Development of Assessment Methods of Target-Driven Systems Resilience

Alexander Bochkov
Department of Analysis and Ranking of Monitored Facilities, LLC Gazprom gaznadzor, Novocheremushkinskaya Street, 65, Moscow, 117418, Russia.
Corresponding author: a.bochkov@gmail.com

Valeriy Lesnykh
Safety and Security Adviser, Administration, LLC Gazprom gaznadzor, Novocheremushkinskaya Street, 65, Moscow, 117418, Russia; RUDN University, Miklukho-Maklay Street, 6, Moscow, 117198, Russia.
E-mail: lesnykh@gaznadzor.gazprom.ru

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Abstract
Existing approaches to formation of quantitative indicator of resilience of objects of different complexity operating under conditions of external impacts are discussed. Advantages and disadvantages of approaches under discussion are specified. As general basis for application of resilience concept, the class of so-called target-driven systems oriented to achieving some preset target in a certain period is marked out. It is proposed to use the value of the indicator constructed using methodical apparatus "difficulties of achieving target" of I. B. Russman for quantitative assessment of resilience of similar systems. Methods of calculation and results of model assessments of resilience using this approach are presented. Directions of Russman’s approach development are proposed based on application of so-called method of trajectory analysis of behavior of target-driven systems, described by dynamics of integral indicator characterizing object of study in coordinates of rate and acceleration of its changes.

Keywords: Critical infrastructure, Dynamic corridor, Difficulty of achieving target, Resilience criterion, Risk, Reference vectors.

1. Introduction
The term "resilience" began to be used in scientific publications in the late 1990s. Initially it was applied in rather narrow areas of psychology and ecology and was primarily related to recoverability (rehabilitation) of a person or natural ecosystem after stressful situation. The following decade there was a significant increase in the volume of publications where this term was used. For example, according to Fraccascia et al. (2018), 93 articles on ecology (1998-2017), 53 articles on technical sciences (2007-2017), 16 articles on business and management (2009-2017) were published in indexed scientific magazines in different periods, where the term "resilience" was applied. In recent years (2015-2019), frequency and areas of use of this term have increased significantly.

Analysis of definitions given in number of works, for example, Malinetskiy and Kochkarov (2005) and Hosseini et al. (2016), allows to identify as main features of "resilience" of object or system: ability to resist external negative effects; ability to recover in acceptable time interval and reasonable costs (material, financial, psychological, etc.).
A separate direction of research is connected to justification of quantitative measure (criterion) of resilience. As rightly noted in the work Bouncing (2013), a criterion should consider the following properties: technical and economic efficiency; stability of development; safety of social, economic and environmental components. The need to use a quantitative measure of resilience is connected to the need to solve tasks of assessing the adequacy of resilience level of the system(s), choosing the option of systems (territories) development taking into account acceptable resilience level, etc.

2. Overview of Existing Approaches to Resilience Level Assessment

The analysis of publications allows to make out three approaches to assess the resilience level of system or territory:
- approach using functional indicators in absolute or relative value as a criterion;
- approach using index criterion(s);
- combined approach (combining functional and index criteria).

The approach using functional indicators is usually used for assessment of resilience of individual system. For such life support systems as electric power system, gas supply system, heat supply system as resilience level you can use power, gas and heat ratios before the accident and after the restoration Francis and Bekera (2014). For water supply system, such indicator as water pressure in the system prior to accident is used Cassottana et al. (2019). For interacting power supply and water supply systems, as a criterion it is proposed to use the ratio of post-recovery system productivity to total losses during the recovery period Jin-Zhu and Baroud (2019).

Rather detailed analysis of methods for quantity-related assessment of resilience for transport system is done in the work of Sun et al. (2019). It is also proposed to take into account functional and social and economic aspects of resilience, and as a criterion for assessment to use such indicators as pre-disaster and post-recovery system productivity, recovery duration, social and economic losses during recovery.

Functional indicators are proposed to use for cyber system resilience at Cyber resilience metrics, measures of effectiveness and scoring. The criterions are connected with: assess level of performance before, during and after disruption; assess time between disruption, detection, response and recovery.

The indicator (grade) approach is often used when assessment of resilience is performed for several interacting systems. In Khazai et al. (2015), it is proposed to use Disaster Resilience Index (DRI). This index was developed as a self-assessment tool with ranking corresponding to five target levels for achieving resilience indicators of urbanized territory. It is proposed to consider five levels of DRI:

1) lack of awareness or absence of awareness,
2) awareness of needs,
3) engagement and commitment,
4) engagement in policy and decision making, and
5) full integration.

Technical Report (2013) proposes to use three indices to assess three phases of development of accidents or disasters (pre-accident, accident, post-accident). The first index relating to pre-accident phase considers elements characterizing physical safety, safety management, security forces,
exchange of information and external environment on safety. The second index describes sustainability of critical infrastructure and possibility of mitigating maximum impacts. This index is key indicator of efficiency that characterizes sustainability of object (system) and can be used to decide of critical infrastructure owners/operators by comparison with similar objects. This index is applicable to all types of critical infrastructure sectors/subsectors and considers all types of hazards (anthropogenic, natural, and cyber), dependencies and capabilities of objects with regard to emergency management. The third index is connected to assessment of consequences, characterizes maximum consequences related to possible adverse events at object (system). This index includes information on health and safety of population, economic, psychological, and managerial consequences and consequences resulting from loss of object or part of system.

Two approaches to assessing resilience level are described in Guideline (2019). In the first method, sustainability cycle is considered as a conceptual model. Indicators are used for indirectly measurement of stability in each phase, that is, without considering the type of recovery curve. Current indicator values (at the time of measurement) are indicators grades that aggregate in six-level hierarchical model, ending with assessment of general level of resilience of critical infrastructure or entire area (such as a city). The second method is direct measurement (modelling/prediction) of the shape of recovery curve or some characteristics of the curve called "macroindicators" (e.g. maximum loss of functionality, downtime, etc.) this method is used for stress testing by comparing recovery curve with acceptable level of stability.

In the work of IMPROVER D2.2 Report of criteria for evaluating resilience (2016), they consider indicative approach to the assessment of resilience considering dynamic component. It is proposed to assess corresponding indices for extended set of phases of life cycle of resilience triangle (Risk assessment, prevention, preparedness, warning, response, recovery, leaning) and to perform convolution of these indices to assess integral level of resilience for the whole cycle.

Several works discuss index approaches to assessment of resilience of critically important systems. For example, in the article of Rehak et al. (2019) they propose an approach to assessment of resilience of the system as arithmetic mean of resilience level of objects (elements) included in the system. Therein resilience of elements is defined as sum of values weighted on 5-grade scale of such components of resilience as Robustness, Adaptability, Recoverability.

A set of indicators for urban resilience assessment is presented in Sharifi and Yamagata (2016). Selected indicators cover multiple dimensions of urban resilience. They are divided into five main categories, namely, materials and environmental resources, society and well-being, economy, built environment and infrastructure, and governance and institutions. It is argued that resilience indicators should be used to help planners understand how best to enhance the abilities to plan/prepare for, absorb, recover, and adapt to disruptive events. The authors proposed a matrix to relate resilience indicators with the main underlying characteristics of urban resilience that are namely, robustness, stability, flexibility, resourcefulness, redundancy, coordination capacity, diversity, foresight capacity, independence, connectivity, collaboration, agility, adaptability, self-organization, creativity, efficiency, and equity.

Another approach includes collective usage of functional and index indicators to assess resilience level. For example, the work of Kilanitis and Sextos (2019), in connection with transport system in combination with natural hazards (earthquake), jointly considers the system of functional indicators and integral index of resilience of territory prone to earthquakes. As functional indicators relative
economic losses are used, relative losses. Related to loss of accessibility of vital facilities (hospitals, power plants, ports, administrative buildings, etc.) relative ecological damage.

Rather interesting approach was presented in Pagano et al. (2018). TOSE-approach based on combination of four dimensions of resilience (technical, organizational, social, ecological) and four performance criteria (robustness, redundancy, resourcefulness, rapidity). The result of the combination are 16 indicators, including functional and qualitative indicators.

Despite such a wide range of interpretations, the concept of resilience in the most general sense characterizes effectiveness of target-driven system in achieving a set target with minimal deviations caused by negative effects, from optimal trajectory with given amount of available resources and requirements for the final result.

Indeed, since efficiency characterizes processes and impacts of a purely managerial nature and reflects, above all, the degree of achievement of set targets, so only target-driven interaction is effective. Assessment of effectiveness is in development of so-called assessment judgment regarding the solution of defined problems over system (set targets of activities) by existing staff of performers and under condition of presence of temporary and material resources enough for this.

Formalized representation of such judgment in our opinion is resilience. From this point of view, works of the group of Voronezh mathematicians Russman et al. (1991) are breakthrough in development of apparatus for analyzing the behavior of target-driven systems. They have introduced into scientific circulation the concept of "difficulties of achieving target" $d_k$, which is assessed with existing assessments of resource quality ($\mu_k$) and requirements for this quality ($\varepsilon_k$).

3. Assessment of Resilience using Difficulties Apparatus

Let us assume that in some time the system needs to achieve a result, the quantitative expression of which is the value at point C. At the same time, it is known that there is a minimum rate of production of the result in time and maximum rate. It is important to find points of control over the state of object (system), which may be moments of time when it is necessary to decide on the control effect or revision of target parameters.

If during movement the object (system) enters the hatched area (Figure 1), it will not be possible to achieve the target at specified time. So, this area becomes forbidden, and its approach should be considered as threat of failure to perform the task. Control of object must be organized in such a way that it is possible to intervene in time in the activity of object (system) if its condition approaches dangerous zone.

For quantitative value of resilience, we take the risk of failure to achieve target $R$ (Figure 1).

Geometric interpretation of system movement can serve to characterize the reliability of achieving target. The task of monitoring system is to assess the current state of system, as well as its proximity to critical area, from where it will not be possible to achieve the target at any permissible costs. Thus, at the same time, such control system can also serve as a rescheduling system (operational management decisions).

Under assumptions made, resilience characterizes probability of achieving target, i.e. probability that the system will return to state of negative impact, considering resource constraints.
Graphically, this probability is taken as ratio of length of section of possible rates to length of section of acceptable rates (moving with which constantly, it is possible to achieve the target not later than assigned term).

![Figure 1. Main calculated ratios of Russman method](image)

For general reasons Russman supposed that the difficulty $d_k$ obtaining result should have the following basic properties:

If $\mu_k = \varepsilon_k$ is maximum, i.e. equal to one (indeed, the difficulty of obtaining result is maximum at the lowest permissible value of quality);

If $\mu_k = 1$ and $\mu_k > \varepsilon_k$ is minimal, i.e. equal to zero (at the highest possible quality value regardless of requirements (if $\varepsilon_k < 1$) the difficulty should be minimal);

If $\mu_k > 0$ and $\varepsilon_k = 0$ is minimal, i.e. equal to zero (obviously, if there are no demands to quality of resource component and $\mu_k$ is greater than zero, the difficulty of obtaining result for this component should be minimal).

Here, $\mu$ is a dimensionless assessment of quality of some resource; $\varepsilon$ is the lower limit of resource quality requirements. Therein $d \in [0,1]$ if $\mu \geq \varepsilon$.

Three-abovementioned conditions if $\varepsilon_k < \mu_k$ the function of the following form satisfies:

$$d_k = \frac{\varepsilon_k(1-\mu_k)}{\mu_k(1-\varepsilon_k)}$$  \hspace{1cm} (1)

We consider also that $d_k = 0$ if $\mu_k = \varepsilon_k = 0$ and $d_k = 1$ if $\mu_k = \varepsilon_k = 1$.

We will illustrate practical application of Russman’s method on the example of analysis of some indicator dynamics. Values (conditional values) of indicator $\Psi^k$ for the current year in which
assessment is carried out and the preceding year are given in Table 1. Rate of indicator value change is calculated on formula $V = \frac{\Psi_{t+1}^k}{\Psi_t^k}$, where $V_{\text{max}} = 1,8500$, and $V_{\text{min}} = 0,6507$. Accordingly, the inclination angles of rate vectors to abscissa axis are 61.60 and 330.

Table 1. Conditional values of indicator

<table>
<thead>
<tr>
<th>No.</th>
<th>Value of target indicator</th>
<th>Rate change of target indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.950</td>
<td>0.940</td>
</tr>
<tr>
<td>2</td>
<td>0.954</td>
<td>0.960</td>
</tr>
<tr>
<td>3</td>
<td>0.950</td>
<td>0.910</td>
</tr>
<tr>
<td>4</td>
<td>0.960</td>
<td>0.850</td>
</tr>
<tr>
<td>5</td>
<td>0.980</td>
<td>0.982</td>
</tr>
<tr>
<td>6</td>
<td>0.965</td>
<td>0.980</td>
</tr>
<tr>
<td>7</td>
<td>0.982</td>
<td>0.930</td>
</tr>
<tr>
<td>8</td>
<td>0.940</td>
<td>0.840</td>
</tr>
<tr>
<td>9</td>
<td>0.968</td>
<td>0.974</td>
</tr>
<tr>
<td>10</td>
<td>0.985</td>
<td>0.940</td>
</tr>
<tr>
<td>11</td>
<td>0.991</td>
<td>0.990</td>
</tr>
<tr>
<td>12</td>
<td>0.650</td>
<td>0.999</td>
</tr>
<tr>
<td>13</td>
<td>0.999</td>
<td>0.890</td>
</tr>
<tr>
<td>14</td>
<td>0.920</td>
<td>0.840</td>
</tr>
<tr>
<td>15</td>
<td>0.925</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Let also as target value of indicator $\Psi^k$ in 2 years from the moment of system movement start to the target the value 0.96 is established. The current average value of $\Psi^k$ is 0.941.

Figure 2 shows the result of control area construction and indicates target (C) and current (M) point, which corresponds to values of indicator $\Psi^k$.

![Figure 2. Calculated example. Initial state](image)
For the situation given in Figure 2, according to the method proposed by Russman, we have:

\[ \varepsilon_1 = \frac{|E_1E_2|}{|E_1E_3|} = 0.2219; \quad \varepsilon_2 = \frac{|F_1F_2|}{|F_1F_3|} = 0.2151; \]

\[ \mu_1 = \frac{|E_1M|}{|E_1E_3|} = 0.4813; \quad \mu_2 = \frac{|F_1M|}{|F_1F_3|} = 0.7384; \]

\[ d_1 = \frac{\varepsilon_1(1 - \mu_1)}{\mu_1(1 - \varepsilon_1)} = 0.3074; \quad d_2 = \frac{\varepsilon_2(1 - \mu_2)}{\mu_2(1 - \varepsilon_2)} = 0.0971. \]

Assessment of resilience of the system will be obtained as follows:

\[ R_{t+1} = \max \left\{ \ln \left( \frac{1}{1 - d_1} \right); \ln \left( \frac{1}{1 - d_2} \right) \right\} = 0.3673. \]

In this way, we assessed the distance from the current point to limits of control loss (where the rate of change is less than minimum for all observations and where rate must take negative values to return the system to controlled state, to target area). Value \( R_{t+1} = 0.3673 \) quantifies resilience of the system and reflects the risk that target indicator will not be reached in two years from the moment the situation is assessed.

Let us suppose now that a year later, in \( t + 2 \) year, an average value of \( \Psi^k = 0.946 \) was obtained. The situation is illustrated in Figure 3 (it is assumed that rates of change of the indicator remained the same).

Figure 3. Calculated example. Critical condition of the system
For this situation:

\[ \varepsilon_1 = \frac{|E_1E_2|}{|E_1E_3|} = 0.4247; \quad \varepsilon_2 = \frac{|F_1F_2|}{|F_1F_3|} = 0.2499; \]

\[ \mu_1 = \frac{|E_1M|}{|E_1E_3|} = 0.5338; \quad \mu_2 = \frac{|F_1M|}{|F_1F_3|} = 0.3528; \]

\[ d_1 = \frac{\varepsilon_1(1 - \mu_1)}{\mu_1(1 - \varepsilon_1)} = 0.6448; \quad d_2 = \frac{\varepsilon_2(1 - \mu_2)}{\mu_2(1 - \varepsilon_2)} = 0.6110, \]

and, respectively,

\[ R_{t+2} = \max \left\{ \ln \frac{1}{1 - d_1}; \ln \frac{1}{1 - d_2} \right\} \cong 1. \]

The value \( R_{t+2} \cong 1 \) shows that if rates of change of the indicator remain, in a year it will not be possible to reach target value. It can be seen (in Figure 3) that the point \( M \), which indicates the current position of the indicator, has approached the dangerous boundaries, \( AD_1C \), beyond which it is highly probable that the controllability and target setting may not be fulfilled, i.e. failure to return to the specified development path. The current situation requires the attention of decision maker and corrective measures.

For comparison, let us consider the third situation, when in \( t + 2 \) year the value of the indicator increased significantly (Figure 4).

Figure 4. Calculated example. Operated condition of the system
For this situation, respectively:

\[
\begin{align*}
\varepsilon_1 &= \frac{|E_1E_2|}{|E_1E_3|} = 0.3633; \quad \varepsilon_2 = \frac{|F_1F_2|}{|F_1F_3|} = 0.1205; \\
\mu_1 &= \frac{|E_1M|}{|E_1E_3|} = 0.7878; \quad \mu_2 = \frac{|F_1M|}{|F_1F_3|} = 0.4137; \\
d_1 &= \frac{\varepsilon_1(1 - \mu_1)}{\mu_1(1 - \varepsilon_1)} = 0.1537; \quad d_2 = \frac{\varepsilon_2(1 - \mu_2)}{\mu_2(1 - \varepsilon_2)} = 0.1942; \\
R_{t+2} &= \max\left\{\ln\frac{1}{1 - d_1}; \ln\frac{1}{1 - d_2}\right\} = 0.2159.
\end{align*}
\]

The value of \(R_{t+2} = 0.2159\) in such development (it is again assumed that rates of change of the indicator remain the same) shows that if rates of change of the indicator keep safe, in a year the target value is quite achievable - the risk has decreased, the point characterizing the position of the indicator is almost on the optimal trajectory.

Obviously, each target-driven system must also operate in a rapidly changing environment. The logic of circumstances often turns out to be stronger than the logic of intent and as a result, the system ceases to demand optimal values of "output" parameters, satisfying permissible ones, efficiency loses connection with optimality and becomes increasingly connected with guarantee and reliability. Reliability can be considered both as a property of the system functioning process, determining its compliance with a certain norm, and as a property characterizing the system from the point of view of possibility of achieving the set target.

The difficulty of achieving targets is considered as a variable value, inverse to resilience, and representing itself a function relative to the current position of the indicator: it increases as the value of the indicator approaches some permissible boundaries, after which it is impossible to reach the target value.

Specified approach to creation of the system for monitoring the movement of object to given target (for example, restoration of functioning, trajectory of development of the system after negative impact) allows to solve the following two management tasks:

- the task of monitoring the "movement" of the indicator along the path to the set target (including determining points of intermediate control of values of the indicator depending on efficiency of the main task);
- the task of determining the permissible level of resilience exceeding which is undesirable.

Optimal trajectory, which will be characterized by minimum of "difficulty in achieving target" and, accordingly, maximum of efficiency of the system in achieving the target, is the diagonal of parallelogram of permissible values of the indicator (direct line \(AC\) in Figures 2-4). The task of control is to keep the system on the optimal path.

This task is not as simple as it may seem at first glance.
First, monitoring (and, respectively, control) is exercised on values of integrated indicator and as it is noted, for example, by Bochkov (2018), Bochkov and Zhigirev (2018) and Bochkov et al. (2019) finished and confirmed with practice methodology of creation of such indicator for systems of different type is not developed now.

Second, Russman’s method does not allow you to track in advance changes in external factors that affect the system’s achievement of the target value of selected integral indicator. Therefore, it is impossible to predict the expected deviation of the indicator from the optimal path of movement to the next point of control. And, the most important thing, it is impossible to say how critical this deviation will be, i.e. if the system still will be able to achieve the target after returning to its original state?

It is possible to solve these problems by introducing another, somewhat modified "target" - a permissible area of change of the indicator - built in coordinates of rate and acceleration of change of its values.

4. Trajectory Analysis Method in Assessment of Resilience
Within so-called trajectory analysis Bochkov (2018), it is proposed to represent intensity of change in specific value of selected indicator with function of time $y(t)$. Its equivalent in physics is instantaneous rate. Then for any period of time $[t_1, t_2]$ the value of the indicator will be defined as $\int_{t_1}^{t_2} y(t) dt$.

As a basis, in the limiting case, we can take three periods: $I_0$ - specific value of the indicator expected (declared, but not actual) for the current period; $I_{-1}$ specific value of the indicator for the previous period (actual, documented by acts) and $I_{-2}$ specific value of the indicator two periods ago (from the current one).

Formally, you can write these three integrals as:

$$
\int_0^1 y(t) dt = I_0; \quad \int_{-1}^{0} y(t) dt = I_{-1}; \quad \int_{-2}^{-1} y(t) dt = I_{-2}.
$$
(2)

Then for each compared system (or considered versions of one system) a trajectory of change of its "capacity" $y(t)$ is built. In the simplest case, the law of changing $y(t)$ values has the form of uniformly accelerating (uniformly slowing) movement:

$$
y(t) = y_0 + v \times t + a \times \frac{t^2}{2},
$$
(3)

where, $y_0$ is assessment of specific value of the indicator at the beginning of measurements in the current period; $v$ is average growth rate of $y(t)$; $a$ is average acceleration (rate of change of value $v$).

If $v > 0$, this means that (according to assessments $I_0$, $I_{-1}$, $I_{-2}$) specific change in target value is increasing; if on the contrary $v < 0$, it falls; if $v = 0$, it is maintained at the same level during three periods in question. Similarly, the acceleration indicator $a$ is interpreted: the value of $a > 0$ means that the system "on take-off" - the rate of change of target indicator value at the end of analyzed three-step period grows faster than upon the average; the value of $a < 0$ may mean that although
the rate of change of target value increases, the rate gain decreases. Formally, we can get an accordance between three values of \( y_0, v, a \) and three values of \( I_0, I_{-1}, I_{-2} \). Substituting (3) in (2) we get:

\[
\begin{align*}
I_0 &= y_0 + \frac{v}{2} + \frac{a}{6}; \\
I_{-1} &= y_0 - \frac{v}{2} + \frac{a}{6}; \\
I_{-2} &= y_0 - \frac{3v}{2} + \frac{7a}{6}.
\end{align*}
\]  

(4)

Solving three equations (4) with three unknowns, define values of \( y_0, v, a \):

\[
\begin{align*}
y_0 &= \frac{2I_0 + 5I_{-1} - I_{-2}}{6}; \\
v &= I_0 - I_{-1}; \\
a &= I_0 - 2I_{-1} + I_{-2}.
\end{align*}
\]  

(5)

The values of \( y_0, v, a \) vary from system to system and make sense of "average" assessments of their trajectory over three periods under consideration. Since these indicators smooth "seasonal" and other diffusion factors, it is desirable to normalize them. When displaying general characteristics, group indicators are interesting firstly, and then relative position of values of each system relative to group indicator. We will introduce two similar indicators. The first, relative acceleration factor \((q_a)\), is calculated as ratio of sum of target value change in the current \((I_0)\) and pre-previous period \((I_{-2})\) to target value change in the previous period \((I_{-1})\). Changes of target value for the previous period \((I_{-1})\) are selected because this is the closest assessment that can be confirmed by the work

\[
q_a = \frac{I_0}{I_{-1}} + \frac{I_{-2}}{I_{-1}} = \frac{a}{I_{-1}} + 2.
\]  

(6)

From (6) it can be seen that \( q_a \) is "fail" measure in changing target value in the previous \((I_{-1})\) \((q_a >> 2)\) or current \((I_0)\) \((q_a << 2)\) periods. That is, indirectly \( q_a \) factor is a factor for assessing trend instability (at large deviations from 2) and, on the contrary, trend stability indicator for values of \( q_a \) close to 2. A priori, it can be considered that instability of \( q_a \) and downward trend are "suspicious," as they are related to reorganization of the system and it is necessary to "find out" whether the system in its new state is successor to qualification and other potentials of predecessor. Besides, a steady "fall" \((q_a \approx 2)\) with negative trend is not exactly what is needed for development, it is important to know the value of the second factor.

Second factor \((q_v)\) shows relative rate of growth of system target over the last two periods:

\[
q_v = \frac{I_0}{I_{-1}} = \frac{v}{I_{-1}} + 1.
\]  

(7)

This indicator is naturally correlated with the first indicator, being part of it. This indicator has a slightly weaker adequacy. In order to visualize change in possible value of factors depending on changes in the state of the system, it is useful to move from indicators of \( q_a \) and \( q_v \) through affine transformations to a pair of related normalized indicators - \( F_a \) and \( F_{va} \), which define geometric image, which will then be called a "target."
First factor $F_a$ is simply normalized factor $q_a$

$$F_a = \frac{q_a - c_a}{L_a}, \quad (-1 \leq F_a \leq 1). \quad (8)$$

Here: $c_a$ is the center of the target along abscissa axis, and $L_a$ is the size of half side of the square of section. Normalized factor $F_{va}$ is a linear combination of acceleration $q_a$ and rate $q_v$ factors:

$$F_{va} = \frac{q_a - c_a - \mu(q_v - c_v)}{L_{va}}, \quad (-1 \leq F_{va} \leq 1). \quad (9)$$

The effect that rate increases with acceleration is taken into account by the fact that this "additive" is subtracted with a coefficient of $\mu > 0$, which is determined, like constants $c_a, c_v, L_a, L_{va}$, on the basis of analysis of group data. Geometrically, the position of square target ($F_a \times F_{va}$) of size $[-1,+1] \times [-1,+1]$ on the plane of factors $q_a$ (abscissa) and $q_v$ (ordinate) will appear as parallelogram ABCD (Figure 5).

![Figure 5. Appearance and parameters of target](image)

Point $A$ corresponds to point $\{F_a = -1; F_{va} = +1\}$ Point $B$ corresponds to point $\{F_a = -1; F_{va} = -1\}$. Point $C$ corresponds to point $\{F_a = +1; F_{va} = -1\}$. Point $D$ corresponds to point $\{F_a = +1; F_{va} = +1\}$. Because transformations are affine, it is obvious that the center of the target with coordinates $(c_a; c_v)$ - the point of intersection of diagonals corresponds to the center of square target, coordinates $\{F_a = 0; F_{va} = 0\}$.

Figure 5 shows that the value of the coefficient $\mu > 0$ is equal to the tangent of the side inclination angle ($DA$) to the ordinate axis. According to the group analysis, size and inclination of the target were determined using modified method of reference vectors. In principle, the target does not have to be a parallelogram, it can be arbitrary polygon according to reference vector method.
Knowing the "reference" path passing through the center of the target, it is possible to assess as a percentage the level of "maximum non-optimality" of paths of the system $\psi$ in the form of "overload" or "underload" to the standard if it were given the opportunity to implement the "optimal loading scenario" in the current period.

$$\psi = \frac{I_0 - I_0^*}{I_0^*}.$$ (10)

Closer than this relative value (percentage), the system will not approach the optimum. Calculating values of deviations of the system path from the optimum in the current period $\delta_0$ and assessing the optimal "expected" deviation of $\delta_0^+$ in the future period (assuming that it is not necessary to "shift" the target)

$$\delta_0 = \frac{l_0 - l_0^*}{l_0^*} - \Theta^*;$$

$$\delta_0^+ = \frac{l_1 - l_0^*}{l_0^*} - \Theta^*$$ (11)

it is possible to produce indicator of "progress" of the system $\xi$ as difference of the above assessments normalized through logistics curves. The type of logistics curves is selected because remaining "leader" is also progress, and a slight improvement of "outsider" is better not to consider significant progress.

Let us illustrate the construction of the above-described target. Figure 6 shows the example of distribution of points on plane $v, a$.

![Figure 6. Area of maximum concentration of positive decisions](image)

Small sample size makes statistical approach to classification difficult. The very possibility of successful classification by complexity of a non-previous criterion in selection requires additional justification and research.

Figure 6 shows that linear separation of positive (black points) and negative (red points) solutions is impossible, since negative solutions are within area of positive ones, even if one strongly spaced
point of positive solution is neglected. As one solution, it was intended to divide using reference vector method, where straightening space is specified using Gaussian kernel.

The result of this automatic classification of objects is shown in Figure 7. The Figure shows that positive area has a ring topology and is not well suited for SVM clustering.

Thus, the task of finding boundaries of positive decision area was reduced to the problem of minimizing errors of the first kind (negative decisions are wrongly recognized as positive) with fixed value of errors of the second kind. The view of positive solutions area (parallelograms) has been specified. Considering this a priori information, the task was divided into parts: first there were left and right boundaries of the parallelogram, then its upper and lower boundaries.

![SVM classification plot](image)

Figure 7. SVM classification, where straightening space specified using Gaussian kernel

To find the right border, based on the task conditions, the function was formed:

\[
f(x) = \begin{cases} 
\sum_i \delta_{\{\text{neg, } x < x_i\}} / N, & \text{если } x \geq \max(\forall x_{\text{pos}}) \\
0, & \text{если } x < \max(\forall x_{\text{pos}})
\end{cases}
\]

where, \(\delta_{\{\text{neg, } x < x_i\}}\) is indicative function that takes 1 or 0, depending on whether or not conditions in the braces are correct, \(N\) is the total number of negative examples. A similar (sign-accurate) function was formed to find the left border. Maximum function \(f(x)\) was the solution for the right (left) border of the area (Figure 8).
The problem of finding the upper and lower boundaries of positive solutions area is to find maximum of the function $f(x)$ when the coordinate system is rotated. This task belongs to the class of multi-variable non-linear extreme tasks with constraints. A genetic algorithm was used as a method of solution. Advantages of this algorithm lie in its ability to simultaneously manipulate many parameters, resistance to the ineligibility of optimized function, and applicability in tasks with a changing environment.

The result of code working written in R language is shown in Figure 9.

In order to determine the identity of the point characterizing the current position of the parallelogram system (or any broken one) limiting the area of stable paths, it is sufficient to
calculate Cauchy integral and compare it with the value of function at the point under test. If they coincide, the point lies in the contour. According to Cauchy's integral theorem, if the point does not lie in the contour, i.e. sub-integral expression does not turn to infinity anywhere, then the integral is zero. Thus, it is enough to check the equality of the integral to zero: equal to zero - the point outside the contour, different from zero - the point lies in the contour.

It is important to understand that described "target" model considers trajectory of indicators of target-driven system without considering the "diffusion" components. Therefore, it is only the basic element for increasing sets and indicators for the future branched system of monitoring the behavior of the system as it moves towards the target value. It is expected that assessments of the current state, and much more assessments of "progress" on different indicators, will partially contradict each other. In this case, it is necessary to treat indicators and parameters as elements of decision-making logic, to carry out analogue-digital conversion of parameters into logical variables, to build a "new" logic for clearing contradictions.

Besides that, in addition to understanding that the point characterizing the state of the system lies in stability contour, for assessment of resilience it is essential to understand how far from the boundaries of stability contour (built "target") the deviations of the parameter are and what factors (signs) of the current situation affect this position. This question requires development of appropriate computational methods.

5. Conclusions
Proposed combination of methods of analysis of the current state of target-driven system makes it possible to solve tasks of assessing the efficiency of its functioning aimed at achieving the target and quantifying its resilience from system position. Indeed, the entering of the indicator characterizing stable movement of the system to the target the area of permitted rate values, which guarantee performance of the task, allows to monitor optimal trajectory of movement and to set periodicity of the system state control. In turn, the entering of the same parameter the area of permitted combinations of rate and acceleration of its change guarantees reversibility of deviation of this parameter from optimal trajectory. In fact, this area is a corridor of stable paths, inside which, despite possible deviations, the system will achieve the target.

The future direction of research is connected with development of methods for removing the above mentioned limitation of the proposed approach in the definition of critical from the point of view of sustainability the trajectory of the system describes its provisions as indicators. As you know, reliable performance of the system's obligations is characterized by the preservation of certain specified characteristics within the established limits. In practice, it is impossible to completely avoid deviations when moving towards the goal, but, as already mentioned, it is necessary to strive to minimize deviations of the current state from a certain set ideal – a goal set, for example, in the form of values of certain indicators.

Considering the measure of the threat of failure to achieve the set values of indicators as a variable that is a function of the current position of the system (it increases when the estimated situation approaches a certain acceptable limit, after reaching which the system cannot fulfill its obligations and achieve the corresponding set target values of indicators) – it is possible to determine how far from the borders of the corridor of stable trajectories is the indicator that characterizes the system at the current time and which factors included in the description of this indicator have the greatest influence on its current position.
In addition, since the true distribution laws of the analyzed random processes and, most importantly, the factors that determine them, will be continuously adjusted (any high-tech system changes faster than adequate statistics accumulate), it is necessary to use the criteria “free from distributions”.

In particular, for example, the criteria for achieving a predictive goal should not be the values of deviations in model and real data, but the criteria used in classification and pattern recognition methods. For example, as a measure of forecast accuracy, you can use the values of prediction errors of the first and second types for different classes and types of situations, and, if possible, depending on the classes of the physical object and depending on the value of the forecast background parameters. The second circumstance is very important, because, for example, it is incorrect to add accident statistics for different seasons of the year, since technological processes occur differently in different seasons.

**Conflict of Interest**
The authors confirm that there is no conflict of interest to declare for this publication.

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