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# Matrix Dimensional Analysis for Electromagnetic Quantities

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**Abstract**
We utilize the electromagnetically-oriented $LTI\emptyset$ dimensional basis in the matrix solution of dimensional-analysis (DA) problems involving mainly electromagnetic quantities, whether these quantities are lumped or distributed. Representations in the $LTI\emptyset$ basis (compared with the standard $MLTI$ basis) are more informative and much simpler. Moreover, matrix DA computations employing the $LTI\emptyset$ basis are more efficient and much less error prone. Extensive discussions of two demonstrative examples expose technical details of a novel DA scheme, and clarify many important facets of modern dimensional analysis.

**Keywords-** Dimensional analysis, Gauss-Jordan algorithm, Bases and regimes, Electromagnetics, Duality, The $LTI\emptyset$ basis, The $MLTI$ basis.

## 1. Introduction

This paper advocates the use of the electromagnetically-oriented $LTI\emptyset$ dimensional basis (instead of the mechanically-oriented $MLTI$ dimensional basis associated with the SI international system of units) for solving Dimensional Analysis (DA) problems involving electromagnetic quantities. We employ the modern DA technique of deriving dimensionless products by using the Gauss-Jordan algorithm to transform the dimensional matrix to its reduced row echelon form (RREF). For comparison purposes, we solve classical DA problems using each of the $LTI\emptyset$ and $MLTI$ dimensional bases. Representations in the $LTI\emptyset$ basis are informative and much simpler, while computations according to the proposed scheme are more efficient and less error prone, especially when electromagnetic (EM) quantities dominate the basis (input) variables of the DA problem, and not necessarily its regime (output) variables.

Any sought product $\pi_j$ of a set of physical variables is dimensionless if and only if the exponents of these variables are a solution of a set of $p$ homogeneous linear equations (not necessarily linearly independent) in $n$ unknowns, expressed in matrix form as (Chen, 1971; Hutter & Jöhnk, 2004; Middendorf, 1986; Oladigbolu & Rushdi, 2020; Palanthandalam-Madapusi et al., 2007; Rushdi & Rushdi, 2016; 2020a; 2020b; Szirtes, 2007)

$$\boldsymbol{Dz} = \boldsymbol{0} \qquad (1)$$

where $\boldsymbol{D}$ is a $p \times n$ matrix, called the dimensional matrix. This matrix has $p$ rows which represent the adopted fundamental reference dimensions (such as the $MLTI$ or $LTI\emptyset$ dimensions), and $n$ columns, which denote the variable exponents in the sought dimensionless product or, with a gross (albeit common and appealing) abuse of notation, designate the variables themselves. We will designate a column twice: (a) by the correct exponent notation, and (b) by the common variable notation. A typical entry of this matrix is the exponent to which a reference dimension (row) is raised in the dimensional product formula representing the particular variable (column). The vector $\boldsymbol{z}$ comprises the $n$ exponents in the sought dimensionless product, which are unknown constants, yet to be inter-related (partially determined). We note that the Gauss-Jordan algorithm has many features that make it an unrivaled choice for handling DA problems, whether in a manual or an automated fashion (Rushdi & Rushdi, 2020a; references therein). Employment of the $LTI\emptyset$ dimensional basis further allows us to make the most of these advantageous features.

The remainder of this paper is structured as follows. Section 2 describes the electromagnetically-oriented $LTI\emptyset$ dimensional basis, and presents the two transformations between the $LTI\emptyset$ basis and the $MLTI$ basis, two illustrative examples handling (a) the transient analysis of current in an RC circuit, and (b) the assessment of volumetric density of energy in an electromagnetic field are then presented in Sections 3 and 4, respectively. The two examples demonstrate the effectiveness of the proposed approach which makes the most of the Gauss-Jordan algorithm through the use of the $LTI\emptyset$ basis for the dimensional analysis of EM problems. The advantages gained utilizing by this novel base are demonstrated by comparing the $LTI\emptyset$-based analysis with the $MLTI$-based one. The issue of partitioning variables into basis (input) and regime (output) ones is also explored in detail. Section 5 concludes the paper.

## 2. The $LTI\emptyset$ Dimensional Basis for Electromagnetics

Dimensional Analysis involving electromagnetic quantities is typically based on the use of the $MLTI$ multidimensional system (Bhaskar & Nigam, 1990; Rushdi & Rushdi, 2016; 2020a; 2020b; Szirtes, 2007). An alternative system using the same number of fundamental dimensions is the $LTI\emptyset$ system, where $\emptyset$ stands for electric potential or voltage (Szirtes, 2007). This system starts as a system covering the two *kinematic* quantities of Length ($L$) and Time ($T$), and augments it with the two *electric* (or *electromagnetic*) quantities of current and potential. Likewise, in the $LTC\Phi$ system that was proposed by Kalantaroff in 1929 (Kinitsky, 1962), electric charge ($C$) and magnetic flux ($\Phi$) are taken as fundamental dimensions, in addition to Length and Time. All these modern multidimensional systems use four fundamental dimensions, but the split of these four dimensions to purely mechanical and purely electromagnetic ones is $3 + 1$ for mechanically-oriented systems (such as the $MLTI$ and $MLTC$ systems) and $2 + 2$ for the electromagnetically-oriented ones (such as the $LTI\emptyset$ and $LTC\Phi$ systems).

If an arbitrary physical quantity $Q$ is expressed in the $MLTI$ and $LTI\emptyset$ bases by the vectors of indices $\boldsymbol{r} = [r_1 \ \ r_2 \ \ r_3 \ \ r_4]^T$ and $\boldsymbol{R} = [R_1 \ \ R_2 \ \ R_3 \ \ R_4]^T$, then these vectors are related by the transformations $\boldsymbol{r} = \boldsymbol{TR}$, and $\boldsymbol{R} = \boldsymbol{T^{-1}}\,\boldsymbol{r}$, or explicitly as

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-3} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix}, \qquad \begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} \mathbf{-2} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix}. \tag{2}$$

The four vectors comprising the transformation matrix $\boldsymbol{T}$ are the vectors of exponents for the variables $L, T, I,$ and $\emptyset$ in the $MLTI$ basis, while the four vectors comprising the inverse transformation matrix $\boldsymbol{T^{-1}}$are the vectors of exponents for the variables $M, L, T,$ and $I$ in the $LTI\emptyset$ basis. For example, If $Q = M$ then $\boldsymbol{r} = [1 \;\; 0 \;\; 0 \;\; 0]^T$ and $\boldsymbol{R} = [-2 \;\; 3 \;\; 1 \;\; 1]^T$, i.e., $M = L^{-2}\, T^3\, I\, \emptyset$. Likewise, if $Q = \emptyset$ then $\boldsymbol{R} = [0 \;\; 0 \;\; 0 \;\; 1]^T$ and $\boldsymbol{r} = [1 \;\; 2 \;\; -3 \;\; -1]^T$, i.e., $\emptyset = M\, L^2\, T^{-3}\, I^{-1}$. The mapping between the $MLTI$ set and the $LTI\emptyset$ set is one-to-one and onto. If the quantity of interest $Q$ is an electromagnetic quantity or if it is a mechanical quantity independent of the Mass ($M$) dimension, then its $LTI\emptyset$-based vector of indices $\boldsymbol{R}$ is much simpler than its $MLTI$-based vector $\boldsymbol{r}$.

## 3. Transient Analysis of an RC Circuit

Middendorf (1986) considered the situation in which a DC voltage source of value $V$ is imposed for time $t \geq 0$ on a series combination of a resistance $R$ and a capacitance $C$. The transient current $i(t)$ is required, and hence the variable $i$ must be a regime variable (Bhaskar & Nigam, 1990), and it is placed last in a proposed dimensionless product $\pi = k\;\; C^c\;\; R^r\;\; V^v\;\; t^\tau\;\; i^\xi$, where $k$ is a dimensionless constant. The most important variable among the remaining variables is $t$, and is placed immediately before $i$. Table 1 demonstrates the Gauss-Jordan procedure for solving this problem in the $LTI\emptyset$ dimensional basis, while Table 2 demonstrates the same procedure for solving this problem in the $MLTI$ dimensional basis. The same final solution is obtained in both tables. However, the $LTI\emptyset$-based solution is obviously more efficient, and hence less error prone. The operations involved in the two tables include the following operations (arranged in decreasing complexity): floating-point row computation ($f$), row summation/differencing ($s$), row negation ($n$), and row assignment ($a$). The $LTI\emptyset$-based solution requires 2 stages beyond the initial stage, and uses $(3s + n + 2a)$ operations. The $MLTI$-based solution requires 3 stages beyond the initial stage, and uses $(3f + 2s + n + 4a)$ operations.

Note that each solution encounters a row whose entries are all 0 (an all-0 row), which we arbitrarily omit in the next stage. This means that the matrix $\boldsymbol{D}$ has a rank $r = 3$, and a nullity or defect $(n - r) = 2$. At the last stage of each solution, the $p \times n$ dimensional matrix $\boldsymbol{D}$ would be changed to an $r \times n$ matrix that is partitioned into an $r \times r$ unit matrix and an $r \times (n - r)$ matrix $\boldsymbol{C}$. We now construct a full-rank $(n - r) \times n$ matrix $\boldsymbol{K}$ of exponents, which is partitioned into two matrices: the negative transpose $-\boldsymbol{C^T}$ of $\boldsymbol{C}$, which is an $(n - r) \times r$ matrix, left juxtapositioned to an $(n - r) \times (n - r)$ unit matrix. The matrix $\boldsymbol{K}$ (called the nullspace or kernel of $\boldsymbol{D}$) has a rank equal to the nullity of $\boldsymbol{D}$. The $(n - r)$ by $p$ matrix $\boldsymbol{KD}^{\mathbf{T}}$ is a zero matrix, and the $(n - r)$ rows of $\boldsymbol{K}$ form a basis for the nullspace of $\boldsymbol{D}$ (Rushdi & Rushdi, 2020a), and they depict a complete set of the dimensionless products as shown at the bottom of Table 1. There are two products: $\pi_1 = t/CR$ and $\pi_2 = iR/V$, which constitute a complete set of dimensionless products. According to the Pi Theorem of Buckingham (1914), these two dimensionless products are related as follows:

$$\Phi\,(\pi_1\,, \pi_2) = 0 \tag{3a}$$

Finally the mathematical model of the transient current $i$ can be stated by expressing its regime $\pi_2$ as an arbitrary function $\Psi$ (to be determined experimentally) of the other regime, namely

$$\pi_2 = \;\Psi\,(\pi_1). \tag{3b}$$

It is well known (outside the scope of dimensional analysis) that the function $\Psi$ is a decaying exponential (Middendorf,1986).

In retrospect, we might have not insisted on taking current and time as regime variables. Table 3 shows an alternative ordering of variables for the $LTI\emptyset$-based solution in Table 1. Here, the Gauss-Jordan algorithm does absolutely nothing beyond constructing its initial tableau. Now, we obtain two products: $\pi_3 = VC/it$ and $\pi_4 = t/CR$ which constitute a complete set of dimensionless products. This new complete set is related to the old one via.

$$\pi_3 = 1/(\pi_1 \pi_2), \;\; \pi_4 = \pi_1, \tag{4a}$$

$$\pi_1 = \pi_4, \pi_2 = 1/(\pi_3 \pi_4), \tag{4b}$$

Table 4 shows yet another ordering of variables for the $LTI\emptyset$-based solution in Table 1. Since the rank of the dimensional matrix is now known to be 3, this ordering suggests that the variables are partitioned into a set $\{t, C, R\}$ of basis variables and a set $\{V, i\}$ of regime variables. An advantage of the Gauss-Jordan algorithm is that it detects the impossibility of this partitioning, and corrects it *en route*. Contrary to widespread belief, the Gauss-Jordan algorithm does not necessarily partition $\boldsymbol{D}$ into two matrices, the first of which is a unit matrix. Generally, the Gauss-Jordan algorithm replaces $\boldsymbol{D}$ by its reduced row echelon form (RREF), an example of which is shown in the second stage of Table 4. In this more general (albeit less intuitive situation), the algorithm employs two correct sets of basis and regime variables as $\{t, C, V\}$ and $\{R, i\}$ by swapping the roles of the variables $R$ and $V$ as basis or regime variables. Now, the three basis variables $t, C,$ and $V$ are assigned to non-consecutive columns, and though the matrix under them is, in fact, a unit matrix, it might not readily appear as such. In the lower part of Table 4, we interchange the columns for $R$ and $V$ so as to place all columns with pivots consecutively at the left to form an identity matrix. Both parts of Table 4 yield the two products: $\pi_5 = CR/t$ and $\pi_6 = ti/CV$ which constitute yet another complete set of dimensionless products, again related to the earlier sets, since $\pi_5$ and $\pi_6$ are the reciprocals of $\pi_4$ and $\pi_3$, respectively. The total number of complete sets of dimensionless products is at most (here strictly less than) the number of choosing two regime variables out of five variables (without order or repetition), which is ten. The non-uniqueness of the complete set of dimensionless products is occasionally cited as a limitation of dimensional analysis (Rushdi & Rushdi, 2020a; 2020b). However, we note that Eq. (3b) is the desirable solution of the problem, and it can be reached in a variety of ways, such as directly from Table 1, or via Table 3 together with equation. (4b).

## 4. Volumetric Density of Energy for the Electromagnetic Field

We consider the problem of estimating the volumetric density of energy ($U$) in an electromagnetic field in a medium characterized by permittivity $\varepsilon$ and permeability $\mu$. As usual, we use $E$ and $H$ to denote the electric and magnetic field intensities, respectively, and use $D$ and $B$ to depict the electric and magnetic flux densities, respectively. A simpler version of this problem was considered in the seminal paper of Buckingham (1914). An expression for the energy per volume ($U$) is required, and hence the variable $U$ must be a regime variable, and it is placed last in a proposed dimensionless product of the form $\pi = k \; \varepsilon^p \; \mu^m \; E^e \; H^h \; D^d \; B^b \; U^u$, where $k$ is a dimensionless constant. Table 5 demonstrates the Gauss-Jordan procedure for solving this problem in the $LTI\emptyset$ dimensional basis, while Table 6 demonstrates the same procedure for solving this problem in the $MLTI$ dimensional basis. The same final solution is obtained in both tables. However, the $LTI\emptyset$-based solution is obviously more efficient, and hence less error prone. The $LTI\emptyset$-based solution requires 3 stages beyond the initial stage, and it involves $(f + 9s + 2n)$ operations. The $MLTI$-based solution also requires 3 stages beyond the initial stage, but it involves $(4f + 3s + 2n + 3a)$ operations. The initial tableau in the $LTI\emptyset$ case has much simpler and self-checking integers. Grouping the electromagnetic quantities into the dual pairs $(\varepsilon, \mu)$, $(E, H)$, and $(D, B)$, we observe that, within the $LTI\emptyset$ dimensional basis, any two dual quantities have identical $L$ and $T$ dimensions, and swapped $I$ and $\emptyset$ dimensions.

**Table 1.** The Gauss-Jordan procedure for solving the circuit problem of Sec. 3 in the $LTI\emptyset$ dimensional basis.

| | $c$ | $r$ | $v$ | $\tau$ | $\xi$ | |
|---|---|---|---|---|---|---|
| | $C$ | $R$ | $V$ | $t$ | $i$ | |
| $E_1^{(0)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_2^{(0)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(0)}$ | 1 | −1 | 0 | 0 | 1 | 0 |
| $E_4^{(0)}$ | −1 | 1 | 1 | 0 | 0 | 0 |
| $E_2^{(1)} \leftarrow E_2^{(0)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(1)} \leftarrow E_3^{(0)} - E_2^{(0)}$ | 0 | −1 | 0 | −1 | 1 | 0 |
| $E_4^{(1)} \leftarrow E_4^{(0)} + E_2^{(0)}$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $E_2^{(2)} \leftarrow E_2^{(1)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(2)} \leftarrow -E_3^{(1)}$ | 0 | 1 | 0 | 1 | −1 | 0 |
| $E_4^{(2)} \leftarrow E_4^{(1)} + E_3^{(1)}$ | 0 | 0 | 1 | 0 | 1 | 0 |
| $\pi_1$ | −1 | −1 | 0 | 1 | 0 | |
| $\pi_2$ | 0 | 1 | −1 | 0 | 1 | |

**Table 2.** The Gauss-Jordan procedure for solving the circuit problem of Sec. 3 in the $MLTI$ dimensional basis.

| | $c$ | $r$ | $v$ | $\tau$ | $\xi$ | |
|---|---|---|---|---|---|---|
| | $C$ | $R$ | $V$ | $t$ | $i$ | |
| $E_1^{(0)}$ | −1 | 1 | 1 | 0 | 0 | 0 |
| $E_2^{(0)}$ | −2 | 2 | 2 | 0 | 0 | 0 |
| $E_3^{(0)}$ | 4 | −3 | −3 | 1 | 0 | 0 |
| $E_4^{(0)}$ | 2 | −2 | −1 | 0 | 1 | 0 |
| $E_1^{(1)} \leftarrow -E_1^{(0)}$ | 1 | −1 | −1 | 0 | 0 | 0 |
| $E_2^{(1)} \leftarrow E_2^{(0)} - 2\,E_1^{(0)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_3^{(1)} \leftarrow E_3^{(0)} + 4\,E_1^{(0)}$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $E_4^{(1)} \leftarrow E_4^{(0)} + 2\,E_1^{(0)}$ | 0 | 0 | 1 | 0 | 1 | 0 |
| $E_1^{(2)} \leftarrow E_1^{(1)} + E_3^{(1)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(2)} \leftarrow E_3^{(1)}$ | 0 | 1 | 1 | 1 | 0 | 0 |
| $E_4^{(2)} \leftarrow E_4^{(1)}$ | 0 | 0 | 1 | 0 | 1 | 0 |
| $E_1^{(3)} \leftarrow E_1^{(2)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(3)} \leftarrow E_3^{(2)} - E_4^{(2)}$ | 0 | 0 | 0 | 1 | −1 | 0 |
| $E_4^{(3)} \leftarrow E_4^{(2)}$ | 0 | 0 | 1 | 0 | 1 | 0 |

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**Table 3.** The Gauss-Jordan procedure for solving the circuit problem of Sec. 3 in the $LTI\emptyset$ dimensional basis with an alternative ordering of variables.

| | $\tau$ | $\xi$ | $v$ | $c$ | $r$ | |
|---|---|---|---|---|---|---|
| | $t$ | $i$ | $V$ | $C$ | $R$ | |
| $E_1^{(0)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_2^{(0)}$ | 1 | 0 | 0 | 1 | 0 | 0 |
| $E_3^{(0)}$ | 0 | 1 | 0 | 1 | −1 | 0 |
| $E_4^{(0)}$ | 0 | 0 | 1 | −1 | 1 | 0 |
| $\pi_3$ | −1 | −1 | 1 | 1 | 0 | |
| $\pi_4$ | 0 | 1 | −1 | 0 | 1 | |

**Table 4**. The Gauss-Jordan procedure for solving the circuit problem of Sec. 3 in the $LTI\emptyset$ dimensional basis with a further changed ordering of variables.

| | $\tau$ | $c$ | $r$ | $v$ | $\xi$ | |
|---|---|---|---|---|---|---|
| | $t$ | $C$ | $R$ | $V$ | $i$ | |
| $E_1^{(0)}$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $E_2^{(0)}$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $E_3^{(0)}$ | 0 | 1 | −1 | 0 | 1 | 0 |
| $E_4^{(0)}$ | 0 | −1 | 1 | 1 | 0 | 0 |
| $E_2^{(1)} \leftarrow E_2^{(0)} - E_3^{(0)}$ | 1 | 0 | **1** | 0 | **−1** | 0 |
| $E_3^{(1)} \leftarrow E_3^{(0)}$ | 0 | 1 | **−1** | 0 | **1** | 0 |
| $E_4^{(1)} \leftarrow E_4^{(0)} + E_3^{(0)}$ | 0 | 0 | **0** | 1 | **1** | 0 |
| $\pi_5$ | **−1** | **1** | 1 | **0** | 0 | |
| $\pi_6$ | **1** | **−1** | 0 | **−1** | 1 | |

| | $\tau$ | $c$ | $v$ | $r$ | $\xi$ | |
|---|---|---|---|---|---|---|
| | $t$ | $C$ | $V$ | $R$ | $i$ | |
| | 1 | 0 | 0 | **1** | **−1** | 0 |
| | 0 | 1 | 0 | **−1** | **1** | 0 |
| | 0 | 0 | 1 | **0** | **1** | 0 |
| $\pi_5$ | **−1** | **1** | **0** | 1 | 0 | |
| $\pi_6$ | **1** | **−1** | **−1** | 0 | 1 | |

We now construct the null space matrix $\boldsymbol{K}$ at the bottom of Table 5. For convenience, we arbitrarily multiply each of the first and third rows of $\boldsymbol{K}$ by 2, and obtain the four products $\pi_1 = \mu H^2/(\varepsilon E^2)$, $\pi_2 = D/(\varepsilon E)$, $\pi_3 = B^2/(\varepsilon \mu E^2)$, and $\pi_4 = U/(\varepsilon E^2)$, which constitute a complete set of dimensionless products. A more interesting complete set for this problem would include besides $\pi_2 = D/(\varepsilon E)$, and $\pi_4 = U/(\varepsilon E^2)$, their "dual" products $\pi_5 = (\pi_3/\pi_1)^{1/2} = B/(\mu H)$, and $\pi_6 = \pi_4/\pi_1 = U/(\mu H^2)$. These results can be supplemented by information outside the scope of dimensional analysis to deduce that

$$U = (\varepsilon E^2 + \mu H^2)/2 = (D^2/\varepsilon + B^2/\mu)/2. \tag{5}$$

The expression suggested by Buckingham (1914) divides the one in (5) by a factor of $4\pi$. The reason is that at his time, the value of a complete solid angle was 1 "sphere," while it is now taken as $4\pi$ steradian (Young, 1957).

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**Table 5.** The Gauss-Jordan procedure for solving the problem of volumetric density of energy of Sec. 4 in the $LTI\emptyset$ dimensional basis. In the initial tableau, any two dual quantities have identical $L$ and $T$ dimensions, and swapped $I$ and $\emptyset$ dimensions.

| | $p$ | $m$ | $e$ | $h$ | $d$ | $b$ | $u$ | |
|---|---|---|---|---|---|---|---|---|
| | $\varepsilon$ | $\mu$ | $E$ | $H$ | $D$ | $B$ | $U$ | |
| $E_1^{(0)}$ | −1 | −1 | −1 | −1 | −2 | −2 | −3 | 0 |
| $E_2^{(0)}$ | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| $E_3^{(0)}$ | 1 | −1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $E_4^{(0)}$ | −1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| $E_1^{(1)} \leftarrow -E_1^{(0)}$ | 1 | 1 | 1 | 1 | 2 | 2 | 3 | 0 |
| $E_2^{(1)} \leftarrow E_2^{(0)} + E_1^{(0)}$ | 0 | 0 | −1 | −1 | −1 | −1 | −2 | 0 |
| $E_3^{(1)} \leftarrow E_3^{(0)} + E_1^{(0)}$ | 0 | −2 | −1 | 0 | −1 | −2 | −2 | 0 |
| $E_4^{(1)} \leftarrow E_4^{(0)} - E_1^{(0)}$ | 0 | 2 | 2 | 1 | 2 | 3 | 4 | 0 |
| $E_1^{(2)} \leftarrow E_1^{(1)} + E_3^{(1)}$ | 1 | −1 | 0 | 1 | 1 | 0 | 1 | 0 |
| $E_2^{(2)} \leftarrow E_2^{(1)} - E_3^{(1)}$ | 0 | 2 | 0 | −1 | 0 | 1 | 0 | 0 |
| $E_3^{(2)} \leftarrow -E_3^{(1)}$ | 0 | 2 | 1 | 0 | 1 | 2 | 2 | 0 |
| $E_4^{(2)} \leftarrow E_4^{(1)} + 2\,E_3^{(1)}$ | 0 | −2 | 0 | 1 | 0 | −1 | 0 | 0 |
| $E_1^{(3)} \leftarrow E_1^{(2)} + E_2^{(3)}$ | 1 | 0 | 0 | 0.5 | 1 | 0.5 | 1 | 0 |
| $E_2^{(3)} \leftarrow E_2^{(2)}/2$ | 0 | 1 | 0 | −0.5 | 0 | 0.5 | 0 | 0 |
| $E_3^{(3)} \leftarrow E_3^{(2)} - E_2^{(2)}$ | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| $E_4^{(3)} \leftarrow E_4^{(2)} + E_2^{(2)}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\pi_1$ | −1 | 1 | −2 | 2 | 0 | 0 | 0 | |
| $\pi_2$ | −1 | 0 | −1 | 0 | 1 | 0 | 0 | |
| $\pi_3$ | −1 | −1 | −2 | 0 | 0 | 2 | 0 | |
| $\pi_4$ | −1 | 0 | −2 | 0 | 0 | 0 | 1 | |

**Table 6.** The Gauss-Jordan procedure for solving the problem of volumetric density of energy of Sec. 4 in the $LTI\emptyset$ dimensional basis.

| | $p$ | $m$ | $e$ | $h$ | $d$ | $b$ | $u$ | |
|---|---|---|---|---|---|---|---|---|
| | $\varepsilon$ | $\mu$ | $E$ | $H$ | $D$ | $B$ | $U$ | |
| $E_1^{(0)}$ | −1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| $E_2^{(0)}$ | −3 | 1 | 1 | −1 | −2 | 0 | −1 | 0 |
| $E_3^{(0)}$ | 4 | −2 | −3 | 0 | 1 | −2 | −2 | 0 |
| $E_4^{(0)}$ | 2 | −2 | −1 | 1 | 1 | −1 | 0 | 0 |
| $E_1^{(1)} \leftarrow -E_1^{(0)}$ | 1 | −1 | −1 | 0 | 0 | −1 | −1 | 0 |
| $E_2^{(1)} \leftarrow E_2^{(0)} - 3\,E_1^{(0)}$ | 0 | −2 | −2 | −1 | −2 | −3 | −4 | 1 |
| $E_3^{(1)} \leftarrow E_3^{(0)} + 4\,E_1^{(0)}$ | 0 | 2 | 1 | 0 | 1 | 2 | 2 | 0 |
| $E_4^{(1)} \leftarrow E_4^{(0)} + 2\,E_1^{(0)}$ | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| $E_1^{(2)} \leftarrow E_1^{(1)} + E_2^{(2)}$ | 1 | 0 | 0 | 0.5 | 1 | 0.5 | 1 | 0 |
| $E_2^{(2)} \leftarrow E_2^{(1)}/2$ | 0 | 1 | 0 | −0.5 | 0 | 0.5 | 0 | 0 |
| $E_3^{(2)} \leftarrow E_3^{(1)} + E_2^{(1)}$ | 0 | 0 | −1 | −1 | −1 | −1 | −2 | 0 |
| $E_4^{(2)} \leftarrow E_4^{(1)}$ | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| $E_1^{(3)} \leftarrow E_1^{(2)}$ | 1 | 0 | 0 | 0.5 | 1 | 0.5 | 1 | 0 |
| $E_2^{(3)} \leftarrow E_2^{(2)}$ | 0 | 1 | 0 | −0.5 | 0 | 0.5 | 0 | 0 |
| $E_3^{(3)} \leftarrow -E_3^{(2)}$ | 0 | 0 | 1 | 1 | 1 | 1 | 2 | 0 |
| $E_4^{(3)} \leftarrow E_4^{(2)} + E_3^{(2)}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

## 5. Conclusions

There are many benefits of employing the Gauss-Jordan algorithm for handling DA problems. In particular, the algorithm neither pre-supposes nor pre-calculates the rank of the dimensional matrix. It handles the task of determining this rank as an offshoot of its own processing at no extra cost. The algorithm allows a flexible partitioning of the underlying variables into basis or input variables and regime or output variables. The role of the algorithm is to switch from an initial tableau of a specific dimensional basis to a final tableau whose dimensional basis is the final set of 'basis' variables. The greater the matching between the dimensional bases of the initial and final tableaus, the fewer the computations needed and the less they are susceptible to errors. Such greater matching is achieved herein by employing the electromagnetically-oriented $LTI\emptyset$ basis in the initial tableau, and requiring electromagnetic quantities to constitute most of the basis in the final tableau.

**Conflict of Interest**

The authors assert that no conflict of interest exists.

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