

Bivariate Software Reliability Growth Models under Budget Constraint for Development Management

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(Received April 30, 2019; Accepted August 14, 2019)

Abstract

Software reliability growth is observed by investing not only the testing-time but also the testing-effort in the testing-phase of software development process. If the testing-time (testing-effort) is reduced to some extent, it is possible to observe the software reliability growth by investing the amount of testing-effort (testing-time) which can compensate the insufficiency of the testing-time (testing-effort). However, most of the existing software reliability growth models (SRGMs) are constructed as univariate models and the substitutability between the testing-time and testing-effort is not considered. Additionally, it is necessary to remove many faults efficiently within the budget. In this paper, we develop bivariate Weibull type SRGMs under budget constraint based on the Cobb-Douglas type and CES (constant elasticity of substitution) type testing-time functions. Simultaneously, we evaluate the substitutability between the testing-time and testing-effort factors which are software reliability growth factors. Finally, we conduct the sensitivity analysis and show numerical examples by using actual data sets.

Keywords- Bivariate Weibull type software reliability growth model, Cobb-Douglas type testing-time function, CES (constant elasticity of substitution) type testing-time function, Budget constraint, Sensitivity analysis.

1. Introduction

Enormous testing-effort, such as CPU time, execution time, man-hour, test coverage, and executed test case, is invested in the actual testing-phase. However, it seems that existing univariate SRGMs (Pham, 2000; Yamada, 2014) cannot reflect the actual testing-phase because they assume that the reliability growth depends only on the testing-time. From these background, bivariate SRGMs have been developed in several literatures (Inoue and Yamada, 2008; Inoue et al., 2010, 2017; Ishii and Dohi, 2012; Kapur et al., 2010; Kapur and Pham, 2012). Especially, an approach using the Cobb-Douglas type production function (Inoue and Yamada, 2008) as a testing-time function is often used for expanding to bivariate models from univariate models. Since the Cobb-Douglas type production function is typical as a production/utility function on economics, and it has simple mathematic structure to handle. From previous research, the proposed bivariate SRGMs based on

the Cobb-Douglas type testing-time function are more accurate than the existing univariate models. However, the parameter of the Cobb-Douglas type testing-time function has no economic significance. Therefore, the relationship between software reliability growth factors such as the testing-time and testing-effort cannot be described sufficiently.

Also, actual software development projects are not always proceeded as planned. For example, if enough testing period cannot be secured in the testing-process, the testing-effort which can compensate the insufficient testing-time is needed to be invested in order to observe the software reliability growth. However, the software reliability growth factor cannot substitute all of the other software reliability growth factor, realistically. Furthermore, it is needed to improve software quality/reliability within the development budget. Therefore, we need to develop bivariate SRGMs which can consider multiple software reliability growth factors under budget constraint, and understand the quantitative substitutability between software reliability growth factors.

In this paper, we apply not only the Cobb-Douglas type but also CES type production functions (Minamino et al., 2017, 2019) as testing-time functions and develop the bivariate Weibull type SRGMs based on these testing-time functions. Then, we introduce the concept of the elasticity of substitution between the software reliability growth factors in terms of economics. The elasticity of substitution means a measure of ease for substituting each software reliability growth factor in order to maintain the influence on the software reliability growth. The Cobb-Douglas type production function is greatly restricted with respect to the elasticity of substitution. However, the CES type production function which can be more flexible with respect to the elasticity of substitution. Additionally, the CES type production function generalizes various production functions including the Cobb-Douglas type production function. That is, we can assume the CES type production function as various testing-time functions.

Furthermore, we solve the optimization problem which maximizes the number of detectable faults under budget constraint in terms of software development management. Then, we propose the bivariate Weibull type SRGMs under budget constraint by introducing the optimal testing-time and the optimal amount of testing-effort to the previous bivariate Weibull type SRGMs. Finally, we show the result of sensitivity analysis by using actual data sets as numerical examples.

2. Bivariate Weibull Type SRGMs

2.1 Definition of Two-Dimensional Nonhomogeneous Poisson Process (NHPP)

Let $\{N(s, u), s \geq 0, u \geq 0\}$ denote a counting process representing the total number of faults detected up to the testing-time s and the testing-effort u . The probability mass function that m faults are detected up to the testing-time s and the testing-effort u is derived as

$$\Pr\{N(s, u) = m\} = \frac{\{H(s, u)\}^m}{m!} \exp\{-H(s, u)\} \quad (m = 0, 1, 2, \dots),$$

$$H(s, u) = \int_0^s \int_0^u h(s, u) ds du, \quad (1)$$

where $H(s, u)$ and $h(s, u)$ represent the mean value function and the intensity function of NHPP, respectively (Inoue and Yamada, 2008).

2.2 Several Testing-Time Functions

We assume that the testing-time t for the existing SRGMs is expressed as a software reliability growth factor by using the following Eqs. (2) and (3) (Inoue and Yamada, 2008; Kapur et al., 2010; Kapur and Pham, 2012; Sakaguchi et al., 2018).

$$t_{CD} \equiv s^\alpha u^{1-\alpha} \quad (0 \leq \alpha \leq 1), \quad (2)$$

$$t_{CES} \equiv (\alpha s^{-\delta} + (1 - \alpha)u^{-\delta})^{-\frac{1}{\delta}} \quad (\delta \geq -1), \quad (3)$$

where s is the testing-time factor (calendar time), and u the testing-effort factor (test coverage). α and δ represent the distribution parameter and the substitution parameter, respectively. Also, the CES type testing-time (production) function has the following characteristics: When δ approaches 1, the CES type testing-time function becomes a linear function. When δ approaches 0, it becomes the Cobb-Douglas type function. When δ approaches ∞ , it becomes Leontief function. Next, the elasticity of substitution is defined as

$$e = \frac{1}{1 + \delta} \quad (\delta \geq -1). \quad (4)$$

It expresses the ease of substitution between software reliability growth factors to maintain software reliability growth in terms of economics. Note that the elasticity of substitution for the Cobb-Douglas type testing-time (production) function always has 1. On the other hand, the elasticity of substitution for the CES type testing-time (production) function can take a value except 1.

2.3 Bivariate Weibull Type SRGMs Based on Several Testing-Time Functions

The univariate Weibull type SRGM is given by

$$H(t) = \gamma(t) = \left(\frac{t}{\rho}\right)^\beta \quad (0 < \beta < 1, \rho > 0), \quad (5)$$

where β represents the reliability growth parameter, and ρ represents the scale parameter. Also, $\gamma(0) = 0$ and $\gamma(\infty) = \infty$. It means that the debugging activities is imperfect. From Eq. (5), we provide the following bivariate Weibull type SRGMs based on the Cobb-Douglas type and CES type testing-time functions, respectively.

Bivariate C-D Weibull model (Inoue and Yamada, 2008):

$$H_{CD}(s, u) \equiv \gamma_{CD}(s, u) = \left(\frac{s^\alpha u^{1-\alpha}}{\rho}\right)^\beta, \quad (6)$$

Bivariate CES Weibull Model:

$$\begin{aligned} H_{CES}(s, u) \equiv \gamma_{CES}(s, u) &= \left(\frac{(\alpha s^{-\delta} + (1 - \alpha)u^{-\delta})^{-\frac{1}{\delta}}}{\rho}\right)^\beta \\ &= \rho^{-\beta} [\alpha s^{-\delta} + (1 - \alpha)u^{-\delta}]^{-\frac{\beta}{\delta}} \end{aligned} \quad (7)$$

$$= c[\alpha s^{-\delta} + (1 - \alpha)u^{-\delta}]^{-\frac{\beta}{\delta}}.$$

Note that we define $c = \rho^{-\beta}$ as a scale parameter (constant term) in Eq. (7).

3. Parameter Estimation

Suppose that we have observed (s_k, u_k, y_k) ($k = 0, 1, 2, \dots, K$) where s_k is the testing-time, u_k the amount of testing-effort, y_k the total number of faults detected during $[0, s_k]$, $[0, u_k]$, and K the total number of data set. From Eqs. (6) and (7), we derive the following regression equations by taking the natural logarithm, respectively.

$$\log \gamma_{CD}(s, u) = -\beta \log \rho + \alpha \beta \log s + (1 - \alpha) \beta \log u, \quad (8)$$

$$\begin{aligned} \log \gamma_{CES}(s, u) &= \log c - \frac{\beta}{\delta} \log(\alpha s^{-\delta} + (1 - \alpha)u^{-\delta}) \\ &\cong \log c + \alpha \beta \log s + (1 - \alpha) \beta \log u - \frac{1}{2} \alpha \beta \delta (1 - \alpha) [\log s - \log u]^2. \end{aligned} \quad (9)$$

From Eq. (8), we obtain the following multiple regression equation by rewriting the coefficients.

$$\log \gamma_{CD}(s, u) \equiv Y_k = a_0 + a_1 K_k + a_2 L_k + \varepsilon_k, \quad (10)$$

$$\left\{ \begin{array}{l} Y_k = \log y_k, \\ K_k = \log s_k, \\ L_k = \log u_k, \\ a_0 = -\beta \log \rho, \\ a_1 = \alpha \beta, \\ a_2 = (1 - \alpha) \beta, \end{array} \right. \quad (11)$$

where ε_k is the error term. Similarly, from Eq. (9), we obtain the following multiple regression equation by rewriting the coefficients.

$$\log \gamma_{CES}(s, u) \equiv Y_k = a_0 + a_1 K_k + a_2 L_k + a_3 [K_k - L_k]^2 + \varepsilon_k, \quad (12)$$

$$\left\{ \begin{array}{l} Y_k = \log y_k, \\ K_k = \log s_k, \\ L_k = \log u_k, \\ a_0 = \log c, \\ a_1 = \alpha \beta, \\ a_2 = (1 - \alpha) \beta, \\ a_3 = -\frac{1}{2} \alpha \beta \delta. \end{array} \right. \quad (13)$$

From Eqs. (10) ~ (13), the sum of squares of errors is derived as Eqs. (14) and (15), respectively.

$$S_{CD}(a_0, a_1, a_2) = \sum_{k=1}^K \varepsilon_k = \sum_{k=1}^K \{Y_k - (a_0 + a_1 K_k + a_2 L_k)\}^2, \quad (14)$$

$$S_{CES}(a_0, a_1, a_2, a_3) = \sum_{k=1}^K \varepsilon_k = \sum_{k=1}^K \{Y_k - (a_0 + a_1 K_k + a_2 L_k + a_3 [K_k - L_k]^2)\}^2. \quad (15)$$

The following estimated parameters of the bivariate C-D Weibull model are obtained by solving simultaneous equations.

$$\left\{ \begin{array}{l} \hat{\alpha} = \frac{\hat{a}_1}{\hat{a}_1 + \hat{a}_2}, \\ \hat{\beta} = \hat{a}_1 + \hat{a}_2, \\ \hat{\rho} = \exp\left[-\frac{\hat{a}_0}{\hat{a}_1 + \hat{a}_2}\right]. \end{array} \right. \quad (16)$$

Also, the following estimated parameters of the bivariate CES Weibull model are obtained by solving simultaneous equation.

$$\left\{ \begin{array}{l} \hat{\alpha} = \frac{\hat{a}_1}{\hat{a}_1 + \hat{a}_2}, \\ \hat{\beta} = \hat{a}_1 + \hat{a}_2, \\ \hat{c} = \exp[\hat{a}_0], \\ \hat{\delta} = -\frac{2\hat{a}_3}{\hat{a}_1 - \frac{\hat{a}_1^2}{\hat{a}_1 + \hat{a}_2}}. \end{array} \right. \quad (17)$$

4. Software Reliability Analysis under Budget Constraint

4.1 Optimization Problem Based on the Bivariate C-D Weibull Model

We formulate the optimization problem based on the bivariate C-D Weibull model to estimate the number of detectable faults under the budget constraint.

$$\begin{array}{ll} \max & H_{CD}(s, u) = \left(\frac{s^\alpha u^{1-\alpha}}{\rho}\right)^\beta, \\ \text{s. t.} & p_s s + p_u u \leq I, \\ & s > 0, u > 0, \end{array} \quad (18)$$

where p_s and p_u represent the cost required in a testing-time factor and the cost required in a testing-effort factor, respectively. I represents the budget for the testing-phase. From Eq. (18), we obtain the following optimal solutions (s^*, u^*) by using the Kuhn-Tucker condition.

$$(s^*, u^*) = \left(\frac{\alpha I}{p_s}, \frac{(1-\alpha)I}{p_u}\right). \quad (19)$$

Substituting Eq. (19) into Eq. (6), the maximum number of detectable faults under the budget constraint based on the bivariate C-D Weibull type model is derived as follows (Inoue et al., 2017):

$$H_{CD}(s^*, u^*) = \left\{ \frac{\left(\frac{\alpha I}{p_s}\right)^\alpha \left(\frac{(1-\alpha)I}{p_u}\right)^{1-\alpha}}{\rho} \right\}^\beta. \quad (20)$$

4.2 Optimization Problem Based on the Bivariate CES Weibull Model

Similarly, we formulate the optimization problem based on the bivariate CES Weibull model to

estimate the number of detectable faults under the budget constraint.

$$\begin{aligned} \max \quad & H_{\text{CES}}(s, u) = c[\alpha s^{-\delta} + (1 - \alpha)u^{-\delta}]^{-\frac{\beta}{\delta}}, \\ \text{s. t.} \quad & p_s s + p_u u \leq I, \\ & s > 0, u > 0. \end{aligned} \tag{21}$$

From Eq. (21), we obtain the following optimal solutions (s^*, u^*) by using the Kuhn-Tucker condition.

$$(s^*, u^*) = \left(\frac{I}{p_s \left[\left(\frac{p_u}{p_s} \right)^{\frac{\delta}{\delta-1}} \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1}{\delta-1}} + 1 \right]}, \frac{I}{p_u \left[\left(\frac{p_s}{p_u} \right)^{\frac{\delta}{\delta-1}} \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\delta-1}} + 1 \right]} \right). \tag{22}$$

Substituting Eq. (22) into Eq. (7), the maximum number of detectable faults under the budget constraint based on the bivariate CES Weibull type model is derived as follows:

$$\begin{aligned} H_{\text{CES}}(s^*, u^*) = c & \left[\alpha \left\{ \frac{I}{p_s \left[\left(\frac{p_u}{p_s} \right)^{\frac{\delta}{\delta-1}} \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1}{\delta-1}} + 1 \right]} \right\}^{-\delta} \right. \\ & \left. + (1 - \alpha) \left\{ \frac{I}{p_u \left[\left(\frac{p_s}{p_u} \right)^{\frac{\delta}{\delta-1}} \left(\frac{1-\alpha}{\alpha} \right)^{\frac{1}{\delta-1}} + 1 \right]} \right\}^{-\delta} \right]^{-\frac{\beta}{\delta}}. \end{aligned} \tag{23}$$

5. Numerical Examples

We show numerical examples by using DS1: $(s_k, u_k, y_k) (k = 0, 1, 2, \dots, 24; s_{24} = 24 \text{ (weeks)}, u_{24} = 90.946, y_{24} = 296)$ and DS2: $(s_k, u_k, y_k) (k = 0, 1, 2, \dots, 21; s_{21} = 21 \text{ (weeks)}, u_{21} = 91.981, y_{21} = 212)$ (Fujiwara and Yamada, 2002) as actual data sets. s_k represents the calendar time measured on the basis of weeks, u_k represents the test coverage, y_k represents the total number of faults detected during $[0, s_k]$, $[0, u_k]$. These data sets are obtained in the testing-process for embedded software developed by software development engineers with approximately 7 to 10 years of development experience. These development scale is 1.972×10^5 (LOC: Lines of Code) and 1.630×10^5 (LOC), respectively. In order to apply the test coverage data as a testing-effort factor, we adjust the testing-time data and detected fault count data as follows:

$$\text{The execution rate of the specified testing period: } s \equiv \frac{s_k}{s_K}, \tag{24}$$

$$\text{The expected fault detection rate: } y \equiv \frac{y_k}{y_K}, \tag{25}$$

where K is the total number of data set (s_k, u_k, y_k) .

Table 1 and 2 show the result of parameter estimation for the bivariate C-D Weibull model and the bivariate CES Weibull model, respectively. From the estimated parameter \hat{e} in Table 2, we can evaluate that the testing-environment for DS1 is easier to substitute between the software reliability growth factors than the testing-environment for DS2, relatively. Table 3 shows the result of goodness-of-fit comparison. We apply MSE (mean squared errors) as an evaluation criterion. The MSE is calculated by deriving the squared errors between the observed and estimated number of detected faults and is given by:

$$MSE = \frac{1}{K} \sum_{k=1}^K \{y_k - \hat{H}(s_k, u_k)\}^2. \quad (26)$$

Note that we derived the MSE of existing Weibull type SRGM after adjusting the testing-time data and detected fault count data as shown in Eqs. (24) and (25). From Table 3, we can say that the bivariate CES Weibull model has better performance than the existing Weibull type SRGM and the bivariate C-D Weibull model, and the significance of considering the bivariate model could be confirmed.

Table 1. The result of parameter estimation (bivariate C-D Weibull model)

DATA	\hat{a}_0	\hat{a}_1	\hat{a}_2	\hat{a}_3	\hat{c}	\hat{a}	$\hat{\beta}$	$\hat{\delta}$	$\hat{\rho}$
DS1	0.089	0.17	0.79	-	-	0.18	0.96	-	0.91
DS2	0.051	0.28	0.67	-	-	0.29	0.95	-	0.95

Table 2. The result of parameter estimation (bivariate CES Weibull model)

DATA	\hat{a}_0	\hat{a}_1	\hat{a}_2	\hat{a}_3	\hat{c}	\hat{a}	$\hat{\beta}$	$\hat{\delta}$	\hat{e}
DS1	0.09	0.18	0.78	0.055	1.095	0.18	0.96	-0.76	4.17
DS2	0.058	0.40	0.56	-0.066	1.06	0.42	0.96	0.56	0.64

Table 3. The result of goodness-of-fit comparison based on MSE

DATA	Weibull type SRGM	Bivariate C-D Weibull Model	Bivariate CES Weibull Model:
DS1	1.15	0.54×10^{-3}	0.27×10^{-4}
DS2	1.18	0.41×10^{-2}	0.61×10^{-4}

Table 4 and 5 show the result of sensitivity analysis for the maximum number of detectable faults under the budget constraint based on the bivariate C-D Weibull model. Note that p_s , p_u and I are given. Also, we assume that p_s or p_u , and I are constant in each Table. From Table 4, when the cost of the testing-effort increases, the number of detectable faults decreases and the testing-effort factor decreases. Also, the testing-time factor does not change although the testing-effort changes because the elasticity of substitution for the Cobb-Douglas type production function is 1. That is, the software reliability growth is not maintained because there is no substitution between them. The similar trend is seen in Table 5.

Table 6 and 7 show the result of sensitivity analysis for the maximum number of detectable faults under the budget constraint based on the bivariate CES Weibull model. Similar to Table 4 and 5, p_s , p_u and I are given. Also, we assume that p_s or p_u , and I are constant in each Table. From Table 6, we can see that not only the testing-effort factor but also the testing-time factor changes in comparison with Table 4 because of the substitution. Note that the testing-time factor decreases because of the budget constraint. Furthermore, when the cost of the testing-effort factor decreases, the number of detectable faults increases. It means that the software reliability growth is maintained because of the substitution. The similar trend is seen in Table 7.

Table 4. The result of sensitivity analysis
 (DS1, bivariate C-D Weibull model, the cost of the testing-time factor p_s and budget I : constant)

p_s	p_u	I	s^*	u^*	$H(s^*, u^*)$	Faults
2	6	5	0.440	0.687	0.708	209.647
2	7	5	0.440	0.589	0.627	185.661
2	8	5	0.440	0.515	0.565	167.114
2	9	5	0.440	0.458	0.515	152.298
2	10	5	0.440	0.412	0.474	140.161
2	11	5	0.440	0.375	0.439	130.017
2	12	5	0.440	0.343	0.410	121.399
2	13	5	0.440	0.317	0.385	113.977
2	14	5	0.440	0.294	0.363	107.510
2	15	5	0.440	0.275	0.344	101.820

Table 5. The result of sensitivity analysis
 (DS1, bivariate C-D Weibull model, the cost of the testing-effort factor p_u and budget I : constant)

p_s	p_u	I	s^*	u^*	$H(s^*, u^*)$	Faults
6	5	5	0.147	0.824	0.680	201.215
7	5	5	0.126	0.824	0.662	196.066
8	5	5	0.110	0.824	0.648	191.712
9	5	5	0.098	0.824	0.635	187.952
10	5	5	0.088	0.824	0.624	184.651
11	5	5	0.080	0.824	0.614	181.715
12	5	5	0.073	0.824	0.605	179.075
13	5	5	0.068	0.824	0.597	176.681
14	5	5	0.063	0.824	0.590	174.492
15	5	5	0.059	0.824	0.583	172.480

Table 6. The result of sensitivity analysis
 (DS1, bivariate CES Weibull model, the cost of the testing-time factor p_s and budget I : constant)

p_s	p_u	I	s^*	u^*	$H(s^*, u^*)$	Faults
2	6	5	0.527	0.658	1.700	503.192
2	7	5	0.500	0.571	1.919	568.058
2	8	5	0.478	0.506	2.131	630.915
2	9	5	0.458	0.454	2.338	692.038
2	10	5	0.442	0.412	2.539	751.638
2	11	5	0.427	0.377	2.736	809.881
2	12	5	0.414	0.348	2.929	866.901
2	13	5	0.402	0.323	3.118	922.808
2	14	5	0.391	0.301	3.303	977.695
2	15	5	0.381	0.282	3.485	1031.641

Table 7. The result of sensitivity analysis
 (DS1, bivariate CES Weibull model, the cost of the testing-effort factor p_u and budget I : constant)

p_s	p_u	I	s^*	u^*	$H(s^*, u^*)$	Faults
6	5	5	0.264	0.683	1.793	530.781
7	5	5	0.237	0.668	1.846	546.341
8	5	5	0.215	0.655	1.893	560.400
9	5	5	0.198	0.644	1.937	573.282
10	5	5	0.183	0.633	1.977	585.213
11	5	5	0.171	0.624	2.015	596.356
12	5	5	0.161	0.615	2.050	606.835
13	5	5	0.151	0.606	2.084	616.744
14	5	5	0.143	0.599	2.115	626.158
15	5	5	0.136	0.592	2.146	635.138

6. Conclusions

We discussed on the bivariate Weibull type SRGMs based on the Cobb-Douglas type and the CES type testing-time functions and considered the budget constraint. Also, we conducted the sensitivity analysis and showed the variation of the testing-time and testing-effort factors under the budget constraint. Finally, we evaluated the substitutability for the testing-time and testing-effort factors in terms of economics. As future studies, we still have to check the performance of our proposed models by using more actual data sets. Furthermore, we need to introduce various software reliability growth factors.

Conflict of Interest

The authors declare that they have no conflict of interest.

Acknowledgement

This research was supported in part by The Telecommunications Advancement Foundation in Japan.

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