

Fuzzy Multi-Criteria Decision Making Algorithm under Intuitionistic Hesitant Fuzzy Set with Novel Distance Measure

Rupjit Saikia

Department of Mathematics,
Rajiv Gandhi Institute of Petroleum Technology,
Sivasagar- 785697, India.
E-mail: rsaikia@rgipt.ac.in

Harish Garg

School of Mathematics,
Thapar Institute of Engineering and Technology (Deemed University), Patiala, India.
Corresponding author: harishg58iitr@gmail.com

Palash Dutta

Department of Mathematics
Dibrugarh University, Dibrugarh-786004, India.
E-mail: palash.dtt@gmail.com

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Abstract

Decision making under uncertainty is a crucial issue and most demanding area of research now a days. Intuitionistic hesitant fuzzy set plays important role in dealing with the circumstances in which decision makers judge an alternative with a collection membership grades and a collection of non-membership grades. This paper contributes a novel and advanced distance measure between Intuitionistic Hesitant fuzzy sets (IHFSs). A comparative analysis of the present distance measure with existing measures is performed first. Afterwards, a case study is carried in multi-criteria decision making problem to exhibit the applicability and rationality of the proposed distance measure. The advantage of the proposed distance measure over the existing distance measures is that in case of deficit number of elements in IHFs, a decision maker can evaluate distance measure without adding extra elements to make them equivalent and furthermore, it works in successfully in all the situations.

Keywords- Hesitant fuzzy set, Intuitionistic fuzzy set, Intuitionistic hesitant fuzzy set, Fuzzy multi criteria decision making.

1. Introduction

Most of the problems encountered in the complexity of the real world involve uncertain or unknown data. One of these problems is also decision making (DM) problem. The attempt to solve DM problems is very important and many mathematical approaches have been proposed for this purpose. MCDM (Multiple criteria decision-making) is the central content of strategically process with their intent is to select the best choice by multiple decision-makers under the numerous resources. However, to handle the uncertainties in the data, a concept of the fuzzy set (Zadeh, 1965) plays a vital role. With the development, Bellman and Zadeh (1970) was initiated the concept of the decision-making problems using fuzzy set. After that, intuitionistic fuzzy set (IFS), an extension of the fuzzy set, was introduced by Atanassov (1986) with the specification of a membership degree as well as a non-membership degree for the element of a fuzzy set. A wide applicability of these sets to solve the MCDM problems are investigated by the researchers using distance measures (Garg and Kaur, 2018a; Rani and Garg, 2017), aggregation operators (Xia et

al., 2013; Garg and Kaur, 2018b) etc. All these stated works are widely applicable with continuous universe of discourse. However, in order to define the set of the membership values as a discrete function rather than a single value, a concept of hesitant fuzzy set (HFS) is introduced by Torra (2010). Deepak et al. (2019) discussed the topological structure of the HFS. With their developments, various scholars have put forward the several applications of such sets into the different fields. For example, Beg and Rashid (2014a) explained the relationship between HFS and IFS and imitated an idea of intuitionistic hesitant fuzzy set (IHFS) to represent situations in which we are equipped with several possible membership values as well as non-membership values. Peng et al. (2014) presented a concept of hesitant interval-valued intuitionistic fuzzy sets (HIVIFSs) and stated some operators on it.

With the advance of the structure and complexity of the decision making process, several approaches with describe the process to access the best alternatives are proposed by the researchers. In the context of the MCDM problems with hesitant fuzzy features, Liu et al. (2014) presented an algorithm for decision making process in which each element is accessed in terms of hesitant intuitionistic fuzzy linguistic elements (HIFLEs). Beg and Rashid (2014b) presented some distance measures between two intuitionistic hesitant fuzzy sets. Zhou et al. (2015) presented a preference relation based group decision-making approach with hesitant intuitionistic fuzzy information. Chen et al. (2016) presented distance measures for intuitionistic hesitant fuzzy sets. Chen and Huang (2017) presented a concept of hesitant triangular intuitionistic fuzzy sets and presented an algorithm for DM process with aggregation operators. In addition, they introduced the distance measures on hesitant triangular IFS to measures the degree of dissimilarity between the sets. Nazra et al. (2017) attempted to make an extension of hesitant intuitionistic fuzzy soft sets to the hesitant fuzzy soft sets by reunion of the concepts of soft sets and hesitant intuitionistic fuzzy sets. Faizi et al. (2018) presented the distance measures between the hesitant intuitionistic fuzzy linguistic terms sets. Garg and Kumar (2020) presented a MCDM algorithm based on novel exponential distance measures to compute the separation measurement between two interval-valued IFSs. Garg and Kaur (2018b) presented MCDM approach with aggregation operators and distance measures with probabilistic dual hesitant fuzzy information. Garg and Kumar (2018) presented series of distance measures based on the connection numbers. Garg and Arora (2017) presented distance and similarity measures for dual hesitant fuzzy soft sets. Li and Chen (2018) introduced an idea of D- intuitionistic hesitant fuzzy set for dealing uncertain information.

From the above discussion, it is concluded that distance measures is one of the importance information measures to solve the MCDM problems. However, from this survey, we observe that the computational complexity of the some of the algorithm increases which may leads to give wrong decision to the final results. Also, we observe that, in Chen et al. (2016) approach to measures the degree of dissimilarity between the two IHFSs the decision makers need to add extra elements in the IHFEs called decision makers risk preference. However, it is quite notices that when a decision maker adds extra element by himself/herself or some by methods in the deficit IHFEs, then biasness may occur, intentionally or unintentionally and the original information expressed in IHFSs may get deformed by adding extra elements. As a consequence, decision making may be turned in a wrong way. In addition to this, the distance measure as proposed by Beg and Rashid (2014a) may give some undesirable results in order to distinguish the pairs of given set (explained in section 4). Thus, motivated from these, there is a need to develop some novel distance measures between the pairs of IHFS which can overcome the drawbacks of existing measures.

Keeping the motivation and features of the IHFS to describe the uncertainties in the data, this paper center around the development of new distance measures to compute the degree of dissimilarity between the sets. In the proposed measure, the major advantages of it is that there is a no need to add an extra element, which itself reduce the noise in the process. Further, the properties of the proposed distance measures are examined in detail. Later, a MCDM algorithm based on it has been presented to solving the decision-making problems. The feasibility and superiority of our proposed distance measure is explained through a numerical example.

The rest of the paper is organized as follows. Section 2 describes the basic preliminaries related to hesitant set. In Section 3, a novel distance measure is proposed and the properties related to it are discussed. In Section 4, a comparative analysis with some existing measures is conducted to show the advantage of the proposed distance measure. Section 5 gives the MCDM algorithm followed by a numerical example. Lastly, Section 6 concludes the paper.

2. Preliminaries

In this section, some basic concepts related to hesitant fuzzy sets over the universal set X are reviewed here.

Definition 1. (Zadeh 1965) A fuzzy set A over X is defined as

$$A = \{(x, \mu_A(x)) : x \in X\} \quad (1)$$

where, $\mu_A(x) : X \rightarrow [0,1]$ define the membership function.

Definition 2. (Atanasov, 1986) An IFS A on X is defined as

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\} \quad (2)$$

where, $\mu_A(x)$ and $\nu_A(x)$ is called the degree of membership and degree of non-membership of x in A with the conditions $0 \leq \mu_A(x) \leq 1$ and $0 \leq \nu_A(x) \leq 1$ with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The hesitancy degree of x in A is given as $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

Definition 3. (Xu and Yager, 2006) Let A and B be two IFSs on X , then the following properties holds:

- i) $A \subseteq B$ if $\forall x \in X, \mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$
- ii) $A = B$ if and only if $\forall x \in X, \mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$
- iii) (Complement:) $A^c = \{(x, \nu_A(x), \mu_A(x)) : x \in X\}$
- iv) (Intersection:) $\cap A_i = \{(x, \wedge \mu_{A_i}(x), \vee \nu_{A_i}(x)) : x \in X\}$
- v) (Union:) $\cup A_i = \{(x, \vee \mu_{A_i}(x), \wedge \nu_{A_i}(x)) : x \in X\}$.

Definition 4. (Torra, 2010) A HFS A on X is defined as

$$A = \{ \langle x, H_A(x) \rangle \mid x \in X \} \quad (3)$$

where, $H_A(x) \in [0,1]$ represents a set of membership grades or values of the element $x \in X$ to the set A . In general, $H_A(x)$ is named hesitant fuzzy element (HFE) of the element $x \in X$ to the set A .

Definition 5. (Xia et al., 2013) For HFE H , a score function $s(H)$ is defined as

$$s(H) = \frac{1}{l_H} \sum_{\gamma \in H} \gamma \quad (4)$$

Here l_H represents the number of elements in the H .

To rank the given HFEs H_1 and H_2 , an order relation is defined by Xia (2013) as

- i) $s(H_1) > s(H_2)$ implies that H_1 is superior to H_2 , symbolized by $H_1 > H_2$
- ii) $s(H_1) = s(H_2)$ implies that $H_1 = H_2$.

Later on, Chen et al. (2016) introduced the idea of deviation degree to rank the numbers, which is defined as.

Definition 6. (Chen et al., 2016) The deviation degree of a HFE is defined as

$$\bar{\sigma}(H) = \left(\frac{1}{l_H} \sqrt{\sum_{\gamma \in H} (\gamma - s(H))^2} \right)^{\frac{1}{2}} \quad (5)$$

and defined the comparison rule between two HFEs H_1 and H_2 as

- i) $s(H_1) > s(H_2)$ then $H_1 > H_2$
- ii) $s(H_1) = s(H_2)$ and $\bar{\sigma}(H_1) < \bar{\sigma}(H_2)$ then $H_1 > H_2$
- iii) $s(H_1) = s(H_2)$ and $\bar{\sigma}(H_1) > \bar{\sigma}(H_2)$ then $H_1 < H_2$
- iv) $s(H_1) = s(H_2)$ and $\bar{\sigma}(H_1) = \bar{\sigma}(H_2)$ then $H_1 = H_2$.

Definition 7. The mean deviation among the elements of the HFEs from their mean as:

$$\bar{m}(H) = \frac{1}{l_H} \sum_{\gamma \in H} |\gamma - s(H)| \quad (6)$$

Definition 8. (Beg and Rashid, 2014a) An IHFS on X is given as

$$IHFS = \{(x, H(x), H'(x)): x \in X\} \quad (7)$$

where, $H(x)$ is the set of membership degrees and $H'(x)$ are the set of non-membership degrees of the element $x \in X$ to the set $IHFS$ satisfying the following condition $\max\{H(x)\} + \min\{H'(x)\} \leq 1$ and $\min\{H(x)\} + \max\{H'(x)\} \leq 1$. A pair $(H(x), H'(x))$ can be said to be an intuitionistic hesitant fuzzy element (IHFE).

Remark: It can be observed that the number of values in two IHFEs may not be same. Let, the number of values in $H(x)$ be $l_{H(x)}$ and the number of values of $H'(x)$ be $l_{H'(x)}$. When the values in IHFE are not in ascending or descending order, the values are arranged in an order in such a way that IHFE $A = (H, H')$ satisfy the following property:

$$H_{\sigma(i)} \leq H_{\sigma(i+1)}, i = 1, 2, \dots, l_H - 1 \text{ and } H'_{\sigma(i)} \leq H'_{\sigma(i+1)}, i = 1, 2, \dots, l_{H'} - 1$$

where, $\sigma: (1, 2, \dots, n) \rightarrow (1, 2, \dots, n)$ and $\tau: (1, 2, \dots, m) \rightarrow (1, 2, \dots, m)$ are taken as two permutations.

Also, it has been proposed that for two IHFEs (H_1, H_1') and (H_2, H_2') , if $l_{H_1} = l_{H_2}, l_{H_1'} = l_{H_2'}, H_{1\sigma(i)} = H_{2\sigma(i)}$ & $H'_{1\sigma(i)} = H'_{2\sigma(i)}$ if and only if $(H_1, H_1') = (H_2, H_2')$ for $i = 1, 2, \dots, l_{H_1}$ and $j = 1, 2, \dots, l_{H_1'}$.

Definition 9. (Beg and Rashid, 2014a) Let A and B be two IHFEs such that $A = (h_A, h'_A) = ((a_1, a_2, \dots, a_n), (a'_1, a'_2, \dots, a'_n))$ and $B = (h_B, h'_B) = ((b_1, b_2, \dots, b_n), (b'_1, b'_2, \dots, b'_n))$, then, the distance measure between them is defined by

$$d(x, y) = \max \left\{ \begin{array}{l} \max_{a_i \in h_A} \left\{ \min_{b_i \in h_B} (|a_i - b_i|) \right\}, \max_{b_i \in h_B} \left\{ \min_{a_i \in h_A} (|a_i - b_i|) \right\}, \\ \max_{a'_i \in h'_A} \left\{ \min_{b'_i \in h'_B} (|a'_i - b'_i|) \right\}, \max_{b'_i \in h'_B} \left\{ \min_{a'_i \in h'_A} (|a'_i - b'_i|) \right\} \end{array} \right\} \quad (8)$$

Chen et al. (2016) defined the generalized intuitionistic hesitant fuzzy weighted normalized Hamming and Hausdorff distances between A and B are defined as

$$d_{GIHFWNHD}(A, B) = \left[\sum_{i=1}^n w_i \left(\frac{1}{2l_x} \sum_{j=1}^{l_{x_i}} |h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}} + |h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}} \right) \right]^{1/\tilde{e}} \quad (9)$$

$$d'_{GIHFWNHD}(A, B) = \left[\sum_{i=1}^n w_i \left(\max_j (|h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}}), |h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}} \right) \right]^{1/\tilde{e}} \quad (10)$$

$$d_{GHIHFWND}(A, B) = \left[\sum_{i=1}^n w_i \left(\frac{1}{4l_x} \sum_{j=1}^{l_{x_i}} (|h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}} + |h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}}) + \frac{1}{2} \left(\max_j (|h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}}), |h_{A\delta(j)}^i(x_i) - h_{B\delta(j)}^i(x_i)|^{\tilde{e}}) \right) \right) \right]^{1/\tilde{e}} \quad (11)$$

It is quite obvious that different IHFSs may have different number of elements. A decision maker has to add extra elements in the shorter IHFEs to make them of equal length to evaluate distance measures proposed by Chen et al. (2016). A decision maker may add extra element depending upon his own risk, or for the pessimistic approach, the smallest element can be added in the shorter IHFEs, while for the optimistic approach the largest element can be added to the shorter IHFEs to make them of equal length. In those cases two things happened

- Case 1: Original information expressed in IHFSs is deformed. (Pessimistic approach or optimistic approach).
- Case 2: Decision maker may get biased intentionally or unintentionally. (Add extra element depending upon his/her own risk).

3. A Novel Intuitionistic Hesitant Fuzzy Distance Measure

In this section, we present a novel distance measure on IHFSs and investigated their properties.

Definition 10. Let $X = \{a_1, a_2, \dots, a_n\}$, be the universal set. Consider two IHFSs $A = \{(a_i, H_A(a_i), h_A'(a_i)) | a_i \in X\}$, and $B = \{(a_i, h_B(a_i), h_B'(a_i)) | a_i \in X\}$ where $h_A(a_i) = (\hat{i}_1^A(a_i), \hat{i}_2^A(a_i), \hat{i}_3^A(a_i), \dots, \hat{i}_{p_i}^A(a_i))$, $h'_A(a_i) = (\hat{i}_1^A(a_i), \hat{i}_2^A(a_i), \hat{i}_3^A(a_i), \dots, \hat{i}_{q_i}^A(a_i))$, $h_B(a_i) = (\hat{i}_1^B(a_i), \hat{i}_2^B(a_i), \hat{i}_3^B(a_i), \dots, \hat{i}_{r_i}^B(a_i))$ and $h'_B(a_i) = (\hat{i}_1^B(a_i), \hat{i}_2^B(a_i), \hat{i}_3^B(a_i), \dots, \hat{i}_{s_i}^B(a_i))$.

Let p_i, q_i, r_i, s_i be the number of the elements in membership grades of $h_A(a_i), h'_A(a_i), h_B(a_i)$ and $h'_B(a_i)$ respectively. Then, the new distance between IHFEs A & B is defined as:

$$d(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{f_i + g_i + h_i}{1 + f_i + g_i + h_i} \quad (12)$$

where,

$$f_i = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \hat{i}_j^A(a_i) - \sum_{j=1}^{r_i} \hat{i}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \hat{i}_j^A(a_i) - \sum_{j=1}^{s_i} \hat{i}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \quad (13)$$

$$g_i = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \hat{i}_j^A(a_i)}{p_i} - \frac{\sum_{j=1}^{r_i} \hat{i}_j^B(a_i)}{r_i} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{r_i} \hat{i}_j^B(a_i)}{r_i} - \frac{\sum_{j=1}^{p_i} \hat{i}_j^A(a_i)}{p_i} \right| \quad (14)$$

$$h_i = \sin \frac{\pi}{6} \left| \text{Var}(\hat{i}_j^A(a_i)) - \text{Var}(\hat{i}_j^B(a_i)) \right| + \sin \frac{\pi}{6} \left| \text{Var}(\hat{i}_i^A(a_i)) - \text{Var}(\hat{i}_i^B(a_i)) \right| \quad (15)$$

and $\text{Var}(\hat{i}_j^A(a_i))$ is the variance of the membership grades; $\hat{i}_j^A(a_i)$ from the mean of the membership grades.

Theorem 1. The stated measure defined in Definition 10 is a valid distance measure.

Proof. In order to show the stated measure is a valid distance measures for IHFSs A and B , we need to show the following three conditions:

- i) $0 \leq d(A, B) \leq 1$
- ii) $d(A, B) = d(B, A)$
- iii) $d(A, B) = 0$ if and only if $A = B$.

For two IHFSs $A = \{(a_i, H_A(a_i), h_A'(a_i)) | a_i \in X\}$, and $B = \{(a_i, h_B(a_i), h_B'(a_i)) | a_i \in X\}$, we have

- i) $0 \leq d(A, B) \leq 1$

Proof:

$$\text{Here } 0 \leq \left| \frac{\sum_{j=1}^{p_i} \hat{i}_j^A(a_i) - \sum_{j=1}^{r_i} \hat{i}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \leq 1$$

$$\Rightarrow 0 \leq \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \hat{i}_j^A(a_i) - \sum_{j=1}^{r_i} \hat{i}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \leq \frac{1}{2}$$

$$\text{Similarly, } 0 \leq \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \hat{i}_j^A(a_i) - \sum_{j=1}^{s_i} \hat{i}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \leq \frac{1}{2}$$

$$\text{So, } 0 \leq \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i) - \sum_{j=1}^{r_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \lambda_j^A(a_i) - \sum_{j=1}^{s_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \leq 1$$

$$\Rightarrow 0 \leq f_i \leq 1.$$

Similarly, $0 \leq g_i \leq 1$ and $0 \leq h_i \leq 1$

$$\therefore 0 \leq \frac{f_i + g_i + h_i}{1 + f_i + g_i + h_i} \leq 1$$

$$\Rightarrow 0 \leq \frac{1}{n} \sum_{i=1}^n \frac{f_i + g_i + h_i}{1 + f_i + g_i + h_i} \leq 1$$

$$\Rightarrow 0 \leq d(A, B) \leq 1.$$

Thus, i) holds.

$$\text{ii) } d(A, B) = d(B, A)$$

Proof:

$$\text{We have, } \left| \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i) - \sum_{j=1}^{r_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| = \left| \frac{\sum_{j=1}^{r_i} \lambda_j^B(a_i) - \sum_{j=1}^{p_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i) - \sum_{j=1}^{r_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{r_i} \lambda_j^B(a_i) - \sum_{j=1}^{p_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \quad (16)$$

$$\text{Also, } \left| \frac{\sum_{j=1}^{q_i} \lambda_j^A(a_i) - \sum_{j=1}^{s_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| = \left| \frac{\sum_{j=1}^{s_i} \lambda_j^B(a_i) - \sum_{j=1}^{q_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \lambda_j^A(a_i) - \sum_{j=1}^{s_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{s_i} \lambda_j^B(a_i) - \sum_{j=1}^{q_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \quad (17)$$

From (16) and (17), we get

$$\begin{aligned} & \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i) - \sum_{j=1}^{r_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \lambda_j^A(a_i) - \sum_{j=1}^{s_i} \lambda_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \\ &= \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{r_i} \lambda_j^B(a_i) - \sum_{j=1}^{p_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{s_i} \lambda_j^B(a_i) - \sum_{j=1}^{q_i} \lambda_j^A(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| \end{aligned} \quad (18)$$

Also,

$$\left| \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i)}{p_i} - \frac{\sum_{j=1}^{r_i} \lambda_j^B(a_i)}{r_i} \right| = \left| \frac{\sum_{j=1}^{r_i} \lambda_j^B(a_i)}{r_i} - \frac{\sum_{j=1}^{p_i} \lambda_j^A(a_i)}{p_i} \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \hat{I}_j^A(a_i)}{p_i} - \frac{\sum_{j=1}^{r_i} \hat{I}_j^B(a_i)}{r_i} \right| = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{r_i} \hat{I}_j^B(a_i)}{r_i} - \frac{\sum_{j=1}^{p_i} \hat{I}_j^A(a_i)}{p_i} \right| \quad (19)$$

$$\left| \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} - \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} \right| = \left| \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} - \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} - \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} \right| = \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} - \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} \right| \quad (20)$$

From Eqs. (19) and (20), we get

$$\sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \hat{I}_j^A(a_i)}{p_i} - \frac{\sum_{j=1}^{r_i} \hat{I}_j^B(a_i)}{r_i} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} - \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} \right|$$

$$= \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{r_i} \hat{I}_j^B(a_i)}{r_i} - \frac{\sum_{j=1}^{p_i} \hat{I}_j^A(a_i)}{p_i} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{s_i} \hat{I}_j^B(a_i)}{s_i} - \frac{\sum_{j=1}^{q_i} \hat{I}_j^A(a_i)}{q_i} \right| \quad (21)$$

$$\left| \text{Var}(\hat{I}_j^A(a_i)) - \text{Var}(\hat{I}_j^B(a_i)) \right| = \left| \text{Var}(\hat{I}_j^B(a_i)) - \text{Var}(\hat{I}_j^A(a_i)) \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_j^A(a_i)) - \text{Var}(\hat{I}_j^B(a_i)) \right| = \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_j^B(a_i)) - \text{Var}(\hat{I}_j^A(a_i)) \right| \quad (22)$$

$$\left| \text{Var}(\hat{I}_i^A(a_i)) - \text{Var}(\hat{I}_i^B(a_i)) \right| = \left| \text{Var}(\hat{I}_i^B(a_i)) - \text{Var}(\hat{I}_i^A(a_i)) \right|$$

$$\Rightarrow \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_i^A(a_i)) - \text{Var}(\hat{I}_i^B(a_i)) \right| = \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_i^B(a_i)) - \text{Var}(\hat{I}_i^A(a_i)) \right| \quad (23)$$

From Eqs. (22) and (23), we get

$$\sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_j^A(a_i)) - \text{Var}(\hat{I}_j^B(a_i)) \right| + \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_i^A(a_i)) - \text{Var}(\hat{I}_i^B(a_i)) \right|$$

$$= \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_j^B(a_i)) - \text{Var}(\hat{I}_j^A(a_i)) \right| + \sin \frac{\pi}{6} \left| \text{Var}(\hat{I}_i^B(a_i)) - \text{Var}(\hat{I}_i^A(a_i)) \right| \quad (24)$$

From Eqs. (18), (21) and (24), we can say that $d(A, B) = d(B, A)$

iii) $d(A, B) = 0$ if and only if $A = B$

Proof:

Let $d(A, B) = 0$

$$\Leftrightarrow d(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{f_i + g_i + h_i}{1 + f_i + g_i + h_i}$$

$$\Leftrightarrow \frac{f_i + g_i + h_i}{1 + f_i + g_i + h_i} = 0$$

$$\Leftrightarrow f_i + g_i + h_i = 0$$

$$\Leftrightarrow f_i = g_i = h_i = 0 \quad (\because f_i, g_i, h_i \geq 0)$$

$$\Leftrightarrow \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \dot{\lambda}_j^A(a_i) - \sum_{j=1}^{r_i} \dot{\lambda}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \dot{\lambda}_j^A(a_i) - \sum_{j=1}^{s_i} \dot{\lambda}_j^B(a_i)}{\max(p_i, q_i, r_i, s_i)} \right| = 0,$$

$$\sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{p_i} \dot{\lambda}_j^A(a_i)}{p_i} - \frac{\sum_{j=1}^{r_i} \dot{\lambda}_j^B(a_i)}{r_i} \right| + \sin \frac{\pi}{6} \left| \frac{\sum_{j=1}^{q_i} \dot{\lambda}_j^A(a_i)}{q_i} - \frac{\sum_{j=1}^{s_i} \dot{\lambda}_j^B(a_i)}{s_i} \right| = 0,$$

$$\sin \frac{\pi}{6} \left| \text{Var}(\dot{\lambda}_j^A(a_i)) - \text{Var}(\dot{\lambda}_j^B(a_i)) \right| + \sin \frac{\pi}{6} \left| \text{Var}(\dot{\lambda}_j^A(a_i)) - \text{Var}(\dot{\lambda}_j^B(a_i)) \right| = 0$$

$$\Leftrightarrow \sum_{j=1}^{p_i} \dot{\lambda}_j^A(a_i) = \sum_{j=1}^{r_i} \dot{\lambda}_j^B(a_i) \ \& \ \sum_{j=1}^{q_i} \dot{\lambda}_j^A(a_i) = \sum_{j=1}^{s_i} \dot{\lambda}_j^B(a_i),$$

$$\Leftrightarrow \text{mean}(\dot{\lambda}_j^A(a_i)) = \text{mean}(\dot{\lambda}_j^B(a_i)) \ \& \ \text{mean}(\dot{\lambda}_j^A(a_i)) = \text{mean}(\dot{\lambda}_j^B(a_i)),$$

$$\Leftrightarrow \text{Var}(\dot{\lambda}_j^A(a_i)) = \text{Var}(\dot{\lambda}_j^B(a_i)) \ \& \ \text{Var}(\dot{\lambda}_j^A(a_i)) = \text{Var}(\dot{\lambda}_j^B(a_i))$$

$$\Leftrightarrow h_A(a_i) = h_B(a_i) \ \& \ h'_A(a_i) = h'_B(a_i)$$

$$\Leftrightarrow A = B.$$

As (12) satisfies all the conditions of distance measure, so (12) is distance measure.

4. Advantages of the Proposed Distance Measure

In this section, we present numerical example to show the advantages of the proposed measure over the existing measures as defined by Beg and Rashid (2014a) and Chen et al. (2016). As in these existing measures, there is a need to add extra elements to the IHFEs in the decision making to balance the length of each IHFEs. For that reasons, decision making methods do not produce reliable result. Therefore, we have to check the validity and reliability of the existing distance measure. To show the uniqueness, novelty and advantages of the proposed distance measure, the results are computed by taking five different pairs of IHFEs and are summarized in Table 1.

Now, from the above table it can be observed from our proposed measure that

$$(x_1, y_1) \neq (x_2, y_2) \neq (x_3, y_3) \neq (x_4, y_4) \neq (x_5, y_5)$$

i.e. all the five IHFSs are different from each other, though distance measured proposed by Beg et al. (2014a) are same 0.87. This is not reasonable, since such types of results affect the ranking in decision making process. This is the main drawback of the distance measure proposed by Beg et

al. (2014a). Similarly, distance measured by Hausdorff Distance for $\ddot{e}=1$ and $\ddot{e}=2$ are 0.87 and 0.9327 respectively for each pair of IHFSs. This is not reasonable as stated above. Also, it has the drawback of case 1 from Section 2 (deformation of original information). Even though distance measured by Hamming and Hybrid distance for $\ddot{e}=1$ and $\ddot{e}=2$ are different for each case but these distance measures still have the drawback of case 1 from Section 2 (deformation of original information). It is noted from the table it can be clearly observed that our proposed distance measure produces different results for different sets which are more reliable and logical results. In addition, our proposed distance measure bypasses the drawbacks of existing distance measures of deforming original information by adding extra elements in the deficit IHFEs.

Table 1. Comparison of Distance degrees for different IHFSs using different distance measures

S. No	Different IHFSs	Distance $d(x_i, y_i)$ (Beg and Rashid, 2014a)	Hamming distance for $\ddot{e}=1$ (Chen et al., 2016)	Hausdorff Distance For $\ddot{e}=1$ (Chen et al., 2016)	Hybrid Distance For $\ddot{e}=1$ (Chen et al., 2016)	Hamming Distance For $\ddot{e}=2$ (Chen et al., 2016)	Hausdorff Distance For $\ddot{e}=2$ (Chen et al., 2016)	Hybrid Distance For $\ddot{e}=2$ (Chen et al., 2016)	Proposed distance measure
1	$x_1 = \begin{pmatrix} (0.2, 0.3, 0.01), \\ (0.02, 0.01, 0.03) \end{pmatrix}$ $y_1 = \begin{pmatrix} (0.4, 0.01, 0.03), \\ (0.7, 0.9, 0.5) \end{pmatrix}$	0.87	0.355	0.87	0.6125	0.5958	0.9327	0.764	0.585351
2	$x_3 = \begin{pmatrix} (0.2, 0.3, 0.01), \\ (0.01, 0.03) \end{pmatrix}$ $y_3 = \begin{pmatrix} (0.4, 0.01, 0.03), \\ (0.5, 0.9) \end{pmatrix}$	0.87	0.3533	0.87	0.6117	0.5944	0.9327	0.7636	0.569409
3	$x_4 = \begin{pmatrix} (0.2, 0.25, 0.3), \\ (0.01, 0.03) \end{pmatrix}$ $y_4 = \begin{pmatrix} (0.4, 0.35, 0.01), \\ (0.5, 0.9) \end{pmatrix}$	0.87	0.3733	0.87	0.6217	0.6110	0.9327	0.7719	0.429861
4	$x_5 = \begin{pmatrix} (0.2, 0.3), \\ (0.01, 0.03, 0.019) \end{pmatrix}$ $y_5 = \begin{pmatrix} (0.4, 0.01), \\ (0.5, 0.9, 0.8) \end{pmatrix}$	0.87	0.4368	0.87	0.6534	0.6609	0.9327	0.7968	0.461651
5	$x_6 = \begin{pmatrix} (0.2, 0.3, 0.29), \\ (0.01, 0.03, 0.02) \end{pmatrix}$ $y_6 = \begin{pmatrix} (0.4, 0.39, 0.01), \\ (0.5, 0.9, 0.6) \end{pmatrix}$	0.87	0.3883	0.87	0.6291	0.6231	0.9327	0.7753	0.478054

Advantages of the proposed distance measure:

- i) The proposed distance measure has the ability to evaluate distance degree between two IHFSs in case of deficit number of elements in that IHFS without adding extra elements in the shorter IHFEs. As a consequence, it can reduce the chance of intentional or unintentional biasness of the decision makers during IHFSs decision making situation using distance measure.
- ii) In addition, some distance measures (Beg and Rashid, 2014a; Chen et al., 2016) produces same distance degrees for completely different IHFSs (Table 1) which is obviously not logical in nature. Whereas, the proposed distance measure produces different distance degrees for different IHFSs., which is a clear advantage of the proposed distance measure over distance measures introduced by (Beg and Rashid, 2014a; Chen et al., 2016).

5. Decision Making Methodology

This section presents a multi-criteria decision-making (MCDM) approach for selecting a most appropriate alternative under intuitionistic hesitant fuzzy environment.

5.1 Proposed Method

Let us consider a decision-making problem, which consists of n alternatives and m criteria. Let $A = \{H_1, H_2, \dots, H_n\}$ be the collection of available alternatives and $C = \{C_1, C_2, \dots, C_m\}$ be the collection of criteria of the decision making system under IHFS environment. To solve the decision-making problem, an expert determines the alternatives under IHFS domain and lists the rating values of alternatives in terms of IHFS. The importance or weights of the criteria are represented by the column vector $w = (w_1, w_2, \dots, w_m)^T$ such that $w_i > 0$ & $\sum_{i=1}^m w_i = 1$. To choose the suitable alternative, the proposed distance measure is applied to improve the decision methodology. The following are steps involved in the methodology:

Step 1: Form the IHFS matrix corresponding to the assessment values of alternatives. Now, the IHFS decision matrix can be made as:

$$\begin{matrix}
 & H_1 & H_2 & \dots & H_n \\
 \begin{matrix} C_1 \\ C_2 \\ \vdots \\ C_m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}
 \end{matrix}$$

Here, $a_{ij} = [H^l_{S_{ij}}, H^l_{S_{ij}}]$; $H^l_{S_{ij}}$ represents the performance of the alternative H_j and criteria C_i and $H^l_{S_{ij}}$ represents the non-performance of H_j and criteria C_i .

Step 2: Represent the alternatives A_j by using the criteria as follows

$$H_j = \{ \langle C_i, a_{ij} \rangle \mid C_i \in C \} \text{ where } a_{ij} \rightarrow [0,1]; i = 1,2, \dots, m \text{ and } j = 1,2, \dots, n.$$

Step 3: Let D_b be the set of benefit criteria (i.e., the larger C_i , the greater preference) and D_c be the set of cost criteria (i.e., the smaller C_i , the greater preference). The IHFS positive ideal solution H^+ and the IHFS negative ideal solution H^- are defined as follows:

$$\begin{aligned}
 H^+ &= \left[(x, x') \mid x \in H^l_{S_{ij}}, \&x' \in H^l_{S_{ij}}, \forall j; \max_j(\min H^l_{S_{ij}}) \leq x \right. \\
 &\leq \max_j(\max H^l_{S_{ij}}), \min_j(\min H^l_{S_{ij}}) \leq x' \leq \min_j(\max H^l_{S_{ij}}) \mid i \in D_b \\
 &\left. (x, x') \mid x \in H^l_{S_{ij}}, \&x' \in H^l_{S_{ij}}, \forall j; \min_j(\min H^l_{S_{ij}}) \leq x \right. \\
 &\leq \min_j(\max H^l_{S_{ij}}), \max_j(\min H^l_{S_{ij}}) \leq x' \leq \max_j(\max H^l_{S_{ij}}) \mid i \in D_c \left. \right] \\
 H^- &= \left[(x, x') \mid x \in H^l_{S_{ij}}, \&x' \in H^l_{S_{ij}}, \forall j; \max_j(\min H^l_{S_{ij}}) \leq x \right. \\
 &\leq \max_j(\max H^l_{S_{ij}}), \min_j(\min H^l_{S_{ij}}) \leq x' \leq \min_j(\max H^l_{S_{ij}}) \mid i \in D_c \\
 &\left. (x, x') \mid x \in H^l_{S_{ij}}, \&x' \in H^l_{S_{ij}}, \forall j; \min_j(\min H^l_{S_{ij}}) \leq x \right.
 \end{aligned}$$

$$\leq \min_j \left(\max H^l_{s_{ij}} \right), \max_j \left(\min H^l_{s_{ij}} \right) \leq x' \leq \max_j \left(\max H^l_{s_{ij}} \right) | i \in D_b]$$

$(i = 1, 2, \dots, m; j = 1, 2, \dots, n.$

Step 4: Compute the positive and negative distance measure $d(H_j, H^+)$ and $d(H_j, H^-)$ by using the proposed measure.

Step 5: Rank the alternatives by using the following:

$$R(H_j) = \frac{d(H_j, H^+)}{d(H_j, H^+) + d(H_j, H^-)} \quad (25)$$

Step 6: Rank the alternatives based on the value of $R(H)$ and select the most optimal one(s).

5.2 A Case Study

In this section, a numerical case study will be carried out to show the reliability, validity and usability of the introduced distance measure. Our ranking methodology will be applied to choose the most desirable schools in the same university and rank the schools depending upon the criteria. Five schools have been considered, namely, Business and Economics (H_1), Science and Technology (H_2), Social Science and Humanities (H_3), Communication and Cultural Studies (H_4), and Textile and Design (H_5). The Management of the university has to choose the schools to distribute funds based on their performance. Four criteria are considered for evaluating the performance of the schools. The criteria are taken as: Expenses of school (C_1), Students taken per year by the school (C_2), Publication from the school (C_3) and Covered area of school (C_4). The steps of the proposed methods are implemented here to find the best school.

Step 1: The rating information of each school under the different criteria are accessed by an expert and their ratings are summarized in Table 2.

Table 2. Expert's review about the schools is represented as intuitionistic hesitant fuzzy sets

	C_1	C_2	C_3	C_4
H_1	$((0.5, 0.6, 0.8), (0.1, 0.2))$	$((0.6, 0.8), (0.1, 0.2))$	$((0.1, 0.3), (0.6, 0.7))$	$((0.1, 0.3), (0.5, 0.6))$
H_2	$((0.1, 0.3), (0.3, 0.4, 0.5))$	$((0.5, 0.7, 0.8), (0.1))$	$((0.5, 0.6), (0.1, 0.3))$	$((0.5, 0.6), (0.2, 0.3))$
H_3	$((0.5, 0.7), (0.2, 0.25))$	$((0.5, 0.6), (0.2, 0.35))$	$((0.7, 0.9), (0.05, 0.1))$	$((0.1, 0.2), (0.6, 0.7))$
H_4	$((0.7, 0.9), (0.05, 0.1))$	$((0.1, 0.2), (0.6, 0.7))$	$((0.1, 0.3), (0.6, 0.7))$	$((0.5, 0.6, 0.7), (0.2))$
H_5	$((1), (0))$	$((0.1, 0.3), (0.5, 0.65))$	$((0, 0.2), (0.7, 0.8))$	$((0.4, 0.7), (0.1, 0.2))$

Step 2: Arrange the information as intuitionistic hesitant fuzzy sets given as

$$H_1 = \left\{ C_1((0.5, 0.6, 0.8), (0.1, 0.2)), C_2((0.6, 0.8), (0.1, 0.2)), C_3((0.1, 0.3), (0.6, 0.7)), C_4((0.1, 0.3), (0.5, 0.6)) \right\};$$

$$H_2 = \left\{ C_1((0.1, 0.3), (0.3, 0.4, 0.5)), C_2((0.5, 0.7, 0.8), (0.1)), C_3((0.5, 0.6), (0.1, 0.3)), C_4((0.5, 0.6), (0.2, 0.3)) \right\};$$

$$H_3 = \left\{ C_1((0.5, 0.7), (0.2, 0.25)), C_2((0.5, 0.6), (0.2, 0.35)), C_3((0.7, 0.9), (0.05, 0.1)), C_4((0.1, 0.2), (0.6, 0.7)) \right\};$$

$$H_4 = \left\{ \begin{array}{l} C_1((0.7,0.9), (0.05,0.1)), C_2((0.1,0.2), (0.6,0.7)), \\ C_3((0.1,0.3), (0.6,0.7)), C_4((0.5,0.6,0.7), (0.2)) \end{array} \right\} \text{ and}$$

$$H_5 = \left\{ \begin{array}{l} C_1((1), (0)), C_2((0.1,0.3), (0.5,0.65)), \\ C_3((0,0.2), (0.7,0.8)), C_4((0.4,0.7), (0.1,0.2)) \end{array} \right\}$$

Step 3: The positive idea solution and negative ideal solution are computed as

$$H^+ = \left\{ \begin{array}{l} C_1((0,0.1,0.2), (0.6,0.7)), C_2((0.6,0.7,0.8,0.9,1), (0)), \\ C_3((0.7,0.8,0.9), (0,0.05)), C_4((0,0.1), (0.7,0.8)) \end{array} \right\}$$

and

$$H^- = \left\{ \begin{array}{l} C_1(((0.7,0.8,0.9), (0))), C_2((0.1,0.2), (0.6,0.65,0.7)), \\ C_3((0,0.1), (0.7,0.8)), C_4((1), (0)) \end{array} \right\}$$

Step 4: Utilize the proposed distance measure, we obtain the following distances of the alternatives from the positive ideal solution as

$$d(H_1, H^+) = 0.553748807; d(H_2, H^+) = 0.439560786; d(H_3, H^+) = 0.40137;$$

$$d(H_4, H^+) = 0.716836712; d(H_5, H^+) = 0.729647072.$$

and from the negative ideal solutions as

$$d(H_1, H^-) = 0.515155918; d(H_2, H^-) = 0.65549294; d(H_3, H^-) = 0.655764847;$$

$$d(H_4, H^-) = 0.287149827; d(H_5, H^-) = 0.348419748.$$

Step 5: Compute the ranking order of each alternatives by using Eq. (25) and get

$$R(H_1) = \frac{d(H_1, H^+)}{d(H_1, H^+) + d(H_1, H^-)} = \frac{0.553748807}{0.553748807 + 0.515155918} = 0.481947461$$

$$R(H_2) = \frac{d(H_2, H^+)}{d(H_2, H^+) + d(H_2, H^-)} = \frac{0.439560786}{0.439560786 + 0.65549294} = 0.59859432$$

$$R(H_3) = \frac{d(H_3, H^+)}{d(H_3, H^+) + d(H_3, H^-)} = \frac{0.40137}{0.40137 + 0.655764847} = 0.620322859$$

$$R(H_4) = \frac{d(H_4, H^+)}{d(H_4, H^+) + d(H_4, H^-)} = \frac{0.716836712}{0.716836712 + 0.287149827} = 0.286009639$$

$$R(H_5) = \frac{d(H_5, H^+)}{d(H_5, H^+) + d(H_5, H^-)} = \frac{0.729647072}{0.729647072 + 0.348419748} = 0.323189381.$$

Step 6: Since $R(H_3) > R(H_2) > R(H_1) > R(H_5) > R(H_4)$ and thus, the ranking of the given alternatives are obtained as

$$H_3 < H_2 < H_1 < H_5 < H_4$$

and conclude that School of Communication and Cultural Studies (H_4) is the most desirable school to allocate more fund on the basis of available criteria, which is same found from existing analysis on the same data.

6. Conclusion

In this research work, a novel distance measure has been proposed for Intuitionistic Hesitant Fuzzy Sets. We have applied this distance measure in the MCDM problem under intuitionistic hesitant fuzzy domain. An illustrative example is taken for showing the advantage of our proposed distance over existing distance measures between IHFEs. By studying, we conclude that our proposed distance measure is more generic and suitable for solving MCDM problem, whereas the existing distance measures fail in case of deficit number of elements in IHFEs. Furthermore, our proposed distance measure can reduce the chance of intentional as well as unintentional biasness of the decision maker. In the future, this distance measure can be extended to the decision-making field under the diverse environment to better represent uncertain information (Garg and Kaur, 2020).

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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