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A Solution Approach for Multi-Level Multi-Objective Quadratic Fractional Programming with Trapezoidal Fuzzy Numbers using Rouben Ranking Function

Vandana Goyal
Department of Mathematics,
Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India.
E-mail: vandanagoyal04@gmail.com

Namrata Rani
Department of Mathematics,
Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India.
E-mail: namratagarg55@gmail.com

Deepak Gupta
Department of Mathematics,
Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India.
Corresponding author: guptadeepak20003@gmail.com

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Abstract
The paper proposed a methodology for obtaining a set of efficient solutions for a model which is multi-level multiobjective quadratic with fractional objectives and constraints having trapezoidal fuzzy number (MLMOQFP-TrFN) as coefficients. The model consists of *r*-levels with several objectives involved to be solved under a set of quadratic constraints. The proposed approach starts with the solution process of the top level and other levels are solved in succession but depending on the solution of the previous levels. The solution process of each level comprises mainly three stages. In the beginning, the Rouben Ranking Function is used to convert the *rth*-level of fuzzy model into a deterministic or crisp one. After that, the crisp form is reconstructed to get a non-fractional model with the help of an iterative parametric approach. Further, in the last, non-fractional model which is still having multiple objectivesis reconstructed to form a model having only one objective with ε -constraint method and is lastly solved by following the solution of *(r-1)th-* level to get a desired set of efficient solution. Such programming models are very useful in day to day life such as in economic planning, industrial activities, waste management, neural networking, unmanned aerial and underwater vehicle management, agricultural yield improvement, transportation problems with maximizing profits and minimizing wastage of material and cost and so on. An algorithm depicting all the steps of solution approach is also presented to reflect a clear idea for the approach. In addition, a numerical regarding the presentation of complete approach that is studied is given in the end.

Keywords- **ε**-constraint method, Parametric approach, Rouben ranking function, Multi-level multiobjective quadratic fractional programming model (MLMOQFP), Trapezoidal fuzzy number (TrFN).

1. Introduction

Multiobjective Quadratic Fractional Programming (MOQFP) is amongst the successful decision making processes for practically analysing situations and making best conclusions out of it. When we formulate such models, some particular values for coefficients are required to be considered. However, in most of the situations, these particular values of coefficients are unknown and rather a close approximation to these values can be made. However, in the vast majority of cases, information which is available is ambiguous or expressed as fuzzy numbers which can be either triangular, trapezoidal, intutionistic or interval. Multi-Level MOQFP is one of the hierarchical optimization technique having multiple quadratic objective functions that are fractional in nature which are to be dealt at multiple levels with the respective decision makers (DMs) having their own goals as objectives and different priorities. In these problems, one is the First level decision maker (FLDM) who is the leader and the other one's are second level (SLDM) , third level (TLDM) decision makers and so on who follow the decisions made by FLDM but in a viable range. So, there is a connectivity between the decisions of all the DMs. Due to the involvement of multiple objectives, these programming problems are not optimized with one solution which satisfies all our desired objectives at one time. Hence, there arises the need of pareto optimality which was suggested by Pareto (1971) according to which all the objectives are satisfied by a pareto solution at the same time.

In last few decades, the field of fractional programming has been explored much and several approaches for solution methods have been suggested by numerous researchers. The exploration began with the Charnes and Cooper (1962) work on linear fractional issues when he devised a method for obtaining a linear problem from a fractional one. Martos and Whinston (1964) & Jagannathan (1966) introduced parametric approach for solving linear and fractional problems. Dinkelbach (1967) further extended the parametric approach for quadratic fractional problems and used Newton's method for obtaining optimal solutions. Numerous researchers then followed his method and devised other methods for attaining optimal results. The concept of Bi-Level programming was developed by Candler and Townsley (1982) which was further extended to multi-level programming. Later on, it was extended by abundant of techniques and algorithms by various researchers like Roy and Bhaumik (2018). Mishra and Ghosh (2006) proposed fuzzy technique for bi-level fractional programs. Pal and Moitra (2003) also proposed goal programming for bi-level programs involving quadratic functions. Almogy and Levin (1971) dealt some fractional problems with parametric approach where the objectivere were considered as addition of functions which were fractional. Later on, Falk and Palocsay (1991) found the flaws in their technique and solved the sum and product of the linear fractional functions with a refined parametric approach. Another method based on gradient method was suggested by Tantawy (2008) for handling fractional problems. Parametric approach was further used by Tammer (2005) for solving MOQFPP by determination of the parameters in the problem. Heesterman (1983) also looked at parametric approaches for solving quadratic problems. In addition, Salahi and Fallahi (2016) further used parametric technique for handling problems with quadratic fractional objectives. For bi-level fractional problems with coefficienta as intervals, (Borza et al., 2014) used a parametric technique. Ojha and Biswal (2014) suggested a method of ε-constraint to optimize fractional problems. Emam (2011, 2013) also proposed using the ε-constraint method to solve integer bi-level problems having multiple objectives. Nayak and Ojha (2015, 2019) also later on studied parametric technique and ε-constraint method for solving linear fractional problems. Valipour et al. (2016) considered the distance between two solutions for solving linear fractional problems. Ehrgott and Ruzika (2008) and Chircop and Zammit-Mangion (2013) also worked with problems having several objectives and proposed using ε-constraint method in a different manner. Emmerich and Deutz (2018) also solved similar problems using evolutionary methods. Bhaumik et al. (2017), Bhaumik et al. (2020), Bhaumik and Roy (2021), Bhaumik et al. (2021a) and Bhaumik et al. (2021b) worked with inter valued, neutrosophic and intutionistic numbers and proposed solution methods in numerous fields. Also, Marin (1996) provided evolutionary methods for elasticity in the field of micropolar bodies. Nikas et al. (2020) later on suggested AUGMECON-R to obtain solutions which are exact for MOLPP. Goyal et al. (2020, 2021) also worked with quadratic fractional models having multiple objectives by parametric and ε-constraint technique for optimizing solutions. When all the work was going with the objectives having real coefficients or traingular fuzzy coefficients, (Fortemps and Roubens, 1996) suggested a new technique to handle fuzzy coefficients which were trapezoidal in nature. Their solution approach was further explored by Rath and Dash (2016, 2017) for solving linear and non-linear problems having trapezoidal coefficients with FGP. Rani et al. (2021a, 2021b) also worked with the Rouben Ranking function coupled with fuzzy programming to MOQFP with trapezoidal coefficients.

It is quite clear from the literature review that a lot of work is going in the field of QFP but there is still a research gap in the field of trapezoidal fuzzy numbers and their defuzzification process using Rouben Ranking Function as most of the work is going with the help of FGP using alpha-cut method. So, the authors proposed this work in QFP with Rouben Ranking Function for handling trapezoidal coefficients but a different approach of parametric vectors in combination to ε-constraint method has been used which is very efficient in finding the pareto optimal set of solutions. Numerous quadratic fractional programming problems are handled with parametric approach in addition to various other methods. From the literature review, it can be seen that the work that has been done till now is mostly in the field of linear fractional problems and that too with only one objective function. Also, to the knowledge of authors, quadratic fractional problems having trapezoidal fuzzy coefficients has not yet been handled with parametric approach coupled with ε-constraint method. Thus, authors have tried to couple both of these approaches for optimizing MOQFP models with trapezoidal fuzzy coefficients.

In this paper, an approach has been proposed to obtain optimal solutions for a multi-level MOQFP model with coefficients as trapezoidal fuzzy numbers in the objectives and constraints. The solution technique begins with the use of Rouben Ranking Function for the conversion of the first level of the fuzzy model to the crisp one. Then, the crisp model which is a fractional one is modified further to a non fractional model with the parametric approach by equating each and every fractional objectives equal to a parametric vector and finally, this non fractional model is subjected to an ε-constraint method for obtaining a model with a single objective only from several objectives. For this, a highest priority objective function is regarded for optimization and rest of the objectives are considered as constraints. All of this is decided by the Decision Maker (DM). After the solutions for first level are found, then the same procedure is followed for the other levels by following the previous level solutions and the final solution is obtained which is acceptable to all the DMs.

The proposed work is categorised into several sections with basic definitions and their related properties being given in section 2. The model proposed is formulated in section 3 and various approaches used in solution procedure are given in section 4 and 5. Algorithm for the solution procedure is given in section 7. All the proposed work is clearly shown with the help of an example and comparison with the FGP has also been given in section 8. A practical application of the model and proposed work is shown in section 9. With the help of proposed work, any organisation with multiple objectives of different natures can be benefitted as it provides efficient solutions and it is the choice of the DMs to choose any one of the solutions out of all pareto solutions which are best suitable for the organisation.

2. Preliminaries

R^{n_i} = space of n_i-dimensional vectors
x^T = Transponse of x
$\alpha^{(t)}$ = vector of parameters , $'t'$ represents iteration number.

T_i^r = Termination Constants of rth-level.
S = Set of Constraints
$\tilde{T} = (a, b, c, d)$; $a \le b \le c \le d$ denotes the Trapezoidal Fuzzy Number.
'r' is used as superscript for representing rth-level.

Trapezoidal Fuzzy Number (TrFN)
A number $\tilde{T} = (a, b, c, d)$; $a \le b \le c \le d$ having membership functions given as follows:

$$\mu_{\tilde{T}}(\tilde{T}(x)) = \begin{cases} 0 & d \le x \le a \\ \dfrac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \dfrac{d-x}{d-c} & c \le x \le d \end{cases}$$

is referred to as the Trapezoidal Fuzzy Number.

Trapezoidal Fuzzy Matrix
A matrix $\widetilde{M} = [\ ]_{m \times m}$ with every entry as a trapezoidal fuzzy number is known as a Trapezoidal Fuzzy Matrix.

Properties of TrFN
Consider, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$

(i) $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$,

(ii) $\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$,

(iii) $-\tilde{A} = (-a_4, -a_3, -a_2, -a_1)$,

(iv) for a real number a, we have, $\tilde{a} = (a, a, a, a)$.

Rouben Ranking Function
Fortemps and Roubens (1996) proposed this function for defuzzifying the fuzzy numbers which is defined as:

$$R(\tilde{T}) = 1/2 \sqrt{(\inf \tilde{T}_\delta + \sup \tilde{T}_\delta) d\delta}.$$

For a trapezoidal number $\tilde{T} = (T^l - \delta, T^l, T^u, T^u + \lambda)$, Rouben Ranking Function defuzzifies it into the form,

$$R(\tilde{T}) = \frac{1}{2}\left(T^l + T^u + \frac{1}{2}(\lambda - \delta)\right). \tag{1}$$

Efficient Solution
A point $u \in S$ is said to be an efficient solution if there is no other point $v \in S$ such that $H_j(v) \le H_j(u)$ for all j and $H_j(v) < H_j(u)$ for atleast one j.

3. Multi-Level Multi-Objective Quadratic Fractional Programming Model Having Trapezoidal Fuzzy Numbers (MLMOQFPM-TrFN)

MLMOQFPM-TrFN is basically a multiple level problem where each rth- level decision maker sets his own objectives and provide decisions by controlling one of the variables. It is formulated for obtaining an efficient solution of the real life problemsin which the objectives are quadratic and of clashing nature but are still inter-related to each other. It has been seen in practical and real life situations that the parameters required are not easy to obatin and mostly vague information regarding those parameters is only available and thus the role of fuzziness comes into picture over here. In the model, TrFN are taken as coefficients in the objectives and constraints. MLMOQFPM-TrFN is expressed as follows:

First Level: $\tilde{H}^{I}_{x_1}(x) = \underset{x_1}{Min}\left\{ \tilde{H}^{I}_{1}(x), \tilde{H}^{I}_{2}(x), \tilde{H}^{I}_{3}(x), ..., \tilde{H}^{I}_{q_1}(x) \right\}$

x_1 is the decision variable

and $\tilde{H}^{I}_{i}(x) = \dfrac{\tilde{H}^{I}_{i1}(x)}{\tilde{H}^{I}_{i2}(x)} = \dfrac{\frac{1}{2}x^T \tilde{D}^{I}_{i1} x + \tilde{C}^{I}_{i1} x + \tilde{d}^{I}_{i1}}{\frac{1}{2}x^T \tilde{D}^{I}_{i2} x + \tilde{C}^{I}_{i2} x + \tilde{d}^{I}_{i2}}$; $1 \le i \le q_1$.

Second Level: $\tilde{H}^{II}_{x_2}(x) = \underset{x_2}{Min}\left\{ \tilde{H}^{II}_{1}(x), \tilde{H}^{II}_{2}(x), \tilde{H}^{II}_{3}(x), ..., \tilde{H}^{II}_{q_2}(x) \right\}$

x_2 is the decision variable

and $\tilde{H}^{II}_{i}(x) = \dfrac{\tilde{H}^{II}_{i1}(x)}{\tilde{H}^{II}_{i2}(x)} = \dfrac{\frac{1}{2}x^T \tilde{D}^{II}_{i1} x + \tilde{C}^{II}_{i1} x + \tilde{d}^{II}_{i1}}{\frac{1}{2}x^T \tilde{D}^{II}_{i2} x + \tilde{C}^{II}_{i2} x + \tilde{d}^{II}_{i2}}$; $1 \le i \le q_2$.

r **th-Level:** $\tilde{H}^{r}_{x_r}(x) = \underset{x_r}{Min}\left\{ \tilde{H}^{r}_{1}(x), \tilde{H}^{r}_{2}(x), \tilde{H}^{r}_{3}(x), ..., \tilde{H}^{r}_{q_r}(x) \right\}$

x_r is the decision variable

and $\tilde{H}^{r}_{i}(x) = \dfrac{\tilde{H}^{r}_{i1}(x)}{\tilde{H}^{r}_{i2}(x)} = \dfrac{\frac{1}{2}x^T \tilde{D}^{r}_{i1} x + \tilde{C}^{r}_{i1} x + \tilde{d}^{r}_{i1}}{\frac{1}{2}x^T \tilde{D}^{r}_{i2} x + \tilde{C}^{r}_{i2} x + \tilde{d}^{r}_{i2}}$; $1 \le i \le q_r$

such that $x \in S = \left\{ x \in R^n \,|\, \frac{1}{2}x^T \tilde{A}_k x + \tilde{B}_k x + \tilde{c}_k \begin{pmatrix} \le \\ \ge \\ = \end{pmatrix} 0,\ x \ge 0 \right\}$; $1 \le k \le m$

where,

$\tilde{D}^{I}_{i1}, \tilde{D}^{I}_{i2}, \tilde{D}^{II}_{i1}, \tilde{D}^{II}_{i2}, ..., \tilde{D}^{r}_{i1}, \tilde{D}^{r}_{i2} = [\ \ ]_{n\times n}$; $\tilde{C}^{I}_{i1}, \tilde{C}^{I}_{i2}, \tilde{C}^{II}_{i1}, \tilde{C}^{II}_{i2}, ..., \tilde{C}^{r}_{i1}, \tilde{C}^{r}_{i2} = [\ \ ]_{n\times 1}$; $\tilde{A}_k = [\ \ ]_{m\times n}$; $\tilde{B}_k = [\ \ ]_{m\times 1}$ are all Trapezoidal Fuzzy Matrices and $\tilde{d}^{I}_{i1}, \tilde{d}^{I}_{i2}, \tilde{d}^{II}_{i1}, \tilde{d}^{II}_{i2}, ..., \tilde{d}^{r}_{i1}, \tilde{d}^{r}_{i2}, \tilde{c}_k$ are TrFN. The DMs at all the levels can alter their objectives by adjusting the variables $x_1, x_2, ..., x_r$ respectively.

4. Parametric Approach for Fractional Programming

This approach is designed to remove the fractional nature of the objectives. Here, a parametric vector α_j is equated to every objective $H_j(x)$ and thus a non-fractional parametric Model is reconstructed which is shown as follows:

$MinH(x) = Min\{H_1(x), H_2(x), ..., H_q(x)\}$.

Take each $H_j(x) = \alpha_j$, i.e. $\dfrac{H_{j1}(x)}{H_{j2}(x)} = \alpha_j$

Consider, $P_j(x) = H_{j1}(x) - \alpha_j H_{j2}(x)$

∴ The above model gets changed to the following model which is non-fractional given as:

$$\underset{x \in S}{Min} H(x) = \underset{j}{Min}\{P_j(x)\}$$

where, $P_j(x)$ are non-fractional parametric functions.

5. ε - Constraint Method

It is one of the methods for the conversion of the models with multiple objectives into the one with only a single objective. It makes it easy to deal with one objective function at a time. This is done by optimizing any one of the objectives to its best level possible and transforming other objectives as constraints. The ε -constraint method is expressed as follows:

$$Min\ P_m(x),\ 1 \le m \le r$$

subjected to $P_j(x) \le \varepsilon_j$; for all $1 \le j \le r, j \ne m$ and $x \in S$

where, $\varepsilon_j \in [\varepsilon_j^l, \varepsilon_j^u]$ and ε_j^l and ε_j^u are the lower and upper values of $P_j(x)$. Finally, this model is solved by putting different values of ε_j.

6. Methodology

MLMOQFP is a nested problem where we have multiple objectives which are quadratic and conflicting in nature. Here, few objectives are required to be maximized and few to be minimized. Our objective is to optimize these objectives simultaneously subjected to some constraints and obtain their efficient solutions which are acceptable to all the DMs at all the levels. There are events when we are unaware of exact information regarding objectives and constraints. So, the concept of fuzzy coefficients is considered to handle that uncertainty. In the present work, trapezoidal fuzzy coefficients have been considered in the model.

Model M1: MLMOQFP-TrFN is as follows:

First Level: $\underset{x_1}{Min}\ \tilde{H}^I(x) = \underset{x_1}{Min}\left\{ \tilde{H}_1^I(x), \tilde{H}_2^I(x), \tilde{H}_3^I(x), ..., \tilde{H}_{q_1}^I(x) \right\}$

where, x_1 is the decision variable.

Second Level: $\underset{x_2}{Min}\ \tilde{H}^{II}(x) = \underset{x_2}{Min}\left\{ \tilde{H}_1^{II}(x), \tilde{H}_2^{II}(x), \tilde{H}_3^{II}(x), ..., \tilde{H}_{q_2}^{II}(x) \right\}$

where, x_2 is the decision variable.

r th-Level: $\tilde{H}^r_{x_r}(x) = \underset{x_r}{Min}\left\{\tilde{H}^r_1(x), \tilde{H}^r_2(x), \tilde{H}^r_3(x), ..., \tilde{H}^r_{q_r}(x)\right\}$

where, x_r is the decision variable.

$$\text{subject to } x \in S = \left\{ x \in R^n \,|\, \frac{1}{2} x^T \tilde{A}_k x + \tilde{B}_k x + \tilde{c}_k \begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} 0,\ x \geq 0 \right\};\ 1 \leq k \leq m.$$

The above model has TrFN as coefficients in the objectives and constraints. The solution process begins with the conversion of the fuzzy model to a crisp one using the Rouben Ranking function defined by Fortemps and Roubens (1996).

Model M2: The model obtained from above consists of real coefficients and is given as:

First Level: $\underset{x_1}{Min}\ H^I(x) = \underset{x_1}{Min}\left\{H^I_1(x),\ H^I_2(x),\ H^I_3(x),\ ...,\ H^I_{q_1}(x)\right\}$.

Second Level: $\underset{x_2}{Min}\ H^{II}(x) = \underset{x_2}{Min}\left\{H^{II}_1(x), H^{II}_2(x), H^{II}_3(x), ..., H^{II}_{q_2}(x)\right\}$.

r th-Level: $H^r_{x_r}(x) = \underset{x_r}{Min}\left\{H^r{}_1(x), H^r{}_2(x), H^r{}_3(x), ..., H^r{}_{q_r}(x)\right\}$

subject to $x \in S$, This model is now solved using the technique as proposed by Goyal et al. (2020, 2021).

Let us assume that each $H^I_i(x) = \alpha^{I(t)}_i\ , 1 \leq i \leq q_1$;

$H^{II}_i(x) = \alpha^{II(t)}_i\ , 1 \leq i \leq q_2$;

$H^r_i(x) = \alpha^{r(t)}_i\ , 1 \leq i \leq q_r$ `t' being the iteration no.

Consider $\alpha^{I(t)} = (\alpha^{I(t)}_1, \alpha^{I(t)}_2, ..., \alpha^{I(t)}_{q_1})$ as the parametric vector for $H^I(x)$ and $\alpha^{II(t)} = (\alpha^{II(t)}_1, \alpha^{II(t)}_2, ..., \alpha^{II(t)}_{q_2})$ as the parametric vector for $H^{II}(x)$ and continuing in the same manner, $\alpha^{r(t)} = (\alpha^{r(t)}_1, \alpha^{r(t)}_2, ..., \alpha^{r(t)}_{q_2})$ as the parametric vector for r th-level objective $H^r(x)$.

$$\text{Consider } P^I_i(\alpha^{I(t)}) = H^I_{i1}(x) - \alpha^{I(t)}_i\ H^I_{i2}(x)\ , 1 \leq i \leq q_1,$$

$$P^{II}_i(\alpha^{II(t)}) = H^{II}_{i1}(x) - \alpha^{II(t)}_i\ H^{II}_{i2}(x)\ , 1 \leq i \leq q_2,$$

$$P^r_i(\alpha^{r(t)}) = H^r_{i1}(x) - \alpha^{r(t)}_i\ H^r_{i2}(x)\ , 1 \leq i \leq q_r.$$

Thus, the model 'M2' gets changed into the model 'M3' which is given as below:

Model M3: **First Level:** $\underset{x_1}{Min}\,\tilde{H}^I(x) = \underset{x_1}{Min}\,H^I(x) = \underset{i}{Min}\,P_i^I(\alpha^{I(t)})$,

Second Level: $\underset{x_2}{Min}\,\tilde{H}^{II}(x) = \underset{x_2}{Min}\,H^{II}(x) = \underset{i}{Min}\,P_i^{II}(\alpha^{II(t)})$,

r **th-Level:** $\underset{x_r}{Min}\,\tilde{H}^r(x) = \underset{x_r}{Min}\,H^r(x) = \underset{i}{Min}\,P_i^r(\alpha^{r(t)})$,

subject to $x \in S$, The above model consists of multiple objectives which are not so easy to tackle. Thus, an ε-constraint method is further followed to optimize single objective by converting remaining objectives into constraints. This selection is made by keeping in mind the priorities of the objectives. Therefore, the model 'M4' so obtained is given as:

Model M4: **First Level**: $Min\,P_{l_1}^I(\alpha^{I(t)}) = H_{l_1 1}^I(x) - \alpha_{l_1}^{I(t)}\,H_{l_1 2}^I(x)$

subject to $x \in S$ and $P_i^I(\alpha^{I(t)}) \le \varepsilon_i^I$; $1 \le i \le q_1$, $i \ne l_1$

where, $\varepsilon_i^I \in [\varepsilon_i^{I(l)}, \varepsilon_i^{I(u)}]$ and $T_{l_1}^I < T_i^I$.

Second Level: $\underset{x_2}{Min}\,P_{l_2}^{II}(\alpha^{II(t)}) = H_{l_2 1}^{II}(x) - \alpha_{l_2}^{II(t)}\,H_{l_2 2}^{II}(x)$

subject to $x \in S$ and $P_i^{II}(\alpha^{II(t)}) \le \varepsilon_i^{II}$; $1 \le i \le q_2$, $i \ne l_2$

where, $\varepsilon_i^{II} \in [\varepsilon_i^{II(l)}, \varepsilon_i^{II(u)}]$ and $T_{l_2}^{II} < T_i^{II}$.

r **th-Level:** $Min\,P_{l_r}^r(\alpha^{r(t)}) = H_{l_r 1}^r(x) - \alpha_{l_r}^{r(t)}\,H_{l_r 2}^r(x)$

subject to $x \in S$ and $P_i^r(\alpha^{r(t)}) \le \varepsilon_i^r$; $1 \le i \le q_r$, $i \ne l_r$,

where, $\varepsilon_i^r \in [\varepsilon_i^{r(l)}, \varepsilon_i^{r(u)}]$ and $T_{l_r}^r < T_i^r$.

T_i^r are the tolerances or termination constants for $H_i^r(x)$ which are acceptable to the DMs. All tolerances are decided by the DMs depending upon the priorities corresponding to the objectives and in general are considered very close to zero. Now, the First level is solved and its efficient solution is obatined which has to be further followed by the next level DM and the process keeps going until all the levels are solved.

First Level: Suppose, X_i^I to be the individual solutions for $H_i^I(x)$ subjected to $x \in S$. Table 1 is then formed showing the values of $H_i^I(X_i^I)$, $1 \le i \le q_1$ at each individual solution X_i^I of $H_i^I(x)$.

These are the values of the objectives at the initially obtained set of solutions to the objective functions. These are obtained to find the range of the objective functions in which they lie to.

Table 1. Pay-off table for obtaining values of objective functions.

X_i^I	$H_1^I(X_i^I)$	$H_2^I(X_i^I)$	$H_3^I(X_i^I)$	...	$H_{q_1}^I(X_i^I)$
Individual Solution	Objective 1	Objective 2	Objective 3	…	Objective q_1
X_1^I	$H_1^I(X_1^I)$	$H_2^I(X_1^I)$	$H_3^I(X_1^I)$	...	$H_{q_1}^I(X_1^I)$
X_2^I	$H_1^I(X_2^I)$	$H_2^I(X_2^I)$	$H_3^I(X_2^I)$	...	$H_{q_1}^I(X_2^I)$
⋮	⋮	⋮	⋮	⋮	⋮
$X_{q_1}^I$	$H_1^I(X_{q_1}^I)$	$H_2^I(X_{q_1}^I)$	$H_3^I(X_{q_1}^I)$	...	$H_{q_1}^I(X_{q_1}^I)$

Next, we define $\varepsilon_i^{I(l)}$ and $\varepsilon_i^{I(u)}$ as: $\varepsilon_i^{I(l)} =$ $\min\{P_i^I(X_i^I); 1 \le i \le q_1\}$, $\varepsilon_i^{I(u)} =$ $\max\{P_i^I(X_i^I); 1 \le i \le q_1\}$.

After this, initial optimal solution $X^{I(0)}$ for model M4 is determined by: $X^{I(0)} = \sum_{i=1}^{q_1} w_i^I X_i^I$ where weightage for individual solutions sums up to unity.

Consider $\alpha^{I(t)}$ for $t=1$ as:
$\alpha^{I(t)} = (\alpha_1^{I(t)}, \alpha_2^{I(t)}, ..., \alpha_{q_1}^{I(t)}) = \left(H_1^I(X^{I(0)}), H_2^I(X^{I(0)}), ..., H_{q_1}^I(X^{I(0)})\right)$.

After substituting this value of $\alpha^{I(1)}$ in each $P_i(\alpha^{I(t)})$, we proceed with our proposed approach and check for the termination criteria given as: $\left|P_i^I(\alpha^{I(t)})\right| \le T_i^I; 1 \le i \le q_1$ and obtain efficient solutions for first level objectives.

Consider $x^I = (x_1^I, x_2^I, ..., x_r^I)$, $x_1^I = [(x_1^I)_l, (x_1^I)_u)]$ be the efficient solution where $(x_1^I)_l$ is the least value of x_1^I and $(x_1^I)_u$ is the greatest value of x_1^I.

Second Level: The decision of the FLDM (in the form of solution) is to be followed by the SLDM i.e. the solution x_1^I is required to be considered while solving the second level. Thus, the second level of the model 'M4' is eiquivalent to:

$$\underset{x_2}{Min}\, P_{l_2}^{II}(\alpha^{II(t)}) = H_{l_2 1}^{II}(x) - \alpha_{l_2}^{II(t)}\ H_{l_2 2}^{II}(x)$$

subject to $P_i^{II}(\alpha^{II(t)}) \le \varepsilon_i^{II}$; $1 \le i \le q_2$, $i \ne l_2$

$x_1 \le (x_1^I)_u$, $x_1 \ge (x_1^I)_l$, $x \in S$.

Now, we solve this problem as we did the first level of the model and get the final set of solutions

$(x_1^{II}, x_2^{II}, ..., x_r^{II})$ where $x_2^{II} \in [(x_1^I)_l, (x_1^I)_u)]$ after checking for the termination condition for second level i.e. $\left|P_i^{II}(\alpha^{II(t)})\right| \le T_i^{II}\,;\, 1 \le i \le q_2$. Finally, we test for the final condition to check whether $(x_1^{II}, x_2^{II}, ..., x_r^{II})$ is the solution acceptable to FLDM too and the condition is given as follows:

$$\frac{\left|H_i^I(x_1^I, x_2^I, ..., x_r^I) - H_i^I(x_1^{II}, x_2^{II}, ..., x_r^{II})\right|}{\left|H_i^I(x_1^{II}, x_2^{II}, ..., x_r^{II})\right|} < T_i^I \;\; 1 \le i \le q_1 .$$

If this condition is satisfied, then $(x_1^{II}, x_2^{II}, ..., x_r^{II})$ is the efficient solution acceptable to both level DMs. Otherwise, we look for another values of termination constants and start again. Then, this solution $(x_1^{II}, x_2^{II}, ..., x_r^{II})$ is followed by the TLDM to obtain another solution and continuing in this manner, we proceed to the final r th-Level to obtain the best solution acceptable to all level DMs.

r **th-Level:** $Min\, P_{l_r}^r(\alpha^{r(t)}) = H_{l_r 1}^r(x) - \alpha_{l_r}^{r(t)}\, H_{l_r 2}^r(x)$

subject to $P_i^r(\alpha^{r(t)}) \le \varepsilon_i^r \;;\, 1 \le i \le q_r\,,\; i \ne l_r$

$(x_1^I)_l \le x_1 \le (x_1^I)_u$, $(x_2^{II})_l \le x_2 \le (x_2^{II})_u$, $(x_{r-1}^{r-1})_l \le x_{r-1} \le (x_{r-1}^{r-1})_u$.

$x \in S$, Solving this level in the same way, we obtain $(x_1^r, x_2^r, ..., x_r^r)$ as the efficient solutions after they satisfy the termination conditions given by: $\left|P_i^r(\alpha^{r(t)})\right| \le T_i^r\,;\, 1 \le i \le q_r$. Finally, we test for the final termination condition to check whether $(x_1^r, x_2^r, ..., x_r^r)$ is acceptable to all previous level DMs given as:

$$\frac{\left|H_i^j(x_1^j, x_2^j, ..., x_r^j) - H_i^j(x_1^r, x_2^r, ..., x_r^r)\right|}{\left|H_i^j(x_1^r, x_2^r, ..., x_r^r)\right|} < T_i^j \;\; 1 \le i \le q_1, 1 \le i \le q_2, ..., 1 \le i \le q_r; 1 \le j \le r-1.$$

Then, $(x_1^r, x_2^r, ..., x_r^r)$ is the final set of solutions to the MLMOQFP after all conditions are satisfied.

Assumptions

(i) Equal weightages are given to each individual solution of objective functions while calculating the initial solutions.

(ii) DMs are the deciding holders for the Termination constants which are taken up nearer to zero for every objective.

(iii) Initial optimal solution suitable to model is determined by: $X^{r(0)} = \sum_{i=1}^{q_r} w_i X_i\,;\; w_i > 0$ and $\sum_{i=1}^{q_r} w_i = 1$.

7. Algorithm for Solution Procedure

Step 1. Apply Rouben Ranking Function as $R(\tilde{T}) = \frac{1}{2}\left(T^l + T^u + \frac{1}{2}(\lambda - \delta)\right)$ for every TrFN $\tilde{T} = (T^l - \delta, T^l, T^u, T^u + \lambda)$, to get the crisp model 'M2' from fuzzy model 'M1'.

Step 2. Start with the solution process of first level of model 'M2' and obtain $X^{I(0)} = \sum_{i=1}^{q_1} w_i^I X_i^I$.

Step 3. Begin with $t = 1$ in first level of model 'M2'.
Step 4. Find $\alpha^{I(1)} = (H_1^I(X^{I(0)}), H_2^I(X^{I(0)}), \ldots, H_{q_1}^I(X^{I(0)}))$.
Step 5. Substitute $\alpha^{I(1)}$ in $P_i^I(\alpha^{I(t)})$ and obtain model 'M3'.
Step 6. Select objective $P_{l_1}^I(\alpha^{I(1)})$ to be optimized having least value of the $T_{l_1}^I$ to get an equivalent model 'M4' from model 'M3'.
Step 7. Select $\varepsilon_i^I \in [\varepsilon_i^{I(l)}, \varepsilon_i^{I(u)}]$; $1 \le i \le q_1, i \ne l_1$ as follows:
i) When $[-T_i^I, T_i^I] \cap [\varepsilon_i^{I(l)}, \varepsilon_i^{I(u)}] = \varphi$, select $\varepsilon_i^I \in [\varepsilon_i^{I(l)}, \varepsilon_i^{I(u)}]$.
ii) Otherwise, choose $\varepsilon_i^I \in [-T_i^I, T_i^I]$.
Step 8. Find a set of solutions $x^I = (x_1^I, x_2^I, \ldots, x_r^I)$ for model 'M4' by putting different values of ε_i^I.
Step 9. Check for termination conditions: $\left|P_i^I(\alpha^{I(t)})\right| \le T_i^I$; $1 \le i \le q_1$. If conditions get satisfied, stop the process. Otherwise, proceed for step 10.
Step 10. Find min $\sum_i \left(\left|P_i^I(\alpha^{I(1)})\right| - T_i^I\right)$ for that i where conditions are not satisfied.
Step 11. Obtain $X^{I(1)}$ as the solution where $\sum_i \left(\left|P_i^I(\alpha^{I(1)})\right| - T_i^I\right)$ has got the least value.
Step 12. Proceed with steps 4 to 9 to obtain $x^I = (x_1^I, x_2^I, \ldots, x_r^I)$ as a representative set of efficient solutions to the first level of the model satisfying the termination conditions. If not satisfied, tolerances can be reset by the DM. Then, start the solution process of the second level of model 'M2'.
Step 13. Consider $(x_1^I)_l \le x_1 \le (x_1^I)_u$ as one of the constraints in the second level of M2.
Step 14. Repeat steps 3 to 12 for the second level and find a set of efficient solutions $(x_1^{II}, x_2^{II}, \ldots, x_r^{II})$.
Step 15. Check for $\dfrac{\left|H_i^I(x_1^I, x_2^I, \ldots, x_r^I) - H_i^I(x_1^{II}, x_2^{II}, \ldots, x_r^{II})\right|}{\left|H_i^I(x_1^{II}, x_2^{II}, \ldots, x_r^{II})\right|} < T_i^I$; $1 \le i \le q_1$. If it is satisfied, go to step 16. If not, then reset tolerances and proceed again.
Step 16. Continue the above process for all the r-levels and obtain $(x_1^r, x_2^r, \ldots, x_r^r)$ as the set of efficient solutions for the rth-level.
Step 17. Test for the termination conditions:

$$\frac{\left|H_i^j(x_1^j, x_2^j, \ldots, x_r^j) - H_i^j(x_1^r, x_2^r, \ldots, x_r^r)\right|}{\left|H_i^j(x_1^r, x_2^r, \ldots, x_r^r)\right|} < T_i^j \ ; \ 1 \le i \le q_1, 1 \le i \le q_2, \ldots, 1 \le i \le q_r; 1 \le j \le r-1.$$

Step 18. If satisfied, proceed to step 19, otherwise ask the DM to look for termination constants.
Step 19. End the process.

8. Numerical Example

Consider the MLMOQFPM-TrFN as:

First Level: $\underset{x_1}{Min}\tilde{H}^I(x) = \left\{ \tilde{H}_1^I(x) = \dfrac{0.9\tilde{5}x_1^2 + 0.\tilde{8}x_2^2 - 0.8\tilde{9}x_3}{1.\tilde{1}x_1^2 + 2.\tilde{1}}, \ \tilde{H}_2^I(x) = \dfrac{0.\tilde{9}x_1^2 + 0.93x_3^2 - 0.\tilde{8}x_2}{0.9\tilde{1}x_1^2 + 1.\tilde{9}} \right\}.$

Second Level: $\underset{x_2}{Min}\tilde{H}^{II}(x) = \left\{ \tilde{H}_1^{II}(x) = \dfrac{0.9\tilde{2}x_2^2 + 0.8\tilde{9}x_3^2 - 0.9\tilde{1}x_1}{1.0\tilde{1}x_2^2 + 2.\tilde{9}x_3^2 + 0.9\tilde{5}}, \ \tilde{H}_2^{II}(x) = \dfrac{1.0\tilde{4}x_1^2 + 0.9\tilde{9}x_2^2 - 1.\tilde{1}x_3}{3.0\tilde{2}x_1^2 + 1.0\tilde{2}x_2^2 + 0.8\tilde{7}} \right\}.$

Third Level: $\underset{x_3}{Min}\tilde{H}^{III}(x)=\left\{\tilde{H}_1^{III}(x)=\dfrac{0.\tilde{9}x_1^2+\tilde{1}x_3^2-1.\tilde{1}x_2}{0.9\tilde{5}x_1^2+\tilde{3}x_3^2+1.\tilde{3}},\ \tilde{H}_2^{III}(x)=\dfrac{0.9\tilde{6}x_1^2+1.0\tilde{3}x_2^2+0.9\tilde{8}x_3}{0.9\tilde{3}x_1^2+3.0\tilde{2}x_2^2+1.0\tilde{5}}\right\}$

such that $S=\left\{\begin{array}{l}1.\tilde{2}x_1^2+0.9\tilde{8}x_2^2+0.8\tilde{4}x_3^2\le 1.0\tilde{8}\\ 1.\tilde{3}x_1^2+1.0\tilde{5}x_2+0.8\tilde{8}x_3\le 4.\tilde{1}\\ 0.7\tilde{9}x_2^2+0.9\tilde{6}x_1+1.0\tilde{3}x_3\le 8.\tilde{8}\\ x_1,x_2,x_3\ge 0\end{array}\right\},$

where,

$0.9\tilde{5}=(0.94,0.95,0.98,0.99)$; $4.\tilde{1}=(4.04,4.09,4.1,4.12)$,

$0.\tilde{8}=(0.81,0.84,0.86,0.89)$; $0.7\tilde{9}=(0.79,0.84,0.89,0.94)$,

$0.8\tilde{9}=(0.84,0.89,0.91,0.95)$; $0.9\tilde{6}=(0.93,0.96,0.98,0.99)$,

$1.\tilde{1}=(1.03,1.08,1.1,1.3)$; $1.0\tilde{3}=(1.03,1.04,1.06,1.08)$,

$2.\tilde{1}=(1.9,1.98,2,2.1)$; $8.\tilde{8}=(8.8,9.1,9.13,9.16)$,

$0.\tilde{9}=(0.9,0.92,0.95,0.98)$; $0.9\tilde{2}=(0.91,0.92,0.94,0.99)$,

$0.9\tilde{3}=(0.93,0.96,0.98,1.1)$; $0.8\tilde{9}=(0.89,0.92,0.94,0.96)$,

$0.\tilde{8}=(0.82,0.85,0.9,0.94)$; $0.9\tilde{1}=(0.90,0.905,0.91,0.93)$,

$0.9\tilde{1}=(0.91,0.94,0.97,1)$; $1.0\tilde{1}=(1.01,1.04,1.06,1.08)$,

$1.\tilde{9}=(1.86,1.9,1.93,1.97)$; $2.\tilde{9}=(2.9,2.93,2.95,2.97)$,

$1.\tilde{2}=(1.04,1.09,1.2,1.22)$; $0.9\tilde{5}=(0.93,0.95,0.96,0.98)$,

$0.9\tilde{8}=(0.96,0.99,1.1,1.4)$; $1.0\tilde{4}=(1.02,1.04,1.05,1.07)$,

$0.8\tilde{4}=(0.84,0.89,0.93,0.97)$; $0.9\tilde{9}=(0.96,0.98,0.99,1.01)$,

$10.\tilde{8}=(10.8,10.9,11,11.1)$; $1.\tilde{1}=(1.07,1.09,1.1,1.12)$,

$1.\tilde{3}=(1.06,1.1,1.2,1.3)$; $3.0\tilde{2}=(3.01,3.02,3.04,3.06)$,

$1.0\tilde{5}=(1.05,1.06,1.08,1.1)$; $1.0\tilde{2}=(1.02,1.04,1.07,1.08)$,

$0.8\tilde{8}=(0.88,0.9,0.93,0.94)$; $0.8\tilde{7}=(0.88,0.9,0.93,0.95)$.

We first use Rouben Ranking Function for every TrFN as given by Fortemps and Roubens (1996) to get a crisp model from the above model. It is shown as below:

$R(0.9\tilde{5})=0.965$, $R(0.9\tilde{8})=1.1125$, $R(0.9\tilde{2})=0.94$, $R(\tilde{1})=1$, $R(0.\tilde{8})=0.85$,

$R(0.8\tilde{4})=0.9075$, $R(0.8\tilde{9})=0.9275$, $R(\tilde{3})=3$, $R(0.8\tilde{9})=0.8975$,

$R(10.\tilde{8})=10.95$, $R(0.9\tilde{1})=0.91125$, $R(1.\tilde{1})=1.1275$, $R(1.\tilde{3})=1.165$,

$R(1.0\tilde{1})=1.0475$, $R(2.\tilde{1})=1.995$, $R(1.0\tilde{5})=1.0725$, $R(2.\tilde{9})=2.9375$

$R(0.\tilde{9})=0.9375,\ \ R(0.8\tilde{8})=0.9125,\ \ R(0.9\tilde{5})=0.955, R(0.9\tilde{3})=0.9925,$
$R(4.\tilde{1})=4.0875,\ \ R(1.0\tilde{4})=1.045,\ \ R(0.\tilde{8})=0.8775,\ \ R(0.7\tilde{9})=0.865,$
$R(0.9\tilde{9})=0.985,\ \ R(0.9\tilde{1})=0.955,\ \ R(0.9\tilde{6})=0.965,\ \ R(1.\tilde{1})=1.095$
$R(1.\tilde{9})=1.915,\ \ R(1.0\tilde{3})=1.0525,\ \ R(3.0\tilde{2})=3.0325,\ \ R(1.\tilde{2})=1.1375,$
$R(8.\tilde{8})=9.0475,\ \ R(1.0\tilde{2})=1.0525,\ \ R(0.8\tilde{8})=0.915.$

Thus, the fuzzy model ‘M1’ gets converted to the crisp model ‘M2’ which is given as below **M2:**
First Level:

$$\underset{x_1}{Min}\, H^{I}(x)=\left\{H_{1}^{I}(x)=\frac{0.965x_1^{2}+0.85x_2^2-0.8975x_3}{1.1275x_1^{2}+1.995}\ ,\ H_{2}^{I}(x)=\frac{0.9375x_1^{2}+0.9925x_3^2-0.8775x_2}{0.955x_1^{2}+1.915}\right\}.$$

Second Level:

$$\underset{x_2}{Min}\, H^{II}(x)=\left\{H_{1}^{II}(x)=\frac{0.94x_2^{2}+0.9275x_3^2-0.91125x_1}{1.0475x_2^{2}+2.9375x_3^2+0.955}\ ,\ H_{2}^{II}(x)=\frac{1.045x_1^{2}+0.985x_2^2-1.095x_3}{3.0325x_1^{2}+1.0525x_2^2+0.915}\right\}.$$

Third Level:

$$\underset{x_3}{Min}\, H^{III}(x)=\left\{H_{1}^{III}(x)=\frac{0.9375x_1^{2}+1x_3^2-1.1275x_2}{0.965x_1^{2}+3x_3^2+1.0525}\ ,\ H_{2}^{II}(x)=\frac{0.965x_1^{2}+1.0525x_2^2-1.1125x_3}{0.9925x_1^{2}+3.0325x_2^2+1.0725}\right\}.$$

such that

$$1.1375x_1^{2}+1.1125x_2^{2}+0.9075x_3^2\le 10.95,$$
$$1.165x_1^{2}+1.0725x_2+0.9125x_3\le 4.0875,$$
$$0.865x_2^{2}+0.965x_1+1.0525x_3\le 9.0475,$$
$$x_1, x_2, x_3\ge 0.$$

We initiate by solving the first level of M2.

First Level: First, we find individual solutions of $H_{1}^{I}(x)$ and $H_{2}^{I}(x)$ using Lingo 15 software given as:
$X_1^{I}=(0.000083, 0.0000878, 3.473631)$; $X_2^{I}=(0.0000523, 3.137307, 0.00005156)$.

Table 2. Pay-off table at initial solutions of first level.

X_j^I	$H_1^I(X_j^I)$	$H_2^I(X_j^I)$
X_1^I	-1.5627	6.253545
X_2^I	4.193606	-1.43759

Table 2 shows the values of the objectives at initial individual solutions. Hence, initial solution for the Model 'M2' is given by: $X^{I(0)} = w_1 X_1^I + w_2 X_2^I$

$$X^{I(0)} = (0.00006765, 1.5686974, 1.73684128).$$

Next, initial parametric vector is found as:

$$\alpha^{I(1)} = (H_1^I(X^{I(0)})),\ H_2^I(X^{I(0)}) = (0.267105, 0.844627).$$

The model 'M2' is further converted to Model 'M3'(non-fractional model) with the help of this parametric vector which is given as below:

M3: $MinH^I(x) = Min\left\{\ P_1^I(\alpha_1^{I(1)}),\ P_2^I(\alpha_2^{I(1)})\ \right\}$,

such that $x \in S$, where, $P_1^I(\alpha^{I(1)}) = 0.6638391125x_1^2 + 0.85x_2^2 - 0.8975x_3 - 0.532874475$ and $P_2^I(\alpha^{I(1)}) = 0.130881215x_1^2 - 0.9925x_3^2 - 0.8775x\ - 1.617460705$.

Initial optimal solutions to model 'M3' are given by:

$X_1^I = (0.00007759655, 0.000068629, 3.473631)$, $\quad X_2^I = (0.0001935, 3.137307, 0.00009559)$.

Table 3. Pay-off table of $P_j^I(X_j^I)$.

X_j^I	$P_1^I(X_j^I)$	$P_2^I(X_j^I)$
X_1^I	-3.6	10.4
X_2^I	7.8	-4.37

Table 3 shows the values of $P_j^I(X_j^I)$. Next, DM sets the termination constants for the functions as $T_1^I = 0.2,\ T_2^I = 0.3$.

As $T_1^I < T_2^I$, therefore, Model 'M4' (a single objective model) is obtained from Model 'M3' by using ε-constraint method shown as below **M4:**

Min $P_1^I(\alpha^{I(1)}) = 0.6638391125x_1^2 + 0.85x_2^2 - 0.8975x_3 - 0.532874475$,

such that $P_2^I(\alpha^{I(1)}) = 0.130881215x_1^2 - 0.9925x_3^2 - 0.8775x\ - 1.617460705 \le \varepsilon_2^I$

$x \in S$, $\varepsilon_2^{I(l)} = -4.37045$ and $\varepsilon_2^{I(u)} = 10.3581$.

Thus, we select $\varepsilon_2^I \in [-0.3, 0.3]$ and by taking different values of ε_2^I, we get a set of efficient solutions as shown in Table 4.

Table 4. Efficient solutions for the first level.

ε_2^I	x_1	x_2	x_3	P_1^I	P_2^I	$H_1^I(x)$	$H_2^I(x)$
-0.3	0.0000328	0.2119	1.101	0.11	-0.3	-0.47624	0.531312
-0.24	0.0000329	0.2072	1.126	0.09	-0.24	-0.48845	0.562643
-0.18	0.0000323	0.2027	1.151	0.06	-0.18	-0.50039	0.593975
-0.12	0.0000319	0.1985	1.175	0.04	-0.12	-0.51208	0.625306
-0.06	0.0000312	0.1946	1.199	0.02	-0.06	-0.52353	0.656637
0.06	0.0000672	0.1807	1.246	-0.02	0.06	-0.54577	0.719301
0.12	0.0000616	0.1839	1.269	-0.04	0.12	-0.55658	0.750633
0.18	0.0000662	0.1807	1.291	-0.06	0.18	-0.5672	0.781964
0.24	0.0000601	0.1776	1.313	-0.08	0.4	-0.57764	0.813295
0.3	0.0000639	0.1747	1.335	-0.11	0.3	-0.5879	0.844627

It is clear that termination conditions are satisfied for $P_1^I(\alpha^{I(1)})$ and $P_2^I(\alpha^{I(1)})$.

So, $x^I = (x_1^I, x_2^I)$; $0.0000312 \le x_1^I \le 0.0000672$ are the efficient solutions which are required to be followed for second and third level DMs.

Second Level: The Model 'M2' of second Level is therefore equivalent with:

$$\underset{x_2}{Min}\, H^{II}(x) = \left\{ H_1^{II}(x) = \frac{0.94x_2^2 + 0.9275x_3^2 - 0.91125x_1}{1.0475x_2^2 + 2.9375x_3^2 + 0.955},\ H_2^{II}(x) = \frac{1.045x_1^2 + 0.985x_2^2 - 1.095x_3}{3.0325x_1^2 + 1.0525x_2^2 + 0.915} \right\}$$

Subject to

$$1.1375x_1^2 + 1.1125x_2^2 + 0.9075x_3^2 \le 10.95,$$

$$1.165x_1^2 + 1.0725x_2 + 0.9125x_3 \le 4.0875,$$

$$0.865x_2^2 + 0.965x_1 + 1.0525x_3 \le 9.0475,$$

$$0.0000312 \le x_1 \le 0.0000672,$$

$$x_1, x_2, x_3 \ge 0.$$

We now solve this reconstructed second level of our model with an additional constraint by the same method as followed in first level and set of efficient solutions is obtained given as in Table 5.

Table 5. Efficient solutions for the second level.

ε_2^{II}	x_1	x_2	x_3	P_1^{II}	P_2^{II}	$H_1^{II}(x)$	$H_2^{II}(x)$
-0.3	0.0000672	0.0000452	2.01	0.09	-0.3	0.292	-2.41
-0.24	0.0000672	0.0000467	1.95	0.07	-0.24	0.291	-2.34
-0.18	0.0000672	0.0000483	1.90	0.05	-0.18	0.289	-2.28
-0.12	0.0000672	0.0000499	1.85	0.04	-0.12	0.288	-2.21
-0.06	0.0000672	0.0000515	1.79	0.02	-0.06	0.287	-2.14
0.06	0.0000672	0.0000464	1.68	-0.016	0.06	0.283	-2.01
0.12	0.0000672	0.0000491	1.63	-0.03	0.12	0.281	-1.95
0.18	0.0000672	0.0000505	1.57	-0.05	0.18	0.279	-1.88
0.24	0.0000672	0.0000521	1.52	-0.06	0.24	0.277	-1.82
0.3	0.0000672	0.0000477	1.46	-0.08	0.3	0.274	-1.75

Thus, termination conditions are satisfied for $P_1^{II}(\alpha^{II(1)})$ and $P_2^{II}(\alpha^{II(1)})$ and so, $x^{II}=(x_1^{II},x_2^{II})$ are the efficient solutions of lower level. Now, it is left to verify if it is acceptable to ULDM and the conditions are given as:

(i) $$\frac{\left|H_1^I(x^I)-H_1^I(x^{II})\right|}{\left|H_1^I(x^{II})\right|}=\frac{|0.5879-0.65811|}{|0.90462|}<T_1^I,$$

(ii) $$\frac{\left|H_2^I(x^I)-H_2^I(x^{II})\right|}{\left|H_2^I(x^{II})\right|}=\frac{|0.844627-1.109085|}{|1.109085|}<T_2^I.$$

So, $x^{II}=(x_1^{II},x_2^{II})$; $0.0000312\le x_1^I\le 0.0000672$, $0.0000452\le x_2^{II}\le 0.0000521$ are the efficient solutions to be further referred by the DM at the third level.

Third Level: The Model 'M2' of third level is finally equivalent with:

$$\underset{x_3}{Min}\,H^{III}(x)=\left\{H_1^{III}(x)=\frac{0.9375x_1^2+1x_3^2-1.1275x_2}{0.965x_1^2+3x_3^2+1.0525},\ H_2^{II}(x)=\frac{0.965x_1^2+1.0525x_2^2-1.1125x_3}{0.9925x_1^2+3.0325x_2^2+1.0725}\right\}$$

subject to

$$1.1375x_1^2+1.1125x_2^2+0.9075x_3^2\le 10.95,$$
$$1.165x_1^2+1.0725x_2+0.9125x_3\le 4.0875,$$
$$0.865x_2^2+0.965x_1+1.0525x_3\le 9.0475,$$
$$0.0000312\le x_1\le 0.0000672,$$
$$0.0000452\le x_2\le 0.0000521,$$
$$x_1,x_2,x_3\ge 0.$$

We now solve this reconstructed third level of our model with additional constraints by the same method as followed in first and second levels and a set of efficient solutions is obtained given as in Table 6.

Table 6. Efficient solutions for the third level.

ε_2^{III}	x_1	x_2	x_3	P_1^{III}	P_2^{III}	$H_1^{III}(x)$	$H_2^{III}(x)$
-0.3	0.0000472	0.0000521	2.01	0.12	-0.3	0.302	-2.08
-0.24	0.0000472	0.0000521	1.95	0.09	-0.24	0.301	-2.03
-0.18	0.0000473	0.0000521	1.89	0.07	-0.18	0.299	-1.97
-0.12	0.0000473	0.0000521	1.85	0.04	-0.12	0.298	-1.91
-0.06	0.0000474	0.0000521	1.79	0.02	-0.06	0.295	-1.86
0.06	0.0000476	0.0000521	1.68	-0.02	0.06	0.291	-1.75
0.12	0.0000476	0.0000521	1.63	-0.04	0.12	0.288	-1.69
0.18	0.0000471	0.0000521	1.58	-0.06	0.18	0.286	-1.63
0.24	0.0000477	0.0000521	1.52	-0.08	0.24	0.283	-1.58
0.3	0.0000480	0.0000521	1.47	-0.1	0.3	0.280	-1.52

Thus, termination conditions are satisfied for $P_1^{III}(\alpha^{III(1)})$ and $P_2^{III}(\alpha^{III(1)})$ and so, $x^{III}=(x_1^{III},x_2^{III})$ are the efficient solutions of third level. Now, it is left to verify if it is acceptable to FLDM and SLDM and the conditions are given as:

(i) $$\frac{|H_1^I(x^I)-H_1^I(x^{III})|}{|H_1^I(x^{III})|}=\frac{|-0.5879+0.66004|}{|0.66004|}<T_1^I,$$

(ii) $$\frac{|H_2^I(x^I)-H_2^I(x^{III})|}{|H_2^I(x^{III})|}=\frac{|0.844627-1.115617|}{|1.115617|}<T_2^I,$$

(iii) $$\frac{|H_1^{II}(x^{II})-H_1^{II}(x^{III})|}{|H_1^{II}(x^{III})|}=\frac{|0.27409-0.27431|}{|0.27431|}<T_1^{II},$$

(iv) $$\frac{|H_2^{II}(x^{II})-H_2^{II}(x^{III})|}{|H_2^{II}(x^{III})|}=\frac{|-1.75065+1.7558|}{|-1.7558|}<T_2^{II}.$$

Thus, finally it is a choice of the DM to choose any solution from Table 6 as the efficient solution for the model. The efficient solutions form a pareto front as shown in Figure1.

Comparison of Above Numerical with Fuzzy Goal Programming (FGP): We solved the above numerical illustration with FGP and it was found that the values of objectives are given by: $H_1^I(x)$ $=-0.00872$ and $H_2^I(x)=-0.0086$; $H_1^{II}(x)=-0.149$ and $H_2^{II}(x)=-0.0149$; $H_1^{III}(x)$ $=-0.024$ and $H_2^{III}(x)=-0.0155$. We can easily see from Table 4, 5 and 6 that in both the approaches, one of the objectives at each level is better optimized. It is better represented by the Figure 2, Figure 3 and Figure 4. Hence, it is concluded that both the approaches are comparable to each other and this verifies our proposed methodology.

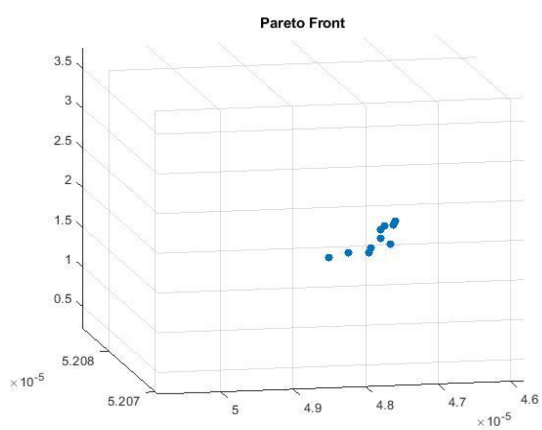

Figure 1. Pareto front formed by efficient solutions.

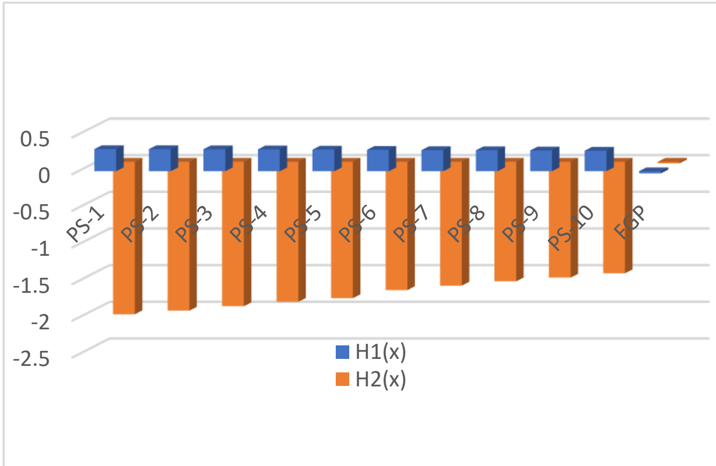

Figure 2. Comparison of objectives of first level with FGP.

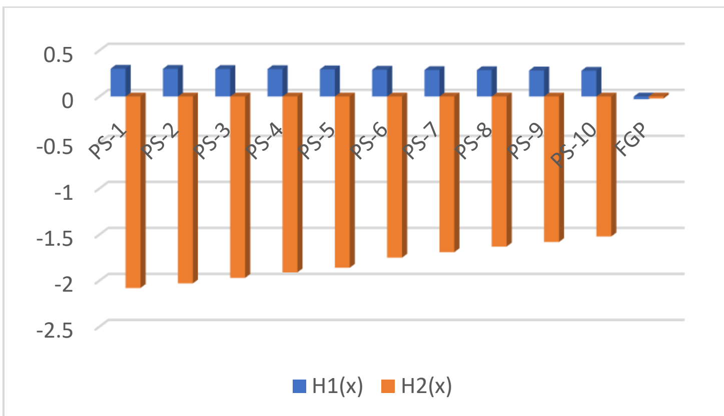

Figure 3. Comparison of objectives of second level with FGP.

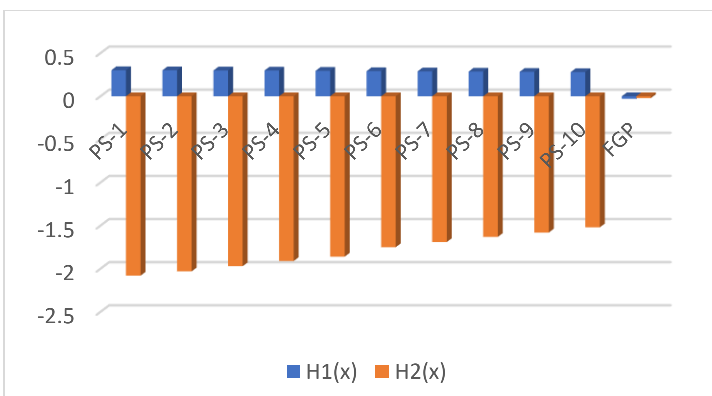

Figure 4. Comparison of objectives of third level with FGP.

9. Application Related to Production Problem

Consider that a car manufacturing unit(MU) is producing three cars $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ with two models-base model $\boldsymbol{A_1}, \boldsymbol{B_1}, \boldsymbol{C_1}$ and top model $\boldsymbol{A_2}, \boldsymbol{B_2}, \boldsymbol{C_2}$ for each car. Suppose, the manufacturing unit decides the selling prices of these cars as Rs. $\boldsymbol{S_A^1}, \boldsymbol{S_B^1}, \boldsymbol{S_C^1}$ for base models and Rs. $\boldsymbol{S_A^2}, \boldsymbol{S_B^2}, \boldsymbol{S_C^2}$ for top models. The unit sets the target of producing at the most $\boldsymbol{T}$ cars altogether. The manufacturing costs of one model of $\boldsymbol{A_1}, \boldsymbol{B_1}, \boldsymbol{C_1}$ are Rs. $\boldsymbol{l_1}, \boldsymbol{m_1}, \boldsymbol{n_1}$ and costs an extra cost of Rs.$\boldsymbol{s}$ per each base model car of all types for time bound completion of target due to extra input charges. Similarly, the manufacturing costs of one model of $\boldsymbol{A_2}, \boldsymbol{B_2}, \boldsymbol{C_2}$ are Rs. $\boldsymbol{l_2}, \boldsymbol{m_2}, \boldsymbol{n_2}$ and costs an extra cost of Rs.$\boldsymbol{t}$ per each top model car. Suppose, $\boldsymbol{x_1}, \boldsymbol{y_1}, \boldsymbol{z_1}$ units of base model cars and $\boldsymbol{x_2}, \boldsymbol{y_2}, \boldsymbol{z_2}$ units of top model cars are manufactured by the unit.

The prices at which base models are sold are Rs. $S_A^1x_1, S_B^1y_1, S_C^1z_1$ and for top models are Rs. $S_A^2x_2, S_B^2y_2, S_C^2z_2$. The costs at which each base model car is manufactured are Rs. $l_1 + sx_1, m_1 + sy_1, n_1 + sz_1$ and for each top model car are Rs. $l_2 + sx_2, m_2 + sy_2, n_2 + sz_2$. Thus, total cost of base model cars are Rs. $(l_1 + sx_1)x_1, (m_1 + sy_1)y_1, (n_1 + sz_1)z_1$ and total cost of top model cars are Rs. $(l_2 + sx_2)x_2, (m_2 + sy_2)y_2, (n_2 + sz_2)z_2$. The objective of every manufacturing unit is to maximise its profit per unit cost of production. Thus, this production problem is finally modelled in the form of QFPP which is presented as:

$$Max\ \ H(x) = \begin{Bmatrix} H_1(x) = \frac{S_A^1x_1+S_B^1y_1+S_C^1z_1-[(l_1+sx_1)x_1+(m_1+sy_1)y_1+(n_1+sz_1)z_1]}{(l_1+sx_1)x_1+(m_1+sy_1)y_1+(n_1+sz_1)z_1} \\ H_2(x) = \frac{S_A^2x_2+S_B^2y_2+S_C^2z_2-[(l_2+sx_2)x_2+(m_2+sy_2)y_2+(n_2+sz_2)z_2]}{(l_2+sx_2)x_2+(m_2+sy_2)y_2+(n_2+sz_2)z_2} \end{Bmatrix}$$

subject to,

$y_1 + x_1 + z_1 + x_2 + y_2 + z_2 + x_3 + y_3 + z_3 \leq T$, $x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3 \geq 0$.

10. Conclusions

An MLMOQFP model using TrFN as coefficients in objectives and constraints is solved in this work to obtain an effective solution. The Rouben Ranking Function is used to cope with TrFN coefficients and build a crisp model. Then, a highly efficient iterative parametric technique for transforming a fractional model to a non-fractional model is provided. Furthermore, this method is paired with the ε-constraint method to tackle a large number of objectives while maintaining a single objective model that is easy to solve. This method alters the viable region, resulting in a more representative set of efficient solutions. Because it swiftly converges on the best optimal solution, the suggested method is very efficient in terms of finding a solution. In addition, a numerical is solved at the end to illustrate the technique's practicality. The proposed method can be used in a variety of disciplines, including industrial optimization, economics, traffic, finance, and waste management. The approach can also be used to solve bi-level and multi-level programming issues, as well as MOQFP issues that involve coefficients as triangular, pentagonal, intuitionistic fuzzy numbers and intervals.

Conflict of Interest

The authors state that they have no financial or other conflicts of interest that are relevant to the content of this study.

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