Optimal allocations
1^{st} stage	2^{nd} stage			
Diesel	Diesel	38730.46	18000.28	$x_{1211}=106.0285$, $x_{1222}=200$, $y_{2211}=60.6060$, $y_{2411}=90.9090$, $y_{2112}=40.7332$, $y_{2312}=75.83418$
Diesel	Petrol	38217.43	18642.57	$x_{1211}=171.6457$, $x_{1221}=134.9974$, $y_{2111}=40.5679$, $y_{2411}=90.9090$, $y_{2212}=60.7902$, $y_{2312}=75.83418$
Diesel	CNG	38730.46	18000.28	$x_{1211}=111.7039$, $x_{1221}=151.1448$, $x_{1222}=43.7992$, $y_{2111}=39.2646$, $y_{2211}=60.6060$, $y_{2411}=90.9090$, $y_{2112}=1.3086$, $y_{2312}=75.83418$
Petrol	Diesel	38612.52	17047.92	$x_{1211}=0.4734$, $x_{1222}=200$, $x_{2212}=105.0214$, $y_{2211}=60.6060$, $y_{2411}=90.9090$, $y_{2112}=40.7332$, $y_{2312}=75.83418$
Petrol	Petrol	38561.41	15913.56	$x_{1222}=185.8989$, $x_{2211}=105.5352$, $x_{2222}=14.1011$, $y_{2411}=90.9090$, $y_{2112}=40.7332$, $y_{2222}=60.7287$, $y_{2312}=75.83418$
Petrol	CNG	38241.35	14161.16	$x_{1222}=143.9072$, $x_{2212}=105.6626$, $x_{2222}=56.09282$, $y_{2111}=40.56795$, $y_{2211}=60.6060$, $y_{2312}=75.8341$, $y_{2422}=91.3705$
CNG	Diesel	38730.46	9930.924	$x_{1211}=106.0285$, $x_{1222}=200$, $y_{2211}=60.6060$, $y_{2411}=90.9090$, $y_{2112}=40.7332$, $y_{2312}=75.83418$
CNG	Petrol	38720.49	9311.438	$x_{1211}=106.0285$, $x_{1222}=200$, $y_{2411}=90.9090$, $y_{2112}=40.7332$, $y_{2222}=60.7287$, $y_{2312}=75.83418$
CNG	**CNG**	**38779.07**	**8517.592**	$x_{1211}=106.0285$, $x_{1222}=122.1558$, $x_{2222}=77.8442$, $y_{2111}=40.5679$, $y_{2211}=60.6060$, $y_{2312}=75.8341$, $y_{2422}=91.3705$

From **Table 10**, it can be observed that, when CNG fuel is used in both stages instead of diesel or petrol, the profit is maximized while carbon emissions are significantly reduced. Sometimes in practical situation, the entrepreneurs may not get conveyance with CNG fuels in either of the stages. In that situation they can prefer the conveyance with CNG fuel in the 1st stage and petrol or diesel in the 2nd stage. If the conveyance with CNG fuel is not available in both the stages then they can prefer the conveyance with petrol fuel in 1st stage and petrol in 2nd stage. The selection of fuel is preferably based on the entrepreneur's budget and availability of fuel.

7. Comparative Study

To measure the efficacy of the solution procedure by utilizing the proposed FHFP Method, we conducted a comparative analysis with the existing Fuzzy Programming Method (Ghosh et al., 2022) for solving the R2 model. The results are shown via **Table 11**.

The optimal compromise solution obtained through the proposed method is better than the existing approach Ghosh et al. (2022) in all the cases which are shown in **Table 11**. For more clarity, the optimal compromise solution of the problem obtained through our solution approach with the existing approach of Ghosh et al. (2022) is shown as pictorial representations in **Figure 4** and **Figure 5**.

Table 11. Comparative study.

Fuel type (stage 1) →		Diesel			Petrol			CNG		
Fuel type (stage 2) → / Methods ↓		Diesel	Petrol	CNG	Diesel	Petrol	CNG	Diesel	Petrol	CNG
Fuzzy programming method (Ghosh et al., 2022)	Max Z_1	37677.22	37716.80	37710.11	37127.51	37176.86	37170.62	38208.02	38221.91	38204.61
	Min Z_2	19040.48	19720.23	19040.48	18085.26	17000.42	15206.18	10982.42	10319.58	9586.27
Proposed Fermatean Hesitant Fuzzy Programming Method	Max Z_1	38730.46	38217.43	38730.46	38612.52	38561.41	38241.35	38730.46	38720.49	38779.07
	Min Z_2	18000.28	18642.57	18000.28	17047.92	15913.56	14161.16	9930.924	9311.438	8517.592

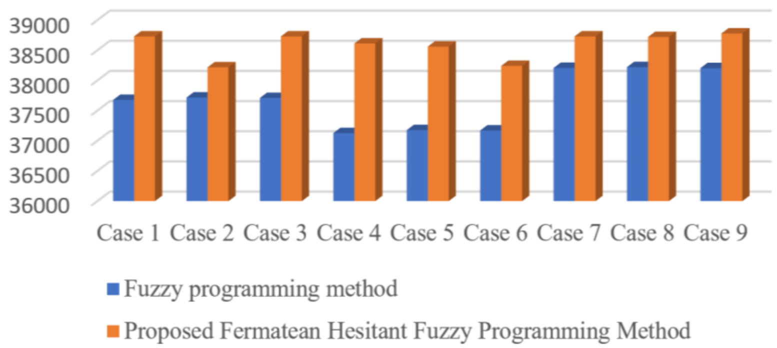

Figure 4. Comparative analysis of solutions of different cases for Z_1.

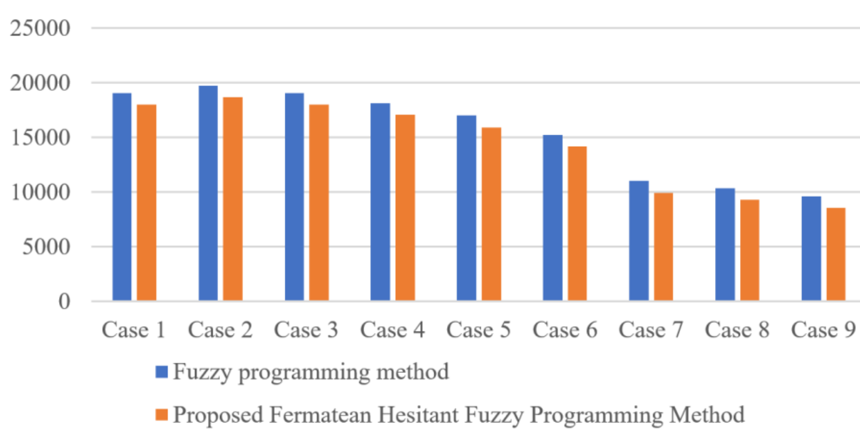

Figure 5. Comparative analysis of solutions of different cases for Z_2.

Table 11, **Figure 4**, and **Figure 5** demonstrate that the solution procedure by using the FHFP method provides better results when compared with the Fuzzy Programming Method (Ghosh et al., 2022) in all the cases. In both approaches, the profit is maximized and carbon emissions are significantly reduced when

CNG fuel is used in both stages rather than diesel or petrol. It can be analyzed in terms of the outcome of all cases. The selection of fuel in stages is purely based on the entrepreneur's choice. So, the optimal compromise solutions in **Table 12** will help the entrepreneur to choose an appropriate case according to their preference.

8. Sensitivity Analysis

The sensitivity analysis is a critical and essential component of optimization problems, as it gives a description and comprehension of the effects of altering the values of the parameters on objective functions. According to Samanta et al. (2024), the process of sensitivity analysis includes altering a particular variable by a certain percent and maintaining the remaining variables as unchanged. This process will show the effect of the variable on the optimal solution. Among all cases, the optimal compromise solution obtained in case 9 is more preferable. So here, we proceed to the sensitivity analysis with respect to the optimum compromise solution attained in case 9 using the Fermatean hesitant fuzzy programming method. **Table 12** demonstrates the change of selling price in the objective functions and how it affects the objective values. In this analysis, a variable in the objective functions is changed for a certain percent (increase or decrease), but all other variables remain unchanged at their original values. The procedure will continue until the basic variables stay consistent, even if their values may have changed.

Table 13. Optimal results of two-stage BOFC4-DTP for different selling price.

% of change in selling price	Z_1	Z_2
-10%	33061.07	8517.592
-5%	36220.07	8517.592
-3%	37483.67	8517.592
-1%	38747.27	8517.592
0%	38779.07	8517.592
+1%	39152.44	7992.373
+3%	39896.70	7674.623
+5%	42540.07	8517.592
+10%	44319.30	7674.623

In order to make understanding easier, **Figure 6** represents a summary of sensitivity analysis for changes in the selling price and its effect on the values of the objective functions.

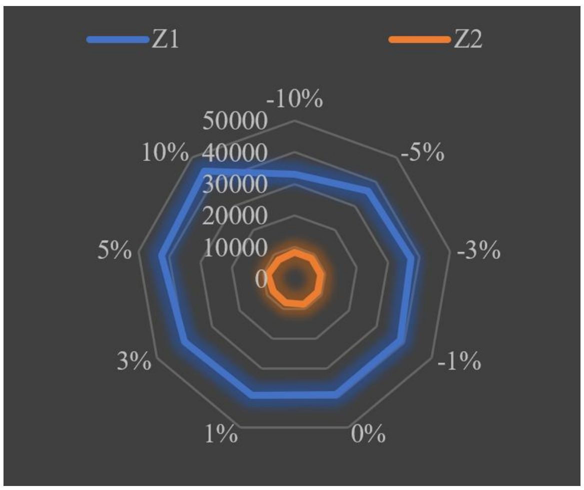

Figure 6. Sensitivity analysis for changes in the selling price.

Table 12 and **Figure 6** show the impact of a change in selling price on the objective functions. **Table 13** shows that it is found that if the range of selling price decreases, then the values of Z_1 decreases but the values of Z_2 remains constant. When the range of selling price increases, then values of Z_1 increases but the values of Z_2 decreases. This can help entrepreneurs to determine the optimum variable values to enhance the opportunity in all aspects by analyzing the effect of varying variables based on financial as well as sustainability concerns.

9. Managerial Implications

This study gives essential managerial information that is important for logistics systems. The article offers important management insights drawn from the findings, which may increase the performance and overall efficacy of the existing supply chain network.

1) The proposed optimization of 4D multi-objective transportation includes the coordination of suppliers, warehouses, and retailers throughout the supply chain in a two-stage process which follows Fermatean fuzzy environment can help the managers to tackle any form of complex uncertain problem with multiple constraints.
2) The implementation of a 4D transportation model demonstrates a commitment to sustainability. So, the managers may devise strategies to optimize the entire system by considering the effects of changes in transportation cost, fixed charges, fuel type, route, supply and demand on the overall supply chain network.
3) The optimal compromise solution obtained from the proposed Fermatean hesitant fuzzy programming will help the managers to conduct an analysis of the impact of vehicle, route, and fuel selection on carbon emissions allow the management to find the scenario that yields the lowest carbon emissions. This research develops solutions that not only improve profit but also reduce emissions. Subsequently, they may enhance their global market reputation by achieving a balance between environmental sustainability and profitability.

10. Conclusions

This article presents a sustainable two-stage BOFC4-DTP model under a fermatean fuzzy environment, which represents a substantial development by including different fuel types into each stage, as mentioned in cases. Today's sustainable transportation system is critical for a cleaner, more environmentally friendly environment. As a result, the main aim of this research focuses on two conflicting objectives: profit maximization and carbon emissions, which can be optimized together by choosing appropriate conveyance, route, and fuel type. The TFFN in the input parameters are converted to deterministic using the (α, β)-cut and accuracy function. To reduce complexity, the deterministic two-stage BOFC4-DTP is converted into a two-stage SOFC4-DTP using the Fermatean hesitant fuzzy programming method, which is solved by Lingo 18.0 software in polynomial time. The results shown in **Table 10** suggest the use of CNG as a fuel in both stages significantly increases profit efficiency and reduces carbon emissions when compared to other fuels. A sensitivity analysis was conducted for coefficients of selling price to assess its impact on objective values. The proposed model provides significant economic and commercial benefits by allowing optimum integration of suppliers, warehouses, and retailers in a two-stage supply chain. Economically, it helps entrepreneurs to minimize the unnecessary transportation costs, to utilize the available resources effectively, and make planned decisions under uncertainty raised by market fluctuations and supply variations. By including sustainability factors, the model supports organizations to assess financially optimum alternatives and environmentally responsible logistics operations. Commercially, implementing such a model demonstrates the improved strategic planning and commitment to green logistics, that not only increases business effectiveness and profit margins but also boosts the company's reputation in international markets by attracting environmentally conscious individuals and organizations.

The present study has several limitations, but by correcting all of the shortcomings and expanding this study, the researchers can get a variety of chances for future studies to expand their scope. The preservation techniques may be used to transport perishable products, and the impact on emissions of carbon during preservation may be added in the model. Our model can be improved for new environments, like type-2 zigzag fuzzy, fuzzy-soft, and so on, using various techniques for ranking on nonlinear membership functions. Another possibility is that the inclusion of industrial waste, mining waste, and various combinations of CO_2 policies, etc., in our proposed model. Our model may also include more complex metaheuristic algorithms. Such avenues of exploration may be followed in future studies.

Conflict of Interest
The authors confirm that there is no conflict of interest to declare for this publication.

Acknowledgments
The authors thank the editors and anonymous referees for their helpful comments.

AI Disclosure
The author(s) declare that no assistance is taken from generative AI to write this article.

References

Akram, M., Shahzadi, G., & Ahmadini, A.A.H. (2020). Decision-making framework for an effective sanitizer to reduce COVID-19 under Fermatean fuzzy environment. *Journal of Mathematics*, *2020*(1), 3263407. https://doi.org/10.1155/2020/3263407.

Akram, M., Shahzadi, S., Shah, S.M.U., & Allahviranloo, T. (2023). An extended multi-objective transportation model based on Fermatean fuzzy sets. *Soft Computing*, 1-23. https://doi.org/10.1007/s00500-023-08117-9.

Aktar, M.S., De, M., Maity, S., Mazumder, S.K., & Maiti, M. (2020). Green 4D transportation problems with breakable incompatible items under type-2 fuzzy-random environment. *Journal of Cleaner Production*, *275*, 122376. https://doi.org/10.1016/j.jclepro.2020.122376.

Aktar, M.S., Kar, C., De, M., Mazumder, S.K., & Maiti, M. (2023). Fixed charge 4-dimensional transportation problem for breakable incompatible items with type-2 fuzzy random parameters under volume constraint. *Advanced Engineering Informatics*, *58*, 102222. https://doi.org/10.1016/j.aei.2023.102222.

Aktar, M.S., Kar, C., De, M., Mazumder, S.K., Maiti, M., & Rathour, L. (2025). Multi-objective green 4-dimensional transportation problem for incompatible damageable items with optimum use of transport vehicles' space. *New Mathematics and Natural Computation*, *21*(01), 245-279. https://doi.org/10.1142/s1793005725500140.

Anuradha, D. (2016). A literature review of transportation problem. *International Journal of Pharmacy and Technology*, *8*(1), 3554-3570.

Anuradha, D., & Sobana, V.E. (2017). A survey on fuzzy transportation problems. *IOP Conference Series: Materials Science and Engineering*, *263*(4), 042105. https://doi.org/10.1088/1757-899x/263/4/042105.

Anuradha, D., Jayalakshmi, M., Deepa, G., & Sujatha, V. (2019). Solution of multi-objective solid transportation problem in fuzzy approach. *Recent Trends in Pure and Applied Mathematics*, *2177*(1), 020009. https://doi.org/10.1063/1.5135184.

Atanassov, K.T. (1999). Intuitionistic fuzzy sets. In: Atanassov, K.T. (ed) *Intuitionistic Fuzzy Sets: Theory and Applications.* Physica-Verlag HD, Heidelberg, pp. 1-137. ISBN: 978-3-7908-2463-6(p), 978-3-7908-1870-3(e). https://doi.org/10.1007/978-3-7908-1870-3_1.

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20*(1), 87-96. https://doi.org/10.1016/S0165-0114(86)80034-3

Bellman, R.E., & Zadeh, L.A. (1970). Decision-making in a fuzzy environment. *Management Science*, *17*(4), B-141-B-273. https://doi.org/10.1287/mnsc.17.4.b141.

Bera, S., Giri, P.K., Jana, D.K., Basu, K., & Maiti, M. (2018). Multi-item 4D-TPs under budget constraint using rough interval. *Applied Soft Computing*, *71*, 364-385. https://doi.org/10.1016/j.asoc.2018.06.037.

Bera, S., Giri, P.K., Jana, D.K., Basu, K., & Maiti, M. (2020). Fixed charge 4D-TP for a breakable item under hybrid random type-2 uncertain environments. *Information Sciences*, *527*, 128-158. https://doi.org/10.1016/j.ins.2020.03.050.

Bind, A.K., Rani, D., Goyal, K.K., & Ebrahimnejad, A. (2023). A solution approach for sustainable multi-objective multi-item 4D solid transportation problem involving triangular intuitionistic fuzzy parameters. *Journal of Cleaner Production*, *414*, 137661. https://doi.org/10.1016/j.jclepro.2023.137661.

Das, S.K., Roy, S.K., & Weber, G.W. (2020). Application of type-2 fuzzy logic to a multi objective green solid transportation–location problem with dwell time under carbon tax, cap, and offset policy: fuzzy versus nonfuzzy techniques. *IEEE Transactions on Fuzzy Systems*, *28*(11), 2711-2725. https://doi.org/10.1109/tfuzz.2020.3011745.

Devnath, S., De, M., Mondal, S.S., & Maiti, M. (2023). Two-stage multi-item 4-dimensional transportation problem with fuzzy risk and substitution. *Journal of Ambient Intelligence and Humanized Computing*, *14*(7), 9469-9496. https://doi.org/10.1007/s12652-023-04614-9.

Devnath, S., Giri, P.K., Mondal, S.S., & Maiti, M. (2021). Multi-item two-stage fixed-charge 4DTP with hybrid random type-2 fuzzy variable. *Soft Computing*, *25*(24), 15083-15114. https://doi.org/10.1007/s00500-021-06371-3.

Devnath, S., Giri, P.K., Mondal, S.S., & Maiti, M. (2022). Fully fuzzy multi-item two-stage fixed charge four-dimensional transportation problems with flexible constraints. *Granular Computing, 7*(4), 779-797. https://doi.org/10.1007/s41066-021-00295-x.

Fakhrzad, M.B., Goodarzian, F., & Golmohammadi, A.M. (2019). Addressing a fixed charge transportation problem with multi-route and different capacities by novel hybrid meta-heuristics. *Journal of Industrial and Systems Engineering*, *12*(1), 167-184. https://dor.isc.ac/dor/20.1001.1.17358272.2019.12.1.9.6.

Gazi, K.H., Mondal, S.P., Chatterjee, B., Ghorui, N., Ghosh, A., & De, D. (2023). A new synergistic strategy for ranking restaurant locations: a decision-making approach based on the hexagonal fuzzy numbers. *RAIRO-Operations Research*, *57*(2), 571-608. https://doi.org/10.1051/ro/2023025.

Ghosh, S., Küfer, K.H., Roy, S.K., & Weber, G.W. (2022). Carbon mechanism on sustainable multi-objective solid transportation problem for waste management in Pythagorean hesitant fuzzy environment. *Complex & Intelligent Systems*, *8*(5), 4115-4143. https://doi.org/10.1007/s40747-022-00686-w.

Giri, B.K., & Roy, S.K. (2022). Neutrosophic multi-objective green four-dimensional fixed-charge transportation problem. *International Journal of Machine Learning and Cybernetics*, *13*(10), 3089-3112. https://doi.org/10.1007/s13042-022-01582-y.

Hameed, Z.A., & Moalla, T.L. (2022). Solving 4D transportation problem using the particle swarm optimization algorithm. *Journal of Mathematical and Computational Science, 12*, Article-ID 181. https://doi.org/10.28919/jmcs/7339.

Hitchcock, F.L. (1941). The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics, 20*(1-4), 224-230. https://doi.org/10.1002/sapm1941201224.

Jana, S.H., & Jana, B. (2020). Application of random triangular and Gaussian type-2 fuzzy variable to solve fixed charge multi-item four dimensional transportation problem. *Applied Soft Computing*, *96*, 106589. https://doi.org/10.1016/j.asoc.2020.106589.

Jana, S.H., Jana, B., Das, B., Panigrahi, G., & Maiti, M. (2019). Constrained FC 4D MITPs for damageable substitutable and complementary items in rough environments. *Mathematics*, *7*(3), 281. https://doi.org/10.3390/math7030281.

Kirişci, M. (2023). Fermatean hesitant fuzzy sets for multiple criteria decision-making with applications. *Fuzzy Information and Engineering*, *15*(2), 100-127. https://doi.org/10.26599/fie.2023.9270011.

Kumar, E., & Dhanapal, A. (2024). Solving multi-objective bi-item capacitated transportation problem with Fermatean fuzzy multi-choice stochastic mixed constraints involving normal distribution. *Contemporary Mathematics*, *5*(4), 4776-4804. https://doi.org/10.37256/cm.5420245217.

Majumder, S., Kundu, P., Kar, S., & Pal, T. (2019). Uncertain multi-objective multi-item fixed charge solid transportation problem with budget constraint. *Soft Computing*, *23*(10), 3279-3301. https://doi.org/10.1007/s00500-017-2987-7.

Midya, S., Roy, S.K., & Yu, V.F. (2021). Intuitionistic fuzzy multi-stage multi-objective fixed-charge solid transportation problem in a green supply chain. *International Journal of Machine Learning and Cybernetics*, *12*(3), 699-717. https://doi.org/10.1007/s13042-020-01197-1.

Mollanoori, H., Tavakkoli-Moghaddam, R., Triki, C., Hajiaghaei-Keshteli, M., & Sabouhi, F. (2019). Extending the solid step fixed-charge transportation problem to consider two-stage networks and multi-item shipments. *Computers & Industrial Engineering*, *137*, 106008. https://doi.org/10.1016/j.cie.2019.106008.

Mondal, A., Roy, S.K., & Midya, S. (2023). Intuitionistic fuzzy sustainable multi-objective multi-item multi-choice step fixed-charge solid transportation problem. *Journal of Ambient Intelligence and Humanized Computing*, *14*(6), 6975-6999. https://doi.org/10.1007/s12652-021-03554-6.

Pandian, P., & Anuradha, D. (2010). A new approach for solving solid transportation problems. *Applied mathematical sciences*, *4*(72), 3603-3610.

Pandian, P., & Anuradha, D. (2011). A new method for solving bi-objective transportation problems. *Australian Journal of Basic and Applied Sciences*, *5*(10), 67-74.

Rani, D., & Rizk-Allah, R.M. (2024). A study of uncertain 4D transportation problems with rough interval parameters and additional real-life factors. *Applied Soft Computing*, *163*, 111920. https://doi.org/10.1016/j.asoc.2024.111920.

Sahoo, L. (2021). A new score function based Fermatean fuzzy transportation problem. *Results in Control and Optimization*, *4*, 100040. https://doi.org/10.1016/j.rico.2021.100040.

Sahoo, P., Jana, D.K., Pramanik, S., & Panigrahi, G. (2023). Implement an uncertain vector approach to solve entropy-based four-dimensional transportation problems with discounted costs. *International Journal of Machine Learning and Cybernetics*, *14*(1), 3-31. https://doi.org/10.1007/s13042-021-01457-8.

Samanta, S., Chakraborty, D., & Jana, D.K. (2024). Neutrosophic multi-period two stage four-dimensional transportation problem for breakable items. *Expert Systems with Applications*, *246*, 123266. https://doi.org/10.1016/j.eswa.2024.123266.

Samanta, S., Jana, D.K., Panigrahi, G., & Maiti, M. (2020). Novel multi-objective, multi-item and four-dimensional transportation problem with vehicle speed in LR-type intuitionistic fuzzy environment. *Neural Computing and Applications*, *32*(15), 11937-11955. https://doi.org/10.1007/s00521-019-04675-y.

Senapati, T., & Yager, R.R. (2020). Fermatean fuzzy sets. *Journal of Ambient Intelligence and Humanized Computing*, *11*(2), 663-674. https://doi.org/10.1007/s12652-019-01377-0.

Sharma, M.K., & Chaudhary, S. (2024). A multi-objective solid transportation problem for the sustainable transit of hazardous waste in the complex Fermatean hesitant fuzzy environment. In: Ali, I., Modibbo, U.M., Bolaji, A.L., Garg, H. (eds) *Optimization and Computing using Intelligent Data-Driven Approaches for Decision-Making*. CRC Press, Boca Raton, pp. 17-37. ISBN: 9781003536796.

Shell, E. (1955). Distribution of a product by several properties, directorate of management analysis. *Proceedings of the Second Symposium in Linear Programming*, *2*, 615-642.

Shivani, & Rani, D. (2024). An extended Vogel's approximation algorithm for efficiently solving Fermatean fuzzy solid transportation problems. *Soft Computing*, *28*(17), 9711-9734. https://doi.org/10.1007/s00500-024-09812-x.

Singh, A., Arora, R., & Arora, S. (2025). A new Fermatean fuzzy multi-objective indefinite quadratic transportation problem with an application to sustainable transportation. *International Transactions in Operational Research*, *32*(4), 1977-2002. https://doi.org/10.1111/itor.13513.

Sobana, V.E., & Anuradha, D. (2018). On solving bi-objective fuzzy transportation problem. In: Madhu, V., Manimaran, A., Easwaramoorthy, D., Kalpanapriya, D., Mubashir Unnissa, M. (eds) *Advances in Algebra and Analysis* (pp. 233-242). Springer International Publishing, Cham. https://doi.org/10.1007/978-3-030-01120-8_27.

Sobana, V.E., & Anuradha, D. (2019). Solving two stage fuzzy transportation problem. *AIP Conference Proceedings*, *2177*(1), 020091. https://doi.org/10.1063/1.5135266.

Torra, V., & Narukawa, Y. (2009, August). On hesitant fuzzy sets and decision. In *2009 IEEE International Conference on Fuzzy Systems* (pp. 1378-1382). IEEE. Jeju, Korea (South). https://doi.org/10.1109/fuzzy.2009.5276884.

Yager, R.R. (2013). Pythagorean fuzzy subsets. In *2013 Joint IFSA World Congress and NAFIPS Annual Meeting* (pp. 57-61). IEEE. Edmonton, AB, Canada. https://doi.org/10.1109/ifsa-nafips.2013.6608375.

Yang, L., Liu, P., Li, S., Gao, Y., & Ralescu, D.A. (2015). Reduction methods of type-2 uncertain variables and their applications to solid transportation problem. *Information Sciences*, *291*, 204-237. https://doi.org/10.1016/j.ins.2014.08.044.

Yu, V.F., Bera, A., Das, S.K., Manna, S., Jhulki, P.K., Dey, B., & Ali, S.A. (2024). Optimizing green solid transportation with carbon cap and trade: a multi-objective two-stage approach in a type-2 Pythagorean fuzzy context. *Soft Computing*, *28*(19), 11015-11039. https://doi.org/10.1007/s00500-024-09864-z.

Zadeh, L.A. (1965). Fuzzy sets. *Inform Control*, *8*(3), 338-353. https://doi.org/10.1016/s0019-9958(65)90241-x.

Zhang, B., Peng, J., Li, S., & Chen, L. (2016). Fixed charge solid transportation problem in uncertain environment and its algorithm. *Computers & Industrial Engineering*, *102*, 186-197. https://doi.org/10.1016/j.cie.2016.10.030.