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7th -Order Caudrey-Dodd-Gibbon Equation and Fisher-Type Equation by Homotopy Analysis Method

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Abstract
In this paper, we first describe the methodology of the Homotopy Analysis Method (HAM) which is an analytical technique and then employ it to some of the non-linear problems which are used in different fields of sciences like plasma physics, fluid dynamics, laser optics, biology, chemical kinetics, nucleation kinetics, physiology, etc. Approximate series solutions have been obtained and the results are compared with the closed form solutions of the equations, which show that this technique gives high accurate results. HAM is a reliable technique, easy to use and is widely applicable to a large class of non-linear differential equations. MATHEMATICA software package has been used for computations.

Keywords- Homotopy analysis method, 7th -order Caudrey-Dodd-Gibbon equation, Fisher-type equation.

1. Introduction

Generally, the non-linear equations are not very easy to solve explicitly. Some techniques like perturbation techniques which involve some parameter (small or large), also known as the perturbation quantity, play an important role in solving the non-linear differential equations to some extent. But like other non-linear analytical methods, these perturbation techniques too, have their own drawbacks, viz., the dependency of these techniques on the perturbation quantities, which restricts the applications of them to a wide class of non-linear differential equations because it is not necessary that there always exists a perturbation quantity in every non-linear problem. The other drawback is that it seems to be a special art that requires a special technique in determining the small parameters. If an appropriate value of the small parameter is chosen, it will lead to the ideal results but if not then it may create a critical problem. Moreover, it has been observed that in most of the cases the approximated solutions obtained by these perturbation techniques are valid only for the smaller values of the perturbation quantities. Also, as the non-linearity becomes stronger, the analytic approximations start breaking down, thus, makes the perturbation approximations valid only for non-linear problems with weak non-linearity. Thus, in view of all above limitations one can conclude that it all arises due to the assumption of that small parameter which is known as the perturbation quantity. In order to overcome these limitations there were developed some non-perturbation techniques, e.g., Sinc-Collocation method (Zakeri and Navab, 2010), Sinc-Galerkin method (Rashidinia and Nabati, 2013), finite element method

(Jin, 2014), finite difference method (Li and Ding, 2014), Eulerian Lagrangian method (Chertock et al., 2014) and so on. The advantage of these techniques is that they don't require any parameter. In spite of that, both the perturbation and non-perturbation techniques themselves do not provide the facility of controlling the rate of approximate series and the convergence region.

In 1992, a new analytical method was proposed by Liao (Liao, 1992), called the Homotopy Analysis Method (HAM), to solve linear and non-linear problems. HAM provides an analytic approach to obtain a series solution to different kinds of linear and non-linear equations, be it coupled, algebraic, ordinary or partial differential equation. One of the most important advantages of this method is that both the rate of approximate series and the convergence region can be controlled and adjusted according to the necessity. The other advantage of this method is that it can be applied to a high range of non-linear problems as it does not depend on any type of physical parameter. Moreover, it ensures the convergence of the solution series and thus becomes valid even for strong non-linear problems. It also facilitates us with the liberty of choosing the auxiliary things, viz., non-zero auxiliary function and non-zero auxiliary parameter. One may refer to his book (Liao, 2003) for the better understanding of this method and to know how this method differs from perturbation/non-perturbation techniques. Also, in his new edition (Liao, 2014), he has described current advances of the HAM technique and its relationships to other methods. A substantial amount of work has been done by a number of researchers on this method to solve a large number of non-linear problems arising in different fields of sciences (Abbasbandy, 2007; Arora and Kumar, 2013; Daniel and Daniel, 2015; Arora and Sharma, 2018; Sharma and Arora, 2019).

In this paper, the HAM method has been successfully employed on non-linear partial differential equations, namely, the seventh-order Caudrey-Dodd-Gibbon equation and a Fisher-type equation. Their approximate analytical and numerical solutions have been found out. The results have been computed using the Mathematica package. The computed results are then compared with the known exact solutions of the corresponding differential equations, and it is shown that the absolute errors between these solutions are very less, that is, the HAM provides highly accurate results. This method is very simple, effective and is being widely used to study the differential equations to a large scale.

2. Basic Concepts of the Homotopy Analysis Method

In order to describe the HAM, let us take the non-linear equation of the form:

$$N[u(x,t)] = 0, \tag{1}$$

where, N is a non-linear operator, u is an unknown function depending on space variable x and time variable t.

Definition: Let ψ be a function of the homotopy parameter q, then

$$D_n(\psi) = \frac{1}{n!}\frac{\partial^n \psi(x,t;q)}{\partial q^n}\bigg|_{q=0}, \quad \text{where} \quad n \geq 0, \tag{2}$$

is called the nth-order homotopy derivative of ψ (Liao, 1992).

Now, construct the zero-order deformation equation as:

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$$(1-q)\ L[\psi(x,t;q)-u_0(x,t)]=\hbar q\ N[\psi(x,t;q)]H(x,t), \tag{3}$$

where, L is an auxiliary linear operator having the property $L\,f=0$ when $f=0$, H is a non-zero auxiliary function, $\hbar$ is a non-zero auxiliary parameter, $\psi(x,t;q)$ denotes an unknown function, u_0 is an initial guess of u and q is an embedding parameter whose value lies in $[0,1]$. One of the advantages of this method is that it contains some auxiliary things which facilitate us in choosing them with full liberty. Therefore, when the values of q are $q=0$ and $q=1$, we get

$$\psi(x,t;0)=u_0(x,t), \qquad \psi(x,t;1)=u(x,t), \tag{4}$$

i.e. the solution ψ ranges from u_0 to u of the given problem. Now, the function ψ is expanded in the Taylor's series about $q=0$ in the following way:

$$\psi(x,t;q)=u_0(x,t)+\sum_{n=1}^{\infty}u_n(x,t)\,q^n, \tag{5}$$

where,

$$u_n(x,t)=D_n(\psi). \tag{6}$$

The series (5) converges at $q=1$, if we choose the initial guess u_0, the auxiliary linear operator L, the non-zero auxiliary function H and parameter $\hbar$, appropriately. Then the series (5) will take the form

$$\psi(x,t;1)=u_0(x,t)+\sum_{n=1}^{\infty}u_n(x,t)\ . \tag{7}$$

This equation should be one of the solutions of the original differential equation (1). Now, we differentiate *n*-times the zero-order deformation equation (3) w.r.t. q, put $q=0$ and then divide the so-obtained equation throughout by $n!$, we obtain the nth-order deformation equation, given by

$$L[u_n(x,t)-\sigma_n\ u_{n-1}(x,t)]=\hbar\ H\,R_n(u_{n-1}), \tag{8}$$

where,

$$R_n(u_{n-1})=D_{n-1}\big(N[\psi(x,t;q)]\big) \quad \text{and} \quad \sigma_n=\begin{cases}0, & n\le 1,\\ 1, & n>1.\end{cases} \tag{9}$$

The nth-order deformation equation (8) is linear and it can be solved easily by using computation software package like Mathematica, Matlab, Maple etc.

3. Application of the Homotopy Analysis Method (HAM)

We shall apply the HAM on non-linear partial differential equations with the given initial conditions.

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3.1 The Caudrey-Dodd-Gibbon Equation

Let us consider the seventh-order Caudrey-Dodd-Gibbon equation (Wazwaz, 2012)

$$u_t + 420\,u^3\,u_x + 210\,u^2\,u_{3x} + 420\,u\,u_x\,u_{2x} + 28\,u\,u_{5x} + 28\,u_x\,u_{4x} + 70\,u_{2x}\,u_{3x} + u_{7x} = 0, \tag{10}$$

subject to the initial condition

$$u(x,0) = \frac{2\,e^x}{(1+e^x)^2}, \tag{11}$$

where, u, x and t are the wavelength, space and time variables, repectively. This equation (10) represents the dispersive effects of the non-linear waves in the areas of science like laser optics, chemical kinetics, fluid dynamics, plasma physics, etc.

To apply the HAM, we first define the linear operator L as:

$$L[\psi(x,t;q)] = \frac{\partial \psi(x,t;q)}{\partial t}, \tag{12}$$

having the property $L(c) = 0$ [$c = \text{constant}$].

Now, from the equation (10), the non-linear operator becomes

$$N[\psi(x,t;q)] = \psi_t + 420\,\psi^3\,\psi_x + 210\,\psi^2\,\psi_{xxx} + 420\,\psi\,\psi_x\,\psi_{xx} + 28\,\psi\,\psi_{xxxxx} + 28\psi_x\,\psi_{xxxx} + 70\psi_{xx}\,\psi_{xxx} + \psi_{xxxxxxx}. \tag{13}$$

Also, the zero-order deformation equation is defined as follows:

$$(1-q)L[\psi(x,t;q) - u_0(x,t)] = H\,\hbar\,q\,N[\psi(x,t;q)], \tag{14}$$

and when $q = 0$ and $q = 1$, we have the following conditions:

$$\psi(x,t;0) = u_0(x,t), \qquad \psi(x,t;1) = u(x,t). \tag{15}$$

We, now, differentiate *n*-times the deformation equation (14) w.r.t. q, put $q = 0$ and then divide the so-obtained equation throughout by $n!$, we get

$$L[u_n - \sigma_n\,u_{n-1}] = H\,\hbar\,R_n(u_{n-1}), \tag{16}$$

subject to the initial condition

$$u_n(x,0) = 0, \tag{17}$$

where,

$$R_n(u_{n-1}) = \frac{\partial u_{n-1}}{\partial t} + 420\sum_{j=0}^{n-1} u_j^3\,\frac{\partial u_{n-1-j}}{\partial x} + 210\sum_{j=0}^{n-1} u_j^2\,\frac{\partial^3 u_{n-1-j}}{\partial x^3} + 420\sum_{j=0}^{n-1}\alpha_j\,\frac{\partial^2 u_{n-1-j}}{\partial x^2} + 28\sum_{j=0}^{n-1} u_j\,\frac{\partial^5 u_{n-1-j}}{\partial x^5} + 28\sum_{j=0}^{n-1}\frac{\partial u_j}{\partial x}\,\frac{\partial^4 u_{n-1-j}}{\partial x^4} + 70\sum_{j=0}^{n-1}\frac{\partial^2 u_j}{\partial x^2}\,\frac{\partial^3 u_{n-1-j}}{\partial x^3} + \frac{\partial^7 u_{n-1}}{\partial x^7}, \tag{18}$$

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and $\alpha_j = \sum_{k=0}^{j} u_k \frac{\partial u_{j-k}}{\partial x}$.

For convenience, we choose the value of $H = 1$ and $\hbar = h$, then the solution of the equation (16), for $n \geq 1$, becomes

$$u_n = \sigma_n u_{n-1} + h\ L^{-1}\left[R_n(u_{n-1})\right]. \tag{19}$$

The initial approximation is given by the equation (11) as $u_0(x,t) = u(x,0)$, followed by the approximations:

$$\begin{aligned} u_0 + u_1 = {} & 2e^x\beta + 14\,e^{2x}\beta + 42\,e^{3x}\beta + 70\,e^{4x}\beta + 70\,e^{5x}\beta + 42\,e^{6x}\beta + 14\,e^{7x}\beta \\ & + 2\,e^{8x}\beta + 2e^x\,ht\beta + 10e^{2x}\,ht\beta - 3342\,e^{3x}\,ht\beta + 16810\,e^{4x}\,ht\beta \\ & - 16810\,e^{5x}\,ht\beta + 3342\,e^{6x}\,ht\beta - 10e^{7x}\,ht\beta - 2e^{8x}\,ht\beta^{-6}, \end{aligned} \tag{20}$$

where, $\beta = \frac{1}{\left(1+e^x\right)^9}$, and so on.

Due to the large expressions of the consecutive terms, we have not mentioned them here. On solving the equations (11) and (20), simultaneously, we may obtain the expression for $u_1(x,t)$. In the same way, by giving the values to n as 2, 3,... in (19), we obtain $u_i(x,t)$ for $i \geq 2$. Therefore, the nine-terms approximation series solution for the equation (10) is

$$u(x,t) = \sum_{i=0}^{8} u_i(x,t). \tag{21}$$

3.1.1 Evaluation of Convergence and Numerical Results

Here, we observe that the series solution of equation (10) contains the auxiliary parameter $\hbar = h$. This parameter is also known as the, "convergence control parameter" of the Homotopy Analysis Method, and, thus, governs the convergence of the series and rates of the approximation of this method. Now, in order to control the convergence of the approximation series, the value of the auxiliary parameter h is chosen appropriately with the help of the h-curve. In the h-curve, the valid region for the admissible values of h corresponds to the line segment which is either parallel or nearly parallel to the horizontal axis. In the Figure 1, the h-curve of $u_{tt}(1,0)$ of the equation (10) is drawn which is obtained by the nine-terms approximation solution of the HAM, and a parallel line segment can easily be seen which yields the range for the admissible values of h. We can certainly determine the better approximations in the initial few terms only, if we choose a good enough initial guess and auxiliary linear operator. In case if these values are not good enough chosen but chosen moderately then also convergent results can be obtained just by choosing the value of the auxiliary parameter h appropriately.

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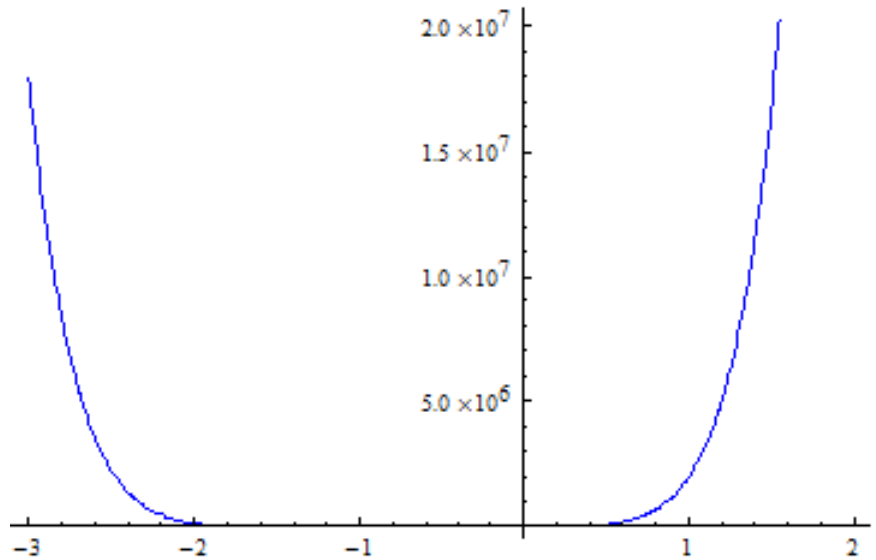
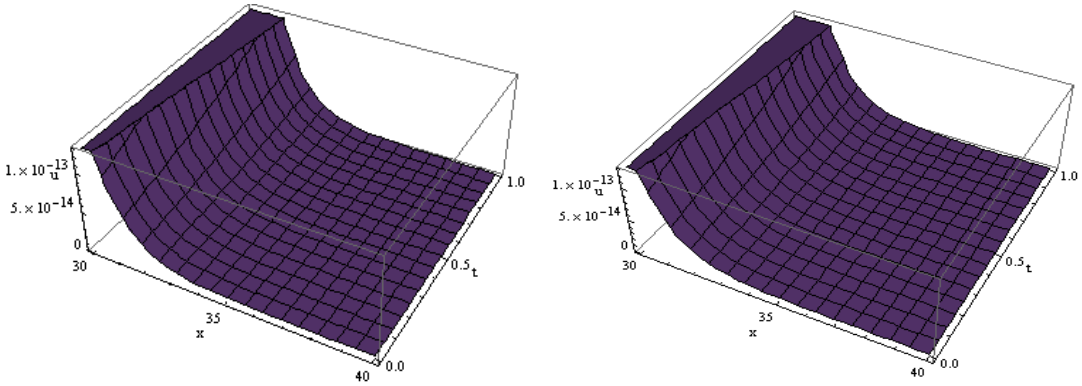


Figure 1. The h -curve of $u_{tt}(1,0)$ of the equation (10) obtained by the nine-terms approximation solution of the Homotopy analysis method

Table 1. Absolute errors for u(x, t) obtained by the nine-terms approximate solution of the HAM for $h=-0.9$

	t					
x	0	0.2	0.4	0.6	0.8	1.0
20	7.445E-24	1.890E-10	4.802E-08	1.227E-06	1.225E-05	7.295E-05
30	6.058E-28	1.759E-20	2.833E-19	2.054E-18	1.012E-17	3.912E-17
40	2.465E-32	7.977E-25	1.265E-23	8.785E-23	4.056E-22	1.456E-21
50	1.316E-36	3.622E-29	5.743E-28	3.989E-27	1.842E-26	6.608E-26
60	2.296E-41	1.644E-33	2.607E-32	1.811E-31	8.361E-31	3.000E-30



(a) (b)

Figure 2. (a) $u(x,t)$ exact and (b) $u(x,t)$ computed of the seventh-order Caudrey-Dodd-Gibbon equation (10)

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From the Figure 1, we choose the most appropriate value of h for our problem to be $h=-0.9$. Now, we will show the efficiency of the HAM via Table 1 and Figure 2, by comparing the so-obtained approximate solution (21) with the exact solution of the equation (10), given by

$$u(x,t)=\frac{2e^{x-t}}{\left(1+e^{x-t}\right)^2}\ . \tag{22}$$

3.2 The Fisher's Type Equation

Let us consider the Fisher-type equation (Singh and Arora, 2014)

$$u_t=u_{xx}+u^2(1-u), \tag{23}$$

subject to the initial condition

$$u(x,0)=\frac{1}{(1+e^{x/\sqrt{2}})},$$

(24)

where, u represents the dependent variable; x and t represent the space and time variables, respectively.

This equation is known as the Fisher-type equation as it takes the form of

$$u_t=\alpha u_{xx}+f(u), \qquad x\in(-\infty,\infty),\ \ t\geq 0,$$

where, α is the difffusion-coefficient and $f(u)$ is the non-linear reaction term. Such equations are also known as Fisher-type reaction-diffusion equations. In the physical and biological systems, the interaction between the reaction mechanism and diffusion transport is described by these models. The role of these equations in the dissipative dynamical systems like wall propagation in liquid crystals, nerve propagation in nerve fibres, pattern formation in dissipative systems, neutron action in the reactor and nucleation kinetics, is very important from the physical point of view. These equations are even found in combustion, physiology, ecology, plasma physics and many more.

In order to apply the HAM, we first define the linear operator L as

$$L[\psi(x,t;q)]=\frac{\partial\psi(x,t;q)}{\partial t} \tag{25}$$

having the property $L(c)=0$ [$c=$ constant].

Now, from the given equation (23), the non-linear operator becomes

$$N[\psi(x,t;q)]=\psi_t-\psi_{xx}-\psi^2+\psi^3. \tag{26}$$

Also, the zero-order deformation equation is defined as follows:

$$(1-q)\,L[\psi(x,t;q)-u_0(x,t)]=H\hbar\ q\ N[\psi(x,t;q)] \tag{27}$$

and when $q=0$ and $q=1$, we have the following values

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$$\psi(x,t;0)=u_0(x,t), \qquad \psi(x,t;1)=u(x,t) \tag{28}$$

We, now, differentiate n-times the deformation equation (27) w.r.t. q, put $q=0$ and then divide the so-obtained equation throughout by $n!$, we get

$$L[u_n-\sigma_n u_{n-1}]=H\hbar\, R_n(u_{n-1}) \tag{29}$$

subject to the initial condition

$$u_n(x,0)=0, \tag{30}$$

where,

$$R_n\left(u_{n-1}\right)=\frac{\partial u_{n-1}}{\partial t}-\frac{\partial u_{n-1}^2}{\partial x^2}-u_{n-1}^2+u_{n-1}^3 \tag{31}$$

For convenience, we choose the value of $H=1$ and $\hbar=h$, then the solution of the equation (29), for $n\geq 1$, becomes

$$u_n=\sigma_n u_{n-1}+\hbar\ L^{-1}\left[R_n(u_{n-1})\right]. \tag{32}$$

The initial approximation is given by the equation (24) as $u_0(x,t)=u(x,0)$, followed by the approximations:

$$u_0+u_1=\zeta+e^{x/\sqrt{2}}\zeta-\frac{1}{2}h\,t\,e^{x/\sqrt{2}}\zeta, \tag{33}$$

where $\zeta=\dfrac{1}{\left(1+e^{x/\sqrt{2}}\right)^2}$, and so on.

Due to the large expressions of the consecutive terms, we have not mentioned them here. On solving the equations (24) and (33), simultaneously, we may obtain the expression for $u_1(x,t)$. In the same way, by giving the values to n as $2, 3,\ldots$ in (32), we obtain $u_i(x,t)$ for $i\geq 2$. Therefore, the sixteen-terms approximation series solution for the equation (23) is

$$u(x,t)=\sum_{i=0}^{15}u_i(x,t). \tag{34}$$

3.2.1 Evaluation of Convergence and Numerical Results

Here, we observe that the series solution of equation (23) contains the auxiliary parameter $\hbar=h$. This parameter is also known as the, "convergence control parameter" of the Homotopy Analysis Method, and, thus, governs the convergence of the series and rates of the approximation of this method. Now, in order to control the convergence of the approximation series, the value of the auxiliary parameter h is chosen appropriately with the help of the h-curve. In the h-curve, the valid region for the admissible values of h corresponds to the line segment which is either parallel or nearly parallel to the horizontal axis. In the Figure 3, the h-curve of $u_{tt}(1,0)$ of the equation (23) is drawn which is obtained by the sixteen-terms approximation solution of the

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HAM, and a parallel line segment can easily be seen which yields the range for the admissible values of h. We can certainly determine the better approximations in the initial few terms only, if we choose a good enough initial guess and auxiliary linear operator. In case if these values are not good enough chosen but chosen moderately then also convergent results can be obtained just by choosing the value of the auxiliary parameter h appropriately.

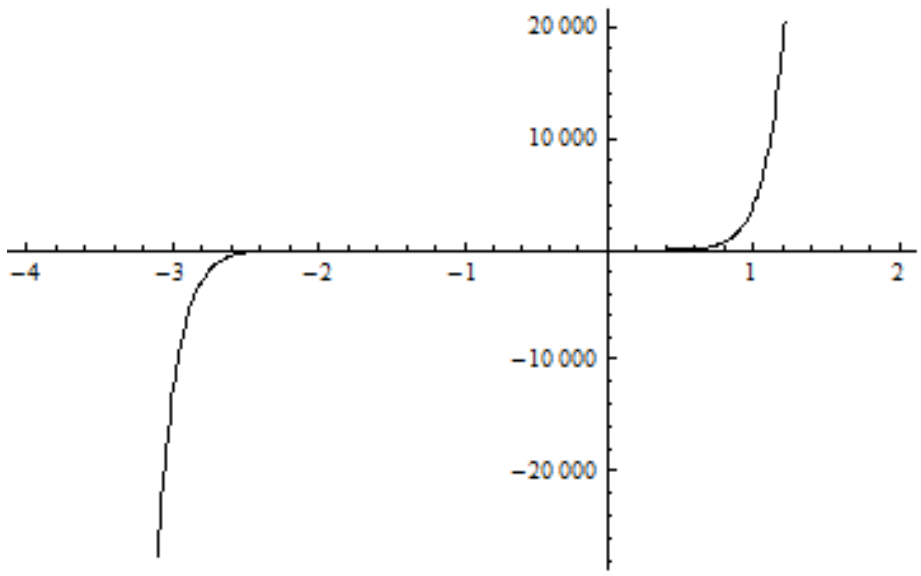

Figure 3. The h-curve of $u_{tt}(1,0)$ of the equation (23) obtained by the sixteen-terms approximation solution of the Homotopy analysis method

From the Figure 3, we choose the most appropriate value of h for our problem to be $h=-2.0$. Now, we will show the efficiency of the HAM via Table 2 and Figure 4, by comparing the so-obtained approximate solution (34) with the exact solution of the Fisher's type equation (23), given by

$$u(x,t)=\frac{1}{\left(1+e^{\frac{1}{\sqrt{2}}\left(x-\frac{t}{\sqrt{2}}\right)}\right)} \tag{35}$$

Table 2. Absolute errors for u(x, t) obtained by the sixteen-terms approximate solution of the HAM for $h=-2.0$

	t					
x	0	0.2	0.4	0.6	0.8	1.0
-60	0	0	0	0	0	0
-40	1.554E-15	1.709E-13	8.213E-13	2.566E-12	6.492E-12	1.441E-11
-20	1.887E-15	2.347E-07	1.136E-06	3.554E-06	8.997E-06	1.998E-05
20	6.353E-22	5.825E-09	2.308E-08	2.481E-08	5.792E-10	3.232E-08
40	1.515E-27	4.202E-15	1.665E-14	1.790E-14	4.178E-16	2.331E-14
60	2.407E-33	3.031E-21	1.201E-20	1.291E-20	3.014E-22	1.682E-20

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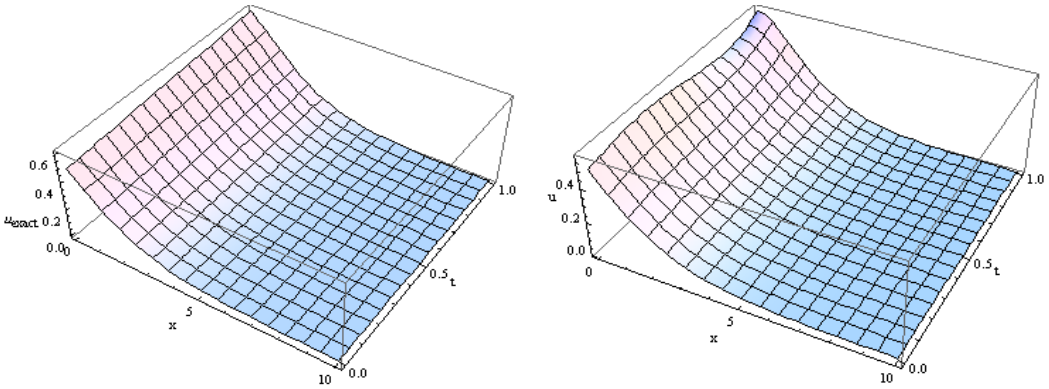

(a) (b)

Figure 4. (a) $u(x,t)$ exact and (b) $u(x,t)$ computed of the Fisher's type equation (23)

4. Conclusion

In the present paper, the HAM technique is employed on 7th-order Caudrey-Dodd-Gibbon equation and Fisher-type equation to obtain an approximate analytical and numerical solution. The software package "MATHEMATICA" is used to compute the numerical results. The comparison of these results with the known exact solution, refer to Tables 1 and 2, shows the accuracy of this method. Since very less absolute errors are obtained, we can conclude that this method gives more efficient results which makes it very effective and opens a wide scope for its applications in different fields of sciences, etc.

Conflict of Interest

The authors confirm no conflict of interests.

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