International Journal of Mathematical, Engineering and Management Sciences
Vol. 10, No. 6, 2083-2102, 2025
https://doi.org/10.33889/IJMEMS.2025.10.6.097

Ram Arti Publishers

Exact Solutions of the Schamel Korteweg-de Vries Equation by Extended Direct Algebraic Method

Bamadev Sahoo
Centre for Data Science,
Siksha `O' Anusandhan (Deemed to be University), Bhubaneswar, 751030, Odisha, India.

Sanjaya K. Mohanty
Department of Mathematics,
Siksha `O' Anusandhan (Deemed to be University), Bhubaneswar, 751030, Odisha, India.
Corresponding author: sanjayamohanty@soa.ac.in

Ambit K. Pany
Department of Mathematics,
Government Women's College, Sambalpur, 768001, Odisha, India.

Sunita Chand
Department of Mathematics,
Siksha `O' Anusandhan (Deemed to be University), Bhubaneswar, 751030, Odisha, India.

(Received on April 9, 2025; Revised on June 4, 2025 & July 10, 2025; Accepted on August 20, 2025)

Abstract
This study examines the analytical investigation of nonlinear wave structures governed by a Schamel-type Korteweg-de Vries (S-KdV) equation, essential in plasma physics for modelling ion-acoustic waves. This work is motivated by the necessity to enhance understanding of the impact of electron trapping effects on solitary wave dynamics. We employ the Extended Direct Algebraic (EDA) method to derive precise wave solutions. The applied method offers a systematic approach to derive various soliton structures and improves our comprehension of their physical properties. In plasma physics, the S-KdV equation is utilised to examine dust ion acoustic waves. Furthermore, it is employed to examine shallow water waves distinguished by steepening and breaking. It is applied in studying the Earth's magnetosphere, the solar wind, the nonlinear plasma turbulence, and the dusty space plasma. Graphical and comparative analyses are presented to validate the results and demonstrate the robustness of the method. The obtained solutions for the S-KdV equation have Kink type, anti-kink type, and multisoliton and solitary wave structures. The properties of the wave structures are demonstrated through the two-dimensional, three-dimensional, and contour plots. Additionally, the impact of the nonlinear term as well as the dispersion term on some of the obtained solutions are discussed through the two dimensional plots.

Keywords- Extended direct algebraic method, Schamel KdV equation, Kink wave, Anti-kink wave, Solitary waves.

1. Introduction

The properties of non-linear evolution equations (NLEEs) and their analytical wave solutions have gained prominence recently, as they provide an easy way to understand the behavior of nonlinear wave occurrences in physical sciences and engineering. One class of important NLEEs is Schamel equations and their variants (Mohanty et al., 2021; Mohanty et al., 2022). In recent years, numerous researchers have generated closed-form solutions for various models using various analytical techniques. These methods have been used for several models, including the following: Qin et al. (2024) used the bilinear neural network method to obtain the exact solution for the (4+1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation. Jafari et al. (2022) used the scaling method to obtain the exact solutions for the modified KdV equation. Similarly, the more recently used methods are as follows: the extended generalized (G'/G) method (Mohanty et al., 2023, 2024), improved tanh method (Sagib, et al., 2024), auxiliary method (Kudryashov and Lavrova, 2024), Kudryashov method (Aljoudi, 2021; Mohanty, 2024), radial boundary element method (Hosseinzadeh and Sedaghatjoo, 2025), spectral element method (Tissaoui et al., 2025), and Nystrom method (Bhat et al., 2025).

Out of the above-mentioned methods, the EDA method (Noor et al., 2024) is a simple and efficient method used to produce exact solutions for NLEEs. Here, the given nonlinear evolution equation is converted to an ordinary differential equation (ODE) by the help of wave transformation. After that the initial approximation solutions for the ODE as $\sum_{i=0}^{n} a_i G^{'}$, where $G = G(\eta)$ satisfies the differential equation $G' = \ln(p)\left(q + rG + sG^2\right)$, where a_i, $i = 0, 1, 2, \cdots n$, $(p \neq 0,1)$, q, r, and s are constants. The EDA technique has been applied recently by many authors, such as Noor et al. (2024), who investigated the Wazwaz--Benjamin Bona--Mahony equation, deriving kink, lump-like singular, trigonometric, hyperbolic, periodic, shock, singular, and non-singular wave solutions. Using the modified-EDA method, (Bilal et al., 2024) getting the analytical solutions of the Kundu-Eckhaus equation. Hussain et al. (2024) examined many nonlinear pseudo-parabolic models using the EDA method and derived analytical solutions expressed as hyperbolic, trigonometric, rational, exponential, and polynomial functions. The discovered solutions define dark, bright, singular, combined dark-bright solitons, dark singular combined solitons, and solitary wave solutions. Using the EDA method, Manzoor et al. (2024) examined the stochastic dynamical equation and obtained rational, hyperbolic, and trigonometric function solutions, including bright and dark periodic solutions and singular periodic solutions. The predator-prey model was studied by Shahzad et al. (2023) using the modified EDA technique, yielding multiple kinds of results: shock, shock-singular, complex solitary-shock, and periodic-singular solutions. Rehman et al. (2023) used the EDA method to study the Zakhrov equation in ionized plasma, deriving periodic singular, dark, singular, and combined dark-singular solutions, as well as dark-bright soliton solutions from trigonometric, rational, and hyperbolic functions. Iqbal et al. (2023) used the EDA approach to study the biofilm model. The solutions they found expressed solitary dark-bright solitons and took the shape of hyperbolic, trigonometric, and rational functions. Ding et al. (2023) used the EDA methodology to solve the equation of KdV-Zakharov-Kuznetsov, getting that the solutions are dark, periodic, singular, and kink-type waves. The Radhakrishnan-Kundu-Lakshmanan equation was solved by Mahmood et al. (2023) using the EDA technique, yielding hyperbolic, periodic, trigonometric, brilliant and dark solitons, combined bright and dark solitons, and W-shaped solutions.

In this paper, we have applied the EDA method to solve the S-KdV equation (Kangalgil, 2016), which is defined as follows:

$$\frac{\partial u}{\partial t} + \left(Au^{\frac{1}{2}} + Bu\right)\frac{\partial u}{\partial x} + C\frac{\partial^3 u}{\partial x^3} = 0 \tag{1}$$

where, A, B and C are arbitrary coefficients and u is the electrostatic potential and the fractional power $u^{\frac{1}{2}}$ is the trapped particle effect.

Fractional power nonlinearity is crucial for the Schamel KdV equation to accurately forecast wave behaviour in plasmas with limited particle populations. Its impact on soliton dynamics highlights the importance of choosing appropriate models for different physical situations.

The S-KdV model is an updated version of the original KdV equation that describes nonlinear wave propagation, particularly in ion-acoustic waves in plasma. The S-KdV equation alters the conventional KdV equation by including a nonlinearity characteristic associated with trapping phenomena. Wave-breaking occurrences and soliton solutions are among the distinctive wave characteristics. Like the well-known KdV equation, the S-KdV equation gives soliton solutions that preserve their structure over propagation. Since the S-KdV model achieves a balance between nonlinear effects and dispersive properties of the medium, it can be used to simulate a wide range of physical entities.

The hyperbolic function and trigonometric function solutions for the S-KdV equation were obtained by Kangalgil (2016) and Taha et al. (2013) using the -expansion method. Using a one-dimensional, collisionless, and unmagnetized plasma with positive and negative ions, trapped electrons exhibiting a vortex-like distribution, and stationary dust grains carrying positive and negative charges, Guo et al. (2016) investigated the acoustic waves of dust ions with small but finite amplitudes for the S-KdV equation. The Bäcklund transformations approach was applied by Kalaawy and Aldenari (2014) to the S-KdV equation, resulting in the derivation of finite amplitude kink-type wave solutions. Tariq et al. (2022a) implemented the modified Kudryashov technique to solve the S-KdV equation, resulting in the derivation of kink-type solitary wave solutions. Lee and Sakthivel (2011) employed the sine-cosine approach and the extended tanh method to derive soliton-like solutions, kink solutions, and many solutions of the S-KdV model. Donmez and Daghan (2017) derived discontinuity and shock waves for the S-KdV equation using various expansion methods. Through the use of the expectational function technique, Lee and Saktivel (2011) obtained precise traveling wave solutions for the S-KdV model. By employing the extended modified auxiliary equation mapping approach and the extended FAN sub-equation method, Tariq et al. (2022b) conducted an analytical study on the S-KdV equation, obtaining solutions for various types of solitons, including dark solitons, light solitons, solitary waves, periodic solitary waves, rational functions, and elliptic functions. Alquran (2024) obtained bidirectional solitons, convex-periodic waves, and cusp-like waves by applying the tanh-coth method, the Kudryashov expansion approach, and the rational sine-cosine method. The S-KdV problem was solved by Karakoc et al. (2023) using the Bernoulli Sub-ODE method and the auxiliary equation method, yielding trigonometric and hyperbolic solutions.

Within the framework of the Schamel KdV equation, the utilization of the EDA approach offers distinct benefits over other contemporary investigations into generic nonlinear evolution equations (NLEEs). The Schamel KdV equation incorporates a fractional power nonlinearity, typically associated with the modeling of ion-acoustic waves in plasma containing trapped electrons. The EDA technique adeptly addresses fractional nonlinearities owing to its versatile solution approach, capable of integrating both polynomial and non-integer power components.

The novelty of this paper is highlighted though the following points:

- Schamel KdV Equation: The model includes electron trapping effects through a non-integer power factor, which is rarely studied in standard KdV research.
- The EDA method: Unlike previous methodologies, the EDA method adopted here gives a straightforward and flexible framework for obtaining accurate solutions to higher-order nonlinear equations.
- Soliton Structures: The resulting solutions offer a more profound understanding of the qualitative behaviour of nonlinear plasma waves and include Kink type, anti-kink type, multisoliton, and solitary wave structures.

- Graphical Analysis: Multiple visualisations that show the impacts of different factors are included in the study to improve interpretability and physical comprehension.

The article arranges as follows: methodology of the algebraic method is provided in Section 2. The technique is applied to the Schamel KdV equation in Section 3. Section 4 offers graphical representations of several solutions explored. The comparison section presents a comparison of the obtained results with some previous literature. These representations highlight the influence of the nonlinear and dispersion variables. Finally, the conclusion section presents the information regarding its conclusions.

2. Methodology of the Extended Direct Algebraic Method

In this section, the details steps of the EDA method are explained.
Consider the NLLEs of the form

$$S\left(u,u_x,u_t,u_{xx},u_{xt},u_{tt},\cdots\right)=0 \tag{2}$$

where, x and t are independent variables and u is dependent on x and t. Further S is function of u and its partial derivatives. Converting, Equation (2) into an ordinary differential equation (ODE) by using $\eta = x - kt$, where k is a constant, let the ODE after integration be as follows:

$$T(u,u',u'',\cdots)=0 \tag{3}$$

where, $u = u(\eta)$, $u' = \dfrac{du}{d\eta}$, and $u'' = \dfrac{d^2u}{d\eta^2}$.

Now assuming the initial approximate solution for the ODE Equation (3) as

$$u=\sum_{i=0}^{n} a_i G^i \tag{4}$$

where, $a_{i,}$ $i = 0,1,2,\cdots,n$ are constants to be determined later and n is a positive integer. Here, the function $G = G(\eta)$ satisfies the following ODE:

$$G' = \ln(p)\left(q + rG + sG^2\right) \tag{5}$$

where, p, q, r, and s are constants provided $p \neq 0$ and $p \neq 1$. The general solutions of Equation (5) are listed below with different conditions.

Family-I: When $r^2 - 4qs < 0$ and $s \neq 0$, then

$$G_1(\eta) = -\frac{r}{2s} + \frac{\sqrt{4qs - r^2}}{2s} tan_p\left(\frac{\sqrt{4qs - r^2}}{2}\eta\right) \tag{6}$$

$$G_2(\eta) = -\frac{r}{2s} - \frac{\sqrt{4qs - r^2}}{2s} cot_p\left(\frac{\sqrt{4qs - r^2}}{2}\eta\right) \tag{7}$$

$$G_3(\eta) = -\frac{r}{2s} + \frac{\sqrt{4qs-r^2}}{2s} tan_p\left(\sqrt{4qs-r^2}\eta\right) \pm \frac{\sqrt{4qs-r^2}}{2s}\sqrt{e_1 e_2}\, sec_p\left(\sqrt{4qs-r^2}\eta\right) \tag{8}$$

$$G_4(\eta) = -\frac{r}{2s} - \frac{\sqrt{4qs-r^2}}{2s} cot_p\left(\sqrt{4qs-r^2}\eta\right) \pm \frac{\sqrt{4qs-r^2}}{2s}\sqrt{e_1 e_2}\, csc_p\left(\sqrt{4qs-r^2}\eta\right) \tag{9}$$

$$G_5(\eta) = -\frac{r}{2s} + \frac{\sqrt{4qs-r^2}}{4s} tan_p\left(\frac{\sqrt{4qs-r^2}}{4}\eta\right) - \frac{\sqrt{4qs-r^2}}{4s} cot_p\left(\frac{\sqrt{4qs-r^2}}{4}\eta\right) \tag{10}$$

Family-II: When $r^2 - 4qs > 0$ and $s \neq 0$, then

$$G_6(\eta) = -\frac{r}{2s} - \frac{\sqrt{r^2-4qs}}{2s} tanh_p\left(\frac{\sqrt{r^2-4qs}}{2}\eta\right) \tag{11}$$

$$G_7(\eta) = -\frac{r}{2s} - \frac{\sqrt{r^2-4qs}}{2s} coth_p\left(\frac{\sqrt{r^2-4qs}}{2}\eta\right) \tag{12}$$

$$G_8(\eta) = -\frac{r}{2s} - \frac{\sqrt{r^2-4qs}}{2s} tanh_p\left(\sqrt{r^2-4qs}\eta\right) \pm \mathrm{i}\sqrt{e_1 e_2}\frac{\sqrt{r^2-4qs}}{2s} sech_p\left(\sqrt{r^2-4qs}\eta\right) \tag{13}$$

$$G_9(\eta) = -\frac{r}{2s} - \frac{\sqrt{r^2-4qs}}{2s} coth_p\left(\sqrt{r^2-4qs}\eta\right) \pm \sqrt{e_1 e_2}\frac{\sqrt{r^2-4qs}}{2s} csch_p\left(\sqrt{r^2-4qs}\eta\right) \tag{14}$$

$$G_{10}(\eta) = -\frac{r}{2s} - \frac{\sqrt{r^2-4qs}}{4s} tanh_p\left(\frac{\sqrt{r^2-4qs}}{4}\eta\right) - \frac{\sqrt{r^2-4qs}}{4s} coth_p\left(\frac{\sqrt{r^2-4qs}}{4}\eta\right) \tag{15}$$

where the generalized hyperbolic and triangular functions are given as

$$tanh_p(\eta) = \frac{e_1 p^{\eta} - e_2 p^{-\eta}}{e_1 p^{\eta} + e_2 p^{-\eta}}, coth_p(\eta) = \frac{e_1 p^{\eta} + e_2 p^{-\eta}}{e_1 p^{\eta} - e_2 p^{-\eta}}, sech_p(\eta) = \frac{2}{e_1 p^{\eta} + e_2 p^{-\eta}}, csch_p(\eta) = \frac{2}{e_1 p^{\eta} - e_2 p^{-\eta}},$$

$$tan_p(\eta) = -\mathrm{i}\frac{e_1 p^{\mathrm{i}\eta} - e_2 p^{-\mathrm{i}\eta}}{e_1 p^{\mathrm{i}\eta} + e_2 p^{-\mathrm{i}\eta}}, cot_p(\eta) = \mathrm{i}\frac{e_1 p^{\mathrm{i}\eta} + e_2 p^{-\mathrm{i}\eta}}{e_1 p^{\mathrm{i}\eta} - e_2 p^{-\mathrm{i}\eta}}, sec_p(\eta) = \frac{2}{e_1 p^{\mathrm{i}\eta} + e_2 p^{-\mathrm{i}\eta}}, csc_p(\eta) = \frac{2\mathrm{i}}{e_1 p^{\mathrm{i}\eta} - e_2 p^{-\mathrm{i}\eta}},$$

where η is an independent variable, e_1 and e_2 are positive constants. The steps of the methodology are listed below:

Step 1: It is required to evaluate the value of n in Equation (4) with the help of the balancing method. In this method, it requires to compare the exponents of the highest order derivative terms with the highest order nonlinear terms of the converted ODE (5). The details are as follows:
Let the initial assumed solutions have degree n i.e. $deg(u(\eta)) = n$. Let, we have highest order partial derivative term like $deg(u_k) = n + k$ and let highest order nonlinear term as $deg(u^s u_r) = sn + r$. Then comparing the exponent of the two terms as $n + k = sn + r$, we get the value of n. Where s is real number and u_k, k times derivative with respect to η.

The positive integer 'n' can be determined by considering the highest order partial derivative and nonlinear terms appearing from the converted ordinary differential equations. If "n" is not positive integer (fraction or negative integer), we make the following transformation:
a) When "$n = p / q$" $q \neq 0$ is a fractional lowest term, we take the transformation $u(\eta) = v^{p/q}(\eta)$.
b) When "n" is negative integer, we take the transformation $u(\eta) = v^{-n}(\eta)$, and return the value of "n" again form the new equation.

As per the steps of the methodology, we have to compare the highest order derivative terms with highest order nonlinear terms.

Step 2: Then, evaluating the required derivative terms and substituting them into Equation (3), we get a polynomial $G(\eta)$. Now, equating the coefficients of the same power of $G(\eta)$ equal to zero, a system of algebraic equations is obtained. Then it is required to solve the system of algebraic equations for the free variables $a_i, i = 0,1,2,\cdots,n$.

Step 3: Now substituting the values of a_i, $i = 0, 1, 2, \cdots, n$, obtained from step 2 in the Equation (4) with general solutions of the Equation (5). Then further back substituting the wave transformation, we can obtain the required solution for Equation (2).

3. Application of the Extended Direct Algebraic Method to the S-KdV Equation

We have applied the EDA method to the S-KdV equation. Using $\eta = x - kt$, in Equation (1), we get

$$-k\frac{du}{d\eta} + \left(Au^{1/2} + Bu\right)\frac{du}{d\eta} + C\frac{d^3u}{d\eta^3} = 0 \tag{16}$$

where, $u = u(\eta)$. Integrating Equation (16), it becomes

$$-ku + \frac{2}{3}Au^{3/2} + \frac{Bu^2}{2} + C\frac{d^2u}{d\eta^2} + c_1 = 0 \tag{17}$$

where, c_1 is the integration constant. Applying $\sqrt{u(\eta)} = v(\eta)$ in Equation (17), it will be

$$-kv^2 + \frac{2}{3}Av^3 + \frac{Bv^4}{2} + 2C\left(\frac{dv}{d\eta}\right)^2 + 2Cv\frac{d^2v}{d\eta^2} + c_1 = 0 \tag{18}$$

where, c_1 is the integration constant. With the help of the balancing between the highest order derivative term and nonlinear term in Equation (18).

We have $deg\left(v\frac{d^2v}{d\eta^2}\right)=2n+2$ and $deg(v^4)=4n$, then comparing its exponents we have $2n+2=4n$, we have $n=1$. The solutions for the S-KdV equation by the help of initial assumption as follows:

$$v(\eta)=a_0+a_1G \tag{19}$$

The required derivatives with respect to η are given below:

$$\frac{dv}{d\eta}=a_1\ln(p)\left(q+rG+sG^2\right),\ \frac{d^2v}{d\eta^2}=a_1\ln^2(p)(r+2sG)\left(q+rG+sG^2\right) \tag{20}$$

Now substituting the values of Equations (19) and (20) in Equation (18), Equation (18) converts to a polynomial of $G(\eta)$. Now, accumulating the same power's co-efficient of G and compare them to zero, the obtained system of equations is listed below:

$$G^4:\ 1/2Ba_1^{\ 4}+6Ca_1^{\ 2}\left(\ln(p)\right)^2s^2=0 \tag{21}$$

$$G^3:2/3Aa_1^{\ 3}+2Ba_0a_1^{\ 3}+4Ca_0a_1\left(\ln(p)\right)^2s^2+10Ca_1^{\ 2}\left(\ln(p)\right)^2sr=0 \tag{22}$$

$$\begin{aligned}G^2:&-ka_1^{\ 2}+2Aa_0a_1^{\ 2}+3Ba_0^{\ 2}a_1^{\ 2}+6Ca_0a_1\left(\ln(p)\right)^2rs+4Ca_1^{\ 2}\left(\ln(p)\right)^2r^2\\&+8Ca_1^{\ 2}\left(\ln(p)\right)^2sq=0\end{aligned} \tag{23}$$

$$\begin{aligned}G^1:&-2ka_0a_1+2Aa_0^{\ 2}a_1+2Ba_0^{\ 3}a_1+2Ca_0a_1\left(\ln(p)\right)^2r^2\\&+4qCa_0a_1\left(\ln(p)\right)^2s+6Ca_1^{\ 2}\left(\ln(p)\right)^2qr=0\end{aligned} \tag{24}$$

$$\begin{aligned}G^0:&-ka_0^{\ 2}+2/3Aa_0^{\ 3}+1/2Ba_0^{\ 4}+2C\left(\ln(p)\right)^2qra_0a_1\\&+2C\left(\ln(p)\right)^2q^2a_1^{\ 2}+c_1=0\end{aligned} \tag{25}$$

Solving the system of Equations (21) – (25) for the free variables a_1, A, k, c_1, the obtained solutions for the S-KdV equation are as follows:

$$\begin{aligned}&C=\frac{4A^2}{\left(300qs-75r^2\right)B\left(\ln(p)\right)^2},k=-\frac{16A^2}{75B},a_0=\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)},\\&a_1=\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right),c_1=0\end{aligned} \tag{26}$$

where, $4qs \neq r^2$, $B \neq 0$, $r \neq 0$. Using the data of Equation (26) and the family-I (6)-(10) and family-II (11)-(15), with $\eta = x - kt$ and $u = v^2$, the simplified solutions are as follows:

Family-I: When $r^2 - 4qs < 0$ and $s \neq 0$, then the solutions have the forms

$$u_1(x,t) = \left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)} + \frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left.\left(-\frac{r}{2s}+\frac{\sqrt{4qs-r^2}}{2s}tan_p\left(\frac{\sqrt{4qs-r^2}}{2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{27}$$

$$u_2(x,t) = \left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left.\left(-\frac{r}{2s}-\frac{\sqrt{4qs-r^2}}{2s}cot_p\left(\frac{\sqrt{4qs-r^2}}{2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{28}$$

$$u_3(x,t) = \left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)} + \frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}+\frac{\sqrt{4qs-r^2}}{2s}tan_p\left(\sqrt{4qs-r^2}\left(x+\frac{16A^2}{75B}t\right)\right)\right.$$
$$\left.\left.\pm\frac{\sqrt{4qs-r^2}}{2s}\sqrt{e_1e_2}sec_p\left(\sqrt{4qs-r^2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{29}$$

$$u_4(x,t) = \left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)} + \frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}-\frac{\sqrt{4qs-r^2}}{2s}cot_p\left(\sqrt{4qs-r^2}\left(x+\frac{16A^2}{75B}t\right)\right)\right.$$
$$\left.\left.\pm\frac{\sqrt{4qs-r^2}}{2s}\sqrt{e_1e_2}csc_p\left(\sqrt{4qs-r^2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{30}$$

$$u_5(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}+\frac{\sqrt{4qs-r^2}}{4s}tan_p\left(\frac{\sqrt{4qs-r^2}}{4}\left(x+\frac{16A^2}{75B}t\right)\right)\right. \tag{31}$$
$$\left.\left.-\frac{\sqrt{4qs-r^2}}{4s}cot_p\left(\frac{\sqrt{4qs-r^2}}{4}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2$$

Family-II:

When $r^2-4qs>0$ and $s\neq 0$, then the solutions have the forms

$$u_6(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left.\left(-\frac{r}{2s}-\frac{\sqrt{r^2-4qs}}{2s}tanh_p\left(\frac{\sqrt{r^2-4qs}}{2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{32}$$

$$u_7(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left.\left(-\frac{r}{2s}-\frac{\sqrt{r^2-4qs}}{2s}coth_p\left(\frac{\sqrt{r^2-4qs}}{2}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2 \tag{33}$$

$$u_8(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}-\frac{\sqrt{r^2-4qs}}{2s}tanh_p\left(\sqrt{r^2-4qs}\left(x+\frac{16A^2}{75B}t\right)\right)\right. \tag{34}$$
$$\left.\left.\pm i\sqrt{e_1e_2}\frac{\sqrt{r^2-4qs}}{2s}sech_p\left(\sqrt{r^2-4qs}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2$$

$$u_9(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}-\frac{\sqrt{r^2-4qs}}{2s}coth_p\left(\sqrt{r^2-4qs}\left(x+\frac{16A^2}{75B}t\right)\right)\right.$$
$$\left.\left.\pm\sqrt{e_1e_2}\frac{\sqrt{r^2-4qs}}{2s}csch_p\left(\sqrt{r^2-4qs}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2$$

(35)

$$u_{10}(x,t)=\left[\frac{2A}{5B}\frac{\left(-4qs+r^2+\sqrt{-4qr^2s+r^4}\right)}{\left(4qs-r^2\right)}+\frac{2sA}{5Br}\left(\frac{-4qs+r^2+\sqrt{-4qr^2s+r^4}}{4qs-r^2}+2\right)\right.$$
$$\left(-\frac{r}{2s}-\frac{\sqrt{r^2-4qs}}{4s}tanh_p\left(\frac{\sqrt{r^2-4qs}}{4}\left(x+\frac{16A^2}{75B}t\right)\right)\right. \qquad (36)$$
$$\left.\left.-\frac{\sqrt{r^2-4qs}}{4s}coth_p\left(\frac{\sqrt{r^2-4qs}}{4}\left(x+\frac{16A^2}{75B}t\right)\right)\right)\right]^2$$

4. Graphical Representation and Discussions

In the section, the graphical representation of some of the obtained solutions for the S-KdV equation is illustrated through the 3-D, contour and 2-D plots. We have obtained the multisoliton wave structure, kink and anti-kink and solitary wave structures. Here, the solutions (27-31) have the multisoliton type structures with the condition $r^2-4qs<0$, whereas the solutions (32-36) are satisfying the condition $r^2-4qs>0$ and having the multisoliton structures, kink, anti-kink and solitary wave structures.

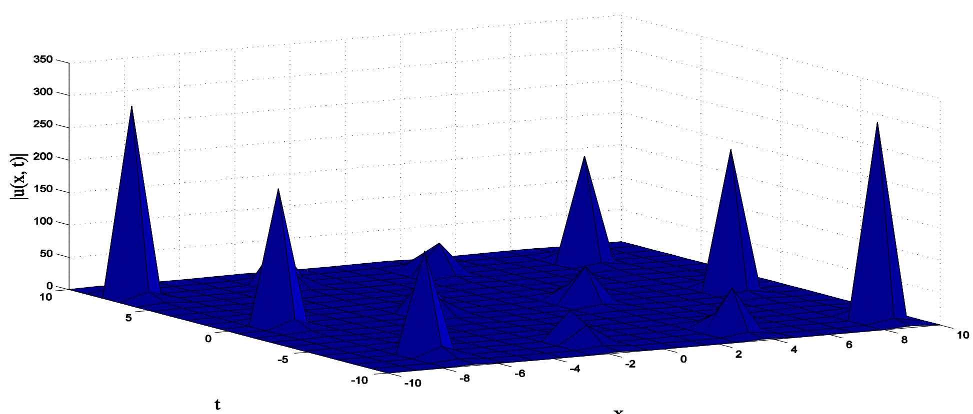

Figure 1. The three-dimensional surface plot of solution $|u(x,t)|$ (27) for the S-KdV equation with A = 1, B = 1.5, s = 1, r = -1, p = 2, q = 1, e_1 = 1, e_2 = 1, which represents multisoliton wave structure. The ploted solutions are evaluated over the interval $-10 \le x \le 10$, $-10 \le t \le 10$.

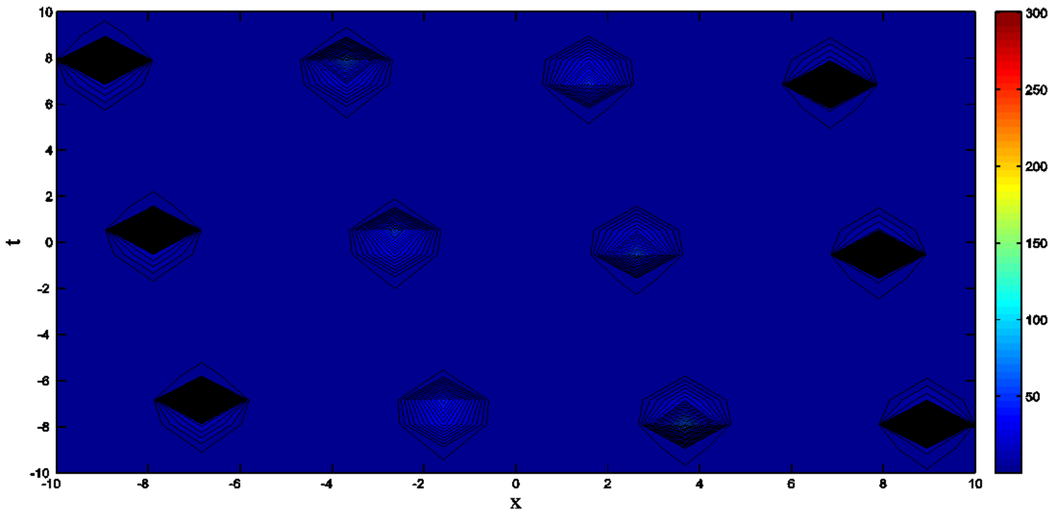

Figure 2. The contour plot of solution $|u(x,t)|$ (27) for the S-KdV equation with A = 1, B = 1.5, s = 1, r = -1, p = 2, q = 1, e_1 = 1, e_2 = 1, which represents multisoliton wave structure. represents the multisoliton wave structure. The ploted solutions are evaluated over the interval $-10 \le x \le 10$, $-10 \le t \le 10$.

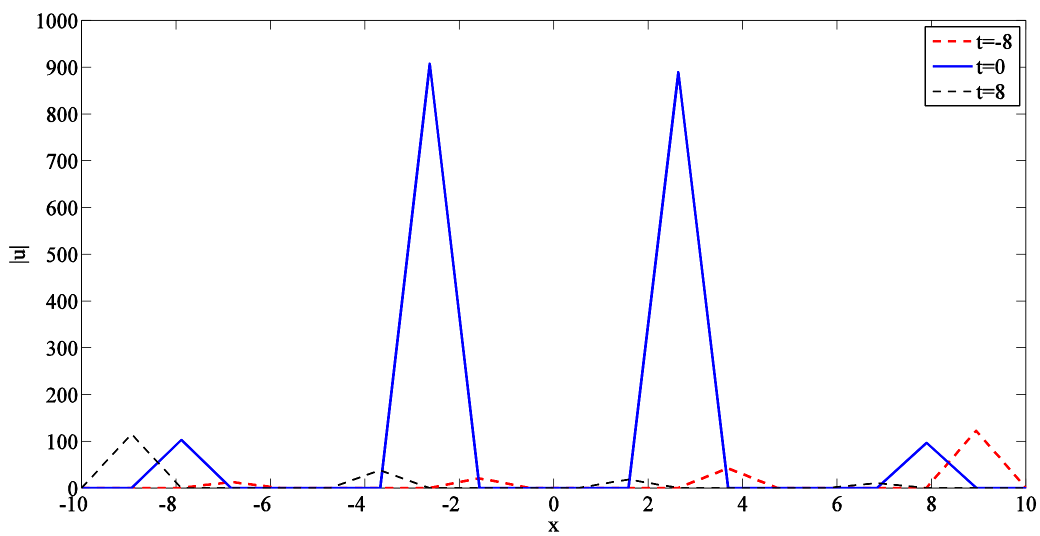

Figure 3. The 2-D plot of solution $|u(x)|$ (27) for the S-KdV equation with A = 1, B = 1.5, s = 1, r = -1, p = 2, q = 1, e_1 = 1, e_2 = 1, represents the multisoliton wave structure. The interval of the solution $-10 \le x \le 10$, with $t = -8$, $t = 0$, $t = 8$.

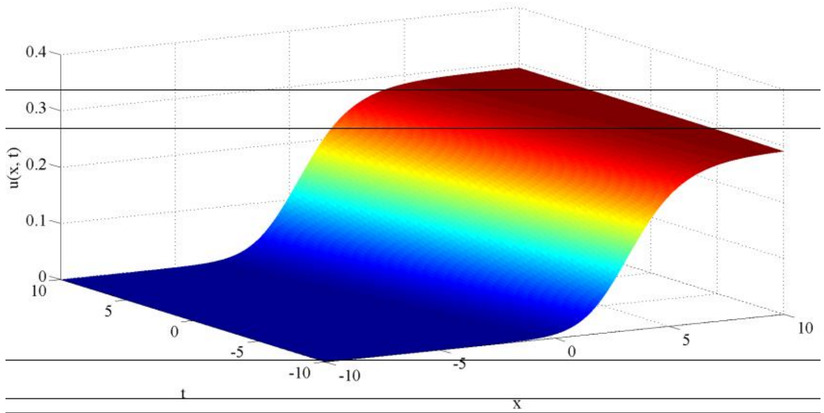

Figure 4. Using the solution 3-D surface plot (32) for the S-KdV equation with A = 1, B = 1.5, s = 1, p = 2, q = 1, r = 2.3, e_1 = 1, e_2 = 2 represents the kink wave. The solutions fall within the interval $-10 \leq x \leq 10$, $-10 \leq t \leq 10$.

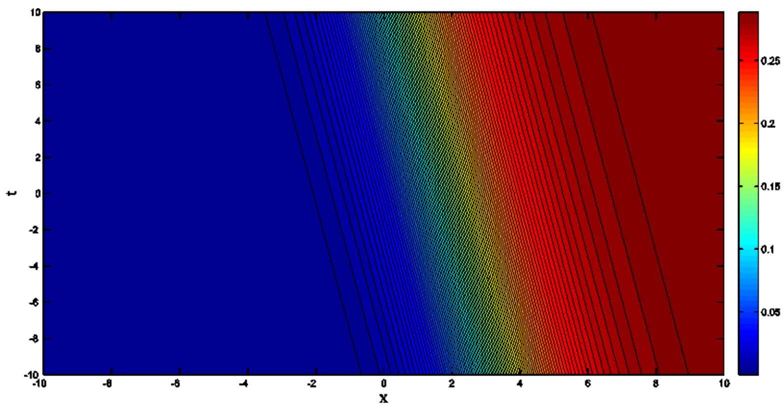

Figure 5. Using the solution the contour plot (32) for the S-KdV equation with A = 1, B = 1.5, s = 1, p = 2, q = 1, r = 2.3, e_1 = 1, e_2 = 2. The solutions lie in between $-10 \leq x \leq 10$, $-10 \leq t \leq 10$.

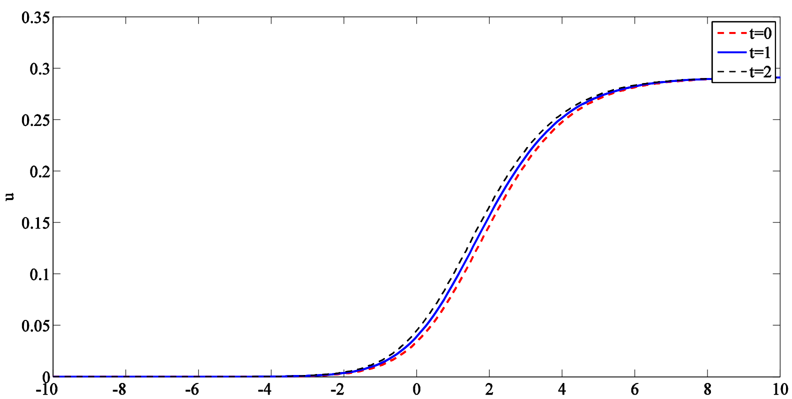

Figure 6. 2-D plot of the solution (32) for the S-KdV equation with A = 1, B = 1.5, s = 1, r = 2.3, p = 2, q = 1, e_1 = 1, e_2 = 2. The solutions fall within the interval $-10 \leq x \leq 10$, with $t = 0$, $t = 1$, and $t = 2$.

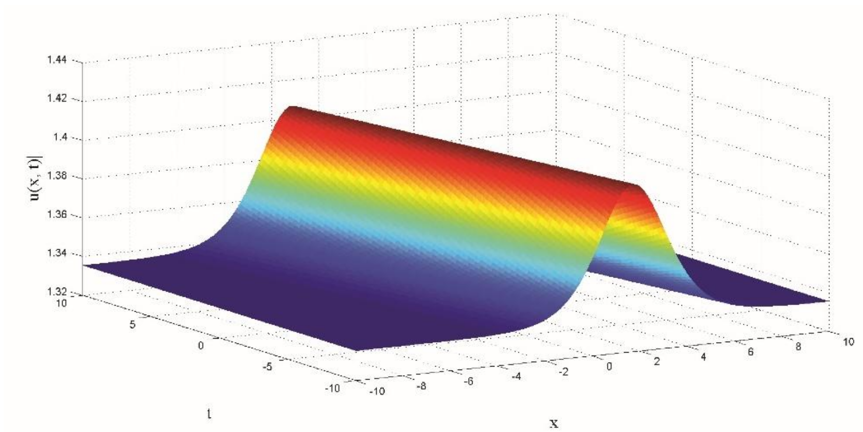

Figure 7. The 3-D surface plot of solution (34) for the S-KdV equation with $A = 1$, $B = 1.5$, $s = 1$, $r = 2.5$, $p = 2$, $q = 1$, $e_1 = 1.5$, $e_2 = 2$. represents the solitary wave. The solutions fall within the interval $-10 \le x \le 10$, $-10 \le t \le 10$.

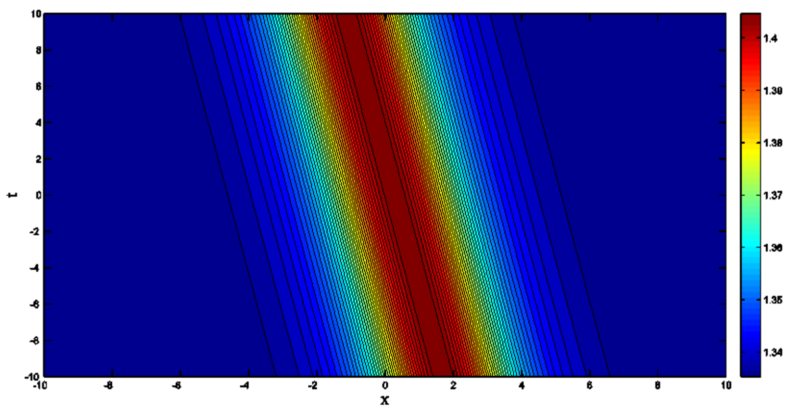

Figure 8. Solution (34) is the contour plot for the S-KdV equation with $A = 1$, $B = 1.5$, $s = 1$, $r = 2.5$, $p = 2$, $q = 1$, $e_1 = 1.5$, $e_2 = 2$. The solutions fall within the interval $-10 \le x \le 10$, $-10 \le t \le 10$.

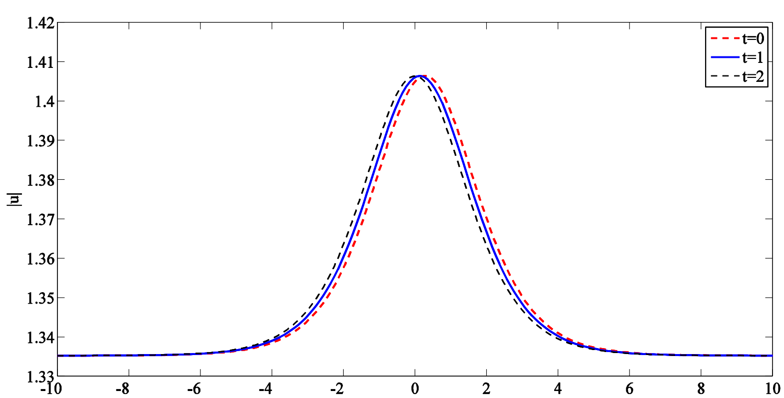

Figure 9. The two dimensional plot of solution (34) for the S-KdV equation with $A = 1$, $B = 1.5$, $s = 1$, $r = 2.5$, $p = 2$, $q = 1$, $e_1 = 1.5$, $e_2 = 2$. The solutions fall within the interval $-10 \le x \le 10$, with $t = 0$, $t = 1$, $t = 2$.

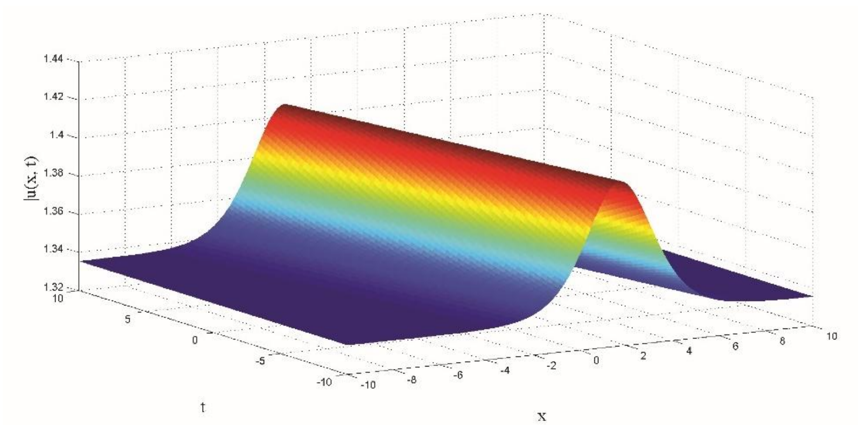

Figure 10. The three-dimensional surface plot of solution (36) for the S-KdV equation with $A = 1$, $B = 1.5$, $s = 1$, $r = 3$, $p = 2$, $q = 1$, $e_1 = 1$, $e_2 = 2$ represents the anti-kink wave, within $-10 \le x \le 10$, $-10 \le t \le 10$.

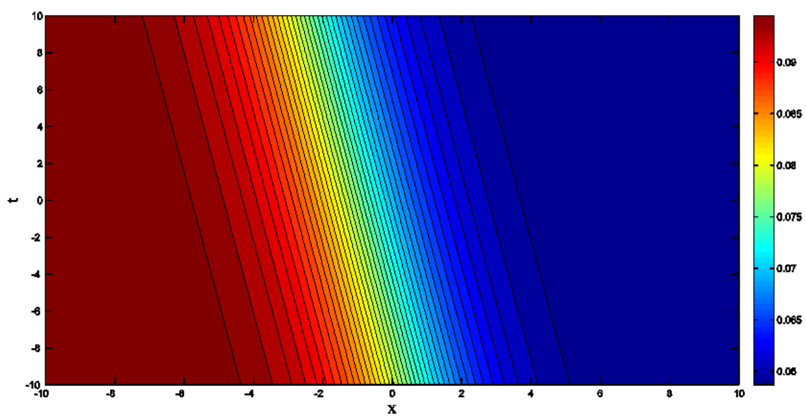

Figure 11. Solution (36) is the contour plot for the S-KdV equation with $A = 1$, $B = 1.5$, $s = 1$, $r = 3$, $p = 2$, $q = 1$, $e_1 = 1$, $e_2 = 2$, within the interval $-10 \le x \le 10$, $-10 \le t \le 10$.

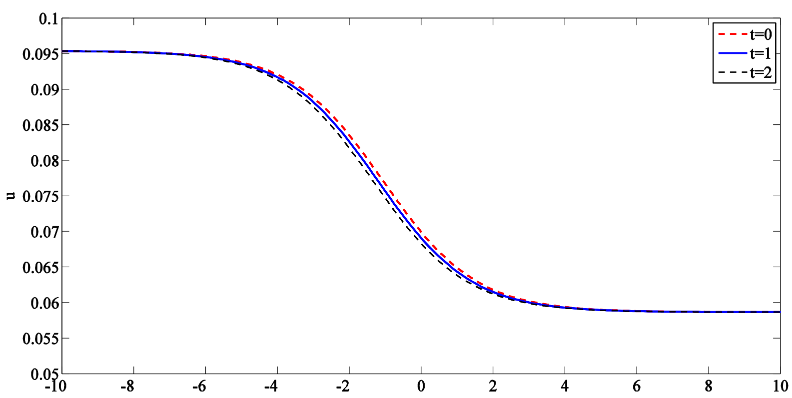

Figure 12. The 2-D plot of solution (36) for the S-KdV equation with $A = 1$, $B = 1.5$, $p = 2$, $q = 1$, $r = 3$, $s = 1$, $e_1 = 1$, $e_2 = 2$. The solutions fall within the interval $-10 \le x \le 10$, with $t = 0$, $t = 1$, $t = 2$.

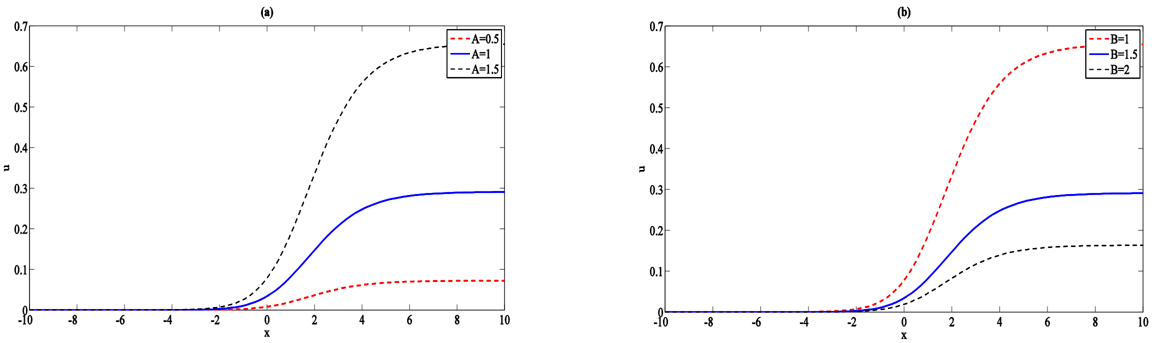

Figure 13. (a) is the two dimensional plot of solution (32) for the S-KdV equation for the free parameter A, with B = 1.5, t = 0.1, p = 2, q = 1, r = 2.3, s = 1, e_1 = 1, e_2 = 2. (b) is the two dimensional plot of solution (32) for the S-KdV equation for the free parameter B, with A = 1, t = 0.5, p = 2, q = 1, r = 2.3, s = 1, e_1 = 1, e_2 = 2.

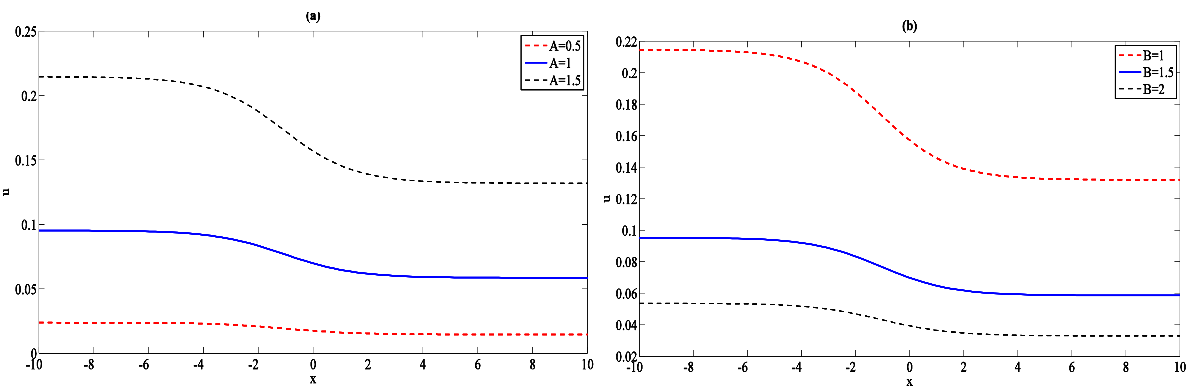

Figure 14. (a) is the two dimensional plot of solution (36) for the S-KdV equation for the free parameter A, with B = 1.5, t = 0.1, p = 2, q = 1, r = 3, s = 1, e_1 = 1, e_2 = 2. (b) is the two dimensional plot of solution (36) for the S-KdV equation for the free parameter B, with A = 1, t = 0.1, p = 2, q = 1, r = 3, s = 1, e_1 = 1, e_2 = 2.

The 3-D, contour, and 2-D plot of the absolute value of the solution (27), where the wave structure is multisoliton, are shown in **Figures 1, 2,** and **3**. Modulus solutions of (30) and (31) also feature multi-soliton wave structures, whereas the modulus of (28) is the double soliton and the modulus of (29) is the triple soliton. Moreover, the multisoliton wave structure without the solution modulus is present in solution (32). The 3-D, contour, and 2-D plots of the solution (32) are shown in **Figures 4, 5**, and **6**. While **Figure 5** displays the matching contour plots and **Figure 6** is the two-dimensional plot with t = 0, t = 1, and t = 2, **Figure 4** depicts the kink type solitary wave. Similarly, the wave structure of (35) is of the kink type. **Figures 7, 8** and **9** display the 3-D, contour, and 2-D plots of the solution (34). **Figure 7** shows solitary wave, whereas **Figure 8** shows the corresponding contour plots, and **Figure 9** is the 2-D plot with t = 0, t = 1, and t = 2. **Figures 10, 11**, and **12** display the solution (36) in 3D, as well as a contour plot and a 2-D plot. **Figure 10** shows the anti-kinkwave, whereas **Figure11** shows the corresponding contour plot. **Figure 12** shows the 2-D plot with t = 0, t = 1, and t = 2. The 2-D plots show the effect of the parameters A and B on the obtained solutions. The effects of the parameters are evident in the kink and anti-kink wave solutions, as illustrated in the graphs. Referencing 13, we observe differing behaviors of the solutions (32) for the non-linear parameters A = 0.5, 1, 1.5 and B = 1, 1.5, 2. and. **Figure 14** illustrates 2-D plots for the anti-kink solutions.

Generally speaking, higher wave amplitude results from increasing the nonlinear coefficients (*A, B*). The fractional nonlinearity's contribution is amplified by increasing *B*, resulting in higher peaks and steeper wavefronts. In the Schamel KdV equation, the soliton velocity depends on amplitude. The soliton speed increases as and *B* rise. This is consistent with classical soliton theory's taller, quicker waves. The velocity-amplitude connection is nonlinear and non-monotonic for fractional nonlinearities. Moderate values of *A, B,* and positive *C* guarantee stable localized structures. Instability or wave breaking may result from a large or tiny *B*. The dispersion term smoothes the profile and stabilizes the wave against collapse.

Localized, non-dispersive waveforms that maintain their shape while propagating at a constant velocity are called solitary waves. Within the Schamel KdV paradigm, these waves Arise from a balance between nonlinearity and dispersion and exhibit a finite amplitude and a localized structure. Unlike normal KdV solitons, they are transformed by electron trapping. Schamel solitary waves define ion-acoustic solitons encompassing imprisoned electrons, a phenomenon commonly observed in astrophysical and experimental plasmas (Schamel, 1973; Schamel, 1986). Trapped particles, particularly electrons, modify the distribution function, leading to a non-quadratic nonlinearity.

The Schamel KdV equation's kink waves, which represent nonlinear transitions between different plasma states, are closely related to the dynamics of trapped particles in the plasma. These waves act as nonlinear electrostatic structures that connect two distinct potential levels; they are important in space and lab plasmas. Show that there are non-Maxwellian velocity distributions because of the trapping of particles in wave potentials, usually electrons. can characterize solitary wavefronts, collisionless shock structures, or electrostatic double layers in a variety of plasma environments, especially when important kinetic effects are not captured by the conventional KdV equation. Show how important it is to strike a balance between dispersion and nonlinearity (as affected by fractional exponents) to allow for the steady propagation of such structures. In space plasma investigations (Verheest, 2002) and laboratory plasmas where electron entrapment significantly alters wave dynamics, these kink-like structures are particularly crucial (EI-Tantawy et al. (2013) from satellites in the magnetosphere or solar wind).

Comparison:

This section provides a concise comparison between our results with the study undertaken by other researchers (Lee and Sakthivel, 2011; Taha et al., 2013; Kangalgil, 2016; Donmez and Daghan, 2017).

- The obtained solutions (27) - (37) and (32) - (36) are in terms of exponential function solutions which are different from (Lee and Sakthivel, 2011; Taha et al., 2013; Kangalgil, 2016; Donmez and Daghan, 2017).
- Kangalgil (2016) used the extended generalized (G'/G)-expansion method to derive trigonometric, hyperbolic and rational solutions to the problem. Taha et al. (2013) used the (G'/G)-expansion method to derived trigonometric, hyperbolic and rational exact solution of the problem. Lee and Sakthivel (2011) used the sine-cosine method and the extended tanh method to obtain sine-cosine functions and tanh functions solutions of the problem. Donmez and Daghan (2017) used the $\left(G'/G, 1/G\right)$-expansion method to obtained the trigonometric and hyperbolic solutions of the problem.
- By selecting suitable parameter values, we use contour and 2-D charts to illustrate the dynamics of kinks, anti-kinks, a multi-soliton, and solitary wave profiles obtained from specific soliton solutions in three dimensions. It suggests that the dynamics that comprise our evolutionary traits are beneficial for comprehending physical phenomena (see **Figures 1** to **14**).

5. Conclusion and Future Scope

We have successfully obtained the exact close form solutions for the S-KdV equation by applying the EDA method. Further, we have concluded that the EDA is an efficient, versatile, and relatively simple method for solving nonlinear differential equations, producing exact solutions that facilitate the analysis of intricate physical systems. The EDA approach is more user-friendly than the Hirota bilinear method or the lie symmetry method for solving nonlinear differential equations. This can elucidate wave stability, dispersion, and energy transmission. Every analytical approach has restrictions; it fails when the coefficient of the highest order term is 0. Furthermore, it cannot directly solve nonlinear equations with variable coefficients.

The obtained solutions have properties like the kink, anti-kink, multi solitons, and solitary wave solitons, which are widely applied in scientific fields, such as in space plasma like black holes (Shapiro and Teukolsky, 1983), in astro physics objects like stars (Chabrier et al., 2006), in Earth's magnetosphere (Chang et al., 2022), and dusty plasma (Shukla and Mamun, 2003). We consider that the solutions obtained in this manuscript and the method utilized might be helpful for research in the above scientific fields, which would enable the researchers to comprehend the physical models.

Highlights of the results:

- The extended direct algebraic method is applied to study the exact solutions of the Schamel KdV equation.
- Soliton solutions for the Schamel KdV equation are explored.
- Kink and solitary wave solutions are obtained for the Schamel KdV equation.

The extended direct algebraic technique is suitable for complicated nonlinear partial differential equations (PDEs) like the Schamel KdV problem with fractional power nonlinearities. To create precise traveling wave solutions, the PDE is transformed into an ODE and a rational ansatz is assumed using known functions such as hyperbolic or trigonometric functions.

Extended Direct Algebraic Method: - Generalizes many procedures and accommodates more solution forms. It provides flexibility for assuming broader ansatzes, especially useful for the Schamel KdV's fractional nonlinearity. Hirota's Method: - Powerful, but potentially difficult for fractional nonlinear equations as the Schamel KdV. It works well for polynomial nonlinearities and may not work for non-integer powers in the Schamel equation. tanh-Method: - The tanh-method is ideal for finding soliton-like solutions via hyperbolic tangent expansions. - Note that this approach is less broad than the extended direct algebraic method and may struggle with non-polynomial nonlinearities. Lie Symmetry Analysis: Finding symmetry generators in Lie symmetry method which may be theoretically complex and computationally demanding.

Several nonlinear evolution equations, such as the Schamel and Korteweg–de Vries (KdV) family, have been solved using the extended direct algebraic method (EDAM). It can be applied to generalized or higher-dimensional versions of these equations, not just the conventional (1+1)-dimensional forms. Various nonlinear phenomena in multi-dimensional physical systems can be investigated with EDAM by correctly converting the dependent variables and using the method's systematic ansatz building. Future research that looks more closely at its effectiveness in more intricate frameworks, including coupled systems or variable-coefficient models, could expand its analytical breadth and usefulness.

Conflict of Interest
The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments
The authors acknowledge that there has been no financial support for this work that could have influenced its outcome. The authors would like to thank the Editor-in-Chief, section editors, and anonymous reviewers for their comments and suggestions that helped to improve the quality of this work.

AI Disclosure
The author(s) declare that no assistance is taken from generative AI to write this article.

References

Aljoudi, S. (2021). Exact solutions of the fractional Sharma-Tasso-Olver equation and the fractional Bogoyavlenskii's breaking soliton equations. *Applied Mathematics and Computation, 405*, 126237. https://doi.org/10.1016/j.amc.2021.126237.

Alquran, M. (2024). Derivation of some bi-wave solutions for a new two-mode version of the combined Schamel and KdV equations. *Partial Differential Equations in Applied Mathematics, 9*, 100641. https://doi.org/10.1016/j.padiff.2024.100641.

Bhat, I.A., Mishra, L.N., & Mishra, V.N. (2025). Comparative analysis of nonlinear Urysohn functional integral equations via Nyström method. *Applied Mathematics and Computation, 494*, 129287. https://doi.org/10.1016/j.amc.2025.129287.

Bilal, M., Iqbal, J., Shah, K., Abdalla, B., Abdeljawad, T., & Ullah, I. (2024). Analytical solutions of the space–time fractional Kundu–Eckhaus equation by using modified extended direct algebraic method. *Partial Differential Equations in Applied Mathematics, 11*, 100832. https://doi.org/10.1016/j.padiff.2024.100832.

Chabrier, G., Saumon, D., & Potekin, A.Y. (2006). Dense plasma in astrophysics: from giant planet to neutron stars. *Journal of Physics A: Mathematics and General, 39*, 4411. https://doi.org/10.1088/0305-4470/39/17/s16.

Chang, C., Huang, K., Lu, Q., Lu, S., Yu, X., Wang, R., Sang, L., & Gao, X. (2022). Electrostatic solitary waves and electron-beam instabilities in the separatrix region of magnetic reconnection. *The Astrophysical Journal, 933*, 67. https://doi.org/10.3847/1538-4357/ac738d.

Ding, B., Xie, X., & Ling, T. (2023). The new extended direct algebraic method for modified KdV-Zakharov-Kuznetsov. *Journal of Applied Science and Engineering, 27*(10), 3397-3406. http://dx.doi.org/10.6180/jase.202410_27(10).0013.

Donmez, O., & Daghan, D. (2017). Analytic solutions of the Schamel-KdV equation by using different methods: application to a dusty space plasma. *Journal of Natural and Applied Sciences, 21*, 208-215. https://doi.org/10.19113/sdufbed.04724.

El-Tantawy, S.A., El-Bedwehy, N.A., Abd El-Razek, H.N., & Mahmood, S. (2013). Large amplitude solitary waves in a warm magnetoplasma with kappa distributed electrons. *Physics of Plasmas*, *20,* 022115. https://doi.org/10.1063/1.4793446.

Guo, S., Mei, L., He, Y., & Li, Y. (2016). Time-fractional Schamel-KdV equation for dust-ion-acoustic waves in pair-ion plasma with trapped electrons and opposite polarity dust grains. *Physics Letters A, 380*(9-10), 1031-1036. https://doi.org/10.1016/j.physleta.2016.01.002.

Hosseinzadeh, H., & Sedaghatjoo, Z. (2025). Radial boundary elements method, a new approach on using radial basis functions to solve partial differential equations, efficiently. *Applied Mathematics and Computation, 493*, 129252. https://doi.org/10.1016/j.amc.2024.129252.

Hussain, A., Ali, H., Zaman, F., & Abbas, N. (2024). New closed form solutions of some nonlinear pseudo-parabolic models via a new extended direct algebraic method. *International Journal of Mathematics and Computer in Engineering, 2*(1), 35-58. https://doi.org/10.2478/ijmce-2024-0004.

Iqbal, M.S., Inc, M., Sohail, S., Khurshid, H., & Chishti, K. (2023). Analysis and soliton solutions of biofilm model by new extended direct algebraic method. *Nonlinear Analysis: Modelling and Control*, *28*(3), 538-553. https://doi.org/10.15388/namc.2023.28.31866.

Jafari, M., Zaeim, A., & Tanhaeivash, A. (2022). Symmetry group analysis and conservation laws of the potential modified KdV equation using the scaling method. *International Journal of Geometric Methods in Modern Physics, 19*(7), 2250098. https://doi.org/10.1142/s0219887822500980.

Kalaawy, O.H.E., & Aldenari, R.B. (2014). Painleve analysis, auto-backlund transformation, and new exact solutions for Schamel and Schamel-Korteweg-de Vries-Burgers equations in dust ion-acoustic waves in plasma. *Physics of Plasmas, 21*(9), 092308. https://doi.org/10.1063/1.4895498.

Kangalgil, F. (2016). Travelling wave solutions of the Schamel-Korteweg-de Vries and the Schamel equations. *Journal of the Egyptian Mathematical Society, 24*(4), 526-531. https://doi.org/10.1016/j.joems.2016.01.007.

Karakoc, S.B.G., Ali, K.K., & Mehanna, M. (2023). Exact traveling wave solutions of the schamel-KdV equation with two different methods. *Universal Journal of Mathematics and Applications, 6*(2), 65-75. https://doi.org/10.32323/ujma.1287524.

Kudryashov, N.A., & Lavrova, S.F. (2024). Traveling wave solutions of the derivative nonlinear Schrödinger hierarchy. *Applied Mathematics and Computation, 477*, 128802. https://doi.org/10.1016/j.amc.2024.128802.

Lee, J., & Sakthivel, R. (2011). Exact travelling wave solutions of the Schamel–Korteweg–de Vries equation. *Reports on Mathematical Physics*, *68*(2), 153-161.

Lee, J., & Saktivel, R. (2011). Exact travelling wave solutions of the Schamel-Korteweg-De Vries equation. *Reports on Mathematical Physics, 68*(2), 153-161. https://doi.org/10.1016/S0034-4877(12)60002-0.

Mahmood, A., Srivastava, H.M., Abbas, M., Abdullah, F.A., Mohammed, P.O., Baleanu, D., & Chorfi, N. (2023). Optical soliton solutions of the coupled Radhakrishnan–Kundu–Lakshmanan equation using the extended direct algebraic approach. *Heliyon*, *9*(10), e20852. https://doi.org/10.1016/j.heliyon.2023.e20852.

Manzoor, Z., Iqbal, M.S., Omer, N., Zakarya, M., Kanan, M., Akgül, A., Hussain, S., & Ashraf, F. (2024). Analyzing multiplicative noise effects on stochastic dynamical ϕ^4 equation using the new extended direct algebraic method. *Alexandria Engineering Journal, 97*, 185-203. https://doi.org/10.1016/j.aej.2024.04.011.

Mohanty, S.K. (2024). Abundant solitons for the generalized Hirota–Satsuma couple KdV system with an efficient technique. *Chinese Journal of Physics, 90*, 945-955. https://doi.org/10.1016/j.cjph.2024.02.017.

Mohanty, S.K., Kravchenko, O.V., & Dev, A.N. (2022). Exact traveling wave solutions of the Schamel Burgers' equation using generalized-improved and generalized (G′/G)-expansion methods. *Results in Physics, 33*, 105124. https://doi.org/10.1016/j.rinp.2021.105124.

Mohanty, S.K., Kravchenko, O.V., Deka, M.K., Dev, A.N., & Churikov, D.V. (2023). The exact solutions of the (2+1)-dimensional Kadomtsev–Petviashvili equation with variable coefficients by extended generalized (G′/G)-expansion method. *Journal of King Saud University - Science, 35*(1), 102358. https://doi.org/10.1016/j.jksus.2022.102358.

Mohanty, S.K., Kumar, S., Deka, M.K., & Dev, A.N. (2021). Dynamics of exact closed form solutions to the Schamel Burgers and Schamel equations with constant coefficients using a novel analytical approach. *International Journal of Modern Physics B, 35*(31), 2150317. https://doi.org/10.1142/s0217979221503173.

Mohanty, S.K., Sagib, M., & Ahmed, M. (2024). Exact solutions to the forced KdV equation via three efficient techniques. *Pramana - Journal of Physics, 98*(3), 83. https://doi.org/10.1007/s12043-024-02774-7.

Noor, S., Alshehry, A.S., Shafee, A., & Shah, R. (2024). Families of propagating soliton solutions for (3+1)-fractional Wazwaz-Benjamin-Bona-Mahony equation through a novel modification of modified extended direct algebraic method. *Physica Scripta*, *99*(4), 045230. https://doi.org/10.1088/1402-4896/ad23b0.

Qin, C.Y., Zhang, R.F., & Li, Y.H. (2024). Various exact solutions of the (4+1)-dimensional Boiti–Leon–Manna–Pempinelli-like equation using the bilinear neural network method. *Chaos, Solitons & Fractals, 187*, 115438. https://doi.org/10.1016/j.chaos.2024.115438.

Rehman, H.U., Hassan, M.U., Saleem, M.S., Nasri, R., Santina, D., & Mlaiki, N. (2023). Soliton solutions of Zakhrov equation in ionized plasma using new extended direct algebraic method. *Results in Physics, 46*, 106325. https://doi.org/10.1016/j.rinp.2023.106325.

Sagib, M., Saha, B.K., Mohanty, S.K., & Rahman, M. (2024). Diverse soliton solutions to the nonlinear partial differential equations related to electrical transmission line. *Physica Scripta*, *99*, 095219. https://doi.org/10.1088/1402-4896/ad69e6.

Schamel, H. (1973). A modified Korteweg-de Vries equation for ion acoustic wavess due to resonant electrons. *Journal of Plasma Physics*, *9*(3), 377-387.

Schamel, H. (1986). Electron holes, ion holes and double layers: electrostatic phase space structures in theory and experiment. *Physics Reports*, *140*(3), 161-191. https://doi.org/10.1016/0370-1573(86)90043-8.

Shahzad, T., Baber, M.Z., Ahmad, M.O., Ahmed, N., Akgül, A., Ali, S.M., Ali, M., & Din, S.M.E. (2023). On the analytical study of predator–prey model with Holling–II by using the new modified extended direct algebraic technique and its stability analysis. *Results in Physics, 51*, 106677. https://doi.org/10.1016/j.rinp.2023.106677.

Shapiro, S.L., & Teukolsky, S.A. (1983). *Black holes, white dwarfs and neutron stars: the physics of compact objects*. John Wiley and Sons, New York. ISBN: 9780471873167 (p), 9783527617661 (e). https://doi.org/10.1002/9783527617661.

Shukla, P.K., & Mamun, A.A. (2003). Solitons, shocks and vortices in dusty plasmas. *New Journal of Physics, 5*, 17. https://doi.org/10.1088/1367-2630/5/1/317.

Taha, W.M., Noorani, M.S.M., & Hashim, I. (2013). New exact solutions of ion-acoustic wave equations by (G′/G)-expansion method. *Journal of Applied Mathematics, 2013*, 810729. https://doi.org/10.1155/2013/810729.

Tariq, K.U., Ahmed, A., & Ma, W.X. (2022a). On some soliton structures to the Schamel–Korteweg-de Vries model via two analytical approaches. *Modern Physics Letters B, 36*, 2250137.

Tariq, K.U., Rezazadeh, H., Zubair, M., Osman, M.S., & Akinyemi, L. (2022b). New exact and solitary wave solutions of nonlinear Schamel-KdV equation. *International Journal of Applied and Computational Mathematics, 8*(3), 114. https://doi.org/10.1007/s40819-022-01315-3.

Tissaoui, Y., Kelly, J.F., & Marras, S. (2025). Efficient spectral element method for the Euler equations on unbounded domains. *Applied Mathematics and Computation, 487*, 129080. https://doi.org/10.1016/j.amc.2024.129080.

Verheest, F., & Lakhina, G.S. (2002). Nonlinear structures in solar system plasmas. *Physica Scripta*, *2002*(T98), 38.