

Atangana-Baleanu and Caputo-Fabrizio Analysis of Fractional Derivatives on MHD Flow past a Moving Vertical Plate with Variable Viscosity and Thermal Conductivity in a Porous Medium

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Abstract

In this paper, a numerical investigation is presented for non-integer order derivatives with Atangana-Baleanu (AB) and Caputo-Fabrizio (CF) fractional derivatives for the variable viscosity and thermal conductivity over a moving vertical plate in a porous medium two dimensional free convection unsteady MHD flow. The effects of radiation have also been considered. The governing partial differential equations along with the boundary conditions are changed to ordinary form by similarity transformations. Hence physical parameters show up in the equations and interpretations on these parameters can be achieved suitably. By using ordinary finite difference scheme the equations are discretized and developed in fractional form. These discretized equations are numerically solved by the approach based on Gauss-seidel iteration scheme. Some numerical strategies are used to find the values of AB and CF approaches on time by developing programming code in MATLAB. The effects of all the physical parameters involved in the problem on velocity, temperature and concentration distribution are compared graphically as well as in tabular form. The effects of each parameter are found to be prominent. We have observed a significant variation of values under different parameters using AB and CF approaches on velocity, temperature and concentration distribution with respect to time.

Keywords- AB and CF derivatives, Viscosity, Thermal conductivity, Porous medium, Radiation.

1. Introduction

The boundary layer and heat transfer flow of a viscous fluid over a moving vertical plate in a porous medium have been investigated in a number of technological approaches such as warm rolling, metallic extrusion, petroleum industries, polymer extrusion, wires drawing and metallic spinning. Natural convection flows driven by temperature differences are of great interest in a number of industrial applications. Bejan and Khair (1985) studied free convection heat and mass transfer in a porous medium. In recent there has been a growing interest in studying the combined application of MHD flow and porous media. Aldoss et al. (1995) investigated combined free and forced convection flow from vertical plate in a porous medium in the presence of magnetic field. Hossain and Munir (2000) analysed a two dimensional mixed convection flow of a viscous incompressible fluid of temperature dependent viscosity past a vertical plate.

Javaherdeh et al. (2015) studied the natural convection heat and mass transfer in MHD fluid flow past a moving vertical plate with variable surface temperature and concentration in a porous medium. However, the impact of variable viscosity and thermal conductivity of a MHD free convection flow past a vertical plate embedded in porous medium has received a little attention. Mukhopadhyay and Layek (2008) presented the effects of variable viscosity through a porous medium over a stretching sheet in presence of thermal radiation.

Recently, fractional calculus has gained tremendous popularity among the researchers because of singular kernel with locality and non-singular kernel with non-locality problem. Caputo and Fabrizio used an exponential function in fractional derivative to avoid the singular kernel problem. Mirza and Vieru (2017) analyzed that the use of the time-fractional derivative without singular kernel is more advantageous than Caputo time-fractional derivative. Nehad et al. (2016) applied CF fractional derivatives to analyze the solutions for heat transfer of second grade fluids over vertical oscillating plates. “A comparative study has been obtained by using the AB and CF fractional derivatives for casson fluid model with chemical reaction and heat generation” by Sheikh et al. (2017). Few works using fractional derivatives have been studied in Histrov (2017), Atangana and Baleanu (2016). A comparative study of Atangana-Baleanu and Caputo-Fabrizio evaluation of fractional derivatives for heat and mass transfer of fluid over a vertical plate has been done by Khan et al. (2017).

The main objective of this paper is to investigate the effects of variable viscosity and thermal conductivity over a moving vertical plate in porous medium and comparing the results with the approaches AB and CF fractional derivatives. The non-dimensional governing equations with the non-dimensional boundary conditions are discretized with ordinary finite-difference kernel solved numerically with the help of AB and CF fractional derivative methods by developing suitable programming code in MATLAB. A comparative analysis under different parameters is represented graphically as well as in tabular form.

2. Mathematical Formulation

Consider a two-dimensional free convection steady heat and mass transfer flow of viscous incompressible electrically conducting fluid past a moving vertical plate in a porous medium (Figure 1). Here a uniform magnetic field is applied in a direction perpendicular to the fluid flow. The \bar{x} -axis is taken along the vertical plate in the direction of the flow and \bar{y} axis is normal to it. At $t = 0^+$, the fluid have gained the temperature \bar{T}_w , and concentration level near the plate is \bar{C}_w . The fluid properties are assumed to be constant except for the fluid viscosity and thermal conductivity which are assumed to vary as an inverse linear function of temperature. A uniform magnetic field of strength B_0 is applied normal to the plate. There is no chemical reaction between the fluid and diffusing species. Plate temperature \bar{T}_w is variable and \bar{T}_∞ is the free stream temperature assumed constant.

The variable viscosity and thermal conductivity are governed by the following equations under the boundary layer approximation:

Equation of Continuity:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

Equation of conservation of momentum:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = g \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \frac{1}{\rho} \frac{\partial \mu}{\partial \bar{y}} \frac{\partial \bar{u}}{\partial \bar{y}} - \frac{\sigma B_0^2 \bar{u}}{\rho} + g\beta_t (\bar{T} - \bar{T}_\infty) + g\beta_c (\bar{C} - \bar{C}_\infty) - \frac{g}{\rho K} \bar{u} + \frac{\bar{F}_m}{\rho} \quad (2)$$

Equation of conservation of energy:

$$\rho C_p \left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = \frac{\partial \lambda}{\partial \bar{y}} \cdot \frac{\partial \bar{T}}{\partial \bar{y}} + \lambda \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 - \frac{\partial \bar{q}_r}{\partial \bar{y}} \quad (3)$$

Equation of concentration:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{C}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = \frac{\partial D_m}{\partial \bar{y}} \frac{\partial \bar{C}}{\partial \bar{y}} + D_m \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (4)$$

The boundary conditions are:

$$\begin{aligned} \bar{t} \leq 0: \bar{u} = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \quad \forall y \\ \bar{t} > 0: \bar{u} = U_0, \bar{T} = \bar{T}_w = \bar{T}_\infty + b\bar{x}^n, \bar{C} = \bar{C}_w = \bar{C}_\infty + a\bar{x}^m \text{ at } \bar{y} = 0 \\ \bar{t} > 0: \bar{u} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \text{ at } \bar{y} \rightarrow \infty \end{aligned} \quad (5)$$

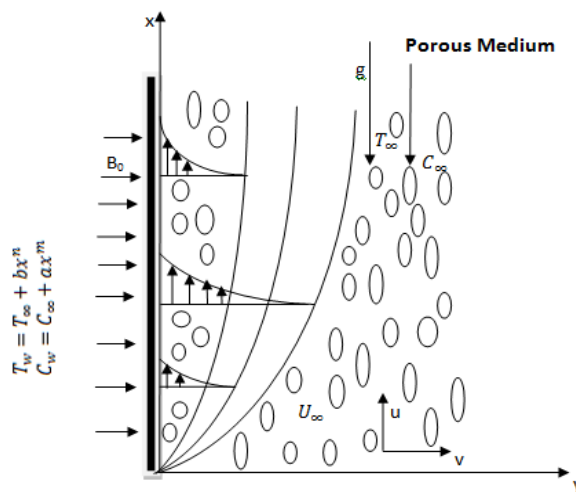


Figure 1. Schematic diagram of the flow.

where, \bar{u} and \bar{v} are the fluid velocities in the direction of \bar{x} and \bar{y} respectively, \mathcal{G} is the kinematic viscosity, \mathcal{G}_∞ is the kinematic viscosity of the fluid in the free stream, ρ is the fluid density, μ is the viscosity of the fluid, σ is the electrical conductivity, g is the acceleration due to gravity, β_t is the volume expansion coefficient for heat transfer, β_c is the volume expansion coefficient for mass transfer, \bar{T} is the fluid temperature within the boundary layer, \bar{T}_∞ is the temperature at free stream of the fluid, \bar{C} is the species concentration of the fluid, \bar{C}_∞ is the concentration at free stream of the fluid, λ is the thermal conductivity of the fluid, C_p is the specific heat at constant pressure, \bar{q}_r is the radiative heat flux, D_m is the mass diffusivity, U_0 is the velocity of the plate, K is permeability of the porous medium, \bar{J} is the electric current density, \bar{a} is the mean absorption coefficient, $\bar{\sigma}$ is the Stefan – Boltzmann constant.

where, $\bar{F}_m = \bar{J} \times \bar{B} = \sigma [\bar{q} \times \bar{B}] \times \bar{B} = \sigma \bar{u} B_0^2$, \bar{q} be fluid velocity at a particular point, $\bar{B} = B_0 \hat{j}$ be the applied magnetic field.

By Rosseland approximation,

$$\text{The radiative heat flux, } \bar{q}_r = -\frac{4\bar{\sigma}}{3\bar{a}} \frac{\partial \bar{T}^4}{\partial \bar{y}} \quad (6)$$

We expand \bar{T}^4 in Taylor's series about \bar{T}_∞ as follows:

$$\bar{T}^4 = \bar{T}_\infty^4 + 4\bar{T}_\infty^3(\bar{T} - \bar{T}_\infty) + 6\bar{T}_\infty^2(\bar{T} - \bar{T}_\infty)^2 + \dots$$

By neglecting the higher order terms beyond 1st degree in $\bar{T} - \bar{T}_\infty$ we have

$$\bar{T}^4 = 4\bar{T}_\infty^3\bar{T} - 3\bar{T}_\infty^4 \quad (7)$$

Using equations (6) and (7), equation (3) reduces to

$$\rho C_p \left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = \frac{\partial \lambda}{\partial \bar{y}} \cdot \frac{\partial \bar{T}}{\partial \bar{y}} + \lambda \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \mu \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right)^2 - \frac{16\bar{\sigma} T_\infty^3}{3\bar{a}} \frac{\partial^2 \bar{T}^4}{\partial \bar{y}^2} \quad (8)$$

Applying the following non dimensional quantities:

$$x = \frac{U_0 \bar{x}}{\mathcal{G}_\infty}, \quad y = \frac{U_0 \bar{y}}{\mathcal{G}_\infty}, \quad u = \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{U_0}, \quad \theta = \frac{\bar{T}_w - \bar{T}}{\bar{T}_w - \bar{T}_\infty}, \quad \phi = \frac{\bar{C}_w - \bar{C}}{\bar{C}_w - \bar{C}_\infty}, \quad t = \frac{U_0^2 \bar{t}}{\mathcal{G}_\infty} \quad (9)$$

According to Lai and Kulacki (1990) the viscosity and thermal conductivity of the fluid are assumed to be inverse linear function of temperature as follows:

$$\frac{1}{\mu} = \frac{1}{\mu_{\infty}} \{1 + \gamma(\bar{T} - \bar{T}_{\infty})\} \quad (10)$$

$$\frac{1}{\lambda} = \frac{1}{\lambda_{\infty}} \{1 + \delta(\bar{T} - \bar{T}_{\infty})\} \quad (11)$$

where, γ and δ are constants which depend on the thermal property of the fluid.

We define two parameters as, $\theta_r = \frac{\bar{T}_w - \bar{T}_r}{\bar{T}_w - \bar{T}_{\infty}}$ is called viscosity parameter and $\theta_c = \frac{\bar{T}_w - \bar{T}_c}{\bar{T}_w - \bar{T}_{\infty}}$ is called thermal Conductivity parameter.

Using these two parameters in (10) and (11), we have the viscosity and thermal conductivity respectively as

$$\mu = \frac{\mu_{\infty}(1 - \theta_r)}{\theta - \theta_r}, \quad \lambda = \frac{\lambda_{\infty}(1 - \theta_c)}{\theta - \theta_c} \quad (12)$$

It is also important to note that θ_r is negative for liquid and positive for gases.

Using the transformations (9) and (12), the non-dimensional forms of (2), (3) and (4) are

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = & \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1 - \theta_r}{(\theta - \theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} \\ & - Mu + Gr(1 - \theta) + Gr_m(1 - \phi) - \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) u K_p \end{aligned} \quad (13)$$

$$\text{Pr} \left(\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} - \frac{n(1 - \theta)}{x} u + v \frac{\partial \theta}{\partial y} \right) = \left(\frac{4}{3} K_r - \frac{1 - \theta_c}{\theta - \theta_c} \right) \frac{\partial^2 \theta}{\partial y^2} - \frac{1 - \theta_r}{(\theta - \theta_r)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 + \left(\frac{1 - \theta_r}{\theta - \theta_r} \right) \text{Pr} . Ec \left(\frac{\partial u}{\partial y} \right)^2 \quad (14)$$

$$\frac{\partial \phi}{\partial t} + u \frac{\partial \phi}{\partial x} - \frac{m(1 - \phi)}{x} u + v \frac{\partial \phi}{\partial y} = - \frac{1 - \theta_r}{(\theta - \theta_r)^2} . Sc \frac{\partial \theta}{\partial y} . \frac{\partial \phi}{\partial y} + \frac{1}{Sc} \left(\frac{1 - \theta_c}{\theta - \theta_c} \right) \frac{\partial^2 \phi}{\partial y^2} \quad (15)$$

The corresponding initial and boundary conditions are transformed to:

$$\begin{aligned} t \leq 0 : u = 0, \quad v = 0, \quad \theta = 0, \quad \phi = 0 \quad \forall y \\ t > 0 : u = 1, \quad v = 0, \quad \theta = 0, \quad \phi = 0 \quad \text{at } y = 0 \\ t > 0 : u \rightarrow 0, \quad v \rightarrow 0, \quad \theta \rightarrow 1, \quad \phi \rightarrow 1 \quad \text{at } y \rightarrow \infty \end{aligned} \quad (16)$$

where, $Ec = \frac{U_o^2}{R_w C_p}$ is the Eckert number, $K_p = \frac{u_\infty^2}{\rho K U_o^2}$ is the Permeability parameter, $Sc = \frac{g_\infty}{D_m}$ is the Schmidt number, $M = \frac{\sigma B_0^2 g_\infty}{\rho U_o^2}$ is the Magnetic field parameter, $Kr = \frac{16\bar{a} g_\infty \bar{\sigma} \bar{T}_\infty^2}{3\lambda_\infty U_o^2}$ is the radiation parameter, $Gr = \frac{g_\infty g \beta_t (\bar{T}_w - \bar{T}_\infty)}{U_o^3}$ is the Grashof Number, $Gr_m = \frac{g_\infty g \beta_c (\bar{C}_w - \bar{C}_\infty)}{U_o^3}$ is the Concentration buoyancy parameter and $Pr = \frac{\rho g_\infty C_p}{\lambda_\infty}$ is the Prandlt number.

2.1 Atangana-Baleanu (AB) Fractional Derivatives

To express AB fractional approach, “The governing partial differential equations can be written with respect to time by the AB fractional operator of the order $0 < \alpha < 1$ ”, equations (13)-(15) become

$$AB \left(\frac{\partial^\alpha u(y,t)}{\partial t^\alpha} \right) = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} + \left(\frac{1-\theta_r}{\theta-\theta_r} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1-\theta_r}{(\theta-\theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} K_p - Mu + Gr(1-\theta) + Gr_m(1-\phi) - \left(\frac{1-\theta_r}{\theta-\theta_r} \right) u \quad (17)$$

$$AB \left(\frac{\partial^\alpha \theta(y,t)}{\partial t^\alpha} \right) = -u \frac{\partial \theta}{\partial x} - v \frac{\partial \theta}{\partial y} + \frac{n(1-\theta)}{x} u + \frac{1}{Pr} \left(\frac{4}{3} Kr - \frac{1-\theta_c}{\theta-\theta_c} \right) \frac{\partial^2 \theta}{\partial y^2} - \frac{1}{Pr} \frac{1-\theta_r}{(\theta-\theta_r)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 + \left(\frac{1-\theta_r}{\theta-\theta_r} \right) Ec \left(\frac{\partial u}{\partial y} \right)^2 \quad (18)$$

$$AB \left(\frac{\partial^\alpha \phi(y,t)}{\partial t^\alpha} \right) = -u \frac{\partial \phi}{\partial x} - v \frac{\partial \phi}{\partial y} + \frac{m(1-\phi)}{x} u - \frac{1-\theta_r}{(\theta-\theta_r)^2} \cdot \frac{1}{Sc} \frac{\partial \theta}{\partial y} \cdot \frac{\partial \phi}{\partial y} + \frac{1}{Sc} \left(\frac{1-\theta_c}{\theta-\theta_c} \right) \frac{\partial^2 \phi}{\partial y^2} \quad (19)$$

where, $\frac{\partial^\alpha u(y,t)}{\partial t^\alpha}$ is the AB fractional operator of order α defined as

$$AB \left(\frac{\partial^\alpha u(y,t)}{\partial t^\alpha} \right) = \frac{1}{1-\alpha} \int_0^t u'(y,t) E_\alpha \left(\frac{-\alpha(z-t)}{1-\alpha} \right) dt \quad (20)$$

and $E_\alpha(-t^\alpha) = \sum_{m=0}^{\infty} \frac{(-t)^{\alpha m}}{\Gamma(1+\alpha m)}$ is the Mittag-Leffler function.

2.2 Caputo-Fabrizio (CF) Fractional Derivatives

To express Caputo-Fabrizio fractional derivatives approach, “The governing partial differential equations can be written with respect to time by the CF fractional operator of the

order $0 < \beta < 1$ ”, equations (13)-(15) become

$$CF\left(\frac{\partial^\beta u(y,t)}{\partial t^\beta}\right) = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} + \left(\frac{1-\theta_r}{\theta-\theta_r}\right) \frac{\partial^2 u}{\partial y^2} - \frac{1-\theta_r}{(\theta-\theta_r)^2} \frac{\partial \theta}{\partial y} \frac{\partial u}{\partial y} - Mu$$

$$+ Gr(1-\theta) + Gr_m(1-\phi) - \left(\frac{1-\theta_r}{\theta-\theta_r}\right) u K_p \quad (21)$$

$$CF\left(\frac{\partial^\beta \theta(y,t)}{\partial t^\beta}\right) = -u \frac{\partial \theta}{\partial x} - v \frac{\partial \theta}{\partial y} + \frac{n(1-\theta)}{x} u + \frac{1}{Pr} \left(\frac{4}{3} Kr - \frac{1-\theta_c}{\theta-\theta_c} \right) \frac{\partial^2 \theta}{\partial y^2} - \frac{1}{Pr} \frac{1-\theta_r}{(\theta-\theta_r)^2} \left(\frac{\partial \theta}{\partial y} \right)^2 + \left(\frac{1-\theta_r}{\theta-\theta_r} \right) Ec \left(\frac{\partial u}{\partial y} \right)^2 \quad (22)$$

$$CF\left(\frac{\partial^\beta \phi(y,t)}{\partial t^\beta}\right) = -u \frac{\partial \phi}{\partial x} - v \frac{\partial \phi}{\partial y} + \frac{m(1-\phi)}{x} u - \frac{1-\theta_r}{(\theta-\theta_r)^2} \cdot \frac{1}{Sc} \frac{\partial \theta}{\partial y} \cdot \frac{\partial \phi}{\partial y} + \frac{1}{Sc} \left(\frac{1-\theta_c}{\theta-\theta_c} \right) \frac{\partial^2 \phi}{\partial y^2} \quad (23)$$

where, $\frac{\partial^\beta u(y,t)}{\partial t^\beta}$ is the CF fractional operator of order β defined as

$$CF\left(\frac{\partial^\beta u(y,t)}{\partial t^\beta}\right) = \frac{1}{1-\beta} \int_0^t u'(y,t) \text{Exp}\left(\frac{-\beta(z-t)}{1-\beta}\right) dt \quad (24)$$

3. Solution of the Problem

Solutions of equations (17) – (20) or (21) – (24) are obtained by using ordinary finite difference scheme. Discretization is performed using the following formulae:

$$\frac{\partial u}{\partial t} = \frac{u_{i+1,j,k} - u_{i,j,k}}{\Delta t}, \quad \frac{\partial u}{\partial x} = \frac{u_{i,j+1,k} - u_{i,j,k}}{\Delta x}, \quad \frac{\partial u}{\partial y} = \frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta y}, \quad \frac{\partial^2 u}{\partial y^2} = \frac{u_{i,j,k+1} - 2u_{i,j,k} + u_{i,j,k-1}}{\Delta y^2}$$

etc.

The fractional derivatives given by (20) or (24) are calculated using numerical integration. Finally the set of equations (17) – (19) or (21) – (23) together with boundary condition (16) completely discretized and the discretized equations are solved by using an iterative method based on Gauss-seidel scheme.

The boundary conditions given in equation (16), now reduces to the form:

$$t \leq 0: u_{i,j,k} = 0, v_{i,j,k} = 0, \theta_{i,j,k} = 0, \phi_{i,j,k} = 0 \quad \forall i, j, k;$$

$$t > 0: u_{i,j,1} = 1, v_{i,j,1} = 0, \theta_{i,j,1} = 0, \phi_{i,j,1} = 0 \quad \text{since } k = 1 \text{ when } y = 0;$$

$$t > 0: u_{i,j,N} \rightarrow 0, v_{i,j,N} \rightarrow 0, \theta_{i,j,N} \rightarrow 1, \phi_{i,j,N} \rightarrow 1 \quad \text{since } y \rightarrow \infty, \text{ means } k \rightarrow N.$$

3.1 Important Physical Parameters

The physical parameter Skin-friction coefficient indicates physical wall shear stress, Nusselt number indicates physical rate of heat transfer and Sherwood number indicates mass transfer.

(a) Coefficient of Skin Friction

By the Newton’s law of viscosity:

$$\text{(Viscous drag)} \tau = \mu \left. \frac{\partial \bar{u}}{\partial \bar{y}} \right)_{\bar{y}=0} = \frac{\mu U_0^2}{y \varrho_\infty} \left. \frac{\partial u}{\partial y} \right)_{y=0} .$$

The non-dimensional skin-friction is given by,

$$C_f = \frac{\tau}{\rho U_0^2} = \frac{\theta_r - 1}{\theta_r} \left. \frac{\partial u}{\partial y} \right)_{y=0} .$$

(b) Nusselt Number

By the Fourier’s law of heat conduction in the form:

$$\text{(Heat flux)} q = -\lambda \left. \frac{\partial \bar{T}}{\partial \bar{y}} \right)_{\bar{y}=0} .$$

The rate of heat transfer coefficient is given by Nusselt Number

$$Nu = \frac{xq}{\lambda(\bar{T}_w - \bar{T}_\infty)} = -\frac{\theta_c - 1}{\theta_c} \left. \frac{\partial \theta}{\partial y} \right)_{y=0} .$$

(c) Sherwood Number

The mass flux q_m from the plate to the fluid is given by the Fick’s law,

$$q_m = -D_m \left. \frac{\partial \bar{C}}{\partial \bar{y}} \right)_{\bar{y}=0} .$$

The rate of mass transfer is given by Sherwood number

$$Sh = \frac{xq_m}{D_m(\bar{C}_w - \bar{C}_\infty)} = -\left(\frac{\theta_r - 1}{\theta_r} \right) \left. \frac{\partial \phi}{\partial y} \right)_{y=0} .$$

4. Results and Discussion

By applying non dimensional quantities, the non dimensional discretized governing equations along with the non-dimensional boundary conditions are solved with the help of AB and CF fractional derivative methods by developing suitable programming code in MATLAB using finite difference scheme. This analysis has been done to study the effects of various parameters such as $\theta_r, \theta_c, M, Kr, Sc, Ec, Pr$ etc. on velocity (u), temperature (θ) and species concentration (ϕ) profiles in presence of time. The numerical results are shown graphically in Figure 2 to

Figure 15. In the following discussion, the initial values of the parameters are considered as $\theta_r = -20$, $\theta_c = -12$, $M = 1$, $\alpha = 0.25$, $Gr_m = 0.1$, $Gr = 0.1$, $\beta = 0.25$, $Kr = 0.05$, $Sc = 0.5$, $Ec = 0.1$, $Pr = 0.25$, $K_p = 0.5$ unless otherwise stated.

4.1 Graphical Representation

In Figure 2, it is seen that with the increasing value of the Hartmann number, velocity decreases. The presence of magnetic field in the normal direction of the flow in an electrically conducting fluid produces Lorentz force which opposes the flow. To overcome this opposing force, some extra work should be done which is transformed to heat energy. Hence temperature increases (Figure 3). This is because that the applied magnetic field opposes the fluid motion and therefore enhancing the temperature for this response. With the increase of M species concentration also increases (Figure 4).

Figure 5 depicts the distribution of velocity with the variation of the thermal conductivity parameter θ_c . Velocity increases with the increasing value of θ_c . In Figure 6 temperature decreases with the increasing value of θ_c . Physically it means that as thermal conduction increases the transportation of heat from hot region to colder region increases. Since temperature within boundary layer is more than the outside so temperature is decreased. Again species concentration increases with the increasing value of θ_c (Figure 7).

The effects of viscosity parameter θ_r on velocity, temperature and species concentration distribution are plotted in Figure 8 to Figure 10. Figure 8 displays that dimensionless velocity decreases with the increase of θ_r . This is due to the fact that with the increase of the viscosity parameter the thickness of the velocity boundary layer decreases. Physically, this is because of that a larger θ_r implies higher temperature difference between the fluid and the surface. Figure 9 shows that temperature increases with the increasing value of θ_r . The viscosity causes a rise in the friction, when friction increases the area of the stretching surface in contact with the flow increases therefore generated heat from the friction on the surface is transferred to the flow which leads to a rise in the surface temperature and the flow is heated. The species concentration decreases for increasing value of θ_r (Figure 10).

The Eckert number Ec signifies the viscous dissipation of the fluid, on temperature it is plotted in Figure 11. It is seen that an increase in viscous dissipation of the fluid tends to increase in fluid temperature with increase of Ec . In Figure 12, it is noticed that with the increase of radiation parameter Kr temperature increases. This is due to the fact that the thermal boundary layer thickness increases with the increase of Kr and hence temperature. Velocity decreases with the increasing value of Prandtl number Pr (Figure 13). This is due to the fact that with the increase of Pr , viscosity increases, so velocity decreases. In Figure 14, it is noticed that with the increasing value of Pr temperature of the fluid decreases. For higher Prandtl number the fluid has a relatively high thermal conductivity which decreases the temperature.

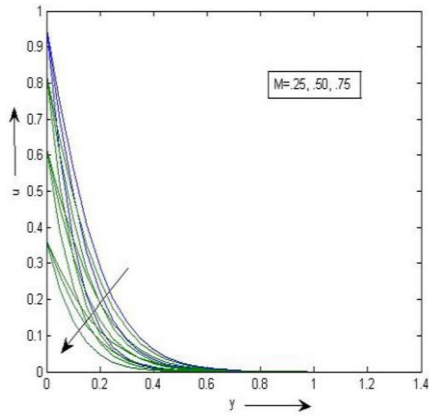


Figure 2. Effect of M on velocity.

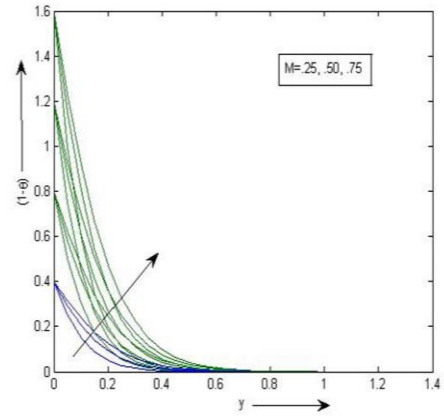


Figure 3. Effect of M on temperature.

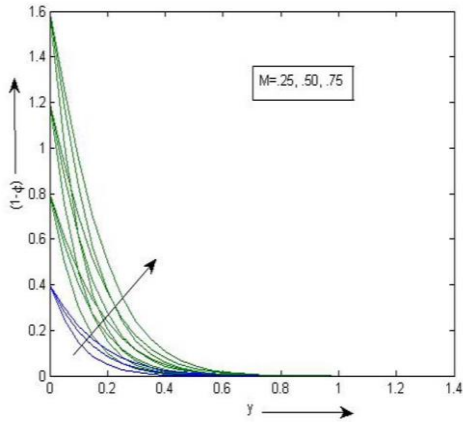


Figure 4. Effect of M on concentration.

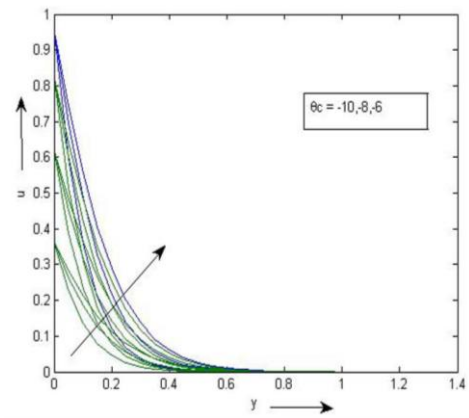


Figure 5. Effect of θ_c on velocity.

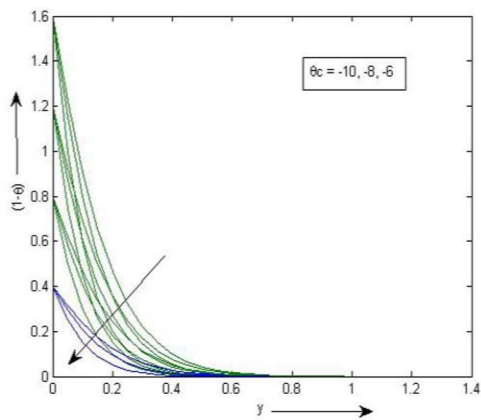


Figure 6. Effect of θ_c on temperature.

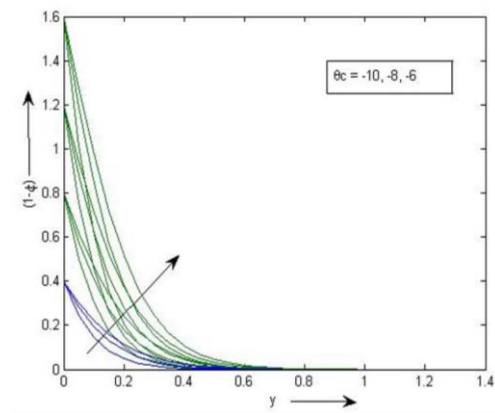


Figure 7. Effect of θ_c on concentration.

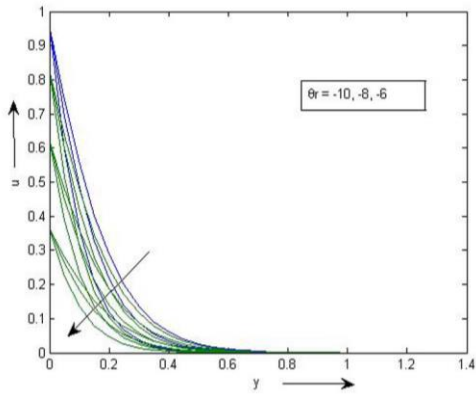


Figure 8. Effect of θ_r on velocity

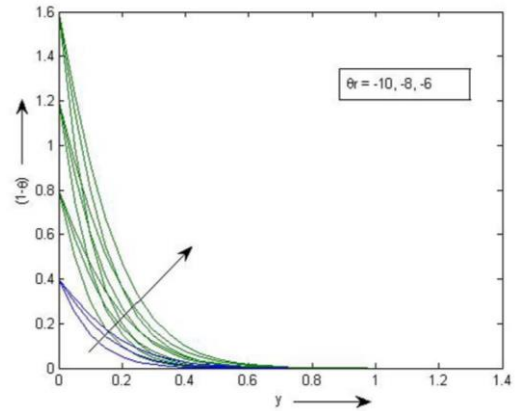


Figure 9. Effect of θ_r on temperature.

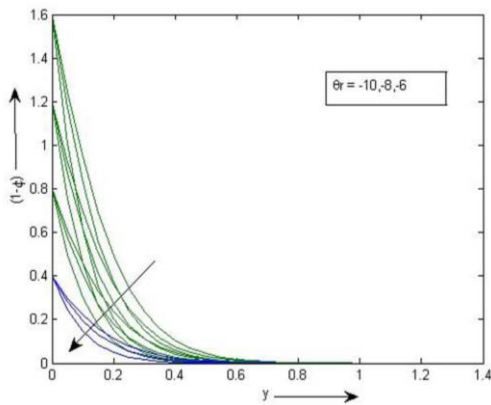


Figure 10. Effect of θ_r on concentration.

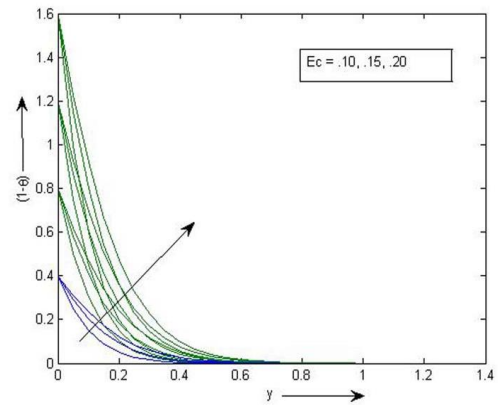


Figure 11. Effect of Ec on temperature.

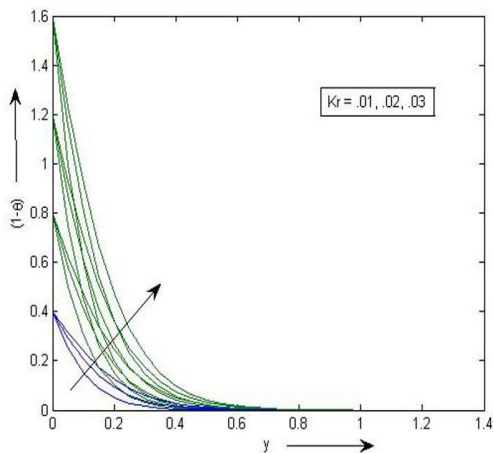


Figure 12. Effect of Kr on temperature.

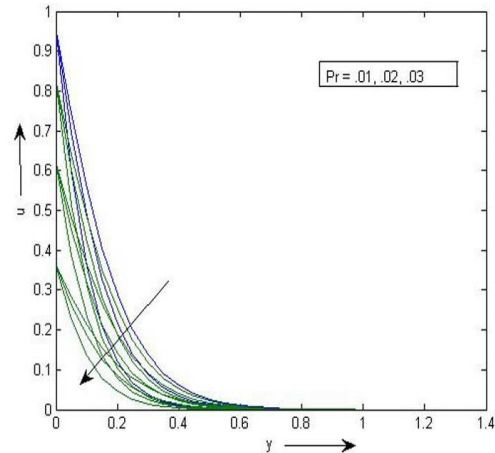


Figure 13. Effect of Pr on velocity.

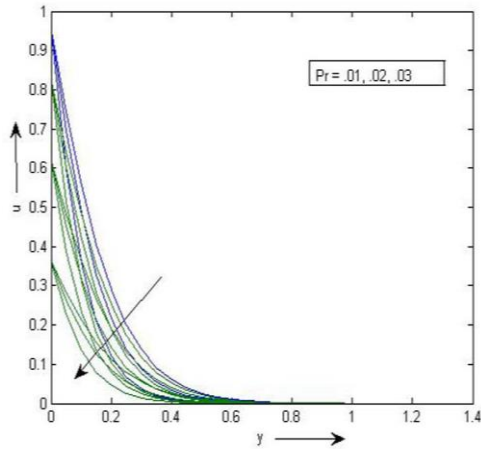


Figure 14. Effect of Pr on temperature.

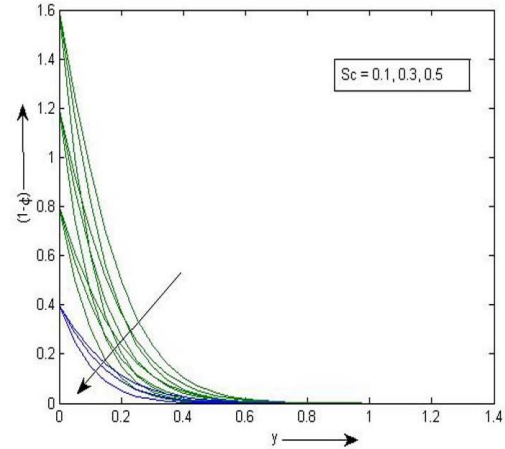


Figure 15. Effect of Sc on concentration.

In Figure 15, Sc is increased the concentration boundary layer becomes thinner than the viscous boundary layer, as a result of which velocity reduces. With thinner concentration boundary layer the concentration gradients are enhanced causing a decrease in concentration of species in the boundary layer.

4.2 Comparison of AB and CF Fractional Derivatives for Various Values of the Parameters in Tabular Form

Here we compare between AB and CF fractional derivative for various values of the parameters $M, \theta_c, \theta_r, Sc, Kr, Pr, Ec$ taking $y = 0.4$ and $t = 0.4$. From the following tables it is found that the values of the velocity, temperature and concentration profiles for various parameters are almost the same for both the methods- AB and CF fractional derivative.

Table 1. Effect of M on u, θ and ϕ .

| M | y | t | u | | θ | | ϕ | |
|------|-----|-----|----------|----------|----------|----------|----------|----------|
| | | | AB | CF | AB | CF | AB | CF |
| 0.25 | 0.4 | 0.4 | 0.036610 | 0.036588 | 0.008659 | 0.008654 | 0.008459 | 0.008439 |
| | | 0.8 | 0.060779 | 0.060758 | 0.017286 | 0.017204 | 0.017143 | 0.017140 |
| | | 1.2 | 0.079966 | 0.079979 | 0.025779 | 0.025793 | 0.025928 | 0.025937 |
| | | 1.6 | 0.092290 | 0.092380 | 0.034333 | 0.034418 | 0.034799 | 0.034889 |
| 0.50 | 0.4 | 0.4 | 0.021852 | 0.021691 | 0.023380 | 0.023378 | 0.021848 | 0.021838 |
| | | 0.8 | 0.036248 | 0.036159 | 0.046098 | 0.046054 | 0.045382 | 0.045347 |
| | | 1.2 | 0.047737 | 0.047815 | 0.068543 | 0.068600 | 0.069422 | 0.069547 |
| | | 1.6 | 0.054954 | 0.054961 | 0.090805 | 0.090810 | 0.094434 | 0.094528 |
| 0.75 | 0.4 | 0.4 | 0.008157 | 0.008086 | 0.039695 | 0.039675 | 0.035782 | 0.035764 |
| | | 0.8 | 0.013610 | 0.013544 | 0.077713 | 0.077694 | 0.075483 | 0.075427 |
| | | 1.2 | 0.017888 | 0.017895 | 0.114888 | 0.114894 | 0.116372 | 0.116454 |
| | | 1.6 | 0.020577 | 0.020582 | 0.150742 | 0.151763 | 0.159960 | 0.159988 |

Table 2. Effect of θ_c on u, θ and ϕ .

| θ_c | y | t | u | | θ | | ϕ | |
|------------|-----|-----|----------|----------|----------|----------|----------|----------|
| | | | AB | CF | AB | CF | AB | CF |
| -10 | 0.4 | 0.4 | 0.020589 | 0.020403 | 0.008648 | 0.008632 | 0.008443 | 0.008443 |
| | | 0.8 | 0.017770 | 0.017698 | 0.017177 | 0.017165 | 0.017163 | 0.017163 |
| | | 1.2 | 0.013406 | 0.013540 | 0.025657 | 0.025702 | 0.025982 | 0.025983 |
| | | 1.6 | 0.007996 | 0.008025 | 0.034096 | 0.034181 | 0.034885 | 0.034886 |
| -8 | 0.4 | 0.4 | 0.054677 | 0.054514 | 0.023351 | 0.023349 | 0.021920 | 0.021909 |
| | | 0.8 | 0.047369 | 0.047246 | 0.045914 | 0.045929 | 0.045696 | 0.045661 |
| | | 1.2 | 0.035772 | 0.035882 | 0.068166 | 0.068223 | 0.070127 | 0.070152 |
| | | 1.6 | 0.021391 | 0.021530 | 0.089819 | 0.090224 | 0.095677 | 0.095679 |
| -6 | 0.4 | 0.4 | 0.091709 | 0.091658 | 0.039482 | 0.039462 | 0.036065 | 0.036048 |
| | | 0.8 | 0.079513 | 0.079426 | 0.077050 | 0.077030 | 0.076657 | 0.076602 |
| | | 1.2 | 0.060184 | 0.060362 | 0.113818 | 0.113824 | 0.118971 | 0.118973 |
| | | 1.6 | 0.036088 | 0.036364 | 0.148869 | 0.149960 | 0.164496 | 0.164498 |

Table 3. Effect of θ_r on u, θ and ϕ .

| θ_r | y | t | u | | θ | | ϕ | |
|------------|-----|-----|----------|----------|----------|----------|----------|----------|
| | | | AB | CF | AB | CF | AB | CF |
| -10 | 0.4 | 0.4 | 0.090985 | 0.090834 | 0.008659 | 0.008654 | 0.035576 | 0.035559 |
| | | 0.8 | 0.078321 | 0.078234 | 0.017286 | 0.017204 | 0.074725 | 0.074669 |
| | | 1.2 | 0.058894 | 0.059070 | 0.025778 | 0.025823 | 0.114768 | 0.114848 |
| | | 1.6 | 0.035137 | 0.035411 | 0.034233 | 0.034318 | 0.157360 | 0.157378 |
| -8 | 0.4 | 0.4 | 0.054312 | 0.054249 | 0.023378 | 0.023376 | 0.021777 | 0.021766 |
| | | 0.8 | 0.046928 | 0.046805 | 0.046082 | 0.046048 | 0.045128 | 0.045093 |
| | | 1.2 | 0.035492 | 0.035500 | 0.068537 | 0.068594 | 0.068887 | 0.068889 |
| | | 1.6 | 0.021233 | 0.021270 | 0.090901 | 0.090906 | 0.093557 | 0.093559 |
| -6 | 0.4 | 0.4 | 0.020453 | 0.020367 | 0.039682 | 0.039562 | 0.008440 | 0.008430 |
| | | 0.8 | 0.017710 | 0.017637 | 0.077489 | 0.077469 | 0.017120 | 0.017110 |
| | | 1.2 | 0.013389 | 0.013393 | 0.114883 | 0.114890 | 0.025877 | 0.025887 |
| | | 1.6 | 0.007996 | 0.007997 | 0.150628 | 0.151718 | 0.034688 | 0.034689 |

Table 4. Effect of Kr and Ec on θ .

| Kr | y | t | θ | | Ec | y | t | θ | |
|------|-----|-----|----------|----------|------|-----|-----|----------|----------|
| | | | AB | CF | | | | AB | CF |
| 0.01 | 0.4 | 0.4 | 0.008670 | 0.008654 | 0.10 | 0.4 | 0.4 | 0.009850 | 0.009755 |
| | | 0.8 | 0.017286 | 0.017205 | | | 0.8 | 0.018486 | 0.018305 |
| | | 1.2 | 0.025880 | 0.025924 | | | 1.2 | 0.025819 | 0.025824 |
| | | 1.6 | 0.034536 | 0.034620 | | | 1.6 | 0.034203 | 0.034219 |
| 0.02 | 0.4 | 0.4 | 0.023385 | 0.023379 | 0.15 | 0.4 | 0.4 | 0.023729 | 0.023627 |
| | | 0.8 | 0.046097 | 0.046052 | | | 0.8 | 0.046189 | 0.046165 |
| | | 1.2 | 0.046087 | 0.046092 | | | 1.2 | 0.068586 | 0.068593 |
| | | 1.6 | 0.090834 | 0.090839 | | | 1.6 | 0.090826 | 0.090831 |
| 0.03 | 0.4 | 0.4 | 0.039670 | 0.039581 | 0.20 | 0.4 | 0.4 | 0.040323 | 0.040293 |
| | | 0.8 | 0.077601 | 0.077582 | | | 0.8 | 0.077999 | 0.077992 |
| | | 1.2 | 0.114864 | 0.114869 | | | 1.2 | 0.115122 | 0.115129 |
| | | 1.6 | 0.151733 | 0.151796 | | | 1.6 | 0.150832 | 0.151833 |

Table 5. Effect of Pr on u, θ and Sc on ϕ .

| Pr | y | t | u | | θ | | Sc | ϕ | |
|------|-----|-----|----------|----------|----------|----------|------|----------|----------|
| | | | AB | CF | AB | CF | | AB | CF |
| 0.01 | 0.4 | 0.4 | 0.091703 | 0.091657 | 0.039570 | 0.039551 | 0.1 | 0.036467 | 0.036446 |
| | | 0.8 | 0.079509 | 0.079420 | 0.077501 | 0.077482 | | 0.075666 | 0.075610 |
| | | 1.2 | 0.060472 | 0.060550 | 0.114864 | 0.114869 | | 0.115655 | 0.115737 |
| | | 1.6 | 0.036469 | 0.036545 | 0.150733 | 0.151796 | | 0.157770 | 0.157788 |
| 0.02 | 0.4 | 0.4 | 0.054574 | 0.054514 | 0.023375 | 0.023373 | 0.3 | 0.022380 | 0.022374 |
| | | 0.8 | 0.047387 | 0.047244 | 0.046087 | 0.046052 | | 0.045518 | 0.045497 |
| | | 1.2 | 0.047294 | 0.047299 | 0.046087 | 0.046092 | | 0.068746 | 0.068801 |
| | | 1.6 | 0.021586 | 0.021595 | 0.090734 | 0.090740 | | 0.092652 | 0.092654 |
| 0.03 | 0.4 | 0.4 | 0.020487 | 0.020403 | 0.008659 | 0.008654 | 0.5 | 0.008556 | 0.008556 |
| | | 0.8 | 0.017770 | 0.017698 | 0.017386 | 0.017205 | | 0.017153 | 0.017153 |
| | | 1.2 | 0.013476 | 0.013490 | 0.025880 | 0.025924 | | 0.025755 | 0.025756 |
| | | 1.6 | 0.008922 | 0.008925 | 0.034316 | 0.034320 | | 0.034360 | 0.034361 |

From Table 1, 2, 3, 4 and 5 we have found some significance regarding velocity, temperature and concentration under the parameters M , θ_c , θ_r , Sc , Kr , Ec and Pr for the approaches AB and CF fractional derivatives. It is observed that, if time is less than 1, the velocity obtained using AB approach is greater than the velocity computed with CF approach. Otherwise if the time is just greater than 1, the velocity obtained by CF is greater than the velocity obtained by AB approach. Again for temperature profile under the parameters the value obtained by CF approach just greater than AB approach but for concentration profile the value obtained by AB approach is just greater than CF approach.

4.3 Comparison of AB and CF method for Coefficient of Skin Friction, Nusselt number and Sherwood Number

We are considering the values of the AB operator (α) and the CF operator (β) as 0.2 and 0.4. The values of C_f , Nu and Sh are calculated by both the AB and CF methods for various values of the involved parameters. The Coefficient of skin- friction (C_f), rate of mass transfer in terms of Sherwood number (Sh) and rate of heat transfer in terms of Nusselt number (Nu) at the plate $y = 0.4$ and time $t = 0.4$ against θ_r , θ_c and M are demonstrated in Table 6 to Table 8.

Table 6. Variation of physical quantities with viscosity parameter for $M = 1$, $\theta_c = -12$, $Gr = 0.51$, $Gr_m = 0.51$, $Pr = 0.70$, $Ec = 0.1$, $Sc = 0.5$, $So = 5$.

| α / β | θ_r | C_f | | Nu | | Sh | |
|------------------|------------|------------|------------|----------|----------|-----------|----------|
| | | AB | CF | AB | CF | AB | CF |
| 0.2 | -10 | -14.510708 | -14.55779 | 7.248705 | 7.237226 | 14.670122 | 14.52998 |
| | -8 | -8.049324 | -8.887118 | 4.305106 | 4.112306 | 9.541315 | 9.072564 |
| | -6 | -6.164552 | -6.555832 | 3.308188 | 3.170818 | 7.654831 | 7.339991 |
| | -4 | -5.123131 | -5.583362 | 2.756970 | 2.646633 | 6.562650 | 6.282060 |
| 0.4 | -10 | -9.621844 | -10.860686 | 5.258340 | 4.994447 | 11.819109 | 10.54274 |
| | -8 | -7.082974 | -7.413013 | 3.720947 | 3.560238 | 8.412463 | 8.058354 |
| | -6 | -5.562100 | -5.992445 | 3.002892 | 2.878940 | 7.760696 | 7.110674 |
| | -4 | -4.652398 | -4.937746 | 2.561298 | 2.459358 | 6.986970 | 6.284284 |

Table 7. Variation of physical quantities with thermal conductivity parameter for $M = 1, \theta_r = -12, Gr = 0.51, Gr_m = 0.51, Pr = 0.70, Ec = 0.1, Sc = 0.5, So = 5$

| α / β | θ_c | C_f | | Nu | | Sh | |
|------------------|------------|-----------|------------|----------|---------|----------|----------|
| | | AB | CF | AB | CF | AB | CF |
| 0.2 | -10 | -14.71153 | -14.75877 | 7.209884 | 7.19839 | 14.75818 | 14.61820 |
| | -8 | -8.267272 | -8.713509 | 4.258720 | 4.06159 | 9.160136 | 9.060340 |
| | -6 | -6.306919 | -6.711279 | 3.238662 | 3.09560 | 7.817970 | 7.61941 |
| | -4 | -5.254050 | -5.533527 | 2.637452 | 2.51804 | 6.814178 | 6.56182 |
| 0.4 | -10 | -9.721226 | -10.959933 | 5.235210 | 4.96659 | 11.88594 | 11.6206 |
| | -8 | -7.166104 | -7.502525 | 3.685785 | 3.52061 | 8.980200 | 8.43676 |
| | -6 | -5.073432 | -6.712769 | 2.945959 | 2.81547 | 7.669259 | 7.03681 |
| | -4 | -4.144791 | -5.841899 | 2.458716 | 2.34579 | 6.858733 | 6.18878 |

Table 8. Variation of physical quantities with magnetic field parameter for $\theta_r = -20, \theta_c = -12, Gr = 0.51, Gr_m = 0.51, Pr = 0.70, Ec = 0.1, Sc = 0.5, So = 5$.

| α / β | M | C_f | | Nu | | Sh | |
|------------------|-----|-----------|------------|----------|----------|----------|----------|
| | | AB | CF | AB | CF | AB | CF |
| 0.2 | 0.4 | -14.72545 | -14.89264 | 7.248048 | 7.236569 | 14.80152 | 14.76111 |
| | 0.8 | -8.733683 | -8.881993 | 4.302752 | 4.108959 | 9.189046 | 9.110243 |
| | 1.2 | -6.700837 | -6.806939 | 3.303619 | 3.165336 | 7.424904 | 7.396263 |
| | 1.6 | -5.599457 | -5.678780 | 2.748759 | 2.636274 | 6.522696 | 6.324539 |
| 0.4 | 0.4 | -10.67705 | -10.916454 | 5.257273 | 4.990260 | 10.94694 | 10.66938 |
| | 0.8 | -7.533452 | -7.671426 | 3.718370 | 3.556140 | 8.120591 | 8.058800 |
| | 1.2 | -6.080710 | -6.720423 | 2.997566 | 2.871148 | 7.896560 | 7.237765 |
| | 1.6 | -5.213385 | -5.908385 | 2.551771 | 2.444825 | 6.502341 | 6.490837 |

In Table 6, it is seen that coefficient of Skin- friction (C_f) increases but Nu and Sh decreases for the increasing values of θ_r . Physically negative values of C_f mean that the surface exerts a drag force on the fluid. In Table 7, it is seen that C_f increases but Nu and Sh decreases for the increasing values of θ_c . It is because viscosity decreases with increasing thermal conductivity which enhances the magnitude of surface velocity gradient and reduces the magnitude of the heat transfer rate. Therefore skin-friction increases and Nusselt number and Sherwood number decreases with increasing θ_c . In Table 8, it is seen that C_f increases but Nu and Sh decreases for the increasing values of M . After comparison we observed that the values of skin-friction and Sherwood number using CF approach is slightly greater than AB approach. But for Nusselt number values of AB approach is just greater than CF approach.

5. Conclusion

In this study, the effects of variable viscosity and thermal conductivity on MHD flow over a moving vertical plate in a porous medium with radiation and viscous dissipation is investigated. Based on the above analysis we can conclude that:

- Velocity, temperature and species concentration increases with the increasing value of AB fractional (α) parameter and CF fractional parameter (β).

- Magnetic field and viscosity have retarding effect in the flow.
- Increasing value of Magnetic field parameter decreases the value of velocity but increases the values of temperature and species concentration.
- When the viscosity parameter increases, the velocity and the species concentration decreases whereas temperature increases.
- With the increasing thermal conductivity parameter, the velocity and the species concentration increases but the temperature decreases.
- Velocity and temperature increases with the increasing value of eckert number and radiation parameter.
- Velocity and temperature increases with the increasing value of prandlt number. Temperature and species concentration decrease with the increasing value of the Schmidt number.
- The values of the velocity, temperature and concentration profiles for various parameters are almost the same for both the methods- AB and CF fractional derivatives. As gamma function is present inside the exponential function in AB fractional derivative method, so the result obtained by it is more accurate over the CF fractional derivative method.

Conflict of Interest

The authors confirm that there is no conflict of interest to declare for this publication.

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