A Joint Return Policy for a Multi-Item Perishable Inventory Model with Deterministic Demands, Return and All-Units Discount

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Abstract
In this paper, we develop a multi-item perishable inventory model with deterministic demands, return and all-units discount. We consider a situation where a retailer sells several products to the customer and orders the products from one supplier. Demands are assumed to be deterministic following an inventory-dependent demand, and the supplier offers all-units discount to the retailer who has an opportunity to return unsold or deteriorated products to the supplier at a certain cost. In order to minimize the total cost for the retailer, the decision variables are the optimal return time and the optimal ordering quantity. Considering a multi-item case as an extension of the model by Setiawan et al. (2018) is the main contribution of this paper. We also develop an algorithm to find the optimal solution of the model. Numerical examples for three items are given to illustrate the model and a sensitivity analysis is performed to study the effect of the changes in parameter values on the optimal solution. We consider two scenarios, one with all-units discount and one with no discount. Within these two scenarios, we consider conditions of individual or joint return time for these three items. It is found that the individual return time with no discount gives the least total inventory cost in the numerical examples. Also, in general increasing the value of holding cost, deteriorating rate, return cost per unit and backorder cost will increase the total inventory cost in all scenarios.

Keywords- Joint return policy, Perishable inventory model, Multi-item, All-units discount.

1. Introduction
Inventory is one of the most important assets for a firm since in general it covers a high percentage of the firm's total capital. Therefore a good inventory management is crucial in order to make the firm's operation more flexible. Having less inventory can reduce cost but it can make customers dissatisfied due to the unmet demands. There are several functions inventory can serve such as anticipating demands from customers, and decoupling the production process, as a result of accepting quantity discounts offered by the supplier and to hedge against price changes and inflation (Heizer and Render, 2014).

For a manufacturing firm, inventory can take the form of raw materials, purchased parts and supplies, partially complete work in process, items being transported, and tools and equipment

(Heizer and Render, 2014; Russell and Taylor III, 2014) classified inventory in terms of raw material, work-in-process, a maintenance/repair/operating supply inventory and a finished-goods inventory. There are some costs associated with inventory, the most common ones of which are ordering cost, carrying (holding) cost and shortage cost (Tersine, 1994; Russell and Taylor III, 2014; Shenoy and Rosas, 2018). Another cost that may be involved in the inventory cost is purchasing cost, although in most of the mathematical models this cost is not included since it is constant, unless there is a quantity discount involved. There are the two most important decisions in the inventory management: when to order and how many items to order. Making these two decisions will help a company in lowering its inventory cost.

Regarding quantity discount offered by the supplier, there are various types of quantity discount such as all-units discount, incremental discount or discount based on some continuous functions. However, in reality the all-units discount is the most common one, giving a reduced price if an item is purchased in larger quantities. Accepting the offer from the supplier to buy an intem in larger quantities can reduce the ordering cost, but at the same time the carrying cost will increase. Managing these two costs wisely through mathematical models can lower the inventory cost.

The situation becomes more complicated when dealing with perishable or deteriorating goods such as vegetables, fruit, bread, milk, fish, blood or chemicals. Ray (2017) defines deterioration as "change, damage, decay, spoilage, obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one product". We (Li et al., 2010) classifies deteriorating items into two categories, deteriorating that is due to time and due to the introduction of new technologies (computers, mobile phones, fashion, seasonal goods and so on). However, deterioration deals with items that have a short life cycle, generally speaking.

Demand for an item in many inventory models can be assumed to be either deterministic (and constant) such as the original Economic Order Quantity (EOQ) model, or probabilistic (having certain distributions) or else deterministic but has some features such as inventory-dependent demand or time-dependent demand or constant and time-dependent demand (Singh et al., 2017). The inventory-dependent demand feature where demand is influenced or depends on the number of the available inventory on the shelf has been widely modelled in the literature and in practice it is common in supermarket, retail industries, and seasonal dresses such as women's dresses or sports clothes during the selling season (Wolfe, 1968; Levin et al., 1972; Silver and Peterson, 1985).

In reality, a company can return some deteriorated items to the supplier at a certain cost, and the supplier will replace them with new items in the next replenishment. This situation along with deteriorating items and the nature of demands will affect two important decisions for a company, the optimal time and quantity to order. Regarding the return policy, the situation becomes complex if a company is dealing with multi-items. The company will face the problem whether to return an item individually or jointly to the supplier. Therefore, the optimal return time will be determined wisely in order to minimize the inventory cost.

In this paper we deal with a condition where a company sells items and the demands of these items that are having inventory-dependent demand features. We assume constant deterioration rate, all-units discounts and return policy. We propose a mathematical model to determine the optimal return time and the optimal order quantity that minimized the total inventory cost. The mathematical model and an algorithm to find the optimal solution for this problem form the main contribution of this paper.

There are several references we used for the formulation of the inventory-dependent demand such as (Nagare and Dutta, 2012; Setiawan et al., 2018 and Loedy et al., 2018). In general, we extend the model of Setiawan et al. (2018) and Loedy et al. (2018) to accommodate the multi-item feature in our model and consider the individual and joint return policy. This is the main contribution of this paper. Also considering a multi-itme model with the above features is relevant to the real problem most of the smalll retailers have to deal with in their daily operation. We also provide an algorithm to find the optimal solution and numerical examples based on several scenarios encountered in our model. Sensitivity analysis is also performed in order to study the effect of changes in parameter values on the optimal solution.

2. Literature Review

As mentioned earlier, if a company wants to lower its inventory cost, it needs a good inventory management. One of the strategies in the inventory management is using mathematical models to describe the situation and find the optimal solution for the problem. Managing deteriorating or perishable goods has been an interesting research topic for decades. Comprehensive reviews on the perishable inventory theory can be found in (Nahmias, 1982; Li et al., 2010 and Ray, 2017). Perishable goods along with their variants such as the nature of demand, deterioration reduction, substitution products and pricing have attracted attention from researchers all over the world. Dye (2012) used particle swarm optimization in dealing with a finite-horizon deteriorating inventory model with two-phase pricing and time-varying demand. A quadratic demand rate and variable holding cost have been proposed by Kumar et al. (2012) in developing a model for deteriorating goods. Another model for the deteriorating inventory model has been developed by Yadav and Vats (2014) for time-dependent quadratic demand and constant holding cost with partial backlogging and inflation. Doung et al. (2015) proposed a multi-criteria inventory management system for perishable and substitutable products. They considered conditions such as multi-period life time, positive lead time and customer service level to build a model where each item is treated separately. When demand is price dependent and the cycle length is variable in the planning period a numerical study is conducted by Prasad Patnaik and Patnaik (2015) for deteriorating items. Sharmila and Uthayakumar (2016) studied a fuzzy inventory model with exponential demand when shortages are fully backlogged. An effort to reduce deterioration for perishable items was studied by Filina-Dawidowicz and Postan (2016) using a generalized Wilson model. Meanwhile, Pervin et al. (2018) considered a model allowing shortage with time-dependent demand, time-varying holding cost and stochastic deterioration.

Another variant in the inventory models that has been developed is the nature of demand, such as inventory-dependent demand and time-dependent demand. The inventory-dependent demand was first studied by many researchers, among others (Wolfe, 1968; Levin et al., 1972 and Silver and Peterson, 1985). In the last few decades, many papers deal with this inventory-dependent demand, such as Baker and Urban (1988a) which proposed single-period inventory demand models where demand is of a polynomial function form to maximize the total profit. By assuming instantaneous replenishments and using the same polynomial function form, in the same year, Baker and Urban (1998b) developed a continuous and deterministic inventory model. Incorporating inventory-dependent demand, variable holding cost and shortages to build a mathematical model were the focus of Rathod and Bhathawala (2013). They assumed that the holding cost is linear with the quantity in storage. In 2017, Singh et al. (2017) considered constant and time-dependent linear demand rate to build a mathematical model to find the optimal policy for deteriorating items. They assumed that the deterioration rate is time-proportional. Kavithapriya and Senbagam (2018)

developed a mathematical model for quadratic time-dependent demand, two parameter Weibull deterioration rate and shortage.

Table 1. Position of this research

Author(s)	Deterioration	Demand depends on	Single Item	Multi Item	Additional Features
Baker and Urban (1988a)		inventory level	√		
Baker and Urban (1988b)		inventory level		√	
Rathod. and Bhathawala (2013)		inventory level	√		variable holding cost and shortages
Li et al. (2010)	√				
Duong et al. (2015)	√			√	
Ray (2017)	√				
Filina-Dawidowicz and Postan (2016)	√		√		additional cost for deterioration reduction
Chang (2004)	√	inventory level	√		non-linear holding cost
Chang et al. (2010)	√	inventory level and price	√		limited shelf space
Dye (2012)	√	time			
Singh et al. (2017)	√	time			
Kumar et al. (2012)	√	quadratic time	√		variable holding cost
Yadav and Vats (2014)	√	quadratic time	√		constant holding cost, partial blocking, and inflation
Kavithapriya and Senbagam (2018)	√	quadratic time	√		shortages
Mishra et al. (2013)	√	time	√		time-varying holding cost
Pervin et al. (2018)	√	time	√		shortages, time-varying holding cost
Prasad Patnaik and Patnaik. (2015)	√	selling price	√		
Sharmila and Uthayakumar (2015)	√	exponential demand	√		shortages
Nagare and Dutta (2012)	√	inventory level	√		
Setiawan et al. (2018)	√		√		return policy
Loedy et al. (2018)	√	inventory level	√		all-units discount, return policy
This paper	√	inventory level		√	all-units discount, return policy

Incorporating deteriorating items and inventory-dependent demand have also been studied for many years. An inventory model with nonlinear holding cost and inventory-dependent demand was studied by Chang (2004). Meanwhile, Chang et al. (2010) developed a model by adding price-dependent demand and limited shelf space. Nagare and Dutta (2012) considered a continuous review model, while Mishra et al. (2013) developed a model with partial backlogging, time-varying holding cost and Weibull deteriorating rate. Considering an economic production quantity (EPQ), Kaliraman et al. (2015) proposed a model with Weibull deterioration and no shortage, while Uthayakhumar and Tharani (2017) proposed an EPQ model for time-dependent demand with rework and multiple production setups. Setiawan et al. (2018) considered a model with constant deterioration rate and return for deteriorated or unsold items.

The contribution of this paper lies in developing a mathematical model for a multi-item deteriorating inventory with inventory-dependent demand, return and all-units discount. The decision variables in the model are the optimal return time and order quantity that minimize the

total inventory cost. We assume that joint return time comes from the joint replenishment policy and individual return time as a result of individual replenishment policy. There is only one supplier in our model setting. The position of this reseach along with its contribution is depicted in Table 1. Our paper differs with other papers in terms of the application of multi-item inventory problem with deterioration factor, inventory-dependent demand, return and all-units discount. To the best of our knowledge, combining all factors in the model is the contribution of this paper and indeed it illustrates the real problem most of the small retailers face in their operation here.

The organization of this paper is as follows. In the next section, we introduce notation and model formulation which is mainly derived from Setiawan et al. (2018) and Loedy et al. (2018). A proposed algorithm to find the optimal solution is also part of the next section. We provide a numerical example in Section 4 along with its optimal solution and analysis. Section 5 is devoted to the sensitivity analysis where we study the effect of changes in parameter values to the optimal solution. Conclusions and further research directions are relegated in the last section.

3. Assumptions, Model Formulation and Algorithm
In developing our model, we use the following notations. The notations are mainly derived from Setiawan et al. (2018) and Loedy et al. (2018) for single item and constant deteriorating rate, while in this context we consider multi-item.

TC_i : total inventory cost for i^{th} item
PC_i : purchasing cost for i^{th} item
OC : ordering cost
HC_i : holding cost for i^{th} item
SC_i : shortage cost for i^{th} item
RC_i : return cost for i^{th} item
T : time between replenishment (months)
$D_i(t)$: demand at time $t(D_i(t) = \alpha_i + \beta_i I(t))$ for i^{th} item
α_i : base demand ($\alpha > 0$) for i^{th} item when $\beta_i = 0$
β_i : inventory-dependent demand factor ($0 < \beta < 1$) for i^{th} item
Q_i : order quantity for i^{th} item
T_{r_i} : return time (months) for i^{th} item
$T_{r_i}^*$: optimum return time for i^{th} item
$I_i(t)$: inventory level at time t for i^{th} item
m_i : amount of inventory during interval $[0, T_r]$ for i^{th} item
S_i : amount of shortage during interval $[T_r, T]$ for i^{th} item
P_i : purchasing cost per unit for i^{th} item
θ : deteriorating rate
h : holding cost per unit per year
A : ordering cost per order
A_r : return cost per return
R : return cost per unit
π : backorder cost per unit per year
Q_{r_i} : inventory level at returns time for i^{th} item

We also have the following assumptions in developing our model:
(i)   There is only a single supplier.

(ii) There is no inventory at hand after the return time for each item, since all unsold or deteriorated items are returned to the supplier and the returned amount will be available in the next shipment.

(iii) The number of returned items is 10% of the order quantity of these items.

(iv) Considering scenarios with joint return time, there are only two possibilities we consider, either all items ordered with all-units discount or without all-units discount.

(v) When items are jointly replenished, the unsold or deteriorated items should also be jointly returned to the supplier.
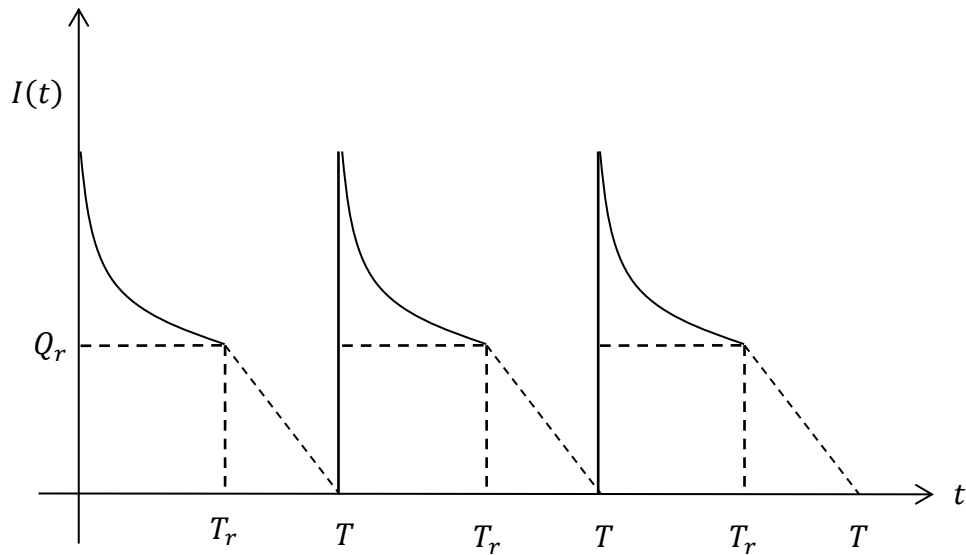

Figure 1. Inventory level

For each item, i=1, 2,…,n demand for the ith item are modeled by the following inventory-dependent demand function.

$$D_i(t) = \begin{cases} \alpha_i + \beta_i I(t) & , 0 \le t \le T_{r_i} \\ B_i & , T_{r_i} \le t \le T \end{cases} \tag{1}$$

Demand for the ith item at time t for $0 \le t \le T_{r_i}$ is a function of the number of inventory at time t, I(t) with a dependent-demand factor β_i and base demand α_i. When $T_{r_i} \le t \le T$, demand for item i at time t is a constant, B_i.

Figure 1 gives a representation of inventory level over time. When $0 \le t \le T_r$, inventory depletes due to demand and deterioration. Then between T_r and T there is no inventory at hand and backorder condition occurs when demand comes. At time $t = T_r$ all unsold or deteriorated items are returned to the supplier, and the supplier will replace them at the next shipment. This cycle is repeated time over time.

The following first order differential equation describes the rate of change of inventory of item i at time t over time during $0 \le t \le T$.

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -D_i(t) \tag{2}$$

The inventory decreases because demand from the customer and deterioration rate. The differential equation represents the rate at which the inventory decreases. Inserting the function for inventory-dependent demand we have the following differential equations.

$$\frac{dI_i(t)}{dt} = -\theta I_i(t) - \left(\alpha_i + \beta_i\, I_i(t)\right), 0 \le t \le T_{r_i} \tag{3}$$

$$\frac{dI_i(t)}{dt} = -\theta I_i(t) - B_i, T_{r_i} \le t \le T \tag{4}$$

We assume that there is no inventory at hand just after the return time (for item i) T_{r_i} since all the remaining inventory is returned to the supplier to be replaced with new ones on the next replenishment. Therefore we have $I(T_{r_i}) = 0$ and the inventory at time t, $I_i(t)$ for $T_{r_i} \le t \le T$ is given by

$$\frac{dI_i(t)}{dt} = -B_i \tag{5}$$

$$I_i(t) = B_i(T_{r_i} - t) \tag{6}$$

However, just before T_{r_i}, there is a certain amount of inventory Q_{r_i} that will be returned to the supplier. This gives another boundary condition of $I(T_{r_i}) = Q_{r_i}$ for $0 \le t \le T_r$, and consequently we have

$$\frac{dI_i(t)}{dt} = -\theta I_i(t) - \left(\alpha_i + \beta_i I_i(t)\right) \tag{7}$$

The solution of these differential equations is given by

$$I_i(t) = -\frac{\alpha_i}{\beta_i + \theta} + \frac{e^{-(\beta_i + \theta)t}\left(Q_{r_i} + \frac{\alpha_i}{\beta_i + \theta}\right)}{e^{-(\beta_i + \theta)T_{r_i}}} \tag{8}$$

The number of inventory items at the beginning of the cycle is Q_i minus the shortage during $T_{r_i} \le t \le T$, so $I_i(0) = Q_i - B_i(T_{r_i} - T)$. Then, we have the following order quantity Q_i

$$Q_i = -\frac{\alpha_i}{\beta_i + \theta} + \frac{e^{-(\beta_i + \theta)t}\left(Q_{r_i} + \frac{\alpha_i}{\beta_i + \theta}\right)}{e^{-(\beta_i + \theta)T_{r_i}}} + B_i(T - T_{r_i}) \tag{9}$$

Assuming that there are 10% of items i returned to the supplier, we can calculate the number of returned goods as follows.

$$10\, Q_{r_i} = -\frac{\alpha_i}{\beta_i + \theta} + \frac{e^{-(\beta_i + \theta)t}\left(Q_{r_i} + \frac{\alpha_i}{\beta_i + \theta}\right)}{e^{-(\beta_i + \theta)T_{r_i}}} + B_i(T - T_{r_i}) \tag{10}$$

$$Q_{r_i} = \frac{B_i T e^{-(\beta_i + \theta)T_{r_i}}\beta_i + B_i T e^{-(\beta_i + \theta)T_{r_i}}\theta - \alpha_i e^{-(\beta_i + \theta)T_{r_i}} + \alpha_i)}{10 e^{-(\beta_i + \theta)T_{r_i}}\beta_i + 10 e^{-(\beta_i + \theta)T_{r_i}}\theta - \beta_i - \theta} \tag{11}$$

The number of inventory items during $[0, T_{r_i}]$ and the number of shortages (that are backordered) during $[T_{r_i}, T]$ is calculated using the following formula

$$m_i = \int_0^{T_{r_i}} I_i(t)dt \tag{12}$$

$$S_i = B_i T_{r_i} \left(T - T_{r_i}\right) - \frac{B_i \left(T^2 - T_{r_i}^2\right)}{2} \tag{13}$$

Based on the above calculations, we are now able to calculate the total inventory cost that includes purchasing, ordering, holding, shortage (backorder) and return costs.

Purchasing cost is defined as the purchase cost per unit times the quantity of purchased item, namely

$$PC_i = \frac{T.P_i.Q_i}{12} \tag{14}$$

The ordering cost usually does not depend on the number of items ordered, but on the order frequency. So the formulation is as follows.

$$OC = \frac{12A}{T} \tag{15}$$

In formulating the holding cost, we need to consider number of stored items (m) and the duration of storage (T). This cost is formulated as follows.

$$HC = \frac{T.h.m_i}{12} \tag{16}$$

Shortage cost occurs when a firm does not have inventory when demands come in. In our model, we assume that all shortages are backlogged and we have the following formulation for the shortage cost.

$$SC_i = \frac{-12\pi \left(B_i T_{r_i}\left(T - T_{r_i}\right) - \frac{B_i\left(T^2 - T_{r_i}^2\right)}{2}\right)}{T} \tag{17}$$

Returning some deteriorated or unsold items to the supplier will induce some cost. This cost is called the return cost in our model and the quantity is given by

$$RC_i = 12 \left(\frac{A_r + Q_{r_i} R}{T}\right) \tag{18}$$

The decision variables in our model are the optimal order quantity and return time that minimize the total inventory cost. We consider two strategies, namely individual return time and joint return time policies to minimize the total inventory cost. The formulations of the total inventory cost for individual return time and joint return time for $i = 1,2,\ldots,n$ are given by the following formula assuming that joint replenishment policy induces joint return time and individual replenishment policy will be followed by individual return time to the single supplier.

Individual Return Time
$$TC = nOC + \sum_{i=1}^{n}(PC_i + HC_i + SC_i + RC_i) \tag{19}$$

Joint Return Time
$$TC = OC - \frac{12(n-1)}{T}A_r + \sum_{i=1}^{n}(PC_i + HC_i + SC_i + RC_i) \tag{20}$$

When items are jointly replenished, the ordering cost only occurred once and the frequency of returning the unsold or deteriorated item will also be reduced. This explains the difference between the above formulas for individual return time and joint return time policy.

We also incorporate the all-units discount offer from the supplier in our model. An algorithm to find the optimal solution of our model considering all the conditions we mentioned is given in Figure 2. First, we evaluate the value of T_{r_i} of each level of purchase price P_i by taking the first derivative of TC with respect to T_{r_i} and equal them to zero. Then we evaluate the order quantity Q_i, whether it is valid or not (according to the all-units discount scheme) and finally we compare the total inventory cost for the joint return time and individual return time. By joint return time, all deteriorated or unsold items are returned at the same time (T_r) while this T_r can be chosen as the same one as one of the T_{r_i}. For individual return time, we use T_{r_i} for each item.
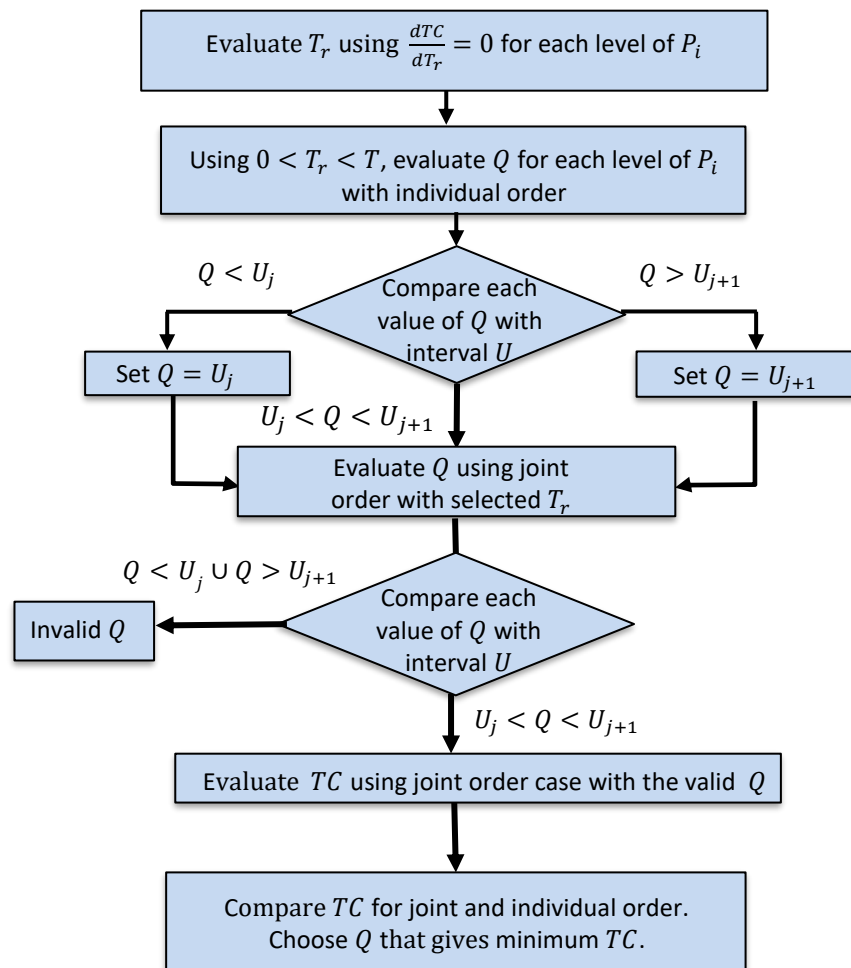

Figure 2. Algorithm to find the optimal solution

4. Numerical Example

We consider the three-item example below in order to illustrate our model. The parameters we use are the following

$$P_1 = \begin{cases} \$6, & Q \le 10,000 \\ \$4, & Q > 10,000 \end{cases}, \quad P_2 = \begin{cases} \$6, & Q \le 10,000 \\ \$3.5, & Q > 10,000 \end{cases}, \quad P_3 = \begin{cases} \$6, & Q \le 10,000 \\ \$4.5, & Q > 10,000 \end{cases}$$

$T = 6$ months, $\alpha_1 = 120$ units, $\alpha_2 = 115$ units, $\alpha_3 = 110$ units, $A = \$10$, $h = \$5$, $\pi = \$2$, $R = \$2$, $A_r = \$10$, $\beta_1 = 0.165$, $\beta_2 = 0.18$, $\beta_3 = 0.175$, $\theta = 0.2$, $B_1 = 4000$ units, $B_2 = 4500$ units, and $B_3 = 3750$ units.

Considering individual and joint return time we develop four scenarios, namely individual return time for Scenario 1 and joint return time for Scenario 2, 3 and 4. Within each scenario, we also consider two conditions, a and b. Condition a for cases with all-units discount and condition b for cases without all-units discount. The difference among Scenario 2, 3 and 4 lies in the joint return time. The joint return time is chosen to be the same as the return time for item 1, item 2 and item 3 for Scenario 2, Scenario 3 and Scenario 4 respectively. Results for the above numerical example are given in Table 2.

Table 2. Scenarios and optimal solution

First Scenario													
T_{r_1}	T_{r_2}	T_{r_3}	a				T_{r_1}	T_{r_2}	T_{r_3}	b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}				TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
3.90	3.72	3.76	173,377	932	935	856	4.99	4.63	5.01	196,416	1,000	1,000	1,000
Second Scenario													
T_{r_1}			a				T_{r_1}			b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}				TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
3.90			173,787	932	935	855	4.99			Invalid	1,000	1,116	992
Third Scenario													
T_{r_2}			a				T_{r_2}			b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}				TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
3.72			173,585	939	935	857	4.63			Invalid	945	1,000	903
Fourth Scenario													
T_{r_3}			a				T_{r_3}			b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}				TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
3.76			173,486	937	934	856	5.01			209,977	1,006	1,127	1,000

Comparing the case without all-units discount (condition a for each scenario), we can see that the first scenario gives the total cost of $173,377, as the most minimal among other scenarios. This means that choosing the return time individually for the cases without discount is superior compared with other scenarios by joining the return time to item 1, item 2 or item 3. However, for the case with all-units discount, again the first scenario, the individual return time gives the least total cost of $196,416. Comparing all scenarios with and without all-units discount, the first scenario with no discount gives the least total inventory cost of $173,377. This means that individual return time with no all-units discount for each item is superior compared to other

scenarios based on the parameters we chose for this numerical experiment. Some scenarios give invalid total inventory cost (in fact this occurs in the number of quantity ordered or quantity returned) since in those cases there is an optimal order quantity that is less than 10,000 units (as a condition to obtain a cheaper price in the all-units discount scheme). For example in Scenario 2b, the total inventory cost is labeled "invalid" since the number of returned item for at least one item is invalid since it is below 1,000 units (to get all-units discount, it should be at least 1,000 units).This is due to our fourth assumption where all items should be ordered together either with or without all-units discount. When one item is ordered with all-units discount, then the other two items should also be ordered with all-units discount, forcing higher order quantity for these two items. Please note that in Table 1, we give the value of Q_r, the quantity of returned goods based on the assumption that 10% of the ordered goods will be returned to the supplier due to deterioration or remaining unsold. The optimal order quantity Q is just 10 times the value of Q_r.

From the managerial point of view, results presented in Table 2 give the interesting insight that the individual return time is preferable to joint retun time, although it cannot be generalized that the individual return time is always preferable. By individual return time, the retailer has more freedom to return the item according to its optimal return time compared to the joint return time. Also, it is found from Table 1 that all-units discount offered by the supplier does not guarantee the retailer to increase its order quantity since the incentive for the discount especially in the ordering and shortage costs cannot offset for the increase in the holding and purchasing costs. Therefore, the individual return time without all-units discount is preferable to individual or joint return time with all-units discount.

When we only consider one item (item 1), the results are given in Table 3 below. We consider conditions without all-units discount (a) and with all-units discount (b), and we found that the all-units discount scheme offered by the supplier does not quite attractive making the condition without all-units discount is favorable with a total inventory cost of $59,243. The optimal returned quantity of the condition (a) is less than the one in condition (b), and also the optimal order quantity. The optimal return time for condition (a) is also shorter than in condition (b) due to the less optimal order quantity.

Table 3. Scenarios and optimal solution for one item (Item 1)

First Scenario					
T_{r_1}	a		T_{r_1}	b	
	TC	Q_{r_1}		TC	Q_{r_1}
3.90	59,243	932	4.99	66,201	1,000

5. Sensitivity Analysis
We perform a sensitivity analysis to study the effect of changes in parameter values on the optimal solution. In each scenario we change the parameter value from -15% to +15% from the original value we considered in Section 4. There are only four parameters under consideration, namely holding cost per unit per year, deteriorating rate, return cost per unit and backorder cost per unit per year. We only change one parameter value at a time while all other parameter values remain

the same as in Section 4. Results of this sensitivity analysis are given in Table 4. We also provide the value of Q_r in Table 2, not Q. The value of Q is just 10 times Q_r.

Generally speaking, increasing the value of holding cost, deteriorating rate, return cost per unit and backorder cost will increase the total inventory cost in all scenarios. For the individual return time (Scenario 1), the all-units discount gives a higher total inventory cost than no discount. We can see that for all-units discount in Scenario 1, all items are ordered with all-units discount, making the amount of returned goods 1,000 units and making the total cost higher compared to cases without discount. Note that decreasing the deterioration rate to -15% will make the condition with all-units discount in the Scenario 1 become invalid. This is due to our fourth assumption that all items should be ordered either with or without discount, meaning that the quantity ordered should be higher than 10,000 units (or the quantity returned is more than 1,000 units).

Table 4. Sensitivity analysis

First Scenario

Parameter	Variance	T_{r_1}	T_{r_2}	T_{r_3}	TC (a)	Q_{r_1}	Q_{r_2}	Q_{r_3}	T_{r_1}	T_{r_2}	T_{r_3}	TC (b)	Q_{r_1}	Q_{r_2}	Q_{r_3}
h	+15%	3.81	3.64	3.67	183,283	935	936	859	4.99	4.63	5.01	215,114	1,000	1,000	1,000
	+10%	3.84	3.67	3.70	180,032	934	935	858	4.99	4.63	5.01	208,881	1,000	1,000	1,000
	−10%	3.97	3.79	3.83	166,510	930	934	856	4.99	4.63	5.01	183,950	1,000	1,000	1,000
	−15%	4.00	3.82	3.86	162,992	929	934	855	4.99	4.63	5.01	177,717	1,000	1,000	1,000
θ	+15%	3.57	3.42	3.45	174,598	1,024	1,015	931	3.67	3.55	4.17	181,124	1,026	1,020	1,000
	+10%	3.67	3.52	3.55	185,270	995	989	908	3.97	3.80	4,45	179,772	1,000	1,000	1,000
	−10%	4.16	3.96	3.99	160,784	859	872	797	5.77	5.32	5.60	228,143	1,000	1,000	1,000
	−15%	4.30	4.09	4.13	154,226	818	837	765	6.16	5.67	5.93	Invalid	1,000	1,000	1,000
R	+15%	3.90	3.72	3.76	174,224	932	935	856	4.99	4.63	5.01	197,813	1,000	1,000	1,000
	+10%	3.90	3.72	3.76	173,942	932	935	856	4.99	4.63	5.01	197,347	1,000	1,000	1,000
	−10%	3.91	3.73	3.77	172,812	931	935	856	4.99	4.63	5.01	195,484	1,000	1,000	1,000
	−15%	3.91	3.73	3.77	172,529	931	935	856	4.99	4.63	5.01	195,019	1,000	1,000	1,000
π	+15%	4.03	3.85	3.89	181,513	928	934	855	4.99	4.63	5.01	198,745	1,000	1,000	1,000
	+10%	3.99	3.82	3.85	178,909	929	934	855	4.99	4.63	5.01	197,969	1,000	1,000	1,000
	−10%	3.80	3.62	3.66	167,341	936	936	858	4.99	4.63	5.01	194,862	1,000	1,000	1,000
	−15%	3.74	3.57	3.60	164,099	938	938	860	4.99	4.63	5.01	194,086	1,000	1,000	1,000

Second Scenario

Parameter	Variance	T_{r_1}	TC (a)	Q_{r_1}	Q_{r_2}	Q_{r_3}	T_{r_1}	TC (b)	Q_{r_1}	Q_{r_2}	Q_{r_3}
h	+15%	3.81	183,673	935	934	856	4.99	Invalid	1,000	1,116	992
	+10%	3.84	180,427	934	934	855	4.99	Invalid	1,000	1,116	992
	−10%	3.97	166,934	930	936	856	4.99	Invalid	1,000	1,116	992
	−15%	4.00	163,423	929	937	856	4.99	Invalid	1,000	1,116	992
θ	+15%	3.57	179,480	1,023	1,021	935	3.67	Invalid	1,026	1,027	940
	+10%	3.67	185,626	995	994	910	3.97	Invalid	1,000	1,015	926
	−10%	4.16	161,252	859	867	792	5.77	147,267	1,000	1,332	1,121
	−15%	4.30	154,726	818	829	757	6.16	Invalid	1,000	1,604	1,264
R	+15%	3.90	174,633	932	935	855	4.99	Invalid	1,000	1,115	992
	+10%	3.90	174,351	932	935	855	4.99	Invalid	1,000	1,115	992
	−10%	3.91	173,221	931	935	855	4.99	Invalid	1,000	1,116	992
	−15%	3.91	172,529	931	935	855	4.99	Invalid	1,000	1,116	992
π	+15%	4.03	182,013	928	938	857	4.99	Invalid	1,000	1,116	992
	+10%	3.99	179,379	929	936	856	4.99	Invalid	1,000	1,116	992
	−10%	3.80	167,690	936	934	856	4.99	Invalid	1,000	1,116	992
	−15%	3.74	164,418	938	934	857	4.99	Invalid	1,000	1,116	992

Table 4 continued …

Parameter	Variance	a					b				

Parameter	Variance	T_{r_2}	a				T_{r_2}	b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}		TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
							Third Scenario				
h	+15%	3.64	183,479	944	936	859	4.63	Invalid	945	1,000	903
	+10%	3.67	180,232	942	935	858	4.63	Invalid	945	1,000	903
	−10%	3.79	166,725	936	934	856	4.63	Invalid	945	1,000	903
	−15%	3.82	163,211	935	934	856	4.63	Invalid	945	1,000	903
θ	+15%	3.42	180,078	1,025	1,015	931	3.55	Invalid	1,024	1,019	934
	+10%	3.52	185,451	998	990	908	3.80	Invalid	996	1,000	915
	−10%	3.96	161,018	872	872	799	5.32	Invalid	874	1,000	885
	−15%	4.08	154,474	836	837	767	5.67	Invalid	823	1,000	871
R	+15%	3.72	174,431	939	935	857	4.63	Invalid	945	1,000	903
	+10%	3.72	174,149	939	935	857	4.63	Invalid	945	1,000	903
	−10%	3.73	173,019	939	935	857	4.63	Invalid	945	1,000	903
	−15%	3.73	172,736	939	935	857	4.63	Invalid	945	1,000	903
π	+15%	3.85	181,772	933	934	855	4.63	Invalid	945	1,000	903
	+10%	3.82	179,151	935	934	856	4.63	Invalid	945	1,000	903
	−10%	3.62	167,514	944	936	859	4.63	Invalid	945	1,000	903
	−15%	3.57	164,255	948	938	861	4.63	Invalid	945	1,000	903

parameter	Variance	T_{r_3}	a				T_{r_3}	b			
			TC	Q_{r_1}	Q_{r_2}	Q_{r_3}		TC	Q_{r_1}	Q_{r_2}	Q_{r_3}
							Fourth Scenario				
h	+15%	3.67	183,385	944	936	859	5.01	230,603	1,006	1,127	1,000
	+10%	3.70	180,136	942	935	858	5.01	223,728	1,006	1,127	1,000
	−10%	3.83	166,623	936	934	856	5.01	196,227	1,006	1,127	1,000
	−15%	3.86	163,107	744	862	855	5.01	189,351	1,006	1,127	1,000
θ	+15%	3.45	179,832	1,024	1,016	931	4.17	189,870	1,065	1,103	1,000
	+10%	3.55	185,359	997	990	908	4.45	194,216	1,049	1,109	1,000
	−10%	3.99	160,914	869	871	797	5.60	Invalid	936	1,159	1,000
	−15%	4.13	154,368	832	836	765	3.38	Invalid	884	1,188	1,000
R	+15%	3.76	174,332	937	934	856	5.01	211,502	1,005	1,127	1,000
	+10%	3.76	174,050	937	934	856	5.01	210,994	1,005	1,127	1,000
	−10%	3.77	172,920	937	934	856	5.01	208,961	1,005	1,127	1,000
	−15%	3.77	172,638	937	934	856	5.01	208,453	1,005	1,127	1,000
π	+15%	3.89	181,654	932	934	855	5.01	211,736	1,005	1,127	1,000
	+10%	3.85	179,039	933	934	855	5.01	211,150	1,005	1,127	1,000
	−10%	3.66	167,427	942	936	858	5.01	208,805	1,005	1,127	1,000
	−15%	3.60	164,176	946	937	860	5.01	208,219	1,005	1,127	1,000

The amount of returned quantity for the case with all-units discount in all scenarios seems insensitive with respect to the changes in parameter values. Despite these changes, the optimal returned quantity is relatively the same, and in Scenario 2 and 3 these values are invalid due to the existence of at least one returned quantity of less than 1,000 units. This is also an indication that there is a limitation to increasing or decreasing the parameter values. Scenario 2 and Scenario 3 give an invalid total inventory cost for the case of all-units discount since at least one item has an order quantity of less than 10,000 units (or a returned quantity of less than 1,000 unit), with the exception of Scenario 2 when we decrease the deterioration rate by 10%. This holds true for all parameter changes (holding cost, deteriorating rate, return cost per unit and backorder cost per unit).

In Scenario 4, the condition without all-units discount gives a lower total inventory cost compared to the condition with all-units discount. This means that condition without discount is preferable in order to minimize the total inventory cost than all-units discount condition. If we look further

into Scenario 4, we can see that increasing or decreasing the holding cost and deterioration rate has a bigger effect on the total inventory cost compared to the return cost per unit and backorder cost. In this situation, based on the choice of parameter values of our model, it is better not to accept the all-units discount scheme offered by the supplier, reduce holding cost and try lowering the deterioration rate in order to minimize the total inventory cost.

6. Conclusions and Further Research

We have developed a mathematical model for the multi-item problem with inventory-dependent demand, return and all-units discount along with the algorithm to find the optimal solution. The multi-item problem here is relevant to the problem where most of small retailers have to deal in their daily operation. There are two decision variables considered in our model, the return time and the order quantity. The objective of our model is to minimize the total inventory cost. We also consider scenarios for the case of three items for individual and joint return time with and without all-units discounts. We found that individual return time gives a minimum total inventory cost in both cases with and without all-units discount. Individual return time with no discount is preferable compared to other scenarios since it gives more freedom to the retailer to return unsold or deteriorated item at its optimal return time. Also the all-units discount offered by the supplier is not quite attractive to the retailer to order more. We also perform a sensitivity analysis to study the impact of changes in parameter values from -15% to +15% on the optimal solution in each scenario. Only one parameter value changes, while the other parameter values are kept the same in each scenario. Some values of order quantities and total inventory costs become invalid since at least one of the order quantities is less than 10,000 units which is the price break in our all-units discount scheme. We found that in general as the holding cost, deterioration rate, return cost and backorder cost increase, the total inventory cost will also increase. In our scenarios, we found that individual return time with no discount gave the least total inventory cost. In practice, individual return time gives the retailer more freedom in returning the unsold (deteriorated) items to the supplier at its optimal return time. Also, all-units discount scheme offered by the supplier can lower the ordering and shortage cost but it cannot offset the return, holding and purchasing cost, making it unfavorable to the retailer. When considering only one item, we also found that the one with no all-units discount gives the least total inventory cost and shorter optimal return time compared with the one with all-units discount.

Our model can be extended in several directions such as considering stochastic deterioration, price-dependent demand, time-dependent demand, two-parameter Weibull deterioration and other discount schemes such as incremental discount. Considering all other possibilities in returning items by grouping items that will be jointly returned to the supplier is also a possible immediate suggestion for further research of this paper.

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