

A Multi Objective Sustainable Queuing Supply Chain Model with Learning Effects and Fuzzy Environment

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Abstract

The present paper deals with a fuzzy-based supply chain model with the effect of learning for the supply chain under queuing theory, where carbon emission policies are allowed. Nowadays, carbon emissions affect the environment and ordering policies of industries. It is considered that storage items emit carbon units, and the carbon emission cost is included in this paper. The effect of learning reduces the total fuzzy cost per shipment during ordering policies, and the backordering cost and setup expenses follow the effect of learning, where the holding cost has been taken as fuzzy input parameters. The queuing concept is included in this model, and the proposed model is classified into three models under the learning effect. The manufacturer produces multiple items due to the multiple demands of the products from several buyers, and the produced items are stored in different warehouses. The total fuzzy cost is defuzzified with the help of the graded mean integration method under the proposed scenario, and the total cost function is compared with and without a fuzzy environment under the learning effect. For the justification of the proposed model, numerical examples have been presented, and a sensitivity analysis has also been presented to know about the changeable effect of inventory parameters that are involved in this model.

Keywords- Learning effect, Carbon emission, Fuzzy environment, Multiproduct, Supply chain.

1. Introduction

A supply chain is a multi-layered and dynamic system that involves the coordinated participation of various organizations, human resources, information flows, and physical resources, through which goods or services are delivered to the final customer. This process includes all the stages of raw material collection (sourcing), manufacturing, inventory management, distribution, and final delivery. An efficient supply chain makes optimal use of resources, reduces costs, and increases customer satisfaction. Although trading systems existed in ancient times, the concept of modern supply chain management evolved after the Industrial Revolution, as transportation and production processes improved greatly during that time. In the 21st century, supply chain optimization has gained special importance. Advanced techniques such as Material Requirement Planning (MRP) are widely used to make production planning and inventory control more efficient. Supply chains have become more complex as markets have expanded due to globalization. As a result, it has become necessary for business organizations to adopt modern and effective production and management techniques to keep inventories low, deal with demand uncertainty, and meet customer expectations on time.

Sustainable supply chain models, developed by incorporating social, ethical, and environmental responsibility into supply chain operations, are considered an important concept in modern industrial management. Such models help industries reduce the adverse impacts of their operations on the environment and society, and use resources responsibly. The fundamental objective of sustainability is to maintain a balance between the three pillars of environmental, social, and economic, thereby achieving long-term value creation. In the wake of environmental problems arising from increasing industrialization, awareness of sustainability has increased significantly at the global level. As a result, industries are becoming more aware and committed to reducing carbon emissions, conserving natural resources, and fulfilling social responsibilities. In today's market, consumer preferences are also shifting towards companies that adopt sustainable practices. Therefore, sustainability has emerged as an essential business strategy rather than an optional concept to maintain long-term competitiveness.

Manufacturers produce a variety of products according to the needs of retailers, taking into account seasonal changes in market demand and specific conditions. In order to distribute the final goods efficiently, they are stored in a balanced manner in several warehouses, thereby achieving flexibility and time saving in the distribution process. In real-world situations, warehouse rental rates are not fixed but vary depending on location, facilities, and storage capacity. Some warehouses charge higher fees for advanced features such as climate control or specialized storage. This variation makes it difficult to accurately measure inventory holding costs, resulting in uncertainty in determining total costs. Considering this uncertainty, the impact of variable warehousing costs on total costs is analyzed in the presented model. For this, a numerical example based on previously published data has been used. Also, crisp and fuzzy inventory models have been created and compared, thereby obtaining more feasible and effective conclusions.

The current literature on EOQ models, supply chains, backorders, steady-state probability, queuing theory, and fuzzy techniques is also reviewed in this paper. Selected literature that is pertinent to the suggested model is included in the next section. The Economic Order Quantity (EOQ) model was initially presented by Harris (1913) in his essay "How Many Parts to Make at Once", which was published in *Factory: The Magazine of Management*. By balancing ordering and holding costs, the EOQ model is a traditional inventory tool that assists businesses in determining the ideal order size to minimize overall

inventory expenses. Harris (1913) established the framework, although Wilson later improved the model on his own Wilson (1934). Over time, many researchers further extended the EOQ framework. Despite its assumptions, such as constant demand and fixed costs, the EOQ model remains widely used, and several extensions have been proposed, including the Economic Production Quantity (EPQ) model. Utama et al. (2023) developed a sustainable production–inventory model that incorporates probabilistic demand, quality deterioration, and multiple raw materials, supported by a bibliometric literature analysis.

Maity et al. (2023) proposed an EOQ model for green products by considering carbon emissions. In their fuzzy model, demand was utilized in a pentagonal sense of fuzzy number, and a new defuzzification approach was introduced. They also compared crisp and fuzzy environments using numerical examples solved with LINGO software. Karmakar and De (2023) examined EOQ models using different triangular fuzzy norms, where demand varies with time and traveler flow. Several researchers have further enhanced EOQ and EPQ models by integrating queuing theory to reflect more realistic supply chain conditions.

By examining resource usage, environmental performance, and system efficiency, queuing theory is a valuable tool for modelling sustainable supply chains. It can be used in fields including waste management, transportation optimization, energy efficiency, emission reduction, and green logistics. Queuing theory is specifically applied to multiproduct sustainable supply chains in the current work.

Teimoury et al. (2010) developed a model based on queuing theory for a multi-item inventory system considering various production techniques. The presented model includes stochastic production durations, alternating production cycles, and setup times. In addition, holding cost, backordering cost, and ordering cost are included in a linear cost function to present a combined cost analysis. Liu et al. (2020) proposed a combined use of a hybrid genetic algorithm and two-phase queuing theory to solve location-inventory problems under supply uncertainty. Their approach can achieve effective decision-making even under uncertain conditions. Nazari-Ghanbarloo and Ghodrathnama (2021) propose a resilient, tri-objective, and multi-period supply chain model based on the queuing theory framework. This model attempts to achieve the best solution by using the p -robust technique to balance various objectives.

Sharma and Pandey (2022) have conducted an in-depth analysis of B2C supply chains using M/M/1 and M/M/C queuing models. In their study, system performance was measured based on important factors such as waiting time and service time, and the efficiency of the service process was evaluated. Rostami et al. (2022) studied a three-tier supply chain structure with cross-docking using G/G/M queue models and metaheuristic optimization techniques. Their approach helps in achieving efficient coordination in complex logistics networks. Nozari and Rahmaty (2023) have focused on the challenges of demand-driven production management by studying make-to-order systems under uncertainty. Similarly, Jahani and Gholizadeh (2022) have proposed a flexible supply chain design through the integrated use of inventory optimization and queuing theory, which can respond more effectively to changing demand conditions. Pandey and Sharma (2024) have incorporated learning effects into a production-inventory model based on queuing theory to account for improvements in production process efficiency over time. Their model allows for a more realistic representation of the dynamics of production and inventory management.

Many researchers have focused on sustainability and green supply chains. Asha et al. (2022) proposed a multi-objective optimization model that integrates economic, environmental, and social objectives. Dey et al. (2022) developed a sustainable supply chain model that includes automated production and intelligent transportation. Also, Maheshwari et al. (2023) conducted an in-depth study of waste management in a three-tier supply chain, and Sharifi et al. (2023) presented a two-stage optimization model for soybean

supply chains under uncertainty. Torabzadeh et al. (2022) analyzed green supply chains using fuzzy parameters for perishable goods. Additionally, Soon et al. (2022) compared robust possibilistic and standard possibilistic programming methods in the context of closed-loop supply chains. Alinezhad et al. (2022) developed a fuzzy, bi-objective inventory network model. Karthick (2023) applied genetic algorithms to a sustainable supply chain model considering uncertain carbon emissions. Also, Panja and Mondal (2023) have proposed a two-level green production–inventory model based on type-2 fuzzy logic. Finally, Ruidas et al. (2022) have studied EOQ models based on incomplete and reworkable items under various uncertain environments.

Learning effects have been studied extensively in inventory and supply chain models. Wright (1936) first introduced the concept of learning curves, explaining how repetition improves efficiency. Learning effects affect ordering costs and holding costs, especially in multi-product systems where there is a high level of repetition of tasks. Li et al. (2018), Yang et al. (2019), Huang and Wang (2020), and Chen et al. (2023) have extensively studied learning effects in the context of decision-making and innovation in supply chains. Additionally, Wang et al. (2022) integrated reinforcement learning with inventory control, while Ganesan and Uthayakumar (2021) used learning effects to determine a retailer's ideal order quantity.

Mahapatra et al. (2012) developed a fuzzy EOQ model for perishable goods considering learning effects and promotional efforts. Bagherinejad and Pishkenari (2026) developed an FM/FM/c fuzzy queueing model by representing arrival and service rates as fuzzy numbers. The study used fuzzy inference and multi-objective decision-making techniques to evaluate system performance and determine the optimal number of servers under uncertainty. Alamri et al. (2023) showed that buyers can achieve higher profits in a fuzzy environment and also incorporated learning effects into an inventory model. Also, Rani et al. (2019) proposed a fuzzy green supply chain model considering carbon-based demand. Singh et al. (2020) analyzed a fuzzy multi-stage EOQ model with backlogging and steady-state probability distributions. Jayaswal et al. (2021) presented a fuzzy EOQ model incorporating credit financing, backordering, and learning effects. In addition, Alamri et al. (2022), Kumar et al. (2023), and Pervin (2024) have further extended EOQ models by including sustainability, learning effects, backordering, and steady-state probability distributions.

This proposed scheme considers some basic fuzzy concepts, which are based on the research work of Kahraman et al. (2006) and Kazemi et al. (2010). Sebatjane (2025) developed a new taxonomy to classify the literature and considers factors such as type of emission regulations, input data, type of goods, shortages, delayed payments, nature of demand, and objective function. The purpose of this research is to review the existing literature on sustainable inventory models under carbon emission regulations from 2013 to 2023. Further, Saurav et al. (2025) developed a sustainable production-storage model incorporating advanced booking, discount incentives, and green investments within a carbon cap-and-trade framework. The model aims to maximize total profits while maintaining environmental sustainability, and also considers time- and price-sensitive demand. The storage cycle is divided into four stages, comparing carbon emissions from production, storage, transportation and degradation to regulatory limits. Penalties are imposed if the limit is exceeded, while additional benefits are obtained through carbon credit trading if emissions are lower. Currently, Sharma et al. (2025) proposed a sustainable inventory model that includes strategic investments in preservation technology and green technologies to reduce waste. This study accounts for price- and advertising-driven demand, as well as time-varying holding costs, all of which are subject to carbon cap-and-trade regulation. In addition, shortages are allowed and treated as fully backlogged, making the model more realistic.

Kazemi et al. (2015) extended the fuzzy EOQ model to include the effect of human learning. In the presented model, demand and lead time are modeled in a fuzzy form, and the concept of learning in fuzziness is adopted, providing a more dynamic approach than traditional static fuzziness. Jayaswal et al. (2025) developed a sustainable EOQ model for deteriorating items, which integrated trade credit, carbon emission, and the learning effect. The main objective of the presented model is to maximize profit, and its realism is enhanced by including preservation technology, shortages, and different demand patterns. Jayaswal and Seikh (2026) proposed a sustainable EOQ model for defective and deteriorating goods, which incorporates trade credit, preservation technology, and carbon emissions. In the presented model, defective and non-defective goods are managed separately. In addition, the model has been made more realistic and practical by including monsoon-based fuzzy demand, inspection processes, and learning effects.

The remainder of this paper is neatly divided into various sections and subsections. Subsection 1.2 discusses the research gaps and highlights the main contributions of the study. Section 2 presents the basic definitions required for the development of the proposed model. Section 3 explains the detailed structure of the model, along with the necessary discussions and numerical examples. Section 4 presents the sensitivity analysis and related findings. Section 5 explains the management implications as well as the social implications. Finally, Section 6 presents the conclusion of the entire study and proposes necessary recommendations for further research. We used some abbreviations, which are (a). FCFS: First Come First Serve (b) SCV: Squared Coefficient of Variation (c) PA: Production Authorization. (d) GMI: Graded Mean Integration method

The overall proposed plan is pictorially presented by the flow chart given below in **Figure 1**.

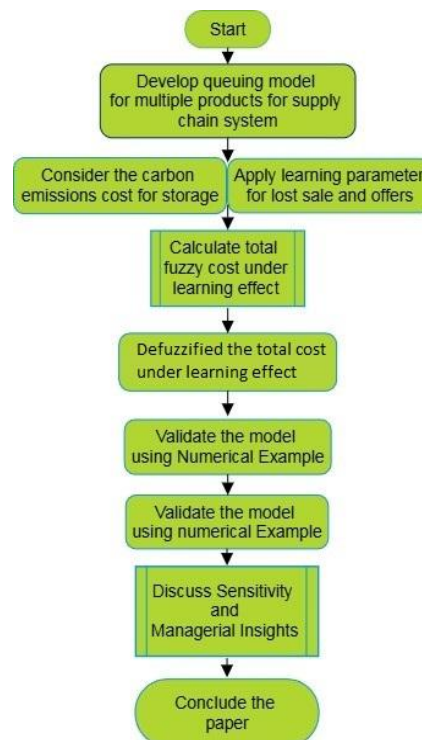


Figure 1. Working process through the flow chart.

2. Research Gap and Novelty

Our in-depth study of the research work has identified some important research gaps. These gaps are particularly useful in situations where holding costs in the supply chain are uncertain across industries, companies, and various industrial sectors. In the proposed study, supply chain-related problems are effectively linked to real-life concepts. These include steady-state probability distributions, queuing theory, as well as fuzzy holding and back-order costs. Notably, these factors have not been fully or comprehensively studied in the previous literature. The identified research gaps and the main contributions of the present study are summarized at the end of **Table 1**.

Table 1. Author's contributions and proposed work.

Authors	Product type taken	Use queuing	Backorder allowed	Fuzzy method	Steady-state probability distribution	Carbon emissions	Learning concepts	Cost structure
Ghalekhondabi and Suer (2018)	General Stock/Inventory	□	×	×	×	×	×	Classical inventory cost
Hum et al. (2018)	General Inventory product	□	×	×	×	×	×	Single-objective cost
Dehghani and Taki (2019)	General Inventory product	□	×	×	×	×	×	Queuing-based
Huang and Wang (2020)	General Inventory product	×	×	×	×	×	□	Learning-based
Liu et al. (2020)	Inventory items	□	×	×	×	×	×	Partial integrated
Daroudi et al. (2021)	Perishable-type multiproducts	×	□□	×	×	×	×	Partial integrated
Jayaswal et al. (2021)	Inventory for Ordering Policy	×	□	×	×	×	□	Learning-based
Karim and Nakade (2021)	Ready to use Inventory product	□	×	×	×	□	×	Queuing-based
Alinezhad et al. (2022)	Bake products	×	×	□	×	×	×	Partial integrated
Aliahmadi et al. (2023)	General Inventory products	□	×	□	×	×	×	Queuing-based
Nozari and Rahmaty (2023)	Ideal Inventory product	×	×	□	×	×	×	Partial integrated
Masruroh et al. (2024)	Green products	□	×	×	×	×	×	Queuing-based
Pandey and Sharma (2024)	Production Inventory	□	×	×	×	□	□	Queuing-based
Pandey et al. (2024)	Sustainable Supply Chain	□	×	□	×	□	□	Learning-based
Jayaswal et al. (2025)	Green Supply Chain	×	×	□	×	□	×	Learning-based
Deepi et al. (2026)	Production Inventory	×	×	□	×	□	×	Learning-based
Meena and Ayyappan (2026)	Queuing Inventory	□	×	×	×	□	×	Queuing-based
Jayaswal and Sheikh (2026)	Sustainable EOQ	×	×	□	×	×	×	Partial integrated
Current study	Production of Multi-Product items	□	□	□	□	□	□	Fully integrated (GMI optimized)

The cost structures in existing studies vary depending on the modelling assumptions and included system effects. Therefore, a classification-based comparison is used instead of a direct numerical cost comparison. The proposed model integrates all major cost components within a unified fuzzy queuing framework.

All of the above factors are very important in terms of multi-product inventory and supply chain system management. However, previous research has not considered all these aspects together in a unified model, as is done in the present study. In the present research, steady-state probability distributions for multiple products in a fuzzy environment are considered. The term “steady-state probability” means the long-run chance of a warehouse being in a congested or stock-out condition. Warehouses with better facilities have faster processing and higher efficiency, so they handle demand more quickly. Because of this, the probability of being in such undesirable states becomes lower compared to other warehouses. In this context, a manufacturer produces various goods according to the demand of retailers and stores them in different warehouses with varying rents. Since holding costs are uncertain in nature, they are represented using triangular fuzzy numbers. Backorder costs are also assumed to vary across products in the event of shortages. In addition, a linear fuzzy cost function is developed that includes transportation costs, holding costs, and backorder costs. This system follows the FCFS (First Come First Serve) rule, and the GI/G/1 queue model has been used, considering the setup time in the production process, as the products are produced one after the other. We also considered the carbon emissions cost due to storage of items in holding cost and the learning effect on lost sale and offers of items, and further, this model has been extended to a three-tier supply chain, and the GMI technique has been adopted to calculate the total fuzzy cost under learning effect.

The model developed in the present study effectively solves various production and manufacturing problems arising in multi-product systems where backorders are allowed in a fuzzy environment. The model is useful in determining the minimum inventory cost in the context of holding costs varying with demand. The uncertainty in holding costs is reduced by using the GMI method. In situations where companies can make or lose money due to fluctuations in demand for their products, the model helps to control the fluctuations in costs by using a steady-state probability distribution. In many existing queue-based supply chain models, holding costs are assumed to be fixed. In contrast, the presented model evaluates the minimum total inventory cost by considering holding costs as a fuzzy value. A distinctive feature of this model is the integrated application of the queue-based supply chain framework to a variety of multi-product scenarios, as shown in **Table 1**. Also, the model is useful for decision makers to gain a practical understanding through numerical examples and sensitivity analysis.

3. Assumptions and Notations

3.1 Assumptions

- (i) We assume that each retailer follows an independent renewal process with a fixed arrival rate $\gamma_i \geq 0$ and a Squared Coefficient of Variation (SCV) D_i^2 . At each retailer, customers demand different types of products. This demand is represented by a probability vector $B_i = (B_{i1}, B_{i2}, \dots, B_{iL})$ ($\sum_{j=1}^L B_{ij} = 1, 0 \leq B_{ij} \leq 1; i = 1, 2, \dots, n$), where the sum of all probabilities equals 1, and each probability lies between 0 and 1. Based on this demand pattern, orders are regularly sent to the warehouse j .
- (ii) In our model, each warehouse stores only one type of product in batches of size B_j . A maximum of O_j Batches can be stored in a warehouse. Therefore, the maximum inventory level of the product j is $I_j = O_j \times B_j$.
- (iii) For each product, we apply a Production Authorization (PA) system. All products are stored in batch form, and each batch has a production authorization card attached to it. In this model, we make the assumption that there are as many batches as there are production authorization cards.
- (iv) When all B_j units in a batch are used up, the production authorization system is activated and sends a signal to start producing a new batch to replenish the inventory.
- (v) The corresponding PA card is sent to the manufacturing unit as a new production order, which begins the next production cycle. The manufacturing unit follows the FCFS (First Come First Serve) rule to

process these orders. After producing B_j units, the finished batch, along with the PA card, is returned to the warehouse. If a customer places an order when no inventory is available, the customer must wait until the product is produced and restocked.

- (vi) The production policy follows a make-to-stock strategy based on demand forecasting. It applies the $(Oj - 1, Oj)$ control rule for product j in the warehouse. This means that when the inventory level falls to $Oj - 1$ batches, production is triggered to restore the stock up to Oj batches.

Based on the above assumptions, the arrival of batches at the manufacturer follows a poisson process. The total arrival rate is

$$\gamma_a(B_L) = \sum_{j=1}^L \frac{\gamma_{aj}}{B_j}.$$

This means that the overall batch arrival rate is obtained by dividing the product arrival rate at each warehouse by its batch size and then summing over all products.

The Squared Coefficient of Variation (SCV) of the batch arrival process at the manufacturer is:

$$D_a^2(B_L) = \frac{\sum_{j=1}^L \frac{\gamma_{aj}}{B_j} D_{aj}^2}{\gamma_a(B_L)}.$$

- (vii) The probability that an arriving batch at the manufacturer belongs to the product j is defined as the proportion of its arrival rate to the total arrival rate.

Whenever the manufacturer switches from producing one product type to another, a setup process is required. Let Pb and SU be random variables representing the batch processing time and the setup time, respectively. For a batch of product j , the setup time is also treated as a random variable with a mean τ_j and variance χ_j . The probability that a batch of product j requires a setup $1 - P_j$. Using this probability, we can calculate the overall mean and standard deviation of the setup time for any batch, which is given by:

$$E[S_U] = \sum_{j=1}^L P_j (1 - P_j) \tau_j \quad (1)$$

$$D_U^2 = \frac{\text{var}[S_U]}{E^2[S_U]} = \frac{\sum_{j=1}^L P_j (1 - P_j) \chi_j}{\left(\sum_{j=1}^L P_j (1 - P_j) \tau_j\right)^2} \quad (2)$$

- (viii) For the product j , we assume that the unit production time at the manufacturer follows a random distribution. It is represented by $1 / \mu_{S_j} = E(B_{L_j})$, with a Squared Coefficient of Variation (SCV) D_U^2 . Therefore, the mean production time for a batch of size B_j is B_j / μ_{S_j} , and the coefficient of variation for that batch becomes D_U^2 / B_j . In the same way, for any batch, we can calculate the overall mean processing time and its SCV using:

$$E[P_b] = \sum_{j=1}^L P_j \frac{Y_j}{\mu_{S_j}} \quad (3)$$

$$D_{P_b}^2 = \sum_{j=1}^L Y_j D_j^2 \quad (4)$$

After finding the effective batch service time S , where $S = SU + Pb$. For any batch, the total service time is the sum of setup time and processing time. Using this combined service time, we can calculate the mean service time and its coefficient of variation based on the mean and variability of both setup time and processing time.

$$E[S] = E[S_U] + E[P_b] \quad (5)$$

$$D_s^2 = D_{S_U}^2 D_{P_b}^2 \quad (6)$$

(ix) The utilization of the manufacturing plant is defined as

$$\Phi = \sum_{j=1}^L \frac{\gamma_{aj}}{B_j} E[S] = \gamma_a(B_L) E[S] \quad (7)$$

This means that plant utilization is equal to the total batch arrival rate multiplied by the average service time of a batch. The system includes three main types of costs under the learning effect, where the carbon emission cost is included.

- Backordering cost $b_{cj} = b_{cjo} + \frac{b_{cj1}}{N^{L_j}}$ per unit of product per unit time.
 - Holding cost H_j (in dollars) per unit of inventory per unit time.
 - Setup cost D_{S_j} per production setup for product j .
- (x) The carbon emission cost is incorporated due to holding items in the inventory system as a holding cost.
- (xi) Since learning plays an important role in the lost sale and various offers provided, we also considered learning parameters on the lost sale and offer as well.

The aim behind the model is to minimize the total supply chain cost and determine the optimal values of O_j (number of batches) and B_j (batch size).

3.2 Notations

The following notations are used in this proposed plan:

B_j : Quantity of units in a single product bucket j , $j = 1, 2, \dots, L$.

L_j : Learning parameter.

O_j : Total number of buckets in the product warehouse j .

H_j : Inventory holding cost (\$/unit time /year).

$\widetilde{H}_j$: Inventory fuzzy holding cost (\$/unit time /year).

c_{ej} : Carbon emission cost per item (\$).

C_j : Maximum carbon emission cost $c_{ej} O_j Y_j$ due to product inventory at the warehouse j ; $O_j Y_j$ (units).

TC : Total inventory cost under the learning effect (in \$).

$\widetilde{TC}$: Total fuzzy inventory cost under learning cost (in \$).

(H_{1j}, H_{2j}, H_{3j}) : Triangular fuzzy number for the holding cost.

X_j : Maximum product inventory at the warehouse j ; $O_j Y_j$ (units).

γ_{aj} : Retailers' demand for product arrival rate $j(\sum_{i=1}^n \gamma_i y_{ij})$ (units/year).

D_{aj}^2 : SCV of retailers' product arrival rate demand $j\left(\frac{1}{\gamma_{aj}} \sum_{i=1}^n \gamma_i B_{ij} D\right)$.

N_{1j} : Quantity of product j orders that reached the manufacturer (units).

μ_{S_j} : In the manufacturing plant, the product j 's service rate (units/unit time).

D_j^2 : In a manufacturing plant, the product j 's SCV of service rate (units/unit time).

P_j : The likelihood that a product will arrive j .

τ_j : Average product setup time j .

χ_j : Variations in the product's setup j .

S_U : Setup time.

P_b : Batch processing time.

- I_{Lj} : Inventory level for product j .
- N_{2j} : Total quantity of product j orders in a $GI / (G / 1)$ queue at the manufacturing facility (in units).
- B_{Lj} : Product j 's backorder level at the warehouse (in units).
- N_{3j} : Numerous orders were received for the product j 's warehouse; however, H_j The inventory holding costs (\$ per unit/unit time) increased after the final batch was released for processing.
- b_{cj} : Costs associated with product backorders j (\$ /unit/unit time). $b_{cj} = b_{cjo} + \frac{b_{cj1}}{N^{L_j}}$ costs associated with product backorders j (\$ /unit/unit time) under the learning effect, where L_j this is the learning effect.
- S_{cj} : Order setup expenses for goods j (\$ / set up). $S_{cj} = S_{cjo} + \frac{S_{cj1}}{N^{L_j}}$, Order setup expenses for goods j (\$ / set up) under the learning effect.
- L_j : Learning parameter.
- ρ : The intensity of the manufacturing plant.
- L_{ji} : Lead time for retailer i to get merchandise from the product j 's warehouse.
- ω_{ji} : Expected order quantity for product j at retailer i , in the steady-state queue $M^{D_j}/M/\infty$.
- Φ : The logistics process's service rate.
- ρ'_j : Strength of the logistics centre $\gamma_{aj}/\Phi < 1$.
- E_{Wj} : Anticipated waiting period for the product j . The warehouse is solely as a result of backorders.
- M_{ji} : Average lead time (including back-ordering delay) for a retailer's order that will be fulfilled from the product warehouse j , $M_{j1} = M_{j2} = \dots = M_{jn} = M_j$.
- $E_{D_{ji}}$: Demand for product j that is expected at the retailer during each item's lead time $i(\gamma_{a,j}M_{ji})$.

4. Model Formulation and its Mathematical Interpretation

Here, we discuss the model formulation and its theoretical aspects. Holding and ordering costs are treated as triangular fuzzy numbers to represent uncertainty. The total cost function is defuzzified using the GMI method. We compare total inventory costs in both fuzzy and non-fuzzy environments. The delivery policy follows the ordered lot size, benefiting sellers, buyers, and customers. Fuzzy theory improves the efficiency of the queuing-based supply chain model and reduces total inventory cost under imprecise conditions. The manufacturer produces multiple products based on retailers' demand and distributes them to different warehouses. Warehouse rents vary over time, and some have better facilities with lower steady-state probabilities. Since holding costs fluctuate, they are considered uncertain. Each product requires a setup time, and production occurs sequentially without overlap. The system is managed using the FCFS rule and a GI/G/1 queuing framework, because general arrival and general service rates with one server are considered in multiproduct production.

In the proposed plan, the GI/G/1 queue is used because both order arrivals and service times vary across products and cannot always be represented by the concepts of fixed distributions. Since all products are processed through a single manufacturing unit, only one server is considered. The First Come First Serve rule is applied because production orders are handled in the order they arrive.

Figures 2 and Figure 3 represent a manufacturing plant operating within a three-echelon supply chain system comprising a manufacturer, L warehouses, and retailers. The network supplies L different products to customers arriving at retailer nodes. Retailers collect customer demand and forward the aggregated product-wise demand to the respective warehouses. Based on warehouse requirements, production authorization is issued to the manufacturer, ensuring each warehouse maintains an order quantity of B_j for product j . The manufacturing unit produces multiple items and maintains separate inventory buffers for

each product. If stock is available, demand is satisfied immediately; otherwise, it is backordered. The production process follows a GI/G/1 queueing mechanism, and completed batches are dispatched to warehouses to fulfil retailer demand.

The GI/G/1 queue is used to represent the manufacturing process, where production orders arrive randomly and are processed by a single server. The queueing system affects inventory levels, waiting times, and backorders. These measures are further incorporated into the cost function together with fuzzy holding costs, learning effects, and carbon emission costs.

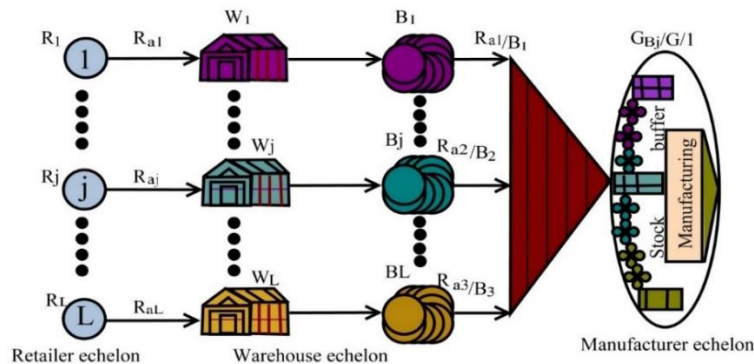


Figure 2. Representation for a 3-level echelon supply chain under a manufacturing system.

4.1 Problem Formulation

This study seeks to minimize the expected total cost of the warehouse system by optimizing the overall cost function. In mathematical terms, the objective can be expressed as follows:

$$\text{Min} \sum_{j=1}^M TC(O_j, B_j) = \sum_{j=1}^M \left(H_j E[I_{Lj}] + \left(b_{cjo} + \frac{b_{cjl}}{N_{Lj}} \right) E[B_{Lj}] + S_c \left(\frac{\gamma_{aj}}{B_j} \right) \right) + C_j \quad (8)$$

Such that $O_j, B_j \in W^+$.

In real-life supply chain systems, holding costs are often uncertain due to different warehouse rent, maintenance, and storage charges may change over time. Therefore, we cannot justify it by using a single fixed value with actual conditions. To handle this situation, the holding cost H_j is expressed as a triangular fuzzy number $\tilde{H}_j = H_{1j}, H_{2j}, H_{3j}$. By replacing the crisp holding cost in Equation (8) with its fuzzy counterpart, the fuzzy total cost function is obtained. The resulting fuzzy cost is then defuzzified using the GMI method to derive an equivalent crisp value for optimization. The fuzzy framework is constructed based on the crisp model and Alsaedi et al. (2023) definitions.

$$\text{Min} \sum_{j=1}^M \tilde{TC}(O_j, B_j) = \sum_{j=1}^M \left(\tilde{H}_j E[I_{Lj}] + \left(b_{cjo} + \frac{b_{cjl}}{N_{Lj}} \right) E[B_{Lj}] + S_c \left(\frac{\gamma_{aj}}{B_j} \right) \right) + C_j \quad (9)$$

Such that $O_j, B_j \in W^+$.

The overall expected fuzzy cost is converted into a crisp value using the GMI method. Consequently, by applying Equation (9), we obtain the following expression:

$$\text{Min } \sum_{j=1}^M \widetilde{\widetilde{TC}}(O_j, B_j) = \sum_{j=1}^M \left(\frac{\int_0^{W_A} H \left(\frac{L^{-1}(H) + R^{-1}(H)}{2} \right)}{\int_0^{W_A} H dH} E[I_{Lj}] + \left(b_{cjo} + \frac{b_{cj1}}{N_{Lj}} \right) E[B_{Lj}] + S_c \left(\frac{\gamma_{aj}}{B_j} \right) \right) \quad (10)$$

Here, $\tilde{H}$ represents a triangular fuzzy number defined as (H_{1j}, H_{2j}, H_{3j}) . Using Equation (10) and simplifying the resulting expression, we obtain:

$$\text{Min } \sum_{j=1}^M \widetilde{\widetilde{TC}}(O_j, B_j) = \sum_{j=1}^M \left(\frac{H_{1j} + 4H_{2j} + H_{3j}}{6} E[I_{Lj}] + \left(b_{cjo} + \frac{b_{cj1}}{N_{Lj}} \right) E[B_{Lj}] + S_c \left(\frac{\gamma_{aj}}{B_j} \right) \right) + C_j \quad (11)$$

The stochastic equations that describe the system characteristics specified in Equation (12), and are used to determine the inventory level and backorders, are given as follows:

$$N_{3j} = N_{1j} - \lfloor N_{1j} B_j^{-1} \rfloor B_j, j = 1, 2, \dots, L \quad (12)$$

$$B_{Lj} = \text{Max} \left[N_{2j} B_j + N_{3j} - K_j B_j, 0 \right], j = 1, 2, \dots, L \quad (13)$$

$$I_{Lj} = \text{Max} \left[K_j B_j - N_{2j} B_j - N_{3j}, 0 \right], j = 1, 2, \dots, L \quad (14)$$

For N_{3j} , N_{2j} , and i_j , the corresponding steady-state probability distributions are given as follows:

The distribution of N_{3j} is uniform from 0 to $B_j - 1$. Thus,

$$P\{N_{3j} = m\} = \frac{1}{B_j}, m = 0, 1, \dots, B_j - 1 \quad (15)$$

To determine the probability distribution of batches in the GI/G/1 queue, we assume a geometric distribution of the following form and apply Equation (15) as given below:

$$P\{N_2 = m\} \approx \begin{cases} 1 - \rho & m = 0 \\ \rho(1 - \sigma)\sigma^{m-1} & m = 1, 2, \dots \end{cases} \quad (16)$$

where, $\sigma = (\hat{N}_2 - \rho / \hat{N}_2)$, $\hat{N}_2 = \gamma_a(B_L)w_0 + \rho$ and $W \approx \left\{ \frac{\rho^2(1 + D_s^2)}{1 + \rho^2 D_s^2} \right\} \left\{ \frac{D_a^2 + \rho^2 D_s^2}{2\gamma_a(1 - \rho)} \right\}$.

Using Equation (16), the approximate order quantity in the GI/G/1 queue for product j is expressed as follows:

$$P_j\{N_{2j} = m_j\} \approx \begin{cases} 1 - \left(\frac{\rho}{\sigma}\right) r_j m_j & m_j = 0 \\ \left(\frac{\rho}{\sigma}\right) (1 - r_j) r_j^{m_j} & m_j = 1, 2, \dots \end{cases} \quad (17)$$

Here, $r_j = P_j \sigma \frac{1}{1 - \sigma(1 - p_j)}$, $p_j = \gamma_{a,j} \frac{1}{Y_j \gamma_a(B_L)}$. The corresponding steady-state probability distributions for I_{Lj} , B_{Lj} are given as follows:

$$P\{I_{Lj} = m\} = \frac{P_{N_{2j}}}{B_j} \left(\left\lfloor \frac{X_j - m}{B_j} \right\rfloor \right); m = 1, 2, \dots, O_j B_j \quad (18)$$

$$P\{B_{Lj} = m\} = \frac{P_{N_{2j}}}{Y_j} \left(\left\lfloor \frac{X_j + m}{B_j} \right\rfloor \right); m = 1, 2, \dots, O_j B_j \quad (19)$$

We can obtain $E[I_{Lj}]$ and $E[B_{Lj}]$ as:

$$E[j] = \sum_{i=1}^{X_j} \frac{X_j - i}{B_j} P_{N_{2j}}(i) \quad (20)$$

$$\text{Total } E[B_{L_i}] = \sum_{i=0}^{\infty} \frac{i}{Y_j} P_{N_{2j}}\left(\left\lfloor \frac{X_j + i}{B_j} \right\rfloor\right) \quad (21)$$

$$\begin{aligned} E[B_{L_i}] &= \sum_{i=0}^{\infty} \frac{i}{B_j} P_{N_{2j}}\left(\left\lfloor \frac{X_j + i}{B_j} \right\rfloor\right) = \frac{1}{2} \left(\frac{\rho}{\sigma}\right) (1 - r_j) r_j^{O_j} \left[\frac{B_j - 1}{1 - r_j} + \frac{2B_j}{1 - r_j} + \frac{r_j}{1 - r_j} \right] \\ &= \frac{1}{2} \left(\frac{\rho}{\sigma}\right) r_j^{O_j} \left[(B_j - 1) + \frac{2B_j}{1 - r_j} \right] \end{aligned} \quad (22)$$

Using Equations (8) and (9), we can determine the optimal inventory level for each product type.

4.2 Warehouse Measures

The stock-out probability represents the fraction of time during which the on-hand inventory of product j at the warehouse is equal to zero. It can be determined using the following expression:

$$\begin{aligned} P\{I_{Lj} = 0\} &= P\{X_j \leq N_{2j}(t)B_j + N_{3j}(t)\} = P\left\{\frac{X_j - N_{3j}(t)}{B_j} \leq N_{2j}(t)\right\} \\ &= P\left\{O_j - \frac{N_{3j}(t)}{Y_j} \leq N_{2j}(t)\right\} \\ &= \frac{1}{B_j} P\left\{O_j \leq N_{2j}(t)\right\} + \frac{B_j - 1}{B_j} P\left\{O_j - \frac{N_{3j}(t)}{B_j} \leq N_{2j}(t)\right\} \\ &= \frac{1}{B} \left(\frac{\rho}{\sigma}\right) r_j^{O_j} + \frac{B_j - 1}{B_j} \left(\frac{\rho}{\sigma}\right) r_j^{O_j - 1} \end{aligned} \quad (23)$$

Furthermore, the fill rate for the product j at the warehouse is defined as the proportion of time during which the on-hand inventory remains positive (i.e., greater than zero):

$$\begin{aligned} P\{I_{Lj} > 0\} &= P\{W_j > N_{2j}B_j + N_{3j}\} = 1 - P\{I_{Lj} = 0\} \\ &= 1 - \frac{1}{B_j} \left(\frac{\rho}{\sigma}\right) r_j^{O_j} - \frac{B_j - 1}{B_j} \left(\frac{\rho}{\sigma}\right) r_j^{O_j - 1} \end{aligned} \quad (24)$$

Moreover, the lead time for the product j , the manufacturing facility is given by the following expression:

$$P_{bsj} = \frac{(B_j - 1)}{2} (1/\gamma_{a,j}) + w_0 + (B_j/\mu_{sj}) \quad (25)$$

Here, the batch formation time is given by $\frac{(Y_j - 1)}{2} (1/\gamma_{a,j})$ and the mean production time for a batch of product j is B_j/μ_{sj} .

4.3 The Squared Coefficient of Variation of the Inter-Departure Times Generated by the Warehouses

In this section, we explain the procedure for deriving the characteristics of the batched departure stream from the production facility using the known parameters $\gamma_d(B_L)$, $D_d^2(B_L)$, which can be expressed as follows:

$$\gamma_d(B_L) = \gamma_a(B_L) = \sum_{j=1}^L \frac{\gamma_{a,j}}{B_j} \quad (26)$$

When we consider batch processing in the (GI/G/1) queue, as given in Equation (12) and explained in Equation (26) for batches coming from the manufacturing unit, we calculate the approximate squared coefficient of variation (SCV) of the time between consecutive departures as follows:

$$D_d^2(B_L) = (1 - \rho)^2 \left\{ \frac{D_a^2(B_L) + \rho^2 D_s^2}{(1 + \rho^2 D_s^2)} \right\} + \rho^2 D_s^2.$$

Moreover, we can apply the following equations to determine the characteristics of product j 's batch departure stream from the manufacturing facility:

$$\gamma_{a,j} = \gamma_{a,j} = \sum_{i=1}^L B_{ij} \gamma_i,$$

$$D_{d,j}^2(B_L) = p_j D_a^2(B_L) + 1 - p_j.$$

We also apply the following approximate formula to estimate the squared coefficient of variation (SCV) of the individual departure times from the warehouse for the product j :

$$D_{d,j}^2(I_L) - B_j + 1 = B_j D_a^2(B_L) \quad (27)$$

Here, B_j represents the batch size of the product type j . The probability B_{ij} , which indicates the chance that a product is sent to a retailer i . After leaving the warehouse, it determines both the average time between arrivals and the SCV of arrivals at the retailer i .

4.4 Logistics Process

Next, we include the logistics process in the model. We assume that transporting goods from warehouses to retailers takes a certain amount of time. To represent this, we use an $(M/M^c/\infty)$ queue in continuous time for a product j , where c_j is the fixed vehicle capacity, and the transportation time follows an exponential distribution. We also assume that the logistics process depends on customer demand during the arrival process.

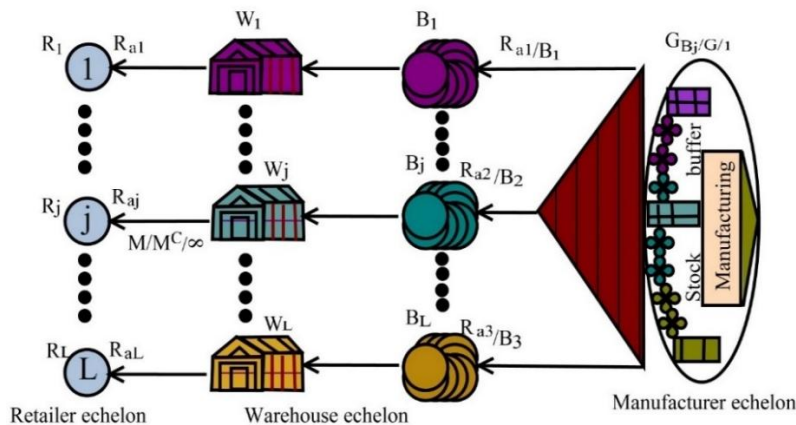


Figure 3. Logistics hub to retailers' supply network process.

We use the results given in Equation (15) and (16) to evaluate the performance of the $M/M^c/\infty$ queue.

By applying Little's Law, the average lead time of product j from its warehouse to the retailer i can be calculated as

$$M_{ji} = L_{ji} + E[W]_j M_{ji} = L_{ji} + E[W]_j M_{ji} = L_{ji} + E[W]_j \quad (28)$$

This can be expressed as follows: $E_{W_j} \gamma_{a,j} = E[B_{L_j}]$,

$$\begin{aligned} M_{ji} &= L_{ji} + E_{W_j}, \\ \omega_{ij} &= \frac{1}{D_j} \frac{y_{ij} \gamma_{a,j}}{\Phi} = \frac{1}{\Phi} (q_{ij} \gamma_{a,j} D_j), \\ L_{ji} &= \frac{\omega_{ij}}{B_{ij} \gamma_{a,j}} = \frac{D_j}{\Phi}. \end{aligned}$$

During the replenishment lead time, the expected demand for the product j at retailer i can be determined as follows:

$$\gamma_{a,j,i} E_{D_{ji}} = M_{ji} \quad (29)$$

5. Numerical Discussion and Managerial Implications

We present a numerical example to justify the proposed model; the numerical data are taken from Teimoury et al. (2010). The demand rates for the products are given as $\gamma_1 = 0.7$ and $\gamma_2 = 0.9$, and the squared coefficients of variation are $D_{a,1}^2 = D_{a,2}^2 = 1$. The probability vectors $y_1 = (0.3, 0.5, 0.7)$ and $y_2 = (0.7, 0.8, 0.9)$ represent customer preferences for the three products at the two retailers. **Table 2** presents the manufacturing unit's production capacity (in units) and the costs of the three warehouses (in dollars). **Table 3** shows the logistics details and the results for the total cost of the system.

Table 2. Decision variables along with their values.

S. No.	Decision variables	For product 1	For product 2	For product 3
1.	μ_{s_j}	0.3	0.7	1.0
2.	D_j^2	0.8	0.9	1.2
3.	τ_j	0.1	0.3	0.6
4.	H_j	2	0.8	0.5
5.	X_j	13	17	21
6.	b_{c_j}	89	112	140
7.	D_{s_j}	10	17	25

5.1 Graphical Representation of Decision Variables

For better understanding, the pictorial interpretation of outcomes of decision variables for product 1, product 2, and product 3 is shown below in **Figure 4**, **Figure 5**, and **Figure 6**, respectively.

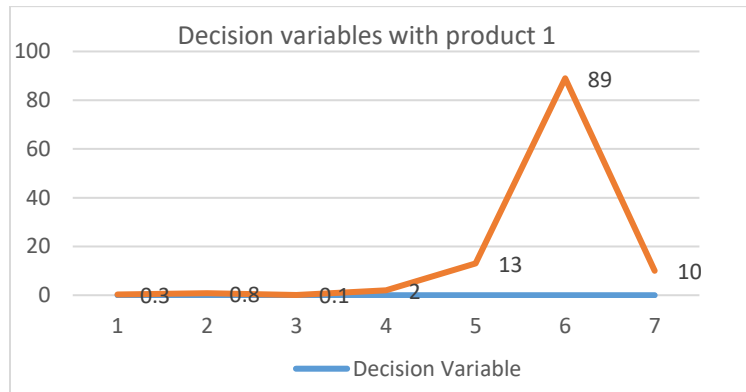


Figure 4. Graphical view of decision variables with product 1.

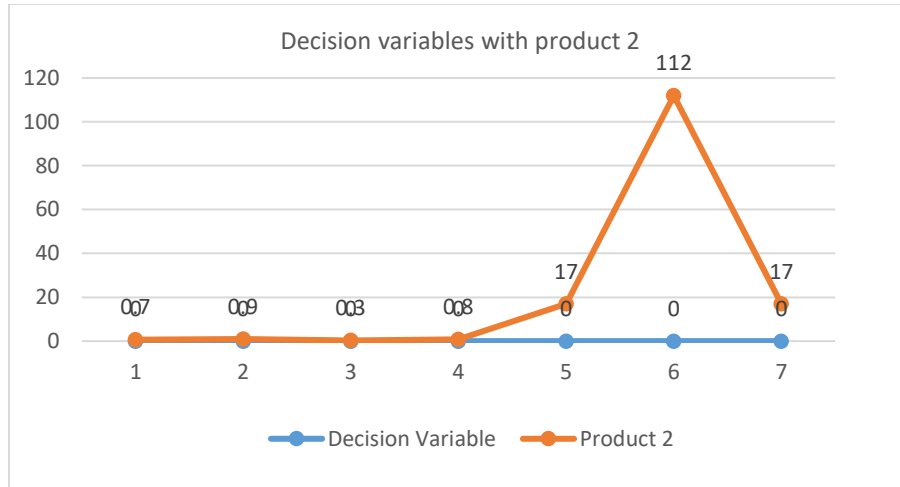


Figure 5. Graphical view of decision variables with product 2.

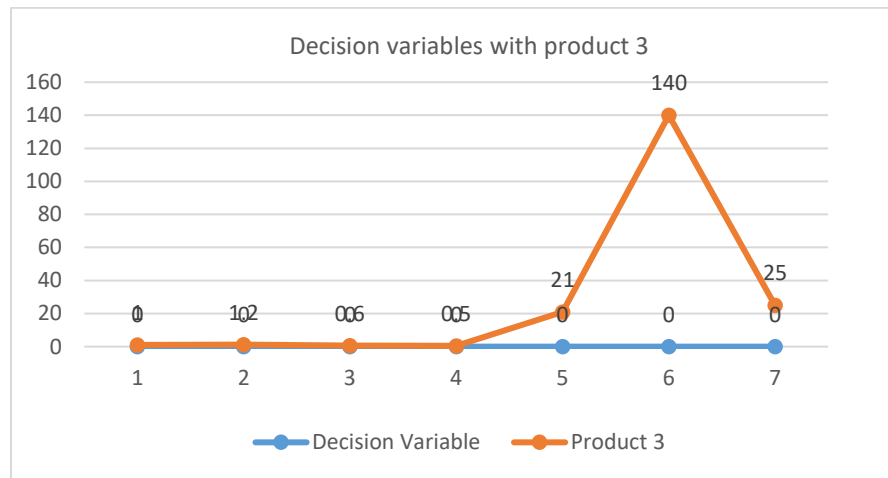


Figure 6. Graphical view of decision variables with product 3.

Figures 4, 5 and 6 illustrate the variation of key decision variables for products 1, 2, and 3 in the system. The variables include production cost, inventory cost, and queueing-related parameters such as μ_{sj} , D_j^2 , τ_j , H_j , χ_j , b_{cj} , and D_{sj} , which collectively capture manufacturing performance, service characteristics, and cost structure. The figures clearly highlight comparative behavior of these parameters across products, reflecting differences in operational efficiency, cost intensity, and system dynamics under learning and stochastic effects.

Using the tabulated data from **Table 2** for all products, we calculate the parameter values of the proposed model, which are reported in **Table 3**. The problem is solved using MATHEMATICA 12, and the total cost is obtained for both the fuzzy and the crisp models. For each of the three products, different batch

sizes are examined to determine the optimal value. We increase B_j , the optimal maximum level of inventory, and analyse the total cost of the products under the condition that $\frac{b_{cjo} + \frac{b_{cj1}}{N_j}}{H_j} - 1 = 0$.

The results indicate that the system maintains higher inventory levels when backorder costs exceed holding costs (**Table 3**).

Table 3. Total fuzzy cost under learning effect by increasing ($\uparrow$) B_j if $\frac{b_{cjo} + \frac{b_{cj1}}{N_j}}{H_j} - 1 = 0$.

Product type	B_j (units)	O_j^*	ρ_j	$E[I_{Lj}]$ (units)	$E[B_{Lj}]$ (units)	$\widetilde{TC}_L$ (in \$)	TC (in \$)
1	4	1	0.8355	38.06	119.04	$2.836e^2$	$3.97e^2$
	6	2	0.4725	504.073	98.34	$2.536e^2$	$3.60e^2$
	8	3	0.1918	1.06	52.26	$2.437e^2$	$3.27e^2$
	10	5	0.0690	4.869	56.38	$2.126e^2$	$3.12e^2$
	12	6	0.0325	1.6678	26.48	$2.056e^2$	$3.06e^2$
2	5	3	0.7500	195.63	82.23	$3.856e^2$	$4.78e^2$
	10	5	0.4000	835.167	72.60	$3.466e^2$	$4.57e^2$
	15	7	0.2000	1.1107	45.30	$3.325e^2$	$4.23e^2$
	20	10	0.0500	$4.95e^3$	32.63	$3.025e^2$	$4.11e^2$
	25	12	0.0250	$1.6878e^3$	39.68	$3.056e^2$	$4.08e^2$
3	7	3	0.5500	$1.740e^3$	43.68	$3.886e^2$	$4.22e^2$
	14	7	0.2600	$2.1104e^3$	29.86	$3.916e^2$	$4.01e^2$
	21	7	0.1300	$9.7860e^3$	27.40	$3.120e^2$	$3.66e^2$
	28	14	0.0650	$1.732e^3$	22.20	$3.015e^2$	$3.30e^2$
	35	17	0.0300	$1.8676e^3$	18.61	$3.106e^2$	$3.08e^2$

5.2 Sensitivity Analysis

To examine the impact of batch size B_j on system performance measures, namely utilization ρ_j , expected inventory level, expected backorder level, and total cost. A sensitivity analysis is carried out for all three product types, which is discussed below separately for each type.

For product type 1, as B_j increases from 4 to 12 units, the utilization factor ρ_j decreases sharply from 0.8355 to 0.0325, indicating reduced system congestion and improved service efficiency. The expected backorder level also shows an overall declining trend (119.04 to 26.48 units). Although the expected inventory level fluctuates across different values of B_j , both fuzzy and crisp total costs decrease gradually. This suggests that increasing the batch size improves cost efficiency while maintaining system stability.

For product type 2, a similar pattern is observed. As B_j increases from 5 to 25 units, the utilization decreases from 0.7500 to 0.0250, and the expected backorder level generally declines. However, the expected inventory level exhibits significant variability, indicating higher sensitivity to batch size. The total cost shows a decreasing trend overall, although minor fluctuations are observed at higher values of B_j .

For product type 3, increasing B_j from 7 to 35 units leads to a reduction in utilization from 0.5500 to 0.0300 and a consistent decrease in expected backorders. The expected inventory level varies

considerably, reflecting sensitivity to parameter changes. The total cost initially fluctuates but eventually decreases, achieving lower values at higher batch sizes.

5.3 Managerial Implications

The proposed model is especially useful in situations where customer demand is not constant and may increase or decrease over time. By using this strategy, decision makers (DMs) can earn higher profits, particularly during seasonal periods. The model can be effectively applied in seasonal industries such as ice cream, umbrellas, sugar, and similar products, as well as in various production and manufacturing firms.

The model is capable of being expanded in further studies by introducing a central warehouse that fulfils customer demand when individual warehouses run out of stock, along with transportation costs. In addition, incorporating pricing decisions into the model would be an important and interesting direction for further study.

This work is particularly beneficial for retailers dealing with seasonal goods in domestic markets. Retailers often face losses when seasonal items remain unsold or become damaged after the season ends, mainly due to insufficient knowledge about demand fluctuations. The proposed model helps retailers manage such situations more effectively. It also recommends including a waste management policy to maintain a profit balance. Such a policy not only reduces losses but also helps protect the environment and contributes to reducing global warming.

6. Conclusions

In this study, a three-echelon model based on queuing-based fuzzy theory for a multi-product supply chain under steady-state probability distributions is developed. A comparative analysis under the same input parameters shows that the fuzzy approach generates a lower total inventory cost compared to the traditional crisp model. The presented model evaluates the efficiency of a warehouse by adopting an inventory policy $(O_j - 1, O_j)$ with the help of a queuing system, and also includes important performance indicators such as lead time, fill rate, and stock-out probability. Also, the analysis of the logistics process is presented using the $(M/M^{cj}/\infty)$ queue model. The conclusion is that the proposed model is effective, and the best efficiency is achieved when the holding cost is represented as a triangular fuzzy number. However, some limitations of this model are also noted. Fuzzy holding cost under the learning effect is limited to a specific structure; the model is mainly limited to multi-item supply chains and is not suitable for all types of ordering strategies. Also, the logistics analysis is limited to the $(M/M^{cj}/\infty)$ queuing systems. Future research has suggested the possibility of extending this model to include imperfect-quality items in a fuzzy environment. In addition, the proposed framework is developed under several simplifying assumptions to maintain analytical tractability. Factors such as dynamic carbon regulations, uncertain lead times, and changing operational conditions are not explicitly considered and may be incorporated in future studies to improve practical applicability.

Conflicts of Interest

There is no conflict of interest.

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AI Disclosure

The author(s) declare that no assistance is taken from generative AI to write this article.

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